Privatization of Knowledge: Did the U.S. Get It Right? (New Version).

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Privatization of Knowledge: Did the U.S. Get It Right?*

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Abstract

To foster innovation and growth should basic research be publicly or privately funded? This paper studies the impact of the gradual shift in the U.S. patent system towards the patentability and commercialization of the basic R&D undertaken by universities. We see this movement as making universities becoming responsive to "market" forces. Prior to 1980, universities undertook research using an exogenous stock of researchers that were motivated by "curiosity." After 1980, universities patent their research and behave as private firms. This move, in a context of two-stage inventions (basic and applied research) has an a priori ambiguous effect on innovation and welfare. We build a Schumpeterian model and match it to the data to assess this important turning point. *Keywords*: R&D and Growth, Sequential Innovation, Basic Research, Patent Laws. *JEL Classification*: O31, O34, O41.

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1 Introduction

Over the last 30 years, U.S. Court decisions switched from the doctrine limiting the patentability of early-stage scientific findings - lacking in current commercial value - to the conception that also fundamental basic scientific discoveries - with no current tradeable application - fall in the general applicability of the patent system.

The year 1980 marked an important turning point in US patentability requirements, as summarized by the following events:

1. the United States Supreme Court's decision on the Diamonds *v.* Chakrabarty case ruled that microorganism produced by genetic engineering could be patented;

2. the Bayh-Dole Act, which facilitated universities in patenting innovations.

After the second world war, universities and public laboratories had always been the main performers of basic R&D in the United States and in Europe. Though an important reason for the relatively low private contribution to basic R&D is often found in the high degree of uncertainty that this activity involves in terms of future commercial application and success, the legal permission to appropriate the fruits of years of investigations makes a big difference, and marks an important change from the pre-1980 to the post-1980 US innovation system. Hence the 1980's jurisprudential and juridical reforms opened the way to a flow of private funds into the academia in search of promising research projects, as well as facilitated professors in patenting their own research without incurring in legal obstacles linked to their direct or indirect involvement in the public system.

Jensen and Thursby (2001) studied the more recent licensing practices of 62 US universities. They found that "Over 75 percent of the inventions licensed were no more than a proof of concept (48 percent with no prototype available) or lab scale prototype (29 percent) at the time of license!". Moreover, most of the inventions licensed were in such an embryonic state of development, that it was difficult to estimate their commercial potential and the inventor's cooperation was required to get a successful commercial development.

In a more general definition of research tools, the US National Institute of Health (1998) is "embracing the full range of tools that scientists use in the laboratory", and includes "cell lines, monoclonal antibodies, reagents, animal models, growth factors, combinatorial chemistry libraries, drugs and drug targets, clones and cloning tools... methods, laboratory equipment and machines, databases and computer software". Nearly all research tools became patentable in the US, thanks to the juridical innovations that took place in the last 30 years.

The agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPs), article 27, encourages countries to extend patentability to"any inventions, whether products or processes, in all fields of technology, provided that they are new, involve an inventive step and are capable of industrial application", and a footnote follows specifying: "For the purposes of this Article, the terms "inventive step" and "capable of industrial application" may be deemed by a Member to be synonymous with the terms "non-obvious" and "useful" respectively." Hence a "useful" research tool should be patentable. Though to ".. make all research activities free of patent infringement would make all research tool patents worthless, and would be contrary to TRIPs", (Thouret-Lemaitre[1], 2006), the adoption of TRIPs by several countries is still controversial, as strong research exemptions to patent infringement are in place in countries such as Japan[2], China[3], Belgium[4], Germany[5], India[6],

[1] Elisabeth Thouret-Lemaitre, Vice President, Head of Patent Operations, Sanofi-Synthelabo, Paris, WIPO Presentation October 11, 2006.

[2] Japan: art 69 (1): " the effects of the patent right shall not extend to the working of the patent right for the purposes of experiment or research."

[3] Article 62 of the Patent Law of the People's Republic of China: "None of the following shall be deemed an infringement of a patent right:...5. Use of the patent in question solely for the purposes of scientific research and experimentation".

[4] Where since 2005 the new Article 28(1)(b) of the Belgian Patent Act states that a patent holder's claims "do not extend to acts that are committed on and/or with the subject of the patented invention for scientific purposes".

[5] The German Constitutional Court (2000) stated that patent holders must "accept such limitations on their rights in view of the development of the state of the art and the public interest". Thus the patent claims become controversial when the commercial interest of the unauthorized use of a patented innovation is not clear.

[6] Section 47 of the Patent Act states that The patented product or process "may be used, by a person for the purpose merely of experiment or research."

Brazil[7], Mexico[8], and Korea[9]. Even if the European Directive on Biotechnology of 1998 aimed at extending patentability to many research tools, it is still being implemented in contradictory ways, leading to a situation in the middle between the pre- and post-1980 US regime. Statutory research exemptions and compulsory licensing render patent claims much weaker.

We believe that an economic analysis of the US turning point may give good insight to start a scientific debate rich of relevant policy implications at least for Europe. This paper, by taking the R&D sequentiality into the Schumpeterian paradigm, investigates the relation between the cumulative uncertainty involved in the two-stages innovation process and the inefficiency in the public research system. Our main theoretical contribution is a theory of endogenous public inefficiency in basic research. Regarding private research, we share the decomposition of each innovation in two stages of *research* and *development* with the oligopolistic patent race literature pioneered by Reinganum (1985), Grossman and Shapiro (1986) and (1987), and, more recently, Denicolò (2000). We contribute with several new insights, by adding free entry, endogenous multisector industrial dynamics and general equilibrium determination of all variables. Our general equilibrium analysis allows a consistent numerical calibration of our theory to the true US data. The main alternative macroeconomic predecessor is Aghion and Howitt (1996), which identified basic research with horizontal innovation[10]. Since in the real world all sectors need basic research not just once, we adopt the complementary view that basic research pervades all sectors, which forces us to substantially modify the standard multisector framework with vertical innovation[11]. We

[7] Article 43 of the Brazilian Industrial Property Law: "The provisions of the preceding Articles shall not apply:...II. to acts carried out for experimental purposes by unauthorized third parties if related to study or to scientific and technological research."

[8] Article 22 of the Industrial Property Law: "The right conferred by a patent shall not have any effect against: (I) a third party who, in the private or academic sphere and for non-commercial purposes, engages in scientific or technological research activities for purely experimental, testing or teaching purposes, and to that end manifactures or uses a product or a process identical to the one patented".

[9] Section 96(1) of the Patent Law states: "The effects of the patent right shall not extend to the following: (i) working of the patented invention for the purpose of research or experiment…".

[10] Gersbach, Sorger, and Amon (2009) extends this framework, with basic research potentially opening more new sectors than applied research manages to complete. Bramoulle' and Saint-Paul's (2010) incorporates a realistic reputation reward system based on citations.

[11] Interestingly, unlike Aghion and Howitt (1996), Leiva-Beltran (2007) constructs a

will assume that basic research can be "curiosity driven", but that it could also be motivated by its potentially socially useful applications.

The rest of this paper is organized as follows. Section 2 explains the modifications in Schumpeterian theory needed to analyse the two-stage innovation process stylizing the innovative mechanism in the presence of research tools. It focusses on the most original aspects of the model, leaving the most standard parts to the Appendix 1, in order to facilitate readability. Section 3 applies this new framework to a stylized pre-1980 US scenario: basic research findings are conceived in public institutions and put into the public domain, triggering patent races by freely entering perfectly competitive private R&D firms aiming at inventing a better quality product. Section 4 models a stylized post-1980 US scenario, where basic R&D achievements are patented and, afterwards, developed into tradable applications within a completely privatized economy. Free entry patent races only occur in the basic research, whereas as soon as a research tool is discovered it will be developed by its patent holder. Section 5 matches the model to the US data prevailing at the time of the jurisprudence and legislative change. We estimate the relevant technological parameter and we undertake numerical simulations in order to assess if the reform could have enhanced innovation. In Section 6, we test the robustness of our findings in an alternative model of privatized basic research, which explicitly includes the debated existence of a "research exemption", which might give birth to reach-through patenting agreements after an infringement suit. Section 7 concludes.

2 The Model

2.1 Overview

Consider an economy with a continuum of differentiated final good sectors with corresponding differentiated research and development (R&D) sectors, along the lines of Grossman and Helpman (1991a and b). In each sector there is a instantaneous price competition, which implies - under the usual constant returns to scale assumption - that at every date there will be a monopolist, that coincides with the owner of the patent on the highest quality product

model in which cost reducing technological progress opens the way to potential applications to specific market. We can interpret the former as basic research and the latter as applied research.

in its industry. Product improvements occur in each consumption good industry, and, within each industry, firms are distinguished by the quality of the final good they produce. When the state-of-the-art quality product in an industry $\omega \in [0,1]$ is $j_t(\omega)$, R&D firms compete in order to learn how to produce the $j_t(\omega)+1$st quality product. We extend the standard quality ladders model by introducing a two-stage innovation path, so first a researcher catches a glimpse of innovation through the $j_t(\omega)+\frac{1}{2}$th inventive half-idea, and then other researchers engage in a patent race to implement it in the $j_t(\omega)+1$st quality product[12]. The best real world interpretation of our "half ideas" are the *research tools*. So, in each industry, the R&D activity is a two stage process by which, first a new idea is invented upstream - a first "half-idea" - and then it is used to find the way to introduce a higher quality product: in the words of Grossman and Shapiro (1987, p.373), the "two stages may be thought of as research and development, respectively."

As in Grossman and Helpman (1991a and b), time is continuous with an unbounded horizon and there is a continuum of infinitely-lived households with identical intertemporally additive preferences. Heterogeneous labour, skilled and unskilled, is the only factor of production. Both labour markets are assumed perfectly competitive. In the final good sectors $\omega \in [0,1]$ monopolistically competitive patent holders of the cutting edge quality good produce differentiated consumption goods by combining skilled and unskilled labour, whereas research firms employ only skilled labour. To facilitate the exposition, the most standard analytical details of the model can be found in the Appendix 1.

2.2 The Mechanics of R&D, and Preliminary Results

In our economy the whole set of industries $\{\omega \in [0,1]\}$ gets partitioned into two subsets of industries: at each date t, there are industries $\omega \in A_0$ with (temporarily) no research tool and, therefore, with one quality leader (the final product patent holder), no applied research and a mass of basic researchers, and the industries $\omega \in A_1 = [0,1] \setminus A_0$, with one research tool and, therefore, one quality leader and a mass of applied researchers directly

[12] Of course, half ideas could be as difficult to get as are Nobel prizes: see, for example, the Cohen-Boyer patents on the basic method and plasmids for gene cloning (granted in 1990).

challenging the incumbent monopolist. Researchers engage in useful[13] basic R&D only in $\omega \in A_0$ industries, while R&D firms engage in applied R&D activity aimed at a final product innovation only in A_1 industries. When a quality improvement occurs in an A_1 industry, the innovator becomes the new quality leader and the industry switches from A_1 to A_0. Similarly, when a discovery arises in an industry $\omega \in A_0$ this industry switches to A_1. Figure 1 illustrates the flow of industries from a condition to the other:

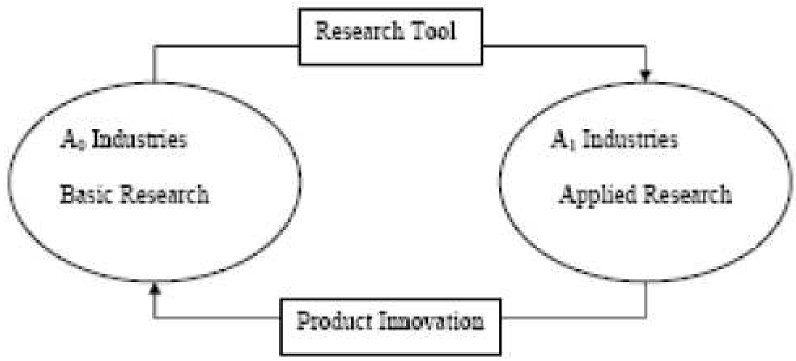

Figure 1 Representation of the economy by flows of industries

Notice that in our multisector two-stage environment with perpetual innovation basic R&D alternates with applied R&D in all sectors of the economy. The two sets A_0 and A_1 change over time, even if the economy will eventually tend to a steady state. At any instant we can measure the mass of industries without any half-idea as $m(A_0) \in [0, 1]$, and the mass of industries with an uncompleted half-idea as $m(A_1) = 1 - m(A_0)$. Clearly, in the steady state these measures will be constant, as the flows in and out will offset each other. However, the endogenous nature of the steady state equilibrium distribution of sectors allows us to study the effects of different institutional scenarios - patentability regimes, public sector inefficiency - on technological dynamics and aggregate innovation. Let index $i = B, A$ denote basic or applied research. $n_i(\omega, t)$,indicates the mass of skilled labor employed in basic, and, respectively, applied research in sector $\omega \in [0, 1]$ at date t. A researcher's

[13] In one of the three economies stylized in this paper, namely the Public Basic Research scenario, some basic research is undertaken also in A_1 industries, but it produces no innovation.

