Prototype Specification for a Real Computable General Equilibrium Model

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Introduction

This document presents a prototype specification for a real computable general equilibrium (CGE) model.[1] The prototype has some key features for assessing structural and poverty impacts:

- Labor markets disaggregated by skill level
- Land and capital markets disaggregated by type of capital/land
- A production structure which differentiates the substitutability of unskilled labor on the one hand, and skilled labor and capital on the other hand
- Differentiation of production of like-goods (e.g. small- and large-scale farms, or public versus private production)
- Detailed income distribution
- Intra-household transfers (e.g. urban to rural), transfers from government, and remittances
- Multiple households
- A tiered structure of trade (differentiating across various trading partners)
- Possibility of influencing export prices
- Internal domestic trade and transport margins
- Various potential factor mobility assumptions
- And simple recursive dynamics.

The model has been adapted to help analyze poverty and trade linkages within the context of the inter-agency task force known as the Integrated Framework (IF), and has also been expanded and articulated for a model of China.

The rest of the document proceeds to describe all of the model details using the standard circular flow description of the economy. It starts with production (*P*), income distribution (*Y*), demand (*D*), trade (*T*), domestic trade and transport margins (*M*), goods market equilibrium (*E*), macro closure (*C*), factor market equilibrium (*F*), macroeconomic identities (*I*), and growth (*G*).

Table 1 describes the indices used in the equations. Note that the model differentiates between production activities, denoted by the index *i*, and commodities, denoted by the index *k*. In many models, the two will overlap exactly. However, this differentiation allows for the same commodity to be produced by one or more sectors, and to differentiate these commodities by source of production. For example, it could be used in a model of economies in transition where commodities produced by the public sector have a different cost structure than commodities produced by the private sector, and the commodities themselves could be differentiated by consumers.[2] Another example, could be small- versus large-scale agricultural producers.

[1] Background information on CGE modeling can be found in Derviş et al (1982), Shoven and Whalley (1984 and 1992), Francois and Reinert (1997) and Hertel (1997).

[2] The model allows for perfect substitution, in which case consumers are indifferent regarding who produces the good. An example might be electricity.

Table 1: Indices used in the model

i	Production activities
k	Commodities
l	Labor skills
ul	Unskilled labor
sl	Skilled labor[a]
kt	Capital types
lt	Land types
e	Corporations
h	Households
f	Final demand accounts[b]
m	Trade and transport margin accounts[c]
r	Trading partners

Notes: a. The unskilled and skilled labor indices, *ul* and *sl*, are subsets of *l*, and their union composes the set indexed by *l*.
b. The standard final demand accounts are '*Gov*' for government current expenditures, '*ZIp*' for private investment, '*ZIg*' for public investment, '*TMG*' for international export of trade and transport services, and '*DST*' for changes in stocks.
c. The standard trade and transport margin accounts are '*D*' for domestic goods, '*M*' for imported goods, and '*X*' for exported goods.

Model Equations

Production

Production, like in most CGE models, relies on the substitution relations across factors of production and intermediate goods. The simplest production structure has a single constant-elasticity-of-substitution (CES) relation between capital and labor, with intermediate goods being used in fixed proportion to output. In the production structure described below, there are multiple types of capital, land and labor, and they are combined in a nested-CES structure intended to represent the various substitution possibilities across these different factors of production. Typically, intermediate goods will enter in fixed proportion to output, though at the aggregate level, the model allows for a degree of substitutability between aggregate intermediate demand and value added.[3] The decomposition of value added has several components (see *figure* 1 for a representation of the multiple nests). First, land is assumed to be a substitute for an aggregate capital labor bundle.[4] The latter is then decomposed into unskilled labor on the one hand, and skilled labor cum capital on the other hand. This conforms to recent observations suggesting that capital and skilled labor are complements, which can substitute for unskilled labor. The four aggregate factors—unskilled and skilled labor, land and capital, are decomposed by type in a final CES nest.

Top-level nest and producer price

The top-level nest has output, *XP*, produced as a combination of value added, *VA*, and an aggregate demand for goods and non-factor services, *ND*. In most cases, the substitution elasticity will be assumed to be zero, in which case the top-level CES nest is a fixed-coefficient Leontief production function. Equations (P-1) and (P-2) represent the optimal demand conditions for the generic CES production function, where *PND* is the price of the *ND* bundle, *PVA* is the aggregate price of value added, *PX* is the unit cost of production, and σ^p is the substitution elasticity. If the latter is zero, both *ND* and *VA* are used

3 Deviations from this structure might include isolating some key inputs, for example energy, or agricultural chemicals in the case of crops, and feed in the case of livestock.

4 In some sectors the model also allows for a sector-specific factor of production, for example, coal mining and oil production require reserves which cannot be used for any other activity. In this case, the nesting follows the same general structure as depicted in Figure 1.

in fixed proportions to output, irrespective of relative prices. Equation (P-3) represents the unit cost function, *PX*. It is derived from the CES dual price formula. The model assumes constant-returns-to-scale and perfect competition in all sectors. Hence, the producer price, *PP*, is equal to the unit cost, adjusted for a producer tax/subsidy, τ^p, equation (P-4).

$$ND_i = \alpha_i^{nd}\left(\frac{PX_i}{PND_i}\right)^{\sigma_i^p} XP_i \qquad \text{(P-1)}$$

$$VA_i = \alpha_i^{va}\left(\frac{PX_i}{PVA_i}\right)^{\sigma_i^p} XP_i \qquad \text{(P-2)}$$

$$PX_i = \left[\alpha_i^{nd} PND_i^{1-\sigma_i^p} + \alpha_i^{va} PVA_i^{1-\sigma_i^p}\right]^{1/(1-\sigma_i^p)} \qquad \text{(P-3)}$$

$$PP_i = \left(1+\tau_i^p\right)PX_i \qquad \text{(P-4)}$$

Second-level production nests

The second-level nest has two branches. The first decomposes aggregate intermediate demand, *ND*, into sectoral demand for goods and services, *XAp*. The model explicitly assumes a Leontief structure. Thus equation (P-5) describes the demand for good *k* by sector *j*, where the coefficient *a* represents the proportion between *XAp* and *ND*. The price of the *ND* bundle, *PND*, is the weighted average of the price of goods and services, *PA*, using the technology coefficients as weights, equation (P-6). The so-called Armington price is multiplied by a sector and commodity specific indirect tax, τ^{cp}.

$$XAp_{k,j} = a_{k,j} ND_j \qquad \text{(P-5)}$$

$$PND_j = \sum_k a_{k,j}\left(1+\tau_{k,j}^{cp}\right)PA_k \qquad \text{(P-6)}$$

The second branch decomposes the aggregate value added bundle, *VA*, into three components: aggregate demand for capital and labor, *KL*, aggregate land demand, TT^d, and a sector-specific resource, *NR*,[5] see equations (P-7) through (P-9). The relevant component prices are *PKL*, *PTT* and *PR*, respectively, and the substitution elasticity is given by σ^v. Equation (P-9) allows for the possibility of factor productivity changes as represented by the λ parameter. The price of value added, *PVA*, is the CES aggregation of the three component prices, as defined by equation (P-10).

[5] The latter will typically be zero in most sectors.

$$KL_i = \alpha_i^{kl}\left(\frac{PVA_i}{PKL_i}\right)^{\sigma_i^v} VA_i \qquad \text{(P-7)}$$

$$TT_i^d = \alpha_i^{tt}\left(\frac{PVA_i}{PTT_i}\right)^{\sigma_i^v} VA_i \qquad \text{(P-8)}$$

$$NR_i^d = \alpha_i^{nr}\left(\lambda_i^{nr}\right)^{\sigma_i^v-1}\left(\frac{PVA_i}{PR_i}\right)^{\sigma_i^v} VA_i \qquad \text{(P-9)}$$

$$PVA_i = \left[\alpha_i^{kl} PKL_i^{1-\sigma_i^v} + \alpha_i^{tt} PTT_i^{1-\sigma_i^v} + \alpha_i^{nr}\left(\frac{PR_i}{\lambda_i^{nr}}\right)^{1-\sigma_i^v}\right]^{1/(1-\sigma_i^v)} \qquad \text{(P-10)}$$

Third-level production nest

The third-level nest decomposes the aggregate capital-labor bundle, *KL*, into two components. The first is the aggregate demand for unskilled labor, *UL*, with an associated price of *PUL*. The second is a bundle composed of skilled labor and capital, *KSK*, with a price of *PKSK*. Equations (P-11) and (P-12) reflect the standard CES optimality conditions for the demand for these two components, with a substitution elasticity given by σ^{kl}. The price of capital-labor bundle, *PKL*, is defined in equation (P-13).

$$UL_i = \alpha_i^{u}\left(\frac{PKL_i}{PUL_i}\right)^{\sigma_i^{kl}} KL_i \qquad \text{(P-11)}$$

$$KSK_i = \alpha_i^{ksk}\left(\frac{PKL_i}{PKSK_i}\right)^{\sigma_i^{kl}} KL_i \qquad \text{(P-12)}$$

$$PKL_i = \left[\alpha_i^{u} PUL_i^{1-\sigma_i^{kl}} + \alpha_i^{ksk} PKSK_i^{1-\sigma_i^{kl}}\right]^{1/(1-\sigma_i^{kl})} \qquad \text{(P-13)}$$

Fourth-level production nest

The fourth-level nest decomposes the capital-skilled labor bundle into a capital component, KT^d, and a skilled labor component, *SKL*. Equations (P-14) and (P-15) represent the optimality conditions where the relevant component prices are *PKT* and *PSKL*, and the substitution elasticity is given by σ^{ks}. Equation (P-16) determines the price of the *KSK* bundle, *PKSK*.

$$SKL_i = \alpha_i^s \left(\frac{PKSK_i}{PSKL_i} \right)^{\sigma_i^{ks}} KSK_i \tag{P-14}$$

$$KT_i^d = \alpha_i^{kt} \left(\frac{PKSK_i}{PKT_i} \right)^{\sigma_i^{ks}} KSK_i \tag{P-15}$$

$$PKSK_i = \left[\alpha_i^s PSKL_i^{1-\sigma_i^{ks}} + \alpha_i^{kt} PKT_i^{1-\sigma_i^{ks}} \right]^{1/(1-\sigma_i^{ks})} \tag{P-16}$$