Poisson process probability of succeeding in inventing a half-idea, or completing one (i.e. introducing the product innovation), is decreasing in the aggregate sectorial R&D labor, $n_i \geq 0$. In particular, we specify the per-unit time Poisson probability intensity to succeed for a basic and an applied research labour unit respectively as

$$\theta_B(\omega,t) \equiv \lambda_0 n_B(\omega,t)^{-a},\ \omega \in A_0 \quad (1)$$
$$\theta_A(\omega,t) \equiv \lambda_1 n_A(\omega,t)^{-a},\ \omega \in A_1 \quad (2)$$

where $\lambda_k > 0$, $k = 0, 1$, are R&D productivity parameters[14] and constant $0 < a < 1$ is an intra-sectorial congestion parameter, capturing[15] the risk of R&D duplications, knowledge theft and other diseconomies of fragmentation in the R&D. Each Poisson process - with arrival rates described by (1)-(2) - governing the assumed two-stage innovative process is supposed to be independent across researchers and across industries. Hence the total amount of probability per unit time of inventing a basic half idea in a sector $\omega \in A_0$ at date t is $n_B(\omega,t)\theta_B(\omega,t)$ and the total amount of probability per unit time of completing a basic research tool in a sector $\omega \in A_1$ is $n_A(\omega,t)\theta_A(\omega,t)$.

Moreover, in all our scenarios, symmetric equilibria exist, allowing us to simplify notation: $n_B(\omega,t) \equiv n_B(t)$ and $n_A(\omega,t) \equiv n_A(t)$.

2.2.1 Manufacturing

So far we have assumed an exogenously given aggregate amount of skilled labour, L, employable in the manufacturing and in the R&D sectors; and an exogenously given aggregate amount of unskilled labour, M, only employable in manufacturing. Adopting the unskilled wage as the numeraire, we will endogenously determine the skill premium, as summarized by the skilled labour (relative) wage w_s.

[14] Eq.s (1)-(2) are build on the assumption of a stationary population. With increasing population, it is easy to recast our model, as done in Appendix 1, in terms of Dinopoulos and Segerstrom's (1999) PEG framework, which captures the difficulty of improving a good in a way that renders a larger population happier. This eliminates the strong scale effect (Jones 2003) that plagued the early generation endogenous growth models, without leading to "semi-endogenous" growth (Jones 1995, Segerstrom 1998), as consistent with recent empirical evidence (e.g. Madsen, 2008). Despite its semplicity, this assumption is equivalent to eliminating the strong scale effect by means of an R&D "dilution effect" over an increasing range of varieties, as proved by Peretto (1998), Young (1998), Dinopoulos and Thompson (1998) and (1999), and Howitt (1999).

[15] As in Jones and Williams' (1998 and 2000) specification of the R&D technology.

In all our equilibria, the per-capita mass of skilled labour employed in manufacturing sector $\omega \in [0, 1]$ at time t, labeled $x(\omega, t)$, will be constant across sectors and equal to $x(\omega, t) = x(t)$. In fact, in the Appendix 1 we prove that the manufacturing employment of the skilled labour obeys the following decreasing function of the relative skilled wage w_s:

$$x(\omega, t) = \frac{1}{w_s(t)} \left(\frac{\alpha}{1-\alpha} \right) M \equiv x(t),$$

where $0 < \alpha < 1$ is the skilled labour elasticity of output. Appendix 2 also show that at any date the profit flows are constant and equal to $\pi = (\gamma - 1) \frac{1}{1-\alpha} M$, where $\gamma > 1$ is the size of each product quality jump.

Since the total mass of sectors in the economy is normalized to 1, $x(t)$ also denotes the aggregate employment of skilled in manufacturing. Hence, these always hold: $x(t)w_s(t) = \alpha Y(t)$ and $M = Mw_u(t) = (1-\alpha)Y(t)$, where $Y(t)$ is aggregate final good production.

In light of the previous discussion, and dropping time indexes for simplicity[16], we can express the skilled labor market equilibrium as:

$$L = \frac{1}{w_s} \left(\frac{\alpha}{1-\alpha} \right) M + m(A_0)n_B + m(A_1)n_A. \tag{3}$$

Eq. (3) states that, at each date, the aggregate supply of skilled labor, L, finds employment in the manufacturing firms of all $[0, 1]$ sectors, x, and in the R&D laboratories of the A_0 sectors, n_B, and of the A_1 sectors, n_A.

3 The Public Basic Research Economy

In this section we assume unpatentable basic scientific results, in order to depict a pre-1980 US normative environment. In our model, public R&D is allocated regardless of profit opportunities: since researchers get paid regardless of the profitability of their discoveries, their activity is "curiosity driven", and their rewards are not aligned to downstream needs. Hence their efforts might, from a social viewpoint, be wrongly targeted. To stylize the partially "un-focussed" research behavior of the public researchers, we assume that

[16] Of course time dependence is implicit, as employment variables, wage, and the mass of sectors in which a half idea is present, respectively absent, keep changing over time, except in the steady state.

public researchers are totally indifferent to sectorial profitability: when in a sector ω that lacked a half-idea, i.e. belonged to A_0, a research tool appears, i.e. it becomes A_1, the public R&D workers keep carrying out basic research in that sector. Given our technological assumptions, this labour is redundant from the economic view point because research tools cannot usefully accumulate.

We will assume from here on that the public researchers are allocated across different industries according to a uniform distribution.

We also make the assumption that the government exogenously sets the fraction, $\bar{L}_G \in [0, L]$, of population of skilled workers to be allocated to the heterogenous research activities conducted by universities and other scientific institutions and funds it by lump sum taxes on consumers. The assumption of lump sum taxation guarantees that government R&D expenditure does not imply additional distortions on private decisions.

Given the mass of sectors normalized to 1, $\bar{L}_G$ is also equal the per sector amount of R&D. Therefore, each basic research labour unit has a probability per unit of time of making a discovery equal to $\theta_O \equiv \lambda_0 \bar{L}_G^{-a}$. Therefore the probability that in any sector $\omega \in A_0$ a useful half idea appears is $\bar{L}_G \theta_B \equiv \bar{L}_G^{1-a} \lambda_0$, whereas the probability that an existing half idea generates a new marketable product is $n_A \theta_A = n_A^{1-a} \lambda_1$.

Let us define v_L^0 the value of a monopolistic firm producing the top quality product in a sector $\omega \in A_0$, and v_L^1 the value of a monopolistic firm producing the top quality product in any sector $\omega \in A_1$. These two types of quality leaders - competing instantaneously a la Bertrand - both earn the same profit flow, π, but the first type has a longer expected life, before being replaced by the new quality leader, i.e. by the patent holder of the next version of the kind of product it is currently producing. In sectors that are currently of type A_0 no applied R&D firms enters because there is no half idea to develop: they shall wait until public researchers invent one, causing that sector to switch into A_1. Instead, in an A_1 sector, applied R&D firms hire skilled workers in order to complete the freely available half idea. Since there is free entry into applied research, the R&D firm's expected profits are dissipated due to our assumption of perfectly efficient financial market that completely diversify the portfolios of risk averse savers, and transferred to the skilled workers. From a welfare perspective, entry into applied R&D could be excessive, thereby generating distortions.

We define r as the relevant real interest rate - the following equations will hold at any date:

$$
\begin{aligned}
w_s &= \lambda_1 n_A^{-a} v_L^0 && \text{(4a)}\\
rv_L^0 &= \pi - \bar{L}_G^{1-a}\lambda_0\left(v_L^0 - v_L^1\right) + \frac{dv_L^0}{dt} && \text{(4b)}\\
rv_L^1 &= \pi - n_A^{1-a}\lambda_1 v_L^1 + \frac{dv_L^1}{dt}. && \text{(4c)}
\end{aligned}
$$

Eq. (4a) is the free entry condition in downstream research in any sector $\omega \in A_1$, equalizing the unit cost of R&D (the skilled wage) to the expected marginal gain - the per unit time probability flow $\lambda_1 n_A^{-a}$ of inventing the next version of the final product multiplied by the value of its patent, v_L^0. Eq. (4b) states that perfectly efficient financial markets lead v_L^0 to the unique value such that the risk free interest income attainable by selling the stock market value of a leader in an A_0 industry, rv_L^0, equals the flow of profit π minus the expected capital loss from being challenged by a half-idea on a better product in the case a follower appears, $\bar{L}_G^{1-a}\lambda_0\left(v_L^0 - v_L^1\right)$, plus gradual appreciation in the case of such event not occurring, $\frac{dv_L^0}{dt}$. In a steady state $\frac{dv_L^0}{dt} = 0$.

Eq. (4c) equals the risk free income per unit time deriving from the liquidation of the stock market value of a leader in an A_1 industry, rv_L^1, and the relative flow of profit π minus the expected capital loss, $n_A^{1-a}\lambda_1 v_L^1$, due to the downstream applied researcher firms' R&D, plus the gradual appreciation if replacement does not occur, $\frac{dv_L^1}{dt}$. In a steady state $\frac{dv_L^1}{dt} = 0$.

All jump processes occurring at the industry level are independent across industries, and the law of large number transforms flow probabilities into deterministic flows. Hence, after aggregating over the set of sectors, the dynamics of the mass of industries is described by the following first order ordinary differential equation:

$$
\frac{dm(A_0)}{dt} = \left(1 - m(A_0)\right) n_A^{1-a}\lambda_1 - m(A_0)\bar{L}_G^{1-a}\lambda_0. \tag{5}
$$

From the skilled labor market clearing condition:

$$
x + \bar{L}_G + (1 - m(A_0))n_A = L, \tag{6}
$$

and the definition of x, we obtain the equilibrium mass of per-sector challengers:

$$
n_A^* = \frac{L - \frac{1}{w_s}\left(\frac{\alpha}{1-\alpha}\right) M - \bar{L}_G}{(1 - m(A_0))}. \tag{7}
$$

Hence the dynamics of this economy is completely characterized by the differential equation system (4a)-(4c) and (5), with cross equation restriction (7).

3.1 Balanced Growth Path

In a balanced growth path equilibrium all variables are constant except the average quality of consumer goods[17], and therefore the instantaneous percapita utility index, which grows at a constant rate[18] $\ln(\gamma)g_{PUBBL}$ proportional to the aggregate innovation rate $g_{PUBBL} = m(A_0)\bar{L}_G^{1-a}\lambda_0 = (1-m(A_0))\,\lambda_1\,(n_A)^{1-a}$. Based on the previous characterization, we can state:

Definition 1. *A balanced growth path equilibrium of the Public Basic Research economy is a vector* $[m(A_0), n_A, v_L^0\ v_L^1, w_s, x, g_{PUBBL}] \in R_+^7$*, satisfying* $m(A_0) \in [0,1]$ *and the following equations:*

$$
\begin{aligned}
w_s &= \lambda_1 n_A^{-a} v_L^0 && \text{(8a)}\\
rv_L^0 &= (\gamma-1)\,\tfrac{1}{1-\alpha}M - \bar{L}_G^{1-a}\lambda_0\left(v_L^0 - v_L^1\right) && \text{(8b)}\\
rv_L^1 &= (\gamma-1)\,\tfrac{1}{1-\alpha}M - n_A^{1-a}\lambda_1 v_L^1 && \text{(8c)}\\
x &= \frac{1}{w_s}\left(\frac{\alpha}{1-\alpha}\right)M && \text{(8d)}\\
(1-m(A_0))\,n_A^{1-a}\lambda_1 &= m(A_0)\bar{L}_G^{1-a}\lambda_0 && \text{(8e)}\\
x + \bar{L}_G + (1-m(A_0))n_A &= L && \text{(8f)}\\
g_{PUBBL} &= \lambda_1\,(1-m(A_0))\,n_A^{1-a}. && \text{(8g)}
\end{aligned}
$$

Given the high non-linearity of system (8a)-(8g), we performed numerical simulations in Matlab[19]. In all simulations a unique economically meaningful

[17] Since we are following Grossman and Helpman's (1991b) framework, it is the geometric average $D(t) = \exp\left[\int_0^1 \ln\left[\gamma^{j_t^*(\omega)} d_{j_t^*(\omega)t}(\omega)\right] d\omega\right]$ that matters. Appendix 1 clarifies these aspects in detail.

[18] This is a usual property of quality ladder models (see e.g. Grossman and Helpman, 1991a and b). Find more on this in the welfare calculations in Appendix 1.

[19] The Matlab and Dynare files used to simulate the model are available from the authors upon request.

steady state equilibrium exists. Moreover, analysing the eigenvalues of the Jacobian matrix of the fully dynamic (out of steady state) system shows that the steady state equilibrium is saddle point stable. Therefore the equilibrium is determinate.

Since in principle there could have multiple steady states, the empirical calibrations and policy conclusions we will obtain in the later sections are credible only if we can first prove analytically the uniqueness of the steady state. In fact, it turns out that this steady state equilibrium is unique, as proved in the following[20]:

Lemma 1. *In the Public Basic Research economy there can exist no more than one balanced growth path equilibrium.*

Proof. See Appendix 2.

4 The Privatized Basic Research Economy

In this section, stylizing a post-1980 US scenario, we assume that once a research tool is invented in an A_0 sector, it gets protected by a patent with infinite legal life. The presence of enforced intellectual property rights on the research tools permits the existence of a market for basic research findings. This implies that, unlike the public researchers of the previous section's scenario, now the basic researchers target their activity only in the A_0 sectors.

Let v_A, denote the present expected value of being a research tool patent holder running a downstream applied R&D firm, operating in an A_1 industry and aiming at becoming a new quality leader. Such a firm - similarly to Grossman and Shapiro's (1986) monopolist - will optimally choose to hire an amount n_A of skilled research labour in order to maximize the difference between its expected gains from completing its own half idea - probability of inventing, $(n_A)^{1-a}\lambda_1$, times the net gain from inventing the final product, $(v_L^0 - v_A)$ - and the implied labour cost $w_s n_A$. From its first order conditions, we easily obtain the optimal applied R&D employment in an A_1 sector:

$$n_A^* = \left[\frac{(1-a)\lambda_1(v_L^0 - v_A)}{w_s}\right]^{\frac{1}{a}}. \tag{9}$$

[20] We are indebted to a Referee for this important analytical point.

Unlike the previous section, now only the research tool patent holder can undertake applied R&D in its industry, whereas free entry is relegated to the basic research stage, where researchers vie for inventing the half idea that will render the winner the only owner of a research tool patent worth v_A. Hence their freely entering and exiting mass will dissipate any excess earning, by equalizing wage to the probability flow $\lambda_0 n_B^{-a}$ times the value of a patent on a half idea[21], v_A. Therefore excessive entry into basic research can cause welfare losses.

Costless arbitraging between risk free loans and firms' equities implies that at each instant the following arbitrage equations must hold in equilibrium:

$$w_s = \lambda_0 n_B^{-a} v_A \tag{10a}$$

$$r v_A = \left(n_A^*\right)^{1-a} \lambda_1 (v_L^0 - v_A) - w_s n_A^* + \frac{dv_A}{dt} \tag{10b}$$

$$r v_L^0 = \pi - \left(n_B\right)^{1-a} \lambda_0 \left(v_L^0 - v_L^1\right) + \frac{dv_L^0}{dt} \tag{10c}$$

$$r v_L^1 = \pi - \left(n_A^*\right)^{1-a} \lambda_1 v_L^1 + \frac{dv_L^1}{dt} \tag{10d}$$

The first equation, (10a), is the free entry condition in the upstream basic research sector. The second equation equalizes the risk free income deriving from the liquidation of the expected present value of the research tool patent in an A_1 industry, rv_A, and the expected increase in value from becoming a quality leader (i.e. completing the product innovation process), $\left(n_A^*\right)^{1-a} \lambda_1(v_L^0 - v_A)$, minus the relative R&D cost, $w_s n_A^*$, plus the gradual appreciation in the case of R&D success not arriving, $\frac{dv_A}{dt}$.

The third and forth equations are as in the previous section.

Plugging $w_s = \lambda_0 n_B^{-a} v_A$ into the expression of the skilled labour wage ratio (eq. 39, in the Appendix 1) and using percapita notation, we obtain:

$$x = \frac{1}{w_s}\left(\frac{\alpha}{1-\alpha}\right) M = \min\left(\frac{n_B^a}{\lambda_0 v_A}, 1\right)\left(\frac{\alpha}{1-\alpha}\right) M. \tag{11}$$

[21] Unlike Grossman and Shapiro (1987), the research tool patent holder has no incentive to license, because in our framework scale diseconomies are assumed at the industry level but not at the firm level.

We have implicitly assumed that $w_s \geq 1$, because skilled workers always have the option to work as unskilled workers. Therefore the skilled labor employment in the manufacturing sector is inversely related to the market value of patented research tools.

The skilled labor market clearing condition states:

$$x + m(A_0)n_B + (1 - m(A_0))\, n_A^* = L \tag{12}$$

Hence, since wages are pinned down by the optimal firm size and by the zero profit conditions in the perfectly competitive basic R&D labor markets, the unique equilibrium per-sector mass of entrant basic R&D firms consistent with skilled labor market clearing (12) is determined by solving equation (12) for n_B:

$$n_B = \frac{L - x - (1 - m(A_0))n_A^*}{m(A_0)}. \tag{13}$$

To complete our analysis, let us look more closely at the inter-industry dynamics depicted by Figure 1. In the set of basic research industries a given number of perfectly competitive (freely entered) upstream researchers, n_B^*, have a flow probability of becoming applied researchers, while in the set of the applied R&D industries each of the n_A^* per-industry applied researchers has a flow probability to succeed. By the law of large numbers, the industrial dynamics of this economy is described by the following first order ordinary differential equation:

$$\frac{dm(A_0)}{dt} = (1 - m(A_0))\, \lambda_1 \left(n_A^*\right)^{1-a} - m(A_0) \left(n_B\right)^{1-a} \lambda_0. \tag{14}$$

System (10b)-(10d) and eq. (14) - jointly with cross equation restrictions (11) and (13) - form a system of four first order ordinary differential equations, whose solution describes the dynamics of this economy for any admissible initial value of the unknown functions of time v_L^0, v_L^1,v_A, and $m(A_0)$.

In a steady state, $\frac{dv_L^1}{dt} = \frac{dv_L^0}{dt} = \frac{dv_A}{dt} = \frac{dm(A_0)}{dt} = 0$.

Remark. The important difference from the unpatentable research tools case, is that here: 1. There is - potentially excessive - endogenous entry into basic research; 2. Congestion in applied research is internalized by the basic patent holder. Therefore both the growth and the welfare comparisons between the two regimes are not obvious and the outcome could depend on the parameter values.

4.1 Balanced Growth Path

In the balanced growth path equilibrium all variables are constant except the average quality of consumer goods, and therefore the instantaneous percapita utility index, which grows at a constant rate $\ln(\gamma) g_{PRIV}$ proportional to the aggregate innovation rate $g_{PRIV} = m(A_0)\left(n_B\right)^{1-a}\lambda_0 = (1-m(A_0))\,\lambda_1\left(n_A^*\right)^{1-a}$. Based on the previous characterization, we can state:

Definition 2. *A balanced growth path equilibrium of the Privatized Basic Research economy is a vector*$[m(A_0), n_B, n_A^*, v_A, v_L^0\ v_L^1, w_s, x, g_{PRIV}] \in R_+^9$ *satisfying* $m(A_0) \in [0,1]$ *and the following equations:*

$$
\begin{aligned}
w_s &= \lambda_0 n_B^{-a} v_A && \text{(15a)}\\
r v_A &= \left(n_A^*\right)^{1-a}\lambda_1 (v_L^0 - v_A) - w_s n_A^* && \text{(15b)}\\
n_A^* &= \left[\frac{(1-a)\lambda_1(v_L^0 - v_A)}{w_s}\right]^{\frac{1}{a}} && \text{(15c)}\\
r v_L^0 &= \pi - \left(n_B\right)^{1-a}\lambda_0\left(v_L^0 - v_L^1\right) && \text{(15d)}\\
r v_L^1 &= \pi - \left(n_A^*\right)^{1-a}\lambda_1 v_L^1 && \text{(15e)}\\
(1-m(A_0))\,\lambda_1\left(n_A^*\right)^{1-a} &= m(A_0)\left(n_B\right)^{1-a}\lambda_0 && \text{(15f)}\\
L &= x + m(A_0) n_B + (1-m(A_0))\, n_A^* && \text{(15g)}\\
x &= \frac{1}{w_s}\left(\frac{\alpha}{1-\alpha}\right) M && \text{(15h)}\\
g_{PRIV} &= m(A_0)\left(n_B\right)^{1-a}\lambda_0. && \text{(15i)}
\end{aligned}
$$

Given the analytical complexity of such system we resorted to numerical analysis. It is worthwhile mentioning also in this case, that in all numerical simulations of the fully dynamical system we have run, the steady state is saddle point stable for any set of parameter values we have tried.

Since, in principle, our numerical simulations could converge to just one of possibly many different steady states, the empirical calibrations and policy conclusions we could obtain would not be credible unless we can first prove analytically the uniqueness of the steady state in the model we are using. This is achieved by the following:

Lemma 2. *In the Privatized Basic Research economy there can exist no more than one balanced growth path equilibrium.*

Proof. See Appendix 2.