Demand for labor by sector and skill

Equations (P-17) and (P-18) decompose the demands for aggregate unskilled and skilled labor, respectively, across their different components. The variable L^d represents labor demand in sector i for labor of skill level l. The relevant wage is given by W which is allowed to be both sector and skill-specific. The respective cross-skill substitution elasticities are σ^u and σ^s. Both equations (P-17) and (P-18) incorporate sector and skill specific labor productivity, represented by the variable λ^l. The aggregate unskilled and skilled price indices are determined in equations (P-19) and (P-20), respectively *PUL* and *PSKL*.

$$L_{i,ul}^d = \alpha_{i,ul}^l \left(\lambda_{i,ul}^l \right)^{\sigma_i^u - 1} \left(\frac{PUL_i}{W_{i,ul}} \right)^{\sigma_i^u} UL_i \qquad \text{for } ul \in \{\text{Unskilled labor}\} \tag{P-17}$$

$$L_{i,sl}^d = \alpha_{i,sl}^l \left(\lambda_{i,sl}^l \right)^{\sigma_i^s - 1} \left(\frac{PSKL_i}{W_{i,sl}} \right)^{\sigma_i^s} SKL_i \qquad \text{for } sl \in \{\text{Skilled labor}\} \tag{P-18}$$

$$PUL_i = \left[\sum_{ul \in \{\text{Unskilled labor}\}} \alpha_{i,ul}^l \left(\frac{W_{i,ul}}{\lambda_{i,ul}^l} \right)^{1-\sigma_i^u} \right]^{1/(1-\sigma_i^u)} \tag{P-19}$$

$$PSKL_i = \left[\sum_{sl \in \{\text{Skilled labor}\}} \alpha_{i,sl}^l \left(\frac{W_{i,sl}}{\lambda_{i,sl}^l} \right)^{1-\sigma_i^s} \right]^{1/(1-\sigma_i^s)} \tag{P-20}$$

Demand for capital and land across types

The aggregate land and capital bundles, KT^d and TT^d respectively, are disaggregated across types, leading to type- and sector-specific capital and land demand, K^d and T^d. The decomposition is represented in equations (P-21) and (P-23), where the respective prices are *R* and *PT* which are both type- and sector-specific. The equations also incorporate productivity factors. Equations (P-22) and (P-24) represent the price indices for aggregate capital and land, respectively *PKT* and *PTT*.

$$K_{i,kt}^{d}=\alpha_{i,kt}^{k}\left(\lambda_{i,kt}^{k}\right)^{\sigma_i^k-1}\left(\frac{PKT_i}{R_{i,kt}}\right)^{\sigma_i^k}KT_i^d \tag{P-21}$$

$$PKT_i=\left[\sum_{kt}\alpha_{i,kt}^{k}\left(\frac{R_{i,kt}}{\lambda_{i,kt}^{k}}\right)^{1-\sigma_i^k}\right]^{1/(1-\sigma_i^k)} \tag{P-22}$$

$$T_{i,lt}^{d}=\alpha_{i,lt}^{t}\left(\lambda_{i,lt}^{t}\right)^{\sigma_i^t-1}\left(\frac{PTT_i}{PT_{i,lt}}\right)^{\sigma_i^t}TT_i^d \tag{P-23}$$

$$PTT_i=\left[\sum_{lt}\alpha_{i,lt}^{t}\left(\frac{PT_{i,lt}}{\lambda_{i,lt}^{t}}\right)^{1-\sigma_i^t}\right]^{1/(1-\sigma_i^t)} \tag{P-24}$$

Commodity aggregation

Each activity produces a single commodity, *XP*, indexed by *i*. Consumption goods, indexed by *k*, are a combination of one or more produced goods. Aggregate domestic supply of good *k*, *X*, is a CES combination of one or more produced goods *i*. In many cases, the CES aggregate is of a single commodity, i.e. there is a one-to-one mapping between a consumed good and its relevant production. There are cases, however, where it is useful to have consumed goods be an aggregation of produced goods, for example when combining similar goods with different production characteristics (e.g. public versus private, commercial versus small-scale, etc.) Equation (P-25) represents the optimality condition of the aggregation of produced goods into commodities. The producer price is *PP*, and the price of the aggregate supply is *P*. The degree of substitutability across produced commodities is σ^c. Equation (P-26) determines the aggregate supply price, *P*. The model allows for perfect substitutability, in which case the law of one price holds and the produced commodities are simply aggregated to form aggregate output.[6]

$$\begin{cases} XP_i=\alpha_{i,k}^{c}\left(\dfrac{P_k}{PP_i}\right)^{\sigma_k^c}X_k & \text{if} \quad \sigma_k^c\neq\infty \\ PP_i=P_k & \text{if} \quad \sigma_k^c=\infty \end{cases} \tag{P-25}$$

$$\begin{cases} P_k=\left[\sum_{i\in K}\alpha_{i,k}^{c}PP_i^{1-\sigma_k^c}\right]^{1/(1-\sigma_k^c)} & \text{if} \quad \sigma_k^c\neq\infty \\ X_k=\sum_{i\in K}XP_i & \text{if} \quad \sigma_k^c=\infty \end{cases} \tag{P-26}$$

6 Electricity is a good example of a homogeneous output but which could be produced by different production technologies, e.g. hydro-electric, nuclear, thermal, etc.

Income distribution

The prototype model has a rich menu of income distribution channels—factor income and intra-household, government and foreign transfers (i.e. remittances). The prototype also includes corporations used as a pass-through account for channeling operating surplus.

Factor income

There are four broad factors—a sector specific resource, land, labor and capital—the latter three which can be sub-divided into various types. Equations (Y-1) through (Y-4) determine aggregate net-income from labor, *LY*, capital, *KY*, land, *TY*, each indexed by their respective sub-types, and the sector specific factor, *RY*. These are net incomes because the model incorporates factor taxes designated by τ^{fl}, τ^{fk}, τ^{ft} and τ^{fr} respectively.[7]

$$LY_l = \sum_i \frac{W_{i,l} L^d_{i,l}}{1+\tau^{fl}_{i,l}} \qquad \text{(Y-1)}$$

$$KY_{kt} = \sum_i \frac{R_{i,kt} K^d_{i,kt}}{1+\tau^{fk}_{i,kt}} \qquad \text{(Y-2)}$$

$$TY_{lt} = \sum_i \frac{PT_{i,lt} T^d_{i,lt}}{1+\tau^{ft}_{i,lt}} \qquad \text{(Y-3)}$$

$$RY = \sum_i \frac{PR_i NR^d_i}{1+\tau^{fr}_i} \qquad \text{(Y-4)}$$

Distribution of profits

All of labor, land and sector-specific factor income is allocated directly to households.[8] Profits (aggregated with income from the sector-specific resouce), on the other hand, are distributed to three broad accounts, enterprises, households, and the rest of the world (ROW). Equation (Y-5) determines the level of profits distributed to enterprises, TR^E. Equation (Y-6) represents the level of profits distributed directly to households, TR^H. And, equation (Y-7) determines the level of factor income distributed abroad, TR^W. Note that the three share parameters, φ^E, φ^H, and φ^W sum to unity.

$$TR^E_{k,kt} = \varphi^E_{k,kt} KY_{kt} \qquad \text{(Y-5)}$$

$$TR^H_{k,kt} = \varphi^H_{k,kt} KY_{kt} \qquad \text{(Y-6)}$$

$$TR^W_{k,kt} = \varphi^W_{k,kt} KY_{kt} \qquad \text{(Y-7)}$$

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7 The factor taxes are type- and sector-specific. Note as well that the relevant factor prices represent the perceived cost to employers, not the perceived remuneration of workers.

8 Depending on the structure of the final SAM, land and or income from the sector-specific resource may also pass through corporate accounts.

Corporate income

Corporate income, TR^E, is split into four accounts. First, the government receives its share through the corporate income tax, κ^c. The residual is split into three: retained earnings, and income distributed to households and the rest of the world. Equation (Y-8) determines corporate income of enterprise *e*, *CY*. It is the sum, over possible capital types, of shares of distributed profits (to corporations).[9] Equation (Y-9) determines retained earnings, i.e. corporate savings, S^c, where the rate of retained earnings is given by s^c. Equations (Y-10) and (Y-11) determine the overall transfers to households and to ROW. Note that the two share parameters, φ^H and φ^W, and the retained earnings rate, s^c, sum to unity.

$$CY_e = \sum_{kt} \varphi^e_{kt,e} TR^E_{k,kt} \qquad \text{(Y-8)}$$

$$S^c_e = s^c_e \left(1 - \kappa^c_e\right) CY_e \qquad \text{(Y-9)}$$

$$TR^H_{c,e} = \varphi^H_{c,e} \left(1 - \kappa^c_e\right) CY_e \qquad \text{(Y-10)}$$

$$TR^W_{c,e} = \varphi^W_{c,e} \left(1 - \kappa^c_e\right) CY_e \qquad \text{(Y-11)}$$

Household income

Aggregate household income, *YH*, is composed of eight elements: labor, land and sector-specific factor remuneration, distributed capital income and corporate profits, transfers from government and households, and foreign remittances, equation (Y-12).[10] Government transfers, in the standard closure, are fixed in real terms and are multiplied by an appropriate price index to preserve model homogeneity. Remittances, are fixed in international currency terms, and are multiplied by the exchange rate, *ER*, to convert them into local currency terms.[11]

9 The share parameters, φ^e, sum to unity.
10 All share parameters within the summation signs sum to unity.
11 *ER* measures the value of local currency in terms of the international currency.