5 Quantitative Analysis

In general, simulating our models[22] suggests that an economy in which public basic research is conducted in a non-profit oriented manner can induce less or more innovations and/or welfare than an economy in which basic R&D is privately carried out. The privatized economy outgrows the public basic research economy when the applied R&D productivity parameter, λ_1, becomes very low: in such cases the equilibrium innovative performance of the private economy with patentable research tools becomes better than the equilibrium growth performance of the economy with a public R&D sector. In fact, if λ_1 is very small or λ_0 is high, the flow out of A_1 will be scarce, whereas the flow out of A_0 will be intense. Therefore in the steady state $m(A_0)$ will be small, thereby exalting the wasteful nature of the public R&D activity uniformly diluted over $[0, 1] - A_0$: in this case the social cost of a public R&D blind to the social needs signalled by the invisible hand would overwhelm the social costs of the restricted entry into the applied R&D sector induced by the patentability of research tools.

While the discussion so far highlights the growth perspective, the aggregate consumer utility - welfare - is also affected negatively by the potentially excessive entry associated with patent races. Since in either regime there is free entry into one of the two types of research activities, this may lead to excessive entry into basic research in the private regime, and excessive entry into development in the public regime. While the lack of commercial focus in basic research can make publicly funded research worse, excessive entry into basic research in the private regime can potentially counter this handicap. Hence, it is not possible a priori to rank the two regimes.

In the next sections we will estimate the unknown parameters and use others taken from the literature, in order to evaluate the alternative patenting regimes. We will undertake our calibrations under the simplifying assumption that the US economy was in an unpatentable research tools balanced growth path from 1963 to 1980. This will deliver the parameter values with which to simulate the alternative scenarios at the last year[23] of the public basic R&D

[22] The codes we have used are available upon request.

[23] Qualitative results would not change if we had chosen another year, or included an average of four years before 1979.

regime (1979). We will not use data from 1980, because we cannot assume that changes from one to the other regime are instantaneous: particularly in the case of basic research, innovation takes many years and its effects should accrue over time.

5.1 Calibration

In this section we calibrate our model to a balanced growth path using U.S. data from 1963 to 1980, obtaining the values of these parameters as well as the endogenous variables in the unpatentable research tools case, which we believe prevailed during that period. Our exercise will obtain an estimation of the difficulty of R&D, summarized inversely by the basic/applied productivity parameters, λ_0 and λ_1. Consistently with our theoretical model, we use only skilled and unskilled labour as inputs and numbers of qualified innovations as R&D output, as represented by patents.

5.2 Description of the Procedure and the Data

Our calibration procedure consists of the following four steps:

1. GMM estimation of the values of the unobservable parameters α, λ_0 and λ_1 based on U.S. 1963-1980 data: results in Table 2.

2. Use of the estimated parameter values $\hat{\alpha}$, $\hat{\lambda}_0$ and $\hat{\lambda}_1$, along with other parameters shown in Table 1 in the system of equations of the balanced growth path equilibrium of the Privatized Basic Research Economy.

3. Use of the previous parameters and of the steady state equilibrium amount of basic research labour, $m(A_0)n_B$, estimated in Step 2 into the Public Basic Research Economy scenario, setting $L_G = m(A_0)n_B$, and simulation of the corresponding Public Basic Research Economy model.

4. Comparison of the steady state innovation rates and welfare levels of the two policy scenarios of steps 2 and 3.

L is the percentage of people who were 25 year old or more and who had completed at least 4 years of college, collected by the U.S. Census (2010a), Current Population Survey, Historical Tables[24].

[24] Available at: www.census.gov/population/socdemo/education/tabA-2.xls

We set the intra-sectorial congestion parameter $a = 0.3$, consistently with Jones and Williams' (1998) and (2000) calibrations.

$\bar{L}_G$ is calculated by dividing the expenditure on basic research by the amount of wages paid to publicly employed scientist and engineers[25]. The relevant series of the expenditure on basic research in our estimations is the total basic R&D expenditure net of the industry performed basic R&D[26].

w_s is the skilled premium estimated by Krusell, Ohanian, Rios-Rull and Violante (2000).

The g_{PUBBL} data (according to our model, the measure of the actual U.S. innovation rate before 1980) are the number of utility patents granted to U.S. residents per million inhabitants[27].

We set the mark-up γ to 1.60, consistently with what estimated by Roeger (1995) and Martins et al. (1996).

As for the real rate of return on consumer assets, we adopt the usual $r = 0.05$, consistently with Mehra and Prescott's (1985) estimates for the pre-1980 period.

The following Table 1 reports the parameters we have utilised and their sources:

---

[25] Source: US Census - Current Population Survey, Annual Social and Economic supplements.

[26] Both series are taken from the NSF Science & Engeneering Indicators (2005).

[27] Source: USPTO (2010).

Table 1

Parameter	Description	Value	Source
γ	Mark-up	1.60	Roeger (1995) and Martins et al. (1996)
a	R&D congestions	0.3	Mehra and Prescott (1985)
L	Skilled Labour (intensity 1979)	0.164	U.S. Census, Current Population Survey
M	Unkilled Labour (intensity 1979)	0.836	U.S. Census, Current Population Survey
ρ	Subjective Rate of Time Preference	0.05	Jones and Williams (1998) and (2000)
α	Skilled Share in Manufacturing	0.098	GMM Estimation
λ_0	Basic Research Productivity	0.25279	GMM Estimation
λ_1	Applied Research Productivity	0.38116	GMM Estimation

We have estimated the R&D and manufacturing technological parameters λ_0 , λ_1 , and α in the Public Basic Research scenario, by using the Generalised Method of Moments[28] (GMM) with data from 1963 to 1981[29]. The reason

[28] The software we have used is E-views 6.

[29] We can safely use the 1981 data to measure the effects of the "pre-1980" regime, because basic research innovation takes many years and its effects accrue over time. Dropping 1981 and even 1980 would not change qualitative results anyway, though reducing the efficiency of the estimates.

why we have also estimated parameter α - the high skilled labour[30] share in manufacturing production - instead of relying on available statistics, is that they fail to single out the fraction of high skilled labour in production[31], consistently with our stylized economy. Since we do not have data on variables $m(A_0)$, n_A, v_L^0, v_L^1, x, we have reduced system (8a)-(8g) by repeated substitutions to only two equations[32], and used these to estimate the parameters with the remaining variables, w_s, M, L, L_G, g_{PUBBL}, on which we have time series from 1963 to 1981. The GMM estimator can deal with such highly non-linear equations, is consistent, and, more importantly, yields results robust to heteroschedasticity and autocorrelation of unknown form (Hansen, 1982). In the estimates reported in Table 2 we had chosen the weighting matrix in GMM-Time Series (HAC), with Newey and West fixed bandwidth[33] Quite reassuringly, our results do not differ substantially[34] when we use the Two-Stage Instrumental Variable (IV) estimator, which may be desirable in small samples in case heteroschedasticity is not present. Similarly for the Three-Stage IV estimators[35]. In all our GMM and IV regressions we have used lagged innovation as an instrument.

In order to check the robustness of our simulations of the alternative scenarios, we have let our estimates vary on their 95% confidence interval. In Table 2 we report the GMM estimated confidence intervals for the estimated parameters:

---

[30] In this paper's restrictive interpretation as highly skilled workers with at least college education, and able to perform R&D activities competently.

[31] For example, the ratio of non-production workers in operating establishments to total employment in 1979 was 0.248 (Berman, Bound, and Griliches, 1994), but this would include a large fraction of not highly skilled workers, as well as people actually undertaking knowledge-related activities.

[32] Reported in Appendix 3.

[33] But results only marginally changed when we used (as a robustness check) Andrews and Variable Newey-West bandwidth selection. These are not reported in the paper to save space, but would not change the rankings of the simulated scenarios.

[34] They are almost identical even with simple Nonlinear Least Squares. Of course, the GMM estimator is more efficient in the presence of arbitrary heteroschedasticity.

[35] With resulting estimates: $\hat{\lambda}_0 = 0.257987$, $\hat{\lambda}_1 = 0.387889$, and $\hat{\alpha} = 0.097176$; and p-values lower than 2%.

Table 2

Method:	GMM Coefficient $\pm t_{0.025}(18-3) * se$
α	$0.09822279 \pm 2.131 * 0.00549361$
λ_0	$0.252794927 \pm 2.131 * 0.007829546$
λ_1	$0.381161461 \pm 2.131 * 0.0897984364$

where $t_{0.975}(18-3)$denotes the 97.5% value of the Student-t random variable with $T-k=15$ degrees of freedom, and se denotes the standard errors of the estimate. All variables are highly significant, and their 5% confidence interval are relatively small. We will use them, along with the other parameters taken from the literature, to compare the alternative policy scenarios.

5.3 Policy Comparisons

In this section we utilise the previously estimated values of the technological parameters as well as all the previously estimated exogenous to compute the hypothetical steady state equilibrium of the two scenarios - unpatentable research tools versus patentable research tools - for the year 1979, i.e. the last year of the non-patentable research tools regime. It is important to remark that the qualitative results do not change if instead we use any combinations of the data in the last 5 years time interval (from 1975 to 1979).

In our exercise, we compared the steady state equilibrium innovative performance of the patentable research tool scenario not only with the actual performance in those years, but also with a hypothetical public scenario constrained to employ the same number of basic researchers as would the privatize system have done. This allowed us to purge the comparison from different levels of employment and allows us to focus on the induced efficiency gains from research tool patentability. In fact, the endogenous public sector inefficiency in channelling researcher's effort only in the sectors where firms need a research tool is weighted against the under-incentive effect of the patented research tools in the downstream research.

The following Table 3 lists the comparative innovation rates in the privatized scenario and in the public basic research scenario, at the estimated coefficient values as well as at the lower and higher bounds of their 95% confidence intervals We have fixed α at its point estimate value, just to economize on space, but results would not change much (certainly not the qualitative ranking) if we had let α take on other values in its 95% confidence interval. However, given their importance, we report the results associated with the technological parameters λ_0 and λ_1. The upper part of Table 3 shows how the balanced growth path aggregate innovation rate of the public basic research economy, g_{PUBBL}, changes over the 95% confidence interval of the parameters λ_0 and λ_1; while the lower part of Table 3 shows how the balanced growth path aggregate innovation rate of the privatized basic research economy, g_{PRIV}, changes over the 95% confidence interval of the parameters λ_0 and λ_1.

Table 3

g_{PUBBL}	Lowest λ_1	Estimated λ_1	Highest λ_1
Lowest λ_0	0.0193	0.0307	0.0371
Estimated λ_0	0.0196	0.0314	0.0381
Highest λ_0	0.0204	0.0320	0.0390

g_{PRIV}	Lowest λ_1	Estimated λ_1	Highest λ_1
Lowest λ_0	0.0203	0.0314	0.0373
Estimated λ_0	0.0207	0.0321	0.0383
Highest λ_0	0.0215	0.0329	0.0393

As the data in the table show, the privatized basic research scenario outgrows the public basic R&D regime for all combinations of the underlying technological parameters along their 95% confidence interval. The simulated privatized economy outgrew the unpatentable R&D scenario in the relevant period immediately before the US turning point, and hence we can say that the 1980 US normative change was the rational institutional response to underlying technological modifications, as if politicians literally simulated the effects of the reformation within their minds before deciding to change the laws.

We have also simulated the welfare levels[36]

$$\begin{aligned} Welf_s &= \int_0^\infty e^{-rt}\left[\log(\gamma)g_s t + \log(x_s^\alpha M^{1-\alpha})\right] dt = \\ &= \frac{\log(\gamma)g_s}{r^2} + \frac{\log(x_s^\alpha M^{1-\alpha})}{r},\ s = PUBBL, \text{ and } PRIV. \qquad (16) \end{aligned}$$

associated with the different IPR scenarios. Notice that both the steady state innovation rate, g_s, and the steady state skilled manufacturing employment, x_s, can differ in different institutional scenarios s. More labour in research would imply less manufacturing, with a negative level effect on welfare, possibly compensated by a positive growth effect. Since the unskilled workers are only employed in manufacturing, its level, M, does not change with s.

The simulated welfare values are shown in Table 4. The upper part of Table 4 shows how the balanced growth path welfare of the public basic research economy, $Welf_{PUBBL}$, changes over the 95% confidence interval of the parameters λ_0 and λ_1; while the lower part of Table 4 shows how the balanced growth path welfare of the privatized basic research economy, $Welf_{PRIV}$, changes over the 95% confidence interval of the parameters λ_0 and λ_1.

Table 4

$Welf_{PUBBL}$	Lowest λ_1	Estimated λ_1	Highest λ_1
Lowest λ_0	-5.7138	-4.4561	-3.7273
Estimated λ_0	-5.6039	-4.2161	-3.3904
Highest λ_0	-5.5003	-3.9873	-3.0656

$Welf_{PRIV}$	Lowest λ_1	Estimated λ_1	Highest λ_1
Lowest λ_0	-4.8690	-3.2804	-2.3743
Estimated λ_0	-4.7366	-3.0160	-2.0207
Highest λ_0	-4.6111	-2.7615	-1.6769

The privatized basic research regime seems to dominate the public regime also in terms of welfare. Therefore the reader can notice from our tables that in 1979 the unpatentability of the basic scientific findings imposed more inefficiency to the US innovation system than would the monopolization of

[36] See Appendix 1 for the derivation of this expression.

applied research would have implied. The policy makers or the courts ended up acting as if they had been aware of this, thereby switching law and doctrine towards the patentability of research tools, inaugurated at the beginning of the Eighties. Therefore our analysis suggests that the policy change in favour of the research tools patentability occurred in the United States from the early Eighties was very likely to be the best institutional reaction to the increase in R&D complexity.

6 The Research Exemption Economy

A patent gives the inventor the exclusive rights to manufacture, use or sell the invention. But it is more important to stress that all these rights are *veto*-rights: hence they can be exercised only if the patent holder is able to observe and sue the infringer of his/her patent. Unlike the production of new final products, which can be easily observed by someone who has a patent on it, the use of a specific research tool in the R&D of a new product can hardly be observed by third parties: its only output is the probability per unit time of innovating. More realistically, only after the innovation has actually appeared - i.e. the corresponding final product gets patented and actually produced - will the research tool patent holder be able to effectively exercise his power to sue, forcing the infringer who succeeded in innovating to share the profits resulting from the sale of the final product. This kind of strategic R&D environment is known as "Research Exemption", and it is subject to intense juridical controversies[37], following the famous Supreme Court decision on Madey *v.* Duke University suit, which practically eliminated the possibility of appealing to it, except under very narrow circumstances. In cases where access to research tools through the marketplace is highly problematic, a research exemption is deemed desirable (Mueller, 2004).

Therefore, the privatized scenario of Section 4 corresponds to a an extreme case of perfect information and verifiability of the unauthorized use of the patented research tool. Here, in order to assess the robustness of our previous numerical results, we simulate would happen in another privatized case, but with imperfectly informed patent holders. With this aim, in this section we develop a third scenario that emphasizes the effect of ex-post bargaining between an upstream patent holder and its downstream developer: an in-

[37] See Mueller (2004) for a detailed discussion of the research exemption debate in the US.

novation (a completed half idea) can be patented and yet infringe another patent (the patented research tool).

The new model of this section is inspired by Green and Scotchmer (1995), which pioneered microeconomic research on this important issue[38]. In order to cast their insight in our general equilibrium framework, we assume that the new final product is patentable but infringes its research tool. Ex post bargaining is rationally expected to transfer to the basic research patent holder a fraction $0 < \beta < 1$ of the value of the final product patent, representing its relative bargaining power. Unlike Green and Scotchmer's (1995) assumption of a unique downstream researcher, we here assume that the downstream unauthorized research with a patented research tools can be carried out by a multitude of freely entrant R&D firms, thereby implying a demand effect on R&D inputs dissipating expected profits, and potentially depressing welfare. Our analysis is also valid in the case of reach-through licensing agreements, which seem pervasive in the US. "For research tools ... [r]oyalities would be pass-through royalties from the product developed to the tool." Maurer and Scotchmer (2004b, p. 236). We first analyze non-exclusive licenses, while the next subsection will study exclusive pass-through licensing agreements. In all our cases, we assume that the ultimate patent on the final product improvement can be granted to only one firm: the first to invent it.

Let v_B,v_L^0, and v_L^1 denote respectively the present expected value of a basic blocking patent (v_B), an A_0 industry quality leader (v_L^0), and an A_1 industry challenged leader (v_L^1).

Costless arbitrage between risk free activities and firms' equities imply that at each instant the following equations shall hold in equilibrium:

$$w_s = \lambda_0 n_B^{-a} v_B \tag{17a}$$

$$r v_B = \lambda_1 n_A^{1-a} \left(\beta v_L^0 - v_B\right) + \frac{dv_B}{dt} \tag{17b}$$

$$w_s = \lambda_1 n_A^{-a} \left(1-\beta\right) v_L^0 \tag{17c}$$

$$r v_L^0 = \pi - n_B^{1-a} \lambda_0 \left(v_L^0 - v_L^1\right) + \frac{dv_L^0}{dt} \tag{17d}$$

$$r v_L^1 = \pi - n_A^{1-a} \lambda_1 v_L^1 + \frac{dv_L^1}{dt} \tag{17e}$$

Equation (17a) is the zero profit condition of a free entrant basic R&D

[38] See Scotchmer (2004, section 5.2) for an accessible exposition of this complex issue.

firm in an A_0 industry, equalizing the skilled wage and the probability $\lambda_0 n_O^{-a}$ of inventing a half idea times the value v_B of the resulting blocking patent.

Equation (17b) states that financial arbitrage pins down the unique value of the blocking patent that equals the risk free income from its sale, rv_B, to the expected present value of maintaining it in an A_1 industry. These are the expected increase in value deriving from someone else's - the n_A downstream researchers' - discovering the industrial application, plus the gradual appreciation in the case of someone else's R&D success not arriving, $\frac{dv_B}{dt}$.

Equation (17c) is the free entry condition for downstream completers that rationally expect to appropriate only fraction $1-\beta$ of the value of the final good monopolist. Notice that unlike in Section 5, the expectation of ex-post bargaining or the presence of reach-through licenses introduces a negative incentive effect of downstream innovation, because the infringer's use of a research tool can appropriate only a fraction of the value of its marginal product.

The last two equations have the usual interpretation.

It is important to note that our results do not hinge on assuming that the first stage patent holder undertakes no applied R&D. In fact, the free entry condition (17c) dissipates all excess profits from doing so: the research tool patent holder, by hiring a marginal unit of skilled labour to complete its patent would increase its expected gains by $\lambda_1 n_A^{-a} (1-\beta) v_L^0 - w_s = 0$. Hence, it would just be equivalent to one of the free entrants into downstream R&D. Therefore, our model is consistent with an indeterminate R&D participation of the first stage blocking patent holder.

It is also important to notice that free entry into downstream research vanifies any attempt to resort to ex ante licensing, which would instead hold if, as Green Scotchmer (1995), Scotchmer (1996), Denicolo (2000), and Aoki and Nagaoka (2007), we had restricted entry to the second stage of R&D to only one completing firm.

As in the previous sections, the industrial dynamics of this economy is described by the following first order ordinary differential equation:

$$\frac{dm(A_0)}{dt} = (1-m(A_0))\,\lambda_1 (n_A)^{1-a} - m(A_0)\,(n_B)^{1-a}\,\lambda_0. \qquad (18)$$

These equations, supplemented with the skilled labour market equilibrium condition

$$x + m(A_0) n_B + (1-m(A_0))\, n_A = L \qquad (19)$$

and by eq. (11) for x determine the equilibrium trajectories.

Since is either regime there is free entry into one of the two types of research activities, this may lead to excessive entry into basic research in the private regime, and excessive entry into development in the public regime, and potentially too in this section's private regime, due to the absence of a market for basic ideas. While the lack of commercial focus in basic research can make publicly funded research worse, excessive entry into basic research in the private regime can potentially counter this handicap. Hence, it is not possible a priori to rank the two regimes. This makes a numerical analysis based on estimated parameters compelling.

We utilise the previously estimated values of the technological parameters as well as all the previously described relevant exogenous parameters to compute the hypothetical steady state equilibrium of the two scenarios - unpatentable research tools versus patentable research tools - for the year 1979, the last year of the non-patentable research tools regime. It is important to remark that our comparisons would not change if instead we use any combinations of the data in the last 5 years time interval (from 1975 to 1979). The aggregate innovation rate in this economy is denoted $g_{REx} \equiv (1 - m(A_0))\, \lambda_1 \left(n_A^*\right)^{1-a}$. This expression will be used in all simulations.

6.1 Balanced Growth Path

Also in this economy, in a balanced growth path equilibrium all variables are constant except the average quality of consumer goods, and therefore the instantaneous percapita utility index, which grows at a constant rate $\ln(\gamma) g_{REx}$ proportional to the aggregate innovation rate $g_{REx} = m(A_0) \left(n_B\right)^{1-a} \lambda_0 = (1 - m(A_0))\, \lambda_1 \left(n_A^*\right)^{1-a}$. Based on the previous characterization, we can state:

Definition 3. *A balanced growth path equilibrium of the Research Exemption economy is a vector* $[m(A_0), n_B, n_A, v_B, v_L^0\ v_L^1, w_s, x, g_{REx}] \in R_+^9$ *satisfying* $m(A_0) \in [0, 1]$ *and the following equations:*

$$
\begin{aligned}
w_s &= \lambda_0 n_B^{-a} v_B && \text{(20a)}\\
rv_B &= \lambda_1 n_A^{1-a}\left(\beta v_L^0 - v_B\right) && \text{(20b)}\\
w_s &= \lambda_1 n_A^{-a}\left(1-\beta\right) v_L^0 && \text{(20c)}\\
rv_L^0 &= \pi - n_B^{1-a}\lambda_0\left(v_L^0 - v_L^1\right) && \text{(20d)}\\
rv_L^1 &= \pi - n_A^{1-a}\lambda_1 v_L^1 && \text{(20e)}\\
\left(1-m(A_0)\right)\lambda_1\left(n_A\right)^{1-a} &= m(A_0)\left(n_B\right)^{1-a}\lambda_0 && \text{(20f)}\\
L &= x + m(A_0)n_B + \left(1-m(A_0)\right)n_A && \text{(20g)}\\
x &= \frac{1}{w_s}\left(\frac{\alpha}{1-\alpha}\right)M && \text{(20h)}\\
g_{REx} &= \left(1-m(A_0)\right)\lambda_1\left(n_A\right)^{1-a}. && \text{(20i)}
\end{aligned}
$$

Due to the analytical complexity of such system, also in this case we resorted to numerical analysis. In all numerical simulations we have run, the steady state exists, and it is saddle point stable for any set of parameter values. Therefore, given an initial condition for $m(A_0)$, there is (locally) only one initial condition for v_L^0, v_L^1, and v_A such that the generated trajectory tends to the steady state vector: the equilibrium is determinate.

Since, in principle, our numerical simulations could converge to just one of possibly many different steady states, the empirical calibrations and policy conclusions we could obtain would not be credible unless we can first prove analytically the uniqueness of the steady state in the model we are using. This is achieved by the following:

Lemma 3. *In the Research Exemption economy there can exist no more than one balanced growth path equilibrium.*

Proof. See Appendix 2.