$$YH_h = \underbrace{\sum_l \varphi^h_{l,h} LY_l}_{\text{Labor}} + \underbrace{\sum_{kt} \varphi^h_{kt,h} TR^H_{k,kt}}_{\text{Capital}} + \underbrace{\sum_{lt} \varphi^h_{lt,h} TY_{lt}}_{\text{Land}} + \underbrace{\varphi^h_{nr,h} RY}_{\text{Sector-specific factor}}$$
$$+ \underbrace{\sum_e \varphi^h_{e,h} TR^H_{c,e}}_{\text{Enterprise}} + \underbrace{PLEV.TR^H_{g,h}}_{\text{Transfers from government}} + \underbrace{\sum_{h'} TR^h_{h,h'}}_{\text{Intra-household transfers}} + \underbrace{ER.TR^h_{W,h}}_{\text{Foreign remittances}} \qquad \text{(Y-12)}$$

$$YD_h = \left(1 - \lambda^h \kappa^h_h\right) YH_h - TR^H_h \qquad \text{(Y-13)}$$

$$TR^H_h = \varphi^H_{h,h} \left(1 - \lambda^h \kappa^h_h\right) YH_h \qquad \text{(Y-14)}$$

$$TR^h_{h,h'} = \varphi^h_{h,h'} TR^H_{h'} \qquad \text{(Y-15)}$$

$$TR^W_h = \varphi^W_h TR^H_h \qquad \text{(Y-16)}$$

Disposable income, *YD*, is equal to after-tax income, less household transfers, equation (Y-13), where the household tax rate is κ^h. It is multiplied by an adjustment factor, λ^h, which is used for model closure. In the standard closure, government savings (or deficit), is held fixed, and the household tax schedule adjusts (uniformly) to achieve the given government fiscal balance. In other words, under this closure rule, the relative tax rates across households remain constant.[12] Aggregate household transfers, TR^H, is a share of after tax income, equation (Y-14). This is transferred to individual households and abroad, respectively TR^h and TR^W, using constant share equations, (Y-15) and (Y-16).

Domestic final demand

Domestic final demand is composed of two broad agents—households and other domestic final demand. The model incorporates multiple households. Household demand has a uniform specification, however, with household-specific expenditure parameters. The other domestic final demand categories, in the standard model, include government current expenditures, *Gov*, private and public investment expenditures, *ZIp* and *ZIg*, exports of international trade and transport services, *TMG*, and changes in stocks, *DST*. The other domestic final demand categories, indexed by *f*, are also assumed to have a uniform expenditure function, but with agent-specific expenditure parameters. Demand at the top-level, reflects demand for the Armington good. The latter are added up across all activities in the economy and split into domestic and import components at the national level.[13]

Household expenditures

Households have a tiered demand structure, see figure 2. At the top-level, households save a constant share of disposable income, with the savings rate given by s^h. At the next level, residual income is allocated across goods and services, *XAc*, using the linear expenditure system (LES).[14] Equation (D-1) represents the LES demand function. Household consumption is the sum of two components. The first, θ,

12 An alternative would be to use an additive factor, which would adjust the average tax rates, not the marginal tax rates.

13 There are few SAMs, which would allow for agent-specific Armington behavior.

14 This class of models often uses the so-called extended linear expenditure system, which integrates household savings directly in the utility function. However, this can create calibration problems for households without savings. On consumer demand theory see Deaton and Muellbauer (1980). On the extended linear expenditure system see Lluch (1973) and Howe (1975).

is referred to as the subsistence minimum, or floor consumption.[15] The second is a share of supernumerary income, or discretionary income. Supernumerary income is equal to residual disposable income, subtracting savings and aggregate expenditures on the subsistence minima from disposable income. The next level, undertaken at the national level, is the decomposition of Armington demand, *XAc*, into its domestic and import components, see below. Equation (D-2) determines household saving, S^h, by residual. The consumer price index, *CPI*, is defined in equation (D-3). Note that the consumer price is equal to the economy-wide Armington price, *PA*, multiplied by a household and commodity specific ad valorem tax, τ^{cc}.

$$XAc_{k,h} = Pop_h\theta_{k,h} + \frac{\mu_{k,h}}{(1+\tau^{cc}_{k,h})PA_k}\left((1-s^h_h)YD_h - \sum_{k'}(1+\tau^{cc}_{k',h})PA_{k'}Pop_h\theta_{k',h}\right) \qquad \text{(D-1)}$$

$$S^h_h = YD_h - \sum_k (1+\tau^{cc}_{k,h})PA_k XAc_{k,h} \qquad \text{(D-2)}$$

$$CPI_h = \frac{\sum_k (1+\tau^{cc}_{k,h})PA_k XAc_{k,h,0}}{\sum_k (1+\tau^{cc}_{k,h,0})PA_{k,0} XAc_{k,h,0}} \qquad \text{(D-3)}$$

Other domestic demand accounts

The other domestic final demand accounts all use a CES expenditure function (with the option of having fixed volume or value expenditure shares with an elasticity of 0 or 1, respectively). Equation (D-4) determines the expenditure share on goods and services, *XAf*. Equation (D-5) defines the expenditure price index, *PF*. And equation (D-6) defines the value of expenditures, *YF*. Model closure is discussed below.

$$XAf_{k,f} = \alpha^f_{k,f}\left(\frac{PF_f}{(1+\tau^{cf}_{k,f})PA_k}\right)^{\sigma^f_f} XF_f \qquad \text{(D-4)}$$

$$PF_f = \left[\sum_k \alpha^f_{k,f}\left((1+\tau^{cf}_{k,f})PA_k\right)^{1-\sigma^f_f}\right]^{1/(1-\sigma^f_f)} \qquad \text{(D-5)}$$

$$YF_f = PF_f XF_f \qquad \text{(D-6)}$$

Trade equations

This section discusses the modeling of trade. There are three sections—import demand, and export supply and demand. The first two use a tiered structure. Import demand is decomposed in two steps. The top tier disaggregates aggregate Armington demand into two components—demand for the domestically produced good and aggregate import demand. At the second tier, the aggregate import demand is

[15] The subsistence minima are scaled by population so that they increase at the same rate as population growth.

allocated across trading partners. Both of these tiers assume that goods indexed by *k* are differentiated by region of origin, i.e. the so-called Armington assumption.[16] A CES specification is used to model the degree of substitutability across regions of origin. The level of the elasticities will often be determined by the level of aggregation. Finely defined goods, such as wheat, would typically have a higher elasticity than more broadly defined goods, such as clothing. At the same time, non-price barriers may also inhibit the degree of substitutability, for example prohibitive transport barriers (inexistent or few transmission lines for electricity), or product and safety standards. Export supply is similarly modeled using a two-tiered constant-elasticity-of-transformation specification. This permits imperfect supply responses to changes in relative prices. Finally, the small-country assumption is relaxed for exports with the incorporation of export demand functions.

Top-level Armington nest

National demand for the Armington good, *XA*, is the sum of Armington demand over all domestic agents: intermediate demand, household and other domestic final demand, and demand generated by the internal trade and transport sector, *XAmg*, equation (T-1). Aggregate Armington demand is then allocated between domestic and import goods using a nested CES structure. Equation (T-2) represents demand for the domestically produced good, XD^d, where the top-level Armington elasticity is given by σ^m. Note that the price of the domestic good is equal to the producer price, *PD*, adjusted by the internal trade and transport margin, τ^{mg}. Demand for aggregate imports, *XMT*, is determined in equation (T-3). The price of aggregate imports is given by *PMT*.[17] The Armington price, *PA*, is defined in equation (T-4), using the familiar CES dual price aggregation formula.

$$XA_k = \sum_j XAp_{k,j} + \sum_h XAc_{k,h} + \sum_f XAf_{k,f} + \sum_m \sum_{k'} XAmg_{k,k',m} \qquad \text{(T-1)}$$

$$XD_k^d = \alpha_k^d \left(\frac{PA_k}{(1+\tau_{k,D}^{mg})\, PD_k} \right)^{\sigma_k^m} XA_k \qquad \text{(T-2)}$$

$$XMT_k = \alpha_k^m \left(\frac{PA_k}{PMT_k} \right)^{\sigma_k^m} XA_k \qquad \text{(T-3)}$$

$$PA_k = \left[\alpha_k^d \left((1+\tau_{k,D}^{mg})\, PD_k \right)^{1-\sigma_k^m} + \alpha_k^m PMT_k^{1-\sigma_k^m} \right]^{1/(1-\sigma_k^m)} \qquad \text{(T-4)}$$

Second-level Armington nest

At the second level, aggregate import demand, *XMT*, is allocated across trading partners using a CES specification. Equation (T-5) defines the domestic price of imports, *PM*.[18] It is equal to the world price (in international currency), *WPM*, multiplied by the exchange rate, and adjusted for by the import tariff, τ^m, i.e. *PM* represents the port-price of imports, tariff-inclusive. The tariff rate is both sector- and region of origin-specific. The tariff rate is also multiplied by a uniform shifter, χ^{tm}, which is normally equal to 1.

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16 The seminal article on product differentiation in trade is Armington (1969). See also de Melo and Robinson (1989).

17 It includes the trade and transport margins, sales tax, and import tariffs.

18 *PM* and *WPM* are indexed by both commodity, *k*, and trading partner, *r*.

The shifter can be modified exogenously for specific simulations, for example setting it to 0.5 would cut tariffs by 50 percent across the board, or it can be rendered endogenous, assuming there is an exogenous target to be achieved. For example, to calculate a uniform revenue neutral tariff rate, one could set all tariffs (or positive-rated tariffs) to 0.1 and endogenize the shift parameter, χ^{tm}. The exogenous variable is the value of initial tariff revenues (in real terms). If the calculated χ^{tm} is 1.5, this indicates that the uniform revenue neutral tariff is 15 percent. Equation (T-6) represents the import of commodity k from region r, XM, where the inter-regional substitution elasticity is given by σ^w. The relevant consumer price includes the internal trade and transport margin, τ^{mg}. The aggregate price of imports, PMT, is defined in equation (T-7).

$$PM_{k,r} = ER.WPM_{k,r}(1+\chi^{tm}\tau^{m}_{k,r}) \tag{T-5}$$

$$XM_{k,r} = \alpha^{w}_{k,r}\left(\frac{PMT_k}{(1+\tau^{mg}_{k,M})\ PM_{k,r}}\right)^{\sigma^w_k} XMT_k \tag{T-6}$$

$$PMT_k = \left[\sum_r \alpha^{w}_{k,r}\left((1+\tau^{mg}_{k,M})\ PM_{k,r}\right)^{1-\sigma^w_k}\right]^{1/(1-\sigma^w_k)} \tag{T-7}$$

Top-level CET nest

Domestic production is allocated across markets using a nested CET specification. At the top nest, producers allocate production between the domestic market and aggregate exports. At the second nest, aggregate exports are allocated across trading partners. The model allows for perfect transformation, i.e. producers perceive no difference across markets. In this case, the law-of-one-price holds. Equation (T-8) represents the link between the domestic producer price, PE, and the world price, WPE. Export prices are both sector- and region-specific. The FOB price, WPE, includes domestic trade and transport margins, τ^{mg}[19], as well as export taxes/subsidies, τ^e. Equations (T-9) and (T-10) represent the CET optimality conditions. The first determines the share of domestic supply, X, allocated to the domestic market, XD^s. The second determines the supply of aggregate exports, XET. PET represents the price of aggregate export supply. The transformation elasticity is given by σ^x. The model allows for perfect transformation. In this case, the optimal supply conditions are replaced by the law-of-one price conditions. Equation (T-11) represents the CET aggregation function. In the case of finite transformation, it is replaced with its equivalent, the CET dual price aggregation function. In the case of infinite transformation, the primal aggregation function is used, where the two components are summed together since there is no product differentiation.