6.2 Numerical Comparisons

In this section, we compare the steady state equilibrium innovative performance of the patentable research tool scenario with a hypothetical public scenario constrained to employ the same number of basic researchers as in the private equilibrium. As in Scotchmer and Green (1995) and Scotchmer (1996), we set parameter $\beta = 0.5$.

As in the previous comparison, we have followed the following four steps.

1. GMM estimation of the values of the unobservable parameters α, λ_0 and λ_1 based on U.S. 1963-1980 data: results in Table 2.

2. Use of the estimated parameter values $\hat{\alpha}$, $\hat{\lambda}_0$ and $\hat{\lambda}_1$, along with other parameters shown in Table 1 in the system of equations of the balanced growth path equilibrium of the Research Exemption Economy.

3. Use of the previous parameters and of the steady state equilibrium amount of basic research labour, $m(A_0)n_B$, estimated in Step 2 into the Public Basic Research Economy scenario, setting $L_G = m(A_0)n_B$, and simulation of the corresponding Public Basic Research Economy scenario.

4. Comparison of the steady state innovation rates and welfare levels of the two policy scenarios of steps 2 and 3.

The following Table 5, lists the comparative innovation rates in the privatized scenario and in the public basic research scenario, at the estimated coefficient values as well as at the lower and higher extremes of their 95% confidence intervals. We report the results associated with the technological parameters λ_0 and λ_1, which the reader can find in Table 2. The upper part of Table 5 shows how the balanced growth path aggregate innovation rate of the public basic research economy, g_{PUBBL}, changes over the 95% confidence interval of the parameters λ_0 and λ_1; while the lower part of Table 5 shows how the balanced growth path aggregate innovation rate of the research exemption economy, g_{REx}, changes over the 95% confidence interval of the parameters λ_0 and λ_1.

Table 5

g_{PUBBL}	Lowest λ_1	Estimated λ_1	Highest λ_1
Lowest λ_0	0.0190	0.0275	0.0304
Estimated λ_0	0.0198	0.0292	0.0346
Highest λ_0	0.0206	0.0307	0.0326

g_{REx}	Lowest λ_1	Estimated λ_1	Highest λ_1
Lowest λ_0	0.0198	0.0282	0.0307
Estimated λ_0	0.0208	0.0300	0.0352
Highest λ_0	0.0216	0.0318	0.0330

As the data in the table show, the privatized basic research scenario with the possibility of downstream researchers carrying out R&D and infringing the upstream patent holder outgrows the public basic R&D scenario for all combinations of the underlying technological parameters over their 95% confidence interval.

Also in this case we have also simulated the welfare levels

$$\begin{aligned} Welf_s &= \int_0^{\infty} e^{-rt}\left[\log(\gamma)g_s t + \log(x_s^{\alpha}M^{1-\alpha})\right]dt = \\ &= \frac{\log(\gamma)g_s}{r^2} + \frac{\log(x_s^{\alpha}M^{1-\alpha})}{r},\ s = PUBBL, \text{ and } REx. \quad (21) \end{aligned}$$

associated with the different IPR scenarios. Notice again that both the steady state innovation rate, g_s, and the steady state skilled manufacturing employment, x_s, can differ in different institutional scenarios s.

The simulated welfare values are shown in Table 6. The upper part of Table 6 shows how the balanced growth path welfare of the public basic research economy, $Welf_{PUBBL}$, changes over the 95% confidence interval of the parameters λ_0 and λ_1; while the lower part of Table 6 shows how the balanced growth path welfare of the research exemption economy, $WelfREx$, changes over the 95% confidence interval of the parameters λ_0 and λ_1.

Table 6

$Welf_{PUBBL}$	Lowest λ_1	Estimated λ_1	Highest λ_1
Lowest λ_0	-5.7076	-4.6454	-4.2454
Estimated λ_0	-5.5944	-4.3939	-3.9093
Highest λ_0	-5.4879	-4.1522	-3.5778

$Welf_{REx}$	Lowest λ_1	Estimated λ_1	Highest λ_1
Lowest λ_0	-4.8461	-3.7253	-3.3586
Estimated λ_0	-4.7024	-3.4328	-2.9923
Highest λ_0	-4.5680	-3.1485	-2.6288

The privatized basic research regime seems better in terms of welfare than the public regime even if patentable research tools could not allow stopping the unauthorized use of patented research tools. We can now confirm that our analysis suggests that the policy change in favour of the research tools patentability occurred in the United States from the early Eighties was very likely to be the best institutional reaction to the increase in R&D difficulty.

This research exemption and/or reach-through agreements analysis is therefore quite important in assessing the robustness of our previous results, in favour of the US policy shift towards the patentability of basic knowledge in 1980.

7 Final Remarks

The debate on the effects of the patentability of research tools on the incentives to innovate is still very controversial, not only in the US but also in Europe and in other important areas of the world. This paper analyzed from a general equilibrium perspective the US policy shift towards the extension of patentability to research tools and basic scientific ideas that took place around 1980. These normative innovations have been modifying the industrial and academic lives in the last three decades, raising doubts on their desirability. The losses from the free entry into basic research and the monopolization of applied research induced by intellectual property of research tools have been compared with the inefficacy of public research institutions to promptly react to downstream market opportunities and the potentially excessive entry into applied R&D.

Results were not a priory unambiguous, which forced us to use the available data and calibrate and simulate our model in order to check if the US did it right in changing their institutions around 1980. We have robustly found that assigning property rights to basic research findings and creating a market for research tools was the best thing the US could do at that time.

We have extended the basic model to incorporate research exemptions and reach-through licensing, without modifying our main policy conclusions.

In light of the current international negotiations on the application of TRIPs, our analysis might be helpful in providing insights from the experience of an important turning point in the US national system of innovation.

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Appendix 1

Model Details

This Appendix explains the details of the quality ladder model used in the main taxt. It may be skipped by most readers familiar with this literature.

Time $t \geq 0$ population $P(t)$ is assumed growing at rate $g_{Pop} \geq 0$ and its initial level is normalized to 1. The representative household preferences are represented by the following intertemporally additive utility functional[39]:

$$U = \int_0^\infty e^{-rt} \ln D(t) dt, \tag{22}$$

[39] We skip starting with an expectational operator in order to save notation. A more general setting of the consumer problem would not change results, as in our framework, due to perfectly diversifiable risks, law of large numbers, and perfect financial markets, the consumer's asset evolves deterministically in equilibrium.

where $r > 0$ is the subjective rate of time preference, and $D(t)$ is an infra-household percapita consumption index reflecting the household's taste for variety and for product quality. Per-family member instantaneous utility is given by:

$$\ln D(t) = \int_0^1 \ln \left(\sum_j \gamma^j d_{jt}(\omega) \right) d\omega, \tag{23}$$

where $d_{jt}(\omega)$ is the individual consumption of a good of quality $j = 1, 2, ...$ (that is, a product that underwent up to j quality jumps) and produced in industry ω at time t. Parameter $\gamma > 1$ measures the size of the quality upgrades. This formulation, the same as in Grossman and Helpman (1991a and b) and Segerstrom (1998), assumes that each consumer prefers higher quality products of different varieties. Since we are not incorporating horizontal innovation, the set of varieties is bounded and normalised to the unit interval. As explained by Grossman and Helpman (1991b, p.88), $D(t) = \exp\left[\int_0^1 \ln \left(\sum_j \gamma^j d_{jt}(\omega) \right) d\omega\right]$ can be interpreted as a CRS production function of a homogenous final product, produced with a range of different intermediate goods of different qualities. Hence, in this model the growth rate of the consumption index $D(t)$ has an immediate interpretation as the growth rate of final production percapita.

The representative consumer is endowed with $L > 0$ units of skilled labor and $M > 0$ units of unskilled labor summing to 1. Since labour bears no disutility it will be inelastically supplied for any level of non negative wages. Since initial population is normalized to 1, L and M will also equal, in equilibrium, the percapita supply of skilled, respectively, unskilled labour. Unskilled labor can only be employed in the final goods production. Skilled labour is able to perform R&D activities.

In the first step of the consumer's dynamic maximization problem, she selects the set $J_t(\omega)$ of the existing quality levels with the lowest quality-adjusted prices. Then, at each instant, the households allocate their income to maximize the instantaneous utility (23) taking product prices as given in the following static (instantaneous) constraint equation:

$$E(t) = \int_0^1 \sum_{j \in J_t(\omega)} p_{jt}(\omega) d_{jt}(\omega)\, d\omega. \tag{24}$$

Here $E(t)$ denotes percapita consumption expenditure and $p_{jt}(\omega)$ is the price of a product of quality j produced in industry ω at time t. Let us define $j_t^*(\omega) \equiv \max\{j : j \in J_t(\omega)\}$ Using the instantaneous optimization results, we can re-write (23) as

$$u(t) = \int_0^1 \ln\left[\gamma^{j_t^*(\omega)} E(t)/p_{j_t^*(\omega)t}(\omega)\right] d\omega = \quad (25)$$

$$= \ln[E(t)] + \ln(\gamma)\int_0^1 j_t^*(\omega)d\omega - \int_0^1 \ln[p_{j_t^*(\omega)t}(\omega)]d\omega \quad (26)$$

The solution to this maximization problem yields the static demand function:

$$d_{jt}(\omega) = \begin{cases} E(t)/p_{jt}(\omega) \text{ for } j = j_t^*(\omega) \\ 0 \text{ otherwise.} \end{cases} \quad (27)$$

Only the good with the lowest quality-adjusted price is consumed, since there is no demand for any other good. We also assume, as usual, that if two products have the same quality-adjusted price, consumers will buy the higher quality product - although they are formally indifferent between the two products - because the quality leader can always slightly lower the price of its product and drive the rivals out of the market. Therefore in equilibrium, only the latest version of the good $j_t(\omega)$ is produced and consumed.

Therefore, given the independent and - in equilibrium and by the law of large numbers - deterministic evolution of the quality jumps and prices, the consumer will only choose the piecewise continuous expenditure trajectory, $E(\cdot)$, of each family member that maximizes:

$$U = \int_0^\infty e^{-rt}\ln[E(t)]dt. \quad (28)$$

Assume that all consumers possess equal shares of all firms at time $t = 0$. Letting $A(0)$ denote the present value of human capital plus the present value of asset holdings at $t = 0$, each household's intertemporal budget constraint is:

$$\int_0^\infty e^{-I(t)}e^{g_{Pop}t}E(t)dt \leqq A(0) \quad (29)$$

where $I(t) = \int_0^t i(s)ds$ represents the equilibrium cumulative real interest rate up to time t.

Finally, the representative consumer chooses the time pattern of consumption expenditure to maximize (28) subject to the intertemporal budget constraint (29). The optimal expenditure trajectory satisfies the Euler equation:

$$\dot{E}(t)/E(t) = i(t) - (r + g_{Pop}) \tag{30}$$

where $i(t) = I(t)$ is the instantaneous market interest rate at time t, and the transversality condition.

Euler equation (30) implies that a constant (steady state) per-capita consumption expenditure is optimal when the instantaneous market interest rate equals the consumer's subjective discount rate r plus the population growth rate g. Since preferences are homothetic, in each industry aggregate demand is proportional to the representative consumer's one. E denotes the aggregate consumption spending and d denotes the aggregate demand.

As for the production side, we assume constant returns to scale technologies in the (differentiated) manufacturing sectors represented by the following production functions:

$$y(\omega) = X^{\alpha}(\omega) M^{1-\alpha}(\omega), \text{ for all } \omega \in [0,1], \tag{31}$$

where $\alpha \in (0,1)$, $y(\omega)$ is the output flow per unit time, $X(\omega)$ and $M(\omega)$ are, respectively, the skilled and unskilled labour input flows in industry $\omega \in [0,1]$. Letting w_s and w_u denote the skilled and unskilled wage rates, in each industry the quality leader seeks to minimize its total cost flow $C = w_s X(\omega) + w_u M(\omega)$ subject to constraint (31). For $y(\omega) = 1$, the solution to this minimization problem yields the conditional unskilled (32) and skilled (33) labour demands (i.e. the per-unit labour requirements):

$$M(\omega) = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \left(\frac{w_s}{w_u}\right)^{\alpha}, \tag{32}$$

$$X(\omega) = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left(\frac{w_u}{w_s}\right)^{1-\alpha}. \tag{33}$$

Thus the (minimum) cost function is:

$$C(w_s, w_u, y) = c(w_s, w_u)y \tag{34}$$

where $c(w_s, w_u)$ is the per-unit cost function:

$$c(w_s, w_u) = \left[\left(\tfrac{1-\alpha}{\alpha}\right)^{-(1-\alpha)} + \left(\tfrac{\alpha}{1-\alpha}\right)^{-\alpha}\right] w_s^{\alpha} w_u^{1-\alpha}. \tag{35}$$

Since unskilled labour is uniquely employed in the final good sectors and all price variables (including wages) are assumed to instantaneously adjust to their market clearing values, unskilled labour aggregate demand $\int_0^1 M(\omega)\,d\omega$ is equal to its aggregate supply, $MP(t)$, at any date. Since industries are symmetric and their number is normalized to 1, in equilibrium[40] $M(\omega) = MP(t)$.

The choice of unskilled labour as numeraire imposes $w_u = 1$, from equations (32) and (33) we get the firm's skilled labour demand negatively depending on skilled (/unskilled) wage (ratio):

$$X(\omega) = \frac{1}{w_s}\left(\frac{\alpha}{1-\alpha}\right) MP(t) \tag{36}$$

In percapita terms,

$$x(\omega) \equiv \frac{X(\omega)}{P(t)} = \frac{1}{w_s}\left(\frac{\alpha}{1-\alpha}\right) M. \tag{37}$$

In each industry, at each instant, firms compete in prices. Given demand function (27), within each industry product innovation is non-drastic[41], hence the quality leader will fix its (limit) price by charging a mark-up γ over the unit cost (remember that parameter γ measures the size of product quality jump):

$$p = \gamma c(w_s, 1) \Rightarrow d = \frac{E}{\gamma c(w_s, 1)}. \tag{38}$$

Hence each monopolist earns a flow of profit, in percapita terms, equal to

$$\begin{aligned} \pi &= \frac{\gamma - 1}{\gamma} E = (\gamma - 1)\frac{w_s x}{\alpha} \\ \pi &= (\gamma - 1)\frac{1}{1-\alpha} M. \end{aligned} \tag{39}$$

[40] More generally, with mass $N > 0$ of final good industries, in equilibrium $M(\omega) = \frac{MP(t)}{N}$.

[41] We are following Aghion and Howitt's (1992) and (1998) definition of drastic innovation as generating a sufficiently large quality jump to allow the new monopolist to maximize profits without risking the re-entry of the previous monopoly. Given the unit elastic demand, here the unconstrained profit maximizing price would be infinitely high: that would induce the previous incumbent to re-enter.

From eq.s (39) follows:

$$\frac{\gamma-1}{\gamma}E=(\gamma-1)\frac{1}{1-\alpha}M\Rightarrow E=\frac{\gamma}{1-\alpha}M. \tag{40}$$

Interestingly, eq. (40) implies that in equilibrium total expenditure is always constant. Therefore, eq. (30) implies a constant real interest rate:

$$i(t)=r+g_{Pop}. \tag{41}$$

Population Growth and Scale Effects

In the main text, we assume constant population. However, introducing growing population would not alter neither our model nor its main empirical results, if we stationarize the growing variables in percapita terms[42]. In particular, we define $n_B(\omega,t)\equiv\frac{N_B(\omega,t)}{P(t)}$ and $n_A(\omega,t)\equiv\frac{N_A(\omega,t)}{P(t)}$ - where $P(t)$ denotes total population at time t - as the skilled labor employment in each basic and, respectively, applied R&D sector. Notice that the assumed utility function in (28) is of Millian type, with households maximizing the utility of consumption percapita.

Notice that as the economies analyzed in the three models of this paper tend to their balanced growth path, the corresponding firm profits and stock market values will tend to evolve at the population growth rate g_{Pop}. To stationarize them we normalize firm values by dividing them by population. Therefore, for example, $v_L^1(\omega,t)\equiv\frac{V_L^1(\omega,t)}{P(t)}$, and similarly for all other firm values. Based on this, the reader can easility re-obtain the equations involving firm values in the main text, because the terms that explicitly contain the growth rate of population cancel out, as for example in

$$iv_L^1=\pi-(n_A^*)^{1-a}\lambda_1 v_L^1+v_L^1 g_{Pop}+\frac{dv_L^1}{dt},$$

which, based on eq. (41), becomes

$$(r+g_{Pop})v_L^1=\pi-(n_A^*)^{1-a}\lambda_1 v_L^1+v_L^1 g_{Pop}+\frac{dv_L^1}{dt}$$

[42]Thereby eliminating the strong scale effect (Jones, 1995 and 2005; Dinopoulos and Thompson, 1998; and Madsen, 2008) as in Dinopoulos and Thompson (1998), Peretto (1998), Young (1998).

and hence

$$rv_L^1 = \pi - (n_A^*)^{1-a}\,\lambda_1 v_L^1 + \frac{dv_L^1}{dt},$$

as it appears in eq.s (4c), (10d), and (17e). For the same reason, g_{Pop} disappears from the other financial market arbitrage equations.

Steady State Welfare

We here derive the equation used in our simulations to assess the steady state welfare associated with each scenario. In equilibrium the instantaneous utility function (23), after reminding that $d_{j_t^*(\omega)t}(\omega) = x^\alpha M^{1-\alpha}$, becomes

$$\ln D(t) = \int_0^1 \ln\left[\gamma^{j_t^*(\omega)} d_{j_t^*(\omega)t}(\omega)\right] d\omega = \log(\gamma)\int_0^1 j_t^*(\omega)d\omega + \log(x^\alpha M^{1-\alpha}). \tag{42}$$

In equilibrium $j_t^*(\omega) = j_t(\omega)$ in all industries. Focussing on balanced growth paths, we can assume[43] that the economy starts from the steady state value of all variables (including $m(A_0)$). Hence:

$$\ln D(t) = \log(\gamma)g_s t + \log(x_s^\alpha M^{1-\alpha}) + \log(\gamma)\int_0^1 j_0^*(\omega)d\omega, \tag{43}$$

with index $s = PUBBL$, $PRIV$, and $RExem$, depending on the institutional scenario chosen. In fact, $\int_0^1 j_t^*(\omega)d\omega = g_s t + \int_0^1 j_0^*(\omega)d\omega$. To understand this, it is important to remember that all processes are independent, all sectors are symmetric within A_0 and A_1, and there is an infinite number of them. Define $\phi(t) \equiv \int_0^1 j_t^*(\omega)d\omega$. Consider a positive and small[44] time increment Δt, and the increment $\phi(t+\Delta t) - \phi(t) = \int_0^1 \left[j_{t+\Delta t}(\omega) - j_t(\omega)\right] d\omega$. Notice that, by the properties of Poisson processes, $j_{t+\Delta t}(\omega) - j_t(\omega) = 0$ or 1, except for events with probability of a zero of higher order than Δt, which we write $o(\Delta t)$. By the law of large numbers the average number of jumps is equal to its expected value. Hence:

$$\begin{aligned}
\phi(t+\Delta t) - \phi(t) &= \int_{A_1(t)} \left[0 * \left(1 - (n_A^*)^{1-a}\lambda_1\Delta t\right) + 1 * (n_A^*)^{1-a}\lambda_1\Delta t\right] d\omega + 0(\Delta t) = \\
&= (1 - m(A_0))\,(n_A^*)^{1-a}\,\lambda_1\Delta t + o(\Delta t).
\end{aligned}$$

[43] Analysing the transition would be arbitrary, and not interesting in a long-run perspective.

[44] Notice that here the two-stage innovation process used in this paper precludes the use of the usual proof of Grossman and Helpman (1991b, p. 97).

Dividing both sides by Δt and taking the limit $\Delta t \to 0$, and remembering that $\lim_{\Delta t \to 0} o(\Delta t)/\Delta t = 0$, gives $\phi'(t) = (1 - m(A_0)) (n_A^*)^{1-a} \lambda_1 \equiv g_s$. Along a steady state g_s is constant, and hence $\phi(t) = g_s t + \phi(0) = g_s t + \int_0^1 j_0^*(\omega) d\omega$. Assuming that the initial value of $\int_0^1 j_0(\omega) d\omega$ is the same under each scenario $s = PUBBL$, PAT, and $RExem$, we can normalise it at zero. Therefore, with no loss of generality, we can use the following simpler expression:

$$Welf_s = \int_0^\infty e^{-rt} \left[\log(\gamma) g_s t + \log(x_s^\alpha M^{1-\alpha}) \right] dt = \tag{44}$$

$$= \frac{\log(\gamma) g_s}{r^2} + \frac{\log(x_s^\alpha M^{1-\alpha})}{r}, \; s = PUBBL, \; PRIV, \text{ and } REx \tag{45}$$

This is the expression we have used in all our numerical welfare comparisons.

As a by-product of our analysis, notice that taking the derivative of both sides of eq. (43) with respect to time gives:

$$\frac{\dot{D}(t)}{D(t)} = \log(\gamma) g_s,$$

which clarifies the link between the aggregate innovation rate g_s and the percapita consumption[45] index growth rate.

Appendix 2

Lemma 1. *In the Public Basic Research economy there can exist no more than one balanced growth path equilibrium.*

Proof. In the steady state, $\frac{dm(A_0)}{dt} = 0$, and hence eq. (5) can be rewritten as:

$$(1 - m(A_0)) n_A^{1-a} \lambda_1 = m(A_0) \bar{L}_G^{1-a} \lambda_0. \tag{46}$$

which defines $m(A_0)$ as an increasing function of n_A:

$$m(A_0) = \frac{n_A^{1-a} \lambda_1}{\bar{L}_G^{1-a} \lambda_0 + n_A^{1-a} \lambda_1}. \tag{47}$$

[45] Or of actual percapita consumption, in the production function interpretation of $D(t)$.

From 47) it is easily seen that $(1-m(A_0))\, n_A$ is an increasing function of n_A.

Eq. (4b) implies that v_L^0 is an increasing function of v_L^1; in turn, (4c) implies that v_L^1 is a decreasing function of n_A. Therefore, also v_L^0 is a decreasing function of n_A. But then, eq. (4a) implies that w_s too will be a decreasing function of n_A.

Let us then rewrite the labour market equilibrium condition (7) as

$$(1-m(A_0))n_A = L - \frac{1}{w_s}\left(\frac{\alpha}{1-\alpha}\right)M - \bar{L}_G. \tag{48}$$

In light of the preceding discussion, the left side of equation (48) is an increasing function of n_A, while the right side is a decreasing function of n_A. The steady state equilibrium value of n_A will be associated with the unique intersection between the curves defined by the two sides of this equation. Since the real values of all the other endogenous variables at the steady state are pinned down by n_A, they will be uniquely determined. Therefore, if a steady state equilibrium exists it will be unique. QED.