[19] Note that the domestic trade and transport margins are differentiated for three different goods: domestically produced goods sold to the domestic market, exported goods, and imported goods.

$$PE_{k,r}\left(1+\tau_{k,X}^{mg}\right)\left(1+\tau_{k,r}^{e}\right)=ER.WPE_{k,r} \tag{T-8}$$

$$\begin{cases} XD_k^s = \gamma_k^d \left(\dfrac{PD_k}{P_k}\right)^{\sigma_k^x} X_k & \text{if} \quad \sigma_k^x \neq \infty \\ PD_k = P_k & \text{if} \quad \sigma_k^x = \infty \end{cases} \tag{T-9}$$

$$\begin{cases} XET_k = \gamma_k^e \left(\dfrac{PET_k}{P_k}\right)^{\sigma_k^x} X_k & \text{if} \quad \sigma_k^x \neq \infty \\ PET_k = P_k & \text{if} \quad \sigma_k^x = \infty \end{cases} \tag{T-10}$$

$$\begin{cases} P_k = \left[\gamma_k^d PD_k^{1+\sigma_k^x} + \gamma_k^e PET_k^{1+\sigma_k^x}\right]^{1/(1+\sigma_k^x)} & \text{if} \quad \sigma_k^x \neq \infty \\ X_k = XD_k^s + XET_k & \text{if} \quad \sigma_k^x = \infty \end{cases} \tag{T-11}$$

Second-level CET nest

The second-level CET nest allocates aggregate export supply, *XET*, across the various export markets, *XE*. Equation (T-12) represents the optimal allocation decision, where σ^z is the transformation elasticity. Equation (T-13) represents the CET aggregation function, where again, the CET dual price formula is used to determine the aggregate export price, *PET*. As above, the model allows the transformation elasticity to be infinite.

$$\begin{cases} XE_{k,r} = \gamma_{k,r}^x \left(\dfrac{PE_{k,r}}{PET_k}\right)^{\sigma_k^z} XET_k & \text{if} \quad \sigma_k^z \neq \infty \\ PE_{k,r} = PET_k & \text{if} \quad \sigma_k^z = \infty \end{cases} \tag{T-12}$$

$$\begin{cases} PET_k = \left[\sum_r \gamma_{k,r}^x PE_{k,r}^{1+\sigma_k^z}\right]^{1/(1+\sigma_k^z)} & \text{if} \quad \sigma_k^z \neq \infty \\ XET_k = \sum_r XE_{k,r} & \text{if} \quad \sigma_k^z = \infty \end{cases} \tag{T-13}$$

Export demand

Export, *ED*, demand is specified using a constant elasticity function, equation (T-14). If the elasticity, η^e, is finite, demand decreases as the international price of exports, *WPE*, increases. The numerator contains an exogenous export price competitive index. If the latter increases relative to the domestic export price, market share of the domestic exporter would increase. The model allows for infinite demand elasticity. This represents the small-country assumption. In this case, the domestic price of exports (in international currency units) is constant. If the two CET elasticities are likewise infinite, then the domestic producer price is also equal to the world price of exports (adjusted for taxes and trade and transportation margins).

$$\begin{cases} ED_{k,r} = \alpha^e_{k,r}\left(\dfrac{\overline{WPE}_{k,r}}{WPE_{k,r}}\right)^{\eta^e_{k,r}} & \text{if} \quad \eta^e_{k,r} \neq \infty \\ WPE_{k,r} = \overline{WPE}_{k,r} & \text{if} \quad \eta^e_{k,r} = \infty \end{cases} \tag{T-14}$$

Domestic trade and transportation margins

The marketing of each good—domestic, imports, and exports—is associated with a commodity specific trade margin.[20] Equations (M-1) through (M-3) define the revenues associated with the domestic trade and transport margins. Domestically produced goods sold domestically generate $Y^{mg}_{.,D}$. Imported goods generate $Y^{mg}_{.,M}$. And exported goods generate $Y^{mg}_{.,X}$. Equation (M-4) defines the volume of margin services. The production of the trade and transport services follows a Leontief technology. Equation (M-5) defines the demand for goods and services. In other words, to deliver commodity *k*' (in either sector *D*, *M*, or *X*) requires an input from commodity *k*, the level of which is fixed in proportions to the overall volume of delivering commodity *k*' in the economy, $XT^{mg}_{k'}$. Equation (M-6) is the expenditure deflator, $PT^{mg}_{k'}$, for individual trade margin activities.

$$YT^{mg}_{k,D} = \tau^{mg}_{k,D} PD_k XD^d_k \tag{M-1}$$

$$YT^{mg}_{k,M} = \sum_r \tau^{mg}_{k,M} PM_{k,r} XM_{k,r} \tag{M-2}$$

$$YT^{mg}_{k,X} = \sum_r \tau^{mg}_{k,X} PE_{k,r} XE_{k,r} \tag{M-3}$$

$$XT^{mg}_{k,m} = YT^{mg}_{k,m} / PT^{mg}_{k,m} \tag{M-4}$$

$$XAmg_{k,k',m} = \alpha^{mg}_{k,k',m} XT^{mg}_{k',m} \tag{M-5}$$

$$PT^{mg}_{k',m} = \sum_k \alpha^{mg}_{k,k',m} PA_k \tag{M-6}$$

Goods market equilibrium

There are three fundamental commodities in the model—domestic goods sold domestically, imports (by region of origin), and exports (by region of destination). All other goods are bundles (i.e. are defined using an aggregation function) and do not require supply/demand balance. The small-country assumption holds for imports, and therefore any import demand can be met by the rest of the world with no impact on the price of imports. Therefore, there is no explicit supply/demand equation for imports.[21] Equation (E-1) represents equilibrium on the domestic goods market, and essentially determines, *PD*, the producer price of the domestic good. Equation (E-2) defines the equilibrium condition on the export market. With a finite

[20] The model does not include international trade and transport margins. A change in the latter could be simulated by a change in the relevant world price index, *WPM* or $\overline{WPE}$.

[21] One could rather easily add an import supply equation and an equilibrium condition.

export demand elasticity, the equation determines *WPE*, the world price of exports. With an infinite export demand elasticity, the equation trivially equates export supply to the given export demand.

$$XD_k^d = XD_k^s \tag{E-1}$$

$$ED_{k,r} = XE_{k,r} \tag{E-2}$$

Macro closure

Macro closure involves determining the exogenous macro elements of the model. The standard closure rules are the following:

- Government fiscal balance is exogenous, achieved with an endogenous direct tax schedule
- Private investment is endogenous and is driven by available savings
- The volume of government current and investment expenditures is exogenous
- The volume of demand for international trade and transport services is exogenous
- The volume of stock changes is exogenous
- The trade balance (i.e. capital flows) is exogenous. The real exchange rate equilibrates the balance of payments.

These are detailed further below.

Government accounts

Equation (C-1) describes nominal tariff revenues, *TarY*, and equation (C-2) defines real tariff revenues, *RTarY*. Equation (C-3) defines total government revenues, *GY*. There are 10 components: revenues from the production tax, sales tax, import tax, export tax, land, capital and wage tax, corporate and household direct taxes, and transfers from the rest of the world. Equation (C-4) defines the government's current expenditures, *GEXP*. It is the sum of three components: expenditures on goods and services, transfers to households, and transfers to ROW. Government savings (on current operations), S^g, is defined in equation (C-5), as the difference between revenues and current expenditures. Real government savings, *RSg*, is defined in equation (C-6). It is this latter which essentially determines the level of direct household taxation since *RSg* is exogenous in the standard closure.