Lemma 2. *In the Privatized Basic Research economy there can exist no more than one balanced growth path equilibrium.*

Proof. Use eq.(10a) to obtain w_s, and plug into (9) to obtain the steady state version of eq. (10b), which, solved for v_A gives:

$$v_A = \left(\frac{a}{r}\right)^a \left(\frac{1-a}{\lambda_0}\right)^{1-a} (n_B)^{1-a}\,\lambda_1(v_L^0 - v_A). \tag{49}$$

Plugging (10a) and (49) into (10b) and solving for v_L^1 gives:

$$v_L^1 = \frac{\pi}{r + \left(\frac{r(1-a)}{a\lambda_0}\right)^{1-a} (n_B)^{a(1-a)}\,\lambda_1},$$

which can be plugged into eq. (10c) to solve for v_L^0 as:

$$v_L^0 = \frac{\pi}{r + (n_B)^{1-a}\,\lambda_0}\left[1 + \frac{(n_B)^{1-a}\,\lambda_0}{r + \left(\frac{r(1-a)}{a\lambda_0}\right)^{1-a} (n_B)^{a(1-a)}\,\lambda_1}\right]. \tag{50}$$

Plugging (50) into eq. (49) and solving for v_A yields:

$$v_A = \frac{\frac{\pi}{r+(n_B)^{1-a}\lambda_0}\left[1+\frac{(n_B)^{1-a}\lambda_0}{r+\left(\frac{r(1-a)}{a\lambda_0}\right)^{1-a}(n_B)^{a(1-a)}\lambda_1}\right]}{1+\frac{r^a\lambda_0^{1-a}}{a^a(1-a)^{1-a}(n_B)^{a(1-a)}\lambda_1}}. \tag{51}$$

As will soon be clear, it is important to study how $\frac{v_A}{n_B^a}$ changes with n_B^a. Based on eq. (49), we can write: $\frac{d}{dn_B}\left(\frac{v_A}{n_B^a}\right) =$

$$= \frac{d}{dn_B}\left[\frac{\pi}{rn_B^a + n_B\lambda_0}\left(1+\frac{n_B^{1-a}\lambda_0}{r+n_B^{a(1-a)}\lambda_1\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{1-a}}\right)\frac{a^a(1-a)^{1-a}n_B^{a(1-a)}\lambda_1}{r^a\lambda_0^{1-a}+a^a(1-a)^{1-a}n_B^{a(1-a)}\lambda_1}\right] =$$

$$
= \quad - \left(\begin{array}{c}
2a^2r^3\lambda_0^2 n_B^{3a^2-a-1}(1-a)^{1-a} + a^2r^4\lambda_0 n_B^{3a^2-2}(1-a)^{1-a} + \\
a^2r^3\lambda_1 n_B^{2a^2+a-2}(1-a)^{2-2a} + a^2r^2\lambda_0^3 n_B^{a(3a-2)}(1-a)^{1-a} + \\
r^2\lambda_0^2\lambda_1 n_B^{2a^2-1}(1-a)^{1-a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{1-a} + \\
a\lambda_0\lambda_1^3 n_B^{2a-1}(1-a)^{2-2a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{2-2a} + \\
2a^2r^2\lambda_1^2 n_B^{a^2+2a-2}(1-a)^{2-2a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{1-a} + \\
a^2r\lambda_1^3 n_B^{3a-2}(1-a)^{2-2a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{2-2a} + \\
a^2r\lambda_0^2\lambda_1 n_B^{a(2a-1)}(1-a)^{2-2a} + (2-a)\,a^2\lambda_0^2\lambda_1^2 n_B^{a^2}(1-a)^{2-2a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{1-a} + \\
2a^2r^2\lambda_0\lambda_1 n_B^{2a^2-1}(1-a)^{2-2a} + 2a^2r^3\lambda_0\lambda_1 n_B^{2a^2+a-2}(1-a)^{1-a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{1-a} + \\
a^2r\lambda_0^2\lambda_1^2 n_B^{a^2+a-1}(1-a)^{1-a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{2-2a} + \\
2a^2r^2\lambda_0^2\lambda_1 n_B^{2a^2-1}(1-a)^{1-a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{1-a} + \\
a^2r^2\lambda_0\lambda_1^2 n_B^{a^2+2a-2}(1-a)^{1-a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{2-2a} + \\
ar\lambda_0^3\lambda_1 n_B^{a(2a-1)}(1-a)^{1-a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{1-a} + \\
ar\lambda_0\lambda_1^2 n_B^{a^2+a-1}(1-a)^{2-2a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{1-a} + \\
(1-a)r\lambda_0^2\lambda_1^2 n_B^{a^2+a-1}(1-a)^{1-a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{2-2a} \\
(3-a)\,a^2r\lambda_0\lambda_1^2 n_B^{a^2+a-1}(1-a)^{2-2a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{1-a}
\end{array} \right).
$$

$$
\cdot \pi a^a \lambda_1 \frac{n_B^{-4a^2+2a+1}}{(\lambda_0 n_B + r n_B^a)^2\left(r^a\lambda_0^{1-a} + a^a\frac{\lambda_1}{n_B^{a(a-1)}}(1-a)^{1-a}\right)^2\left(r + \frac{\lambda_1}{n_B^{a(a-1)}}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{1-a}\right)^2}
$$

which is certainly negative because $0 < a < 1$, that is:

$$
\frac{d}{dn_B}\left(n_B^{-a}v_A\right) < 0. \tag{52}
$$

Plugging (9) into (14), setting $\frac{dm(A_0)}{dt} = 0$, and solving for $m(A_0)$ gives:

$$
m(A_0) = \frac{1}{1 + \frac{\lambda_0^{2+a}}{\lambda_1}\left[\left(\frac{a}{r(1-a)}\right)^{1-a}\right] n_B^{(1-a)^2}}. \tag{53}
$$

Eq. (53) shows that $m(A_0)$ is a decreasing function of n_B, and therefore $1-m(A_0)$ is an increasing function of n_B. However, notice also that $m(A_0)n_B$ is and increasing function of n_B.

Obtaining skilled wage from (10a) and plugging it into (11), and in light of eq.s (9) and (49), we can rewrite the skilled labour market condition (13) as:

$$m(A_0)n_B = L - \frac{\alpha M}{(1-\alpha)\lambda_0 n_B^{-a} v_A} - \frac{(1-m(A_0))\, n_B^a (1-a) r}{\lambda_0 a}. \tag{54}$$

Recalling the discussion after eq. (53), the left side of equation (54) is an increasing function of n_B. From (52) and (53), the right side of (54) is instead a decreasing function of n_B. Therefore there will exist only one intersection between the corresponding curves, and therefore a unique real value of n_B that solves equation (54). Since the real values of all other endogenous variables are uniquely pinned down by n_B, there can exist only a unique steady state equilibrium. QED

Lemma 3. *In the Research Exemption Economy there can exist no more than one balanced growth path equilibrium.*

Proof. From system (20a) and (20h) we easily have:

$$r = \lambda_1 n_A^{1-a} \left(\beta \frac{v_L^0}{v_B} - 1 \right) \tag{55a}$$

$$\frac{v_L^0}{v_B} = \left(\frac{n_A}{n_B} \right)^a \frac{\lambda_0}{\lambda_1 (1-\beta)} \tag{55b}$$

$$\left(\frac{n_A}{n_B} \right)^{1-a} = \frac{m(A_0)\lambda_0}{(1-m(A_0))\,\lambda_1} \tag{55c}$$

The first equation is obtained by dividing both sides of eq. (20b) by v_B. The second is the result of dividing eq.s (20a) and (20c) side by side. The third comes directly from eq. (20f). If there exist more than one steady state, then in one of them the mass $m(A_0)$ will be larger than in the other steady state. But then, eq. (55c) implies that also $\frac{n_A}{n_B}$ will be higher than in the other steady state, and hence - by eq. (55b) - $\frac{v_L^0}{v_B}$ too. Hence by eq. (55a) n_A will be lower, which implies that v_L^1 will be higher due to eq. (20e). Since $\frac{n_A}{n_B}$ is higher, it follows that also n_B is lower. Therefore, by eq. (20d) value v_L^0 will increase.

Notice that eq. (55c) can be rewritten as

$$\left(\frac{n_A}{n_B}\right)^{-a}\frac{\lambda_1}{\lambda_0}=\frac{n_B m(A_0)}{n_A\left(1-m(A_0)\right)},$$

which implis that the larger $m(A_0)$ is associated with a lower $\frac{n_A m(A_0)}{n_B(1-m(A_0))}$. Since, as we know already, n_B and $(1-m(A_0))$ are lower, it must follows that $n_A m(A_0)$ has decreased as well. These and eq. (20g) require x to be larger, and hence, by eq. (20g) that w_s is lower. This and the previously obtained lower value of n_A are consistent with eq. (20c) if and only if v_L^0 is lower, which contradicts what previously obtained. In a similar way we can exclude a BGP with a lower level of $m(A_0)$. QED

Appendix 3

The equations of the system used in our GMM estimation of λ_0, λ_1. and α, obtained from system (8a)-(8g), are the following:

$$\lambda_1^{\frac{1}{a}} = \left[\frac{g_{PUBBL}}{\left(L-\frac{\alpha M}{w_s(1-\alpha)}-L_G\right)}\right]^{\frac{1-a}{a}}\left[\frac{(\gamma-1)\frac{M}{1-\alpha}-}{r\left(\frac{w_s\left[L-\frac{\alpha M}{w_s(1-\alpha)}-L_G\right]}{g_{PUBBL}}-(\gamma-1)\frac{M}{1-\alpha}-r\frac{\frac{w_s\left[L-\frac{\alpha M}{w_s(1-\alpha)}-L_G\right]}{g_{PUBBL}}}{L_G^{1-a}\lambda_0}\right)}\right];$$

and

$$\left[1-g_{PUBBL}\frac{\frac{w_s\left[L-\frac{\alpha M}{w_s(1-\alpha)}-L_G\right]}{g_{PUBBL}}-\frac{w_s\left(\frac{L-\alpha M}{w_s(1-\alpha)}\right)-L_G}{g_{PUBBL}}-\frac{\frac{(\gamma-1)\frac{M}{1-\alpha}-\frac{rw_s}{g_{PUBBL}}\frac{(L-\alpha M)}{w_s(1-\alpha)L_G}}{g_{PUBBL}}}{L_G^{1-a}\lambda_0}}{(\gamma-1)\frac{M}{1-\alpha}-rw_s\frac{\frac{(L-\alpha M)}{w_s(1-\alpha)L_G}}{g_{PUBBL}}}\right]$$

$$\left[(\gamma-1)\frac{M}{1-\alpha}-r\frac{w_s\left[L-\frac{\alpha M}{w_s(1-\alpha)}-L_G\right]}{g_{PUBBL}}\frac{(\gamma-1)\frac{M}{1-\alpha}-r\frac{w_s\left[L-\frac{\alpha M}{w_s(1-\alpha)}-L_G\right]}{g_{PUBBL}}}{L_G^{1-a}\lambda_0}\right]$$

$$= g_{PUBBL}\left[\frac{w_s\left[L-\frac{\alpha M}{w_s(1-\alpha)}-L_G\right]}{g_{PUBBL}}-\frac{(\gamma-1)\frac{M}{1-\alpha}-r\frac{w_s\left[L-\frac{\alpha M}{w_s(1-\alpha)}-L_G\right]}{g_{PUBBL}}}{L_G^{1-a}\lambda_0}\right].$$

As explained in the text., the time series for w_s, M, L, L_G, g_{PUBBL}have

been used in the estimations, while the parameters r, γ, and respectively a have been set equal to 0.05, 1.60, and respectively 0.3.