$$TarY = ER\sum_k \sum_r \chi^{tm} \tau^m_{k,r} WPM_{k,r} XM_{k,r} \tag{C-1}$$

$$RTarY = TarY / PLEV \tag{C-2}$$

$$\begin{aligned}
GY \quad = \quad & \underbrace{\sum_k \sum_j \tau^{cp}_{k,j} PA_k XAp_{k,j}}_{\text{Sales tax on intermediate demand}} + \underbrace{\sum_k \sum_h \tau^{cc}_{k,h} PA_k XAc_{k,h}}_{\text{Sales tax on household demand}} + \underbrace{\sum_k \sum_f \tau^{cf}_{k,f} PA_k XAf_{k,f}}_{\text{Sales tax on other final demand}} \\
+ \quad & \underbrace{TarY + \sum_k \sum_r \tau^e_{k,r} (1 + \tau^{mg}_{k,X}) PE_{k,r} XE_{k,r}}_{\text{Import tariff and export tax revenues}} \\
+ \quad & \underbrace{\sum_{lt} \sum_i \frac{\tau^{ft}_{i,lt} PT_{i,lt} T^d_{i,lt}}{1 + \tau^{ft}_{i,lt}}}_{\text{Land tax}} + \underbrace{\sum_{kt} \sum_i \frac{\tau^{fk}_{i,kt} R_{i,kt} K^d_{i,kt}}{1 + \tau^{fk}_{i,kt}}}_{\text{Capital tax}} + \underbrace{\sum_l \sum_i \frac{\tau^{fl}_{i,l} W_{i,l} L^d_{i,l}}{1 + \tau^{fl}_{i,l}}}_{\text{Wage tax}} + \underbrace{\sum_i \frac{\tau^{fr}_i PR_i NR^d_i}{1 + \tau^{fr}_i}}_{\text{Resource tax}} \\
+ \quad & \underbrace{\sum_i \tau^p_i PX_i XP_i}_{\text{Production tax}} + \underbrace{\sum_e \kappa^c_e CY_e}_{\text{Corporate tax}} + \underbrace{\lambda^h \sum_h \kappa^h_h YH_h}_{\text{Income tax}} + \underbrace{ER.TR^g_W}_{\text{Transfers from ROW}}
\end{aligned} \tag{C-3}$$

$$GEXP = YF_{Gov} + PLEV \sum_h TR^H_{g,h} + ER.TR^W_g \tag{C-4}$$

$$S^g = GY - GEXP \tag{C-5}$$

$$RSg = S^g / PLEV \tag{C-6}$$

Investment and macro closure

Equation (C-7) defines the investment savings balance. In the standard closure, it determines the level of private investment since public investment and stock changes are exogenous. These three components are financed by aggregate savings defined over corporations, households, and the government, and adjusted by foreign savings. The latter is fixed (in international currency terms). Equations (C-8) through (C-11) define the exogenous volumes of public current and investment expenditures, exports of international trade and transport services and stock changes. The aggregate price level, *PLEV*, is the average absorption (Armington) price, equation (C-12). Equation (C-13) represents the balance of payments (in international currency terms). It can be shown to be redundant, and is dropped from the model specification.

$$YF_{ZIp} + YF_{ZIg} + YF_{DST} = \sum_e S_e^c + \sum_h S_h^h + S^g + ER.S^f \tag{C-7}$$

$$XF_{Gov} = \overline{XF}_{Gov} \tag{C-8}$$

$$XF_{ZIg} = \overline{XF}_{ZIg} \tag{C-9}$$

$$XF_{TMG} = \overline{XF}_{TMG} \tag{C-10}$$

$$XF_{DST} = \overline{XF}_{DST} \tag{C-11}$$

$$PLEV = \frac{\sum_k PA_k XA_{k,0}}{\sum_k PA_{k,0} XA_{k,0}} \tag{C-12}$$

$$\begin{aligned} BoP &= \sum_r \sum_k WPE_{k,r} XE_{k,r} + YF_{TMG} + \sum_h TR_{W,h}^h + TR_W^g + S^f \\ &- \sum_r \sum_k WPM_{k,r} XM_{k,r} - \frac{\sum_{kt} TR_{k,kt}^W + \sum_e TR_{c,e}^W + \sum_h TR_h^W}{ER} - TR_g^W \\ &\equiv 0 \end{aligned} \tag{C-13}$$

Factor market equilibrium

The following sections describe the standard factor market equilibrium conditions.[22]

Labor markets

Labor markets are assumed to clear. Equation (F-1) describes the upward sloping labor supply curve, including the two polar cases of a vertical supply curve ($\omega^l = 0$) and a horizontal supply curve, i.e. an infinite elasticity, in which case the real wage is fixed. Equation (F-2) sets aggregate demand, by skill-level, equal to aggregate supply, L^s. This equation determines the equilibrium wage, W^e.[23] Equation (F-3) equates sectoral wages to the equilibrium wage, but allows for a fixed sector-specific relative wage factor, ϕ^l.[24]

22 More detailed analysis may require more market segmentation, e.g. rural versus urban labor markets, though some of this segmentation can be picked up by the data itself.

23 Market structure can emulate perfect market segmentation by an appropriate definition of labor skills. For example, unskilled rural labor can assume to be only employed in rural sectors, whereas unskilled urban labor is only employed in urban sectors. Perfect market segmentation, as modeled here, does not allow for migration.

24 Quite a few alternatives could be used to endogenize relative sector-specific wages, for example union wage bargaining models, efficiency wages, etc.

$$
\begin{cases}
L_l^s = \alpha_l^{ls}\left(\dfrac{W_l^e}{PLEV}\right)^{\omega^l} & \text{if} \quad \omega^l \neq \infty \\
W_l^e = PLEV.W_{l,0}^e & \text{if} \quad \omega^l = \infty
\end{cases} \tag{F-1}
$$

$$
L_l^s = \sum_i L_{i.l}^d \tag{F-2}
$$

$$
W_{i,l} = \phi_{i,l}^l W_l^e \tag{F-3}
$$

Capital market

Equilibrium on the capital market allows for both limiting cases—perfect capital mobility and perfect capital immobility, or any intermediate case. Aggregate capital, K^s, is allocated across sectors and type according to a nested CET system. At the top-level, the aggregate investor allocates capital across types, according to relative rates of return. Equation (F-4) determines the optimal supply decision, where TK^s is the supply of capital of type kt, with an average return of PTK. PK is the aggregate rate-of-return to capital. If the supply elasticity is infinite, the law-of-one-price holds. Equation (F-5) represents the top-level aggregation function, replaced by the CET dual price function in the case of a finite transformation elasticity. Perfect capital mobility is represented by setting ω^{kt} to infinity. Perfect immobility is modeled by setting the transformation elasticity to 0.

$$
\begin{cases}
TK_{kt}^s = \gamma_{kt}^{tks}\left(\dfrac{PTK_{kt}}{PK}\right)^{\omega^{kt}} K^s & \text{if} \quad \omega^{kt} \neq \infty \\
PTK_{kt} = PK & \text{if} \quad \omega^{kt} = \infty
\end{cases} \tag{F-4}
$$

$$
\begin{cases}
PK = \left[\displaystyle\sum_{kt} \gamma_{kt}^{tks} PTK_{kt}^{1+\omega^{kt}}\right]^{1/(1+\omega^{kt})} & \text{if} \quad \omega^{kt} \neq \infty \\
K^s = \displaystyle\sum_{kt} TK_{kt}^s & \text{if} \quad \omega^{kt} = \infty
\end{cases} \tag{F-5}
$$

At the second level, capital by type, TK^s, is allocated across sectors using another CET function. Equation (F-6) determines the optimal allocation of capital of type kt to sector i, K^s, where the transformation elasticity is ω^k. Equation (F-7) represents the CET aggregation function. The equilibrium return to capital, R, is determined by equation capital supply to demand, equation (F-8).[25]

[25] If the transformation elasticity is infinite, equation (F-6) determines the sector- and type-specific rate of return using the law-of-one price, and equation (F-8) trivially sets capital supply equal to capital demand.

$$\begin{cases} K_{i,kt}^{s} = \gamma_{i,kt}^{k}\left(\dfrac{R_{i,kt}}{PTK_{kt}}\right)^{\omega^{k}} TK_{kt}^{s} & \text{if} \quad \omega^{k} \neq \infty \\ R_{i,kt} = PTK_{kt} & \text{if} \quad \omega^{k} = \infty \end{cases} \tag{F-6}$$

$$\begin{cases} PTK_{kt} = \left[\sum_{i} \gamma_{i,kt}^{k} R_{i,kt}^{1+\omega^{k}}\right]^{1/(1+\omega^{k})} & \text{if} \quad \omega^{k} \neq \infty \\ TK_{kt} = \sum_{i} K_{i,kt}^{s} & \text{if} \quad \omega^{k} = \infty \end{cases} \tag{F-7}$$

$$K_{i,kt}^{s} = K_{i,kt}^{d} \tag{F-8}$$

Land market

Land market equilibrium is specified in an analogous way to the capital market with a tiered CET supply system. The first tier allocates total land across types. This could have a zero transformation elasticity if for example land used for rice production could not be used to produce other commodities. Their respective prices are *PLAND* and PTT^{s}.

$$\begin{cases} TT_{lt}^{s} = \gamma_{lt}^{tts}\left(\dfrac{PTT_{lt}^{s}}{PLAND}\right)^{\omega^{tl}} LAND & \text{if} \quad \omega^{tl} \neq \infty \\ PTT_{lt}^{s} = PLAND & \text{if} \quad \omega^{tl} = \infty \end{cases} \tag{F-9}$$

$$\begin{cases} PLAND = \left[\sum_{lt} \gamma_{lt}^{tts}\left(PTT_{lt}^{s}\right)^{1+\omega^{tl}}\right]^{1/(1+\omega^{tl})} & \text{if} \quad \omega^{tl} \neq \infty \\ LAND = \sum_{lt} TT_{lt}^{s} & \text{if} \quad \omega^{tl} = \infty \end{cases} \tag{F-10}$$

Equations (F-11) and (F-12) determine the optimality conditions at the second and final tier, determining land supply (by type and) by sector of use. Land market equilibrium is represented by equation (F-13).

$$\begin{cases} T_{i,lt}^{s} = \gamma_{i,lt}^{t}\left(\dfrac{PT_{i,lt}}{PTT_{lt}^{s}}\right)^{\omega_{lt}^{t}} TT_{lt}^{s} & \text{if} \quad \omega_{lt}^{t} \neq \infty \\ PT_{i,lt} = PTT_{lt}^{s} & \text{if} \quad \omega_{lt}^{t} = \infty \end{cases} \tag{F-11}$$

$$\begin{cases} PTT_{lt}^{s} = \left[\sum_{i} \gamma_{i,lt}^{t} PT_{i,lt}^{1+\omega^{t}}\right]^{1/(1+\omega_{lt}^{t})} & \text{if} \quad \omega_{lt}^{t} \neq \infty \\ TT_{lt}^{s} = \sum_{i} T_{i,lt}^{s} & \text{if} \quad \omega_{lt}^{t} = \infty \end{cases} \tag{F-12}$$

$$T_{i,lt}^{s} = T_{i,lt}^{d} \tag{F-13}$$

Natural resource market

The market for natural resources differs from the others in the sense that there is no inter-sectoral mobility, i.e. this is a sector specific resource. There is therefore a sector specific supply curve (eventually flat).[26] Equation (F-14) describes the sector-specific supply function, or NR^s. Equation (F-15) then determines the equilibrium price, *PR*.

$$\begin{cases} NR_i^s = \gamma_i^{nr}\left(\dfrac{PR_i}{PLEV}\right)^{\omega^{nr}} & \text{if} \quad \omega^{nr} \neq \infty \\ PR_i = PLEV.PR_{i,0} & \text{if} \quad \omega^{nr} = \infty \end{cases} \qquad \text{(F-14)}$$

$$NR_i^d = NR_i^s \qquad \text{(F-15)}$$

Macroeconomic identities

The macroeconomic identities are not normally needed for the model specification, i.e. they could be calculated at the end of a simulation. In the case of dynamic scenarios, one or more of them could be used to calibrate dynamic parameters to a given set of exogenous assumptions. For example, the growth of GDP could be made exogenous. In this case, a growth parameter, typically a productivity factor, would be endogenous and set to target the given growth path of GDP.

Equations (I-1) and (I-2) define nominal and real GDP, respectively, at market prices. Equation (I-3) is the GDP at market price deflator. Similarly, equations (I-4) and (I-5) define nominal and real GDP at factor cost. Note that real GDP at factor cost is evaluated in efficiency units.[27] Equation (I-6) defines the GDP at factor cost deflator.

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26 More realistic models allow for kinked supply curves. It is typically easier to take resources out of production than to bring them online—the latter requiring new investments and/or new exploration. Thus a so-called down supply elasticity would be higher than a so-called up supply elasticity.

27 So is nominal GDP at factor cost, but the efficiency factors cancel out in the equation since the nominal wage is divided by the efficiency factor to derive the efficiency wage.

$$GDPMP = \sum_k \sum_h (1+\tau^{cc}_{k,h}) PA_k XAc_{k,h} + \sum_k \sum_f (1+\tau^{cf}_{k,f}) PA_k XAf_{k,f} + ER \sum_k \sum_r WPE_{k,r} XE_{k,r} - \sum_k \sum_r PM_{k,r} (1+\tau^{mg}_{k,M}) \, XM_{k,r} \quad \text{(I-1)}$$

$$RGDPMP = \sum_k \sum_h (1+\tau^{cc}_{k,c,0}) PA_{k,0} XAc_{k,h} + \sum_k \sum_f (1+\tau^{cf}_{k,f,0}) PA_{k,0} XAf_{k,f} + ER_0 \sum_k \sum_r WPE_{k,r,0} XE_{k,r} - \sum_k \sum_r PM_{k,r,0} (1+\tau^{mg}_{k,M,0}) \, XM_{k,r} \quad \text{(I-2)}$$

$$PGDPMP = GDPGMP / RGDPMP \quad \text{(I-3)}$$

$$GDPFC = \sum_l \sum_i W_{i,l} L^d_{i,l} + \sum_{kt} \sum_i R_{i,kt} K^d_{i,kt} + \sum_{lt} \sum_i PT_{i,lt} T^d_{i,lt} + \sum_i PR_i NR^d_i \quad \text{(I-4)}$$

$$RGDPFC = \sum_l \sum_i W_{i,l,0} \lambda^l_{i,l} L^d_{i,l} + \sum_{kt} \sum_i R_{i,kt,0} \lambda^k_{i,kt} K^d_{i,kt} + \sum_{lt} \sum_i PT_{i,lt,0} \lambda^t_{i,lt} T^d_{i,lt} + \sum_i PR_{i,0} \lambda^r_i NR^d_i \quad \text{(I-5)}$$

$$PGDPFC = GDPGFC / RGDPFC \quad \text{(I-6)}$$

Growth equations

Model equations

In a simple dynamic framework, equation (G-1) defines the growth rate of GDP at market price. Equation (G-2) determines the growth rate of labor productivity. The growth rate has two components, a uniform factor applied in all sectors to all types of labor, γ^l, and a sector- and skill-specific factor, χ^l. In defining a baseline, the growth rate of GDP is exogenous. In this case, equation (G-1) is used to calibrate the γ^l parameter. In policy simulations, γ^l is given, and equation (G-1) defines the growth rate of GDP. Other elements of simple dynamics include exogenous growth of labor supply, exogenous growth rates of capital and land productivity (typically 0), and investment driven capital accumulation.[28]

$$RGDPMP = (1+g^y) \, RGDPMP_{-1} \quad \text{(G-1)}$$

$$\lambda^l_{ip,l} = (1+\gamma^l + \chi^l_{ip,l}) \, \lambda^l_{ip,l,-1} \quad \text{(G-2)}$$

Equations external to the model

The remaining growth equations are external to the model. They involve only exogenous variables which can be determined outside of the model specification. There are four elements driving model dynamics—labor growth, capital accumulation, growth of natural resources, and productivity.

[28] Note that public investment, in this version of the model, has no impact on production technology.

Equation (G-3) determines labor supply growth. It simply applies an exogenous assumption about the growth of labor supply, g^{ls}, to the labor supply shift parameter. If the supply curve is vertical, it will simply move the vertical supply curve by the growth rate. In the absence of independent growth rates for labor, the growth rate of the population tranche of persons aged between 15 and 65 is sometimes used as an approximate growth rate for labor supply. Equation (G-4) updates population (by household). Equations (G-5) and (G-6) are similar growth equations for land and the sector-specific resource, respectively.[29]

$$\alpha_l^{ls} = (1+g_l^{ls})\,\alpha_{l,-1}^{ls} \qquad \text{(G-3)}$$

$$Pop_h = (1+g_h^{Pop})\,Pop_{h,-1} \qquad \text{(G-4)}$$

$$Land = (1+g^t)\,Land_{-1} \qquad \text{(G-5)}$$

$$\gamma_i^{nr} = (1+g_i^{nr})\,\gamma_{i,-1}^{nr} \qquad \text{(G-6)}$$

Capital accumulation is based on the level of investment of the previous period less depreciation. Equation (G-7) represents the motion equation for capital growth, where δ is the rate of depreciation and *KAP* is the capital stock. The variable *KAP* differs from the capital stock described in the model, K^s (see equations (F-4) and (F-5)). *KAP* represents the true volume of the capital stock, the so-called non-normalized value. The variable K^s is a capital stock index, which may be equal to the true value of the capital stock, but is often set equal to the normalized value of the capital stock. The distinction is important in the accumulation equation but is of no consequence for the model specification, i.e. the normalization of the capital stock value does not affect model results. An example may help clarify the distinction. Start with an economy with a GDP of 100 and a 40 percent capital share, i.e. 40 percent of GDP is composed of profits. The normalized value of the capital stock is 40, i.e. it is the value of the capital stock consistent with a rental rate of capital of 1. Assume the rate of return on capital is 20 percent. Then the non-normalized value of the capital stock is 200, i.e. investors receive a return of 40 because 20 percent of 200 is 40. Next assume investment is 30 percent of GDP in this economy, and the rate of depreciation is 8 percent. The capital stock in the following period is 214 (= 0.92*200+30), i.e. an increase of 7 percent. The investment, 30, must be added to the non-normalized value of the capital stock because the units matter in the capital accumulation function. Equation (G-8) determines the capital stock index which simply assumes that the rate of the capital stock index to the non-normalized capital stock remains constant. In other words, the growth rate of the normalized capital stock is equal to the growth rate of the non-normalized capital stock.

$$KAP = (1-\delta)KAP_{-1} + XF_{ZIp,-1} \qquad \text{(G-7)}$$

$$K^s = \left(K_0^s / KAP_0\right)KAP \qquad \text{(G-8)}$$

Equation (G-2) determines labor productivity growth in a subset of sectors, indexed by *ip*. In all other sectors, labor productivity growth is exogenous. The complementary subset is indexed by *np*.

[29] If the sector specific resource is a renewable or non-renewable natural resource, the growth equation should normally be replaced by equations determining the underlying supply of the resource. For example, a depletion module could be used for a non-renewable resource such as crude oil.

Equation (G-9) represents the increase in labor productivity in sectors not subject to the uniform productivity shift factor γ^l. Equations (G-10) through (G-12) update productivity of capital, land and the sector specific factor, respectively. The updating of productivity of these factors, unlike labor, is always assumed to be exogenous. One standard assumption is to isolate agricultural sectors from the others, i.e. to make the subset *ag* a subset of *np*. If agricultural productivity is assumed to be uniform across all factors of production, then the same growth parameter will be applied in formulas (G-9) through (G-12) for all sectors indexed by *ag*. Equation (G-13) determines the change in efficiency in the trade and transport sector. If the parameter γ^{mg} is negative, for example -1 percent, then efficiency is improving.

$$\lambda^l_{np,l} = (1+\chi^l_{np,l})\lambda^l_{np,l,-1} \tag{G-9}$$

$$\lambda^k_{i,kt} = (1+\chi^k_{i,kt})\lambda^k_{i,kt,-1} \tag{G-10}$$

$$\lambda^t_{i,lt} = (1+\chi^t_{i,lt})\lambda^t_{i,lt,-1} \tag{G-11}$$

$$\lambda^r_i = (1+\chi^r_i)\lambda^r_{i,-1} \tag{G-12}$$

$$\tau^{mg}_{k,m} = (1+\gamma^{mg}_{k,m})\tau^{mg}_{k,m,-1} \tag{G-13}$$

The assumption that productivity growth is only labor-augmenting may not be appropriate in all situations. There are two possible alternatives. The first assumes that productivity growth is uniform between capital and labor. In this case equations (G-2) and (G-10) would be replaced with:

$$\lambda^l_{ip,l} = (1+\gamma+\chi^l_{ip,l})\lambda^l_{i,l,-1}$$

$$\lambda^k_{ip,kt} = (1+\gamma+\chi^k_{ip,kt})\lambda^k_{ip,kt,-1}$$

(Equation G-10 would still hold for the sectors indexed by *np*.) Thus in the baseline scenario, with GDP growth fixed, a common productivity factor, γ, would apply to both labor and capital in sectors indexed by *ip*. A third alternative is to introduce an additional target to determine a capital-specific productivity factor. In some applications, the additional target is some formula which expresses so-called balanced growth. One version of balanced growth is that the capital per worker, in efficiency units, remains constant over time. In this alternative, equation (G-2) is maintained, with the uniform factor, γ^l, still determined by the GDP growth rate. The additional equation (target) is the balanced growth expression given by:

$$\frac{\sum_{kt}\sum_i \lambda^k_{i,kt}K^d_{i,kt}}{\sum_l\sum_i \lambda^l_{i,l}L^d_{i,l}} = \frac{\sum_{kt}\sum_i \lambda^k_{i,kt,0}K^d_{i,kt,0}}{\sum_l\sum_i \lambda^l_{i,l,0}L^d_{i,l,0}} = \chi^{kl}_0$$

This expression represents the ratio of capital to labor in efficiency units. The capital productivity equation is replaced by:

$$\lambda_{ip,kt}^{k} = (1+\gamma^{k} + \chi_{ip,kt}^{k})\lambda_{ip,kt,-1}^{k}$$

The expression holds only over sectors indexed by *ip* and includes a productivity factor, γ^k, uniform over all *ip* sectors, but different from γ^l.

Other exogenous variables may require updating for the baseline. One obvious one is government expenditure. This is typically assumed to grow at the same rate as GDP:

$$\overline{XF}_{Gov} = \left(1+g^{y}\right)\overline{XF}_{Gov,-1}$$

Other variables that may need updating include the various transfer variables, foreign savings, exogenous world prices (i.e. the terms of trade), and fiscal policies.

Model variables and parameters

Tables 2-5 provide a complete list of model variables and parameters. Tables 2 and 3 list respectively endogenous and exogenous model variables. Table 4 provides a list of the key model parameters, mostly substitution, demand and supply elasticities. Table 5 provides a list of the model's calibrated parameters. Each table has three columns. The first column represents the symbol of the respective variable or parameter as it is used in this document. The second column shows the equivalent GAMS name with the appropriate indices. The third column provides a brief description.

Table 2: Endogenous variables

Production		
ND_i	`nd(i)`	Demand for aggregate intermediate demand bundle
VA_i	`va(i)`	Demand for value added bundle
PX_i	`px(i)`	Unit cost of production
PP_i	`pp(i)`	Producer price
$XAp_{k,j}$	`xap(k,j)`	Intermediate demand for goods and services
PND_i	`pnd(i)`	Price of aggregate intermediate demand bundle
KL_i	`kl(i)`	Demand for capital-labor bundle
TT_i^d	`ttd(i)`	Demand for aggregate land bundle
NR_i^d	`rd(i)`	Demand for sector-specific resource
PVA_i	`pva(i)`	Price of value added bundle
UL_i	`usk(i)`	Demand for aggregate unskilled labor bundle
KSK_i	`ksk(i)`	Demand for capital/skilled labor bundle
PKL_i	`pkl(i)`	Price of capital-labor bundle
SKL_i	`skl(i)`	Demand for aggregate unskilled labor bundle
KT_i^d	`ktd(i)`	Demand for aggregate capital bundle
$PKSK_i$	`pksk(i)`	Price pf capital/skilled labor bundle
$L_{i,l}^d$	`ld(i,l)`	Sectoral demand for labor by labor type
PUL_i	`pusk(i)`	Price of aggregate unskilled labor bundle
$PSKL_i$	`pskl(i)`	Price of aggregate skilled labor bundle
$K_{i,kt}^d$	`kd(i,kt)`	Sectoral demand for capital by capital type
PKT_i	`pktd(i)`	Price of aggregate capital demand bundle
$T_{i,lt}^d$	`td(i,lt)`	Sectoral demand for land by land type
PTT_i	`pttd(i)`	Price of aggregate land demand bundle
XP_i	`xp(i)`	Aggregate sector output (of activity *i*)
P_k	`p(k)`	Aggregate producer price of commodity *k*

Table 2, continued: Endogenous variables

Income distribution		
LY_l	`ly(l)`	Aggregate net labor remuneration
KY_{kt}	`ky(kt)`	Aggregate after-tax capital income
TY_{lt}	`ty(lt)`	Aggregate after-tax land income
RY	`ry`	Aggregate after-tax income from sector-specific resource
$TR^E_{k,kt}$	`ktre(kt)`	Capital income transferred to enterprises
$TR^H_{k,kt}$	`ktrh(kt)`	Capital income transferred to households
$TR^W_{k,kt}$	`ktrw(kt)`	Capital income transferred abroad
CY_e	`cy(e)`	Corporate income
S^c_e	`savc(e)`	Corporate retained earnings
$TR^H_{c,e}$	`ctrh(e)`	Corporate earnings transferred to households
$TR^W_{c,e}$	`ctrw(e)`	Corporate earnings transferred abroad
YH_h	`yh(h)`	Aggregate household income
YD_h	`yd(h)`	Disposable income net of taxes and transfers
TR_h	`htr(h)`	Aggregate transfers by households
$TR^H_{h,h'}$	`htrh(h,hh)`	Intra-household transfers
TR^W_h	`htrw(h)`	Household transfers abroad
Domestic demand variables		
$XAc_{k,h}$	`xac(k,h)`	Household demand for goods and services
S^h_h	`savh(h)`	Household savings
CPI_h	`cpi(h)`	Household-specific consumer price index
$XAf_{k,f}$	`xaf(k,f)`	Other domestic final demand for goods and services
PF_f	`pf(f)`	Other domestic final demand price deflator
YF_f	`yf(f)`	Other domestic final demand aggregate expenditure level

Table 2, continued: Endogenous variables

Trade variables		
XA_k	`xa(k)`	Economy-wide demand for Armington good
XD_k^d	`xdd(k)`	Domestic demand for domestic production
XMT_k	`xmt(k)`	Domestic demand for aggregate imports
PA_k	`pa(k)`	Price of Armington good
$PM_{k,r}$	`pm(k,r)`	Domestic tariff-inclusive price of imports by region of origin
$XM_{k,r}$	`xm(k,r)`	Sectoral import volume by region of origin
PMT_k	`pmt(k)`	Price of aggregate import bundle
$PE_{k,r}$	`pe(k,r)`	Producer price of exports by region of destination
XD_k^s	`xds(k)`	Domestic output sold domestically
XET_k	`xet(k)`	Aggregate export supply
X_k	`x(k)`	Aggregate output
$XE_{k,r}$	`xe(k,r)`	Export supply by region of destination
PET_k	`pet(k)`	Price of aggregate exports
$ED_{k,r}$	`ed(k,r)`	Demand for exports by region of destination
Domestic trade and transportation margins		
$YT_{k,m}^{mg}$	`ytmg(k,mg)`	Revenues generated by domestic trade and transport margins
$XT_{k,m}^{mg}$	`xtmg(k,mg)`	Trade and transport volumes
$XAmg_{k,k',m}^{mg}$	`xamg(k,kk,mg)`	Demand for good *k* used to transport commodity *kk*
$PT_{k,m}^{mg}$	`ptmg(k,mg)`	Aggregate trade and transport price to transport commodity *k*
Goods price equilibrium		
PD_k	`pd(k)`	Price of domestic goods sold domestically
$WPE_{k,r}$	`wpe(k,r)`	World price of exports by region of destination
Macro variables		
$TarY$	`tary`	Nominal tariff revenues
$RTarY$	`rtary`	Real tariff revenues
GY	`gy`	Government revenues
$GEXP$	`gexp`	Total government current expenditures
S^g	`savg`	Nominal government savings
λ^h	`taxadjh`	Household direct tax schedule shifter
XF_{ZIp}	`xf("invst")`	Volume of private investment
$PLEV$	`Plev`	Absorption price deflator

Table 2, continued: Endogenous variables

Factor market variables		
L_l^s	`ls(l)`	Labor supply
W_l^e	`ewage(l)`	Equilibrium wage rate
$W_{i,l}$	`wage(i,l)`	Sector specific wage rate
TK_{kt}^s	`tks(kt)`	Aggregate capital supply by type
PK	`pk`	Economy-wide aggregate rate of return to capital
$K_{i,kt}^s$	`ks(i,kt)`	Sectoral capital supply by type
PTK_{kt}	`ptks(kt)`	Economy-wide aggregate rate of return to capital by type
$R_{i,kt}$	`rent(i,kt)`	Sectoral rate of return to capital by type
TT_{lt}^s	`tts(lt)`	Aggregate land supply by type
$PLAND$	`pland`	Economy-wide aggregate rate of return to land
$T_{i,lt}^s$	`ts(i,lt)`	Sectoral land supply by type
PTT_{lt}^s	`ptts(lt)`	Economy-wide aggregate rate of return to land by type
$PT_{i,lt}$	`pt(i,lt)`	Sectoral rate of return to land by type
NR_i^s	`rs(i)`	Sectoral supply of sector-specific factor
PR_i	`pr(i)`	Price of sector-specific factor
Macroeconomic variables		
$GDPMP$	`gdpmp`	Nominal GDP at market price
$RGDPMP$	`rgdpmp`	Real GDP at market price
$PGDPMP$	`pgdpmp`	GDP at market price deflator
$GDPFC$	`gdpfc`	Nominal GDP at factor cost
$RGDPFC$	`rgdpfc`	Real GDP at factor cost
$PGDPFC$	`pgdpfc`	GDP at factor cost deflator
Growth variables		
g^y	`ggdp`	Growth rate of real GDP
$\lambda_{ip,l}^l$	`lambdal(ip,l)`	Sector- and labor-specific growth factor

Table 3: Exogenous variables

Growth factors		
γ^l	`gl`	Economy-wide labor productivity growth
$\lambda^k_{i,kt}$	`lambdak(i,kt)`	Capital productivity factor
$\lambda^t_{i,lt}$	`lambdat(i,lt)`	Land productivity factor
λ^{nr}_i	`lambdar(i)`	Sector-specific factor productivity
K^s	`ksup`	Aggregate (normalized) capital stock
$LAND$	`land`	Aggregate land supply
Trade prices		
$WPM_{k,r}$	`wpm(k,r)`	World price of imports (CIF)
$\overline{WPE}_{k,r}$	`wpendx(k,r)`	Export price index of competitors
ER	`er`	Exchange rate and model numéraire
Fiscal variables		
RSg	`rsg`	Government fiscal target (in real terms)
XF_{Gov}	`xf("govnt")`	Volume of government expenditures on goods and services
XF_{ZIg}	`xf("ginvs")`	Volume of public investment
τ^p_i	`tp(i)`	Production tax
$\tau^{cp}_{k,j}$	`tcp(k,j)`	Indirect tax on intermediate demand
$\tau^{cc}_{k,j}$	`tcc(k,h)`	Indirect tax on household consumption
$\tau^{cf}_{k,j}$	`tcf(k,f)`	Indirect tax on other final demand
κ^h_h	`kappah(h)`	Initial marginal direct tax rates
$TR^H_{g,h}$	`gtrh(h)`	Transfers from government to households
κ^c_e	`kappac(e)`	Corporate tax rates
λ^{tm}	`tmadj`	Uniform tariff adjustment factor
$\tau^m_{k,r}$	`tm(k,r)`	Sectoral tariffs by region of origin
$\tau^e_{k,r}$	`te(k,r)`	Sectoral export taxes by region of destination
$\tau^{fl}_{i,l}$	`tfl(i,l)`	Wage tax by sector and labor type
$\tau^{fk}_{i,kt}$	`tfk(i,kt)`	Capital tax by sector and capital type
$\tau^{ft}_{i,lt}$	`tft(i,lt)`	Land tax by sector and land type
τ^{fr}_i	`tfr(i)`	Tax on natural resource

Table 3, continued: Exogenous variables

Miscellaneous exogenous variables		
$TR^H_{W,h}$	`wtrh(h)`	Remittances from abroad
TR^G_W	`wtrg`	Foreign transfers (and grants) to government
TR^W_G	`gtrw`	Transfers from government to rest of the world
XF_{TMG}	`xf("trdmg")`	Volume of exports of international trade and transport services
XF_{DST}	`xf("delst")`	Volume of stock building
S^f	`savf`	Net capital flows, i.e. capital account balance
s^c_e	`csavrate(e)`	Corporate savings rate
$\tau^{mg}_{k,mg}$	`tmg(k,mg)`	Domestic trade margins by transportation node

Table 4: Key model elasticities

Production elasticities		
σ^p_i	`sigmap(i)`	Substitution elasticity between *ND* and *VA* bundles
σ^v_i	`sigmav(i)`	Substitution elasticity between *KL*, *TT* and ND^d bundles
σ^{kl}_i	`sigmakl(i)`	Substitution elasticity between unskilled labor and capital/skilled labor
σ^{ks}_i	`sigmaks(i)`	Substitution elasticity between capital and skilled labor
σ^u_i	`sigmau(i)`	Substitution across unskilled labor categories
σ^s_i	`sigmas(i)`	Substitution across skilled labor categories
σ^k_i	`sigmak(i)`	Substitution across types of capital
σ^t_i	`sigmat(i)`	Substitution across types of land
σ^c_k	`sigmac(k)`	Product aggregation elasticity (converting produced goods to consumed goods)
Demand elasticities		
$\eta_{k,h}$	`eta(k,h)`	Base household income elasticities
σ^f_f	`sigmaf(f)`	Other final demand substitution elasticity
Trade elasticities		
σ^m_k	`sigmam(k)`	Armington elasticity between domestic and aggregate import demand
σ^w_k	`sigmaw(k)`	Armington elasticity for import demand across regions
σ^x_k	`sigmax(k)`	Top-level transformation elasticity between the domestic market and aggregate exports
σ^z_k	`sigmaz(k)`	Transformation elasticity of exports across regions of destination
$\eta^e_{k,r}$	`etae(k,r)`	Transformation elasticity of exports across regions of destination

Table 4, continued: Key model elasticities

Factor market elasticities

ω^{kt}	`omegakt`	Transformation of capital across types
ω_{kt}^{k}	`omegak(kt)`	Transformation of capital by type across sectors
ω^{tl}	`omegakl`	Transformation of land across types
ω_{lt}^{t}	`omegak(kt)`	Transformation of capital by type across sectors
ω_{i}^{nr}	`omegar(i)`	Supply elasticity of sector specific factor

Table 5: Calibrated parameters

Production

α_i^{nd}	`and(i)`	CES share parameter for *ND* bundle
α_i^{va}	`ava(i)`	CES share parameter for *VA* bundle
$a_{k,j}$	`a(k,j)`	Leontief coefficients for intermediate demand
α_i^{kl}	`akl(i)`	CES share parameter for *KL* bundle
α_i^{tt}	`att(i)`	CES share parameter for TT^d bundle
α_i^{nr}	`ar(i)`	CES share parameter for NR^d bundle
α_i^{u}	`au(i)`	CES share parameter for *UL* bundle
α_i^{ksk}	`aksk(i)`	CES share parameter for *KSK* bundle
α_i^{s}	`as(i)`	CES share parameter for *SKL* bundle
α_i^{kt}	`akt(i)`	CES share parameter for KT^d bundle
$\alpha_{i,ul}^{l}$	`al(i,ul)`	CES share parameter for unskilled labor demand
$\alpha_{i,sl}^{l}$	`al(i,sl)`	CES share parameter for skilled labor demand
$\alpha_{i,kt}^{k}$	`ak(i,kt)`	CES share parameter for capital demand
$\alpha_{i,lt}^{t}$	`at(i,lt)`	CES share parameter for land demand
$\alpha_{i,k}^{c}$	`ac(i,k)`	CES share parameter for commodity aggregation

Table 5, continued: Calibrated parameters

Income distribution parameters		
$\varphi^{E}_{k,kt}$	`xket(kt)`	Enterprise share of after-tax capital income
$\varphi^{H}_{k,kt}$	`xkht(kt)`	Household share of after-tax capital income
$\varphi^{W}_{k,kt}$	`xkwt(kt)`	Rest of the world share of after-tax capital income
$\varphi^{e}_{kt,e}$	`xke(kt,e)`	Distribution of capital income across enterprises
$\varphi^{H}_{c,e}$	`xcht(e)`	Household share of after-tax corporate income
$\varphi^{W}_{c,e}$	`xcwt(e)`	Rest of world share of after-tax corporate income
$\varphi^{h}_{l,h}$	`xlh(h,l)`	Distribution of wage income across households
$\varphi^{h}_{kt,h}$	`xkh(kt,h)`	Distribution of capital income across households
$\varphi^{h}_{lt,h}$	`xth(lt,h)`	Distribution of land income across households
$\varphi^{h}_{nr,h}$	`xrh(h)`	Distribution of sector-specific income across households
$\varphi^{h}_{e,h}$	`xch(e,h)`	Distribution of corporate income across households
$\varphi^{H}_{h,h}$	`ahtr(h)`	Transfer share of household after-tax income
$\varphi^{h}_{h,h'}$	`ahtrh(h,hh)`	Distribution of household transfers to households
$\varphi^{W}_{h,h}$	`ahtrw(h)`	Rest of world share of household transfers
Demand parameters		
s^{h}_{h}	`asav(h)`	Household savings rate
$\theta_{k,h}$	`theta(k,h)`	Household consumption floor parameter
$\mu_{k,h}$	`mu(k,h)`	Household marginal consumption (out of discretionary income) parameter
$\alpha^{f}_{k,f}$	`af(k,f)`	Other final demand CES share parameters
Trade parameters		
α^{d}_{k}	`ad(k)`	Domestic share parameter in top-level Armington CES
α^{m}_{k}	`am(k)`	Import share parameter in top-level Armington CES
$\alpha^{w}_{k,r}$	`aw(k,r)`	Regional import share parameter in second-level Armington CES
γ^{d}_{k}	`gd(k)`	Domestic share parameter in top-level CET
γ^{e}_{k}	`ge(k)`	Export share parameter in top-level CET
$\gamma^{x}_{k,r}$	`gx(k,r)`	Regional export share parameter in second-level CET
Domestic trade and transport parameters		
$\alpha^{mg}_{k,k',m}$	`amg(k,kk,mg)`	Leontief coefficients for transporting good *k*'

Table 5, continued: Calibrated parameters

Factor market parameters		
$\phi_{i,l}^{l}$	`phil(i,l)`	Inter-sectoral wage differential parameter
α_{l}^{ls}	`als(l)`	Labor supply shift parameter
γ_{kt}^{tks}	`akst(kt)`	Top-level CET capital allocation share parameters
$\gamma_{i,kt}^{k}$	`aks(i,kt)`	Second-level CET capital allocation share parameters
γ_{lt}^{tts}	`atts(lt)`	Top-level CET land allocation share parameters
$\gamma_{i,lt}^{t}$	`ats(i,lt)`	Second-level CET land allocation share parameters
γ_{i}^{nr}	`ars(i)`	Sector-specific factor supply shifter

Variable count

The number of variables in the model is represented by the following equation:

$$\begin{aligned} NV &= 23N + 3L(1+N) + 3KT(2+N) + 3LT(1+N) + 4.E + H(6+H) + 2F + 18 \\ &+ K\left(N + H + F + 10 + 6R + MG(3+MG)\right) \end{aligned}$$

The variable N is the number of production sectors, K the number of commodities, L the number of labor types, KT the number of capital types, LT the number of land types, E the number of enterprises, H the number of households, F the number of other final demand activities, R the number of trading partners, and MG is the number of transportation nodes (maximum is three). Setting all variables to 1 with the exception of F (=2) and MG (=3) leads to 115 variables in the model (assuming no zero activities). Using GTAP's dimensions, N=57, L=2, KT=1, LT=1, E=0, H=1, F=3, K=57, R=1 and MG=0 leads to 6,430 variables again assuming no zero activities.

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Annex 1: Labor market segmentation

In the standard model labor markets are assumed to be integrated, i.e. there is full labor mobility across sectors with a single economy-wide equilibrating wage rate for each labor type. The model allows for inter-sectoral wage differentials, but these are exogenous in the standard model. This section describes a dual labor market specification where there is imperfect labor mobility between two sectors of the economy—we will call these two sectors rural and urban, though this is not always the best distinction. In many cases the rural sector will be identified with the agricultural sectors and the urban sector with all other sectors, but there may be significant non-agricultural activity occurring in rural areas. The implementation of dual labor markets follows the standard Harris-Todaro specification where the decision to migrate is a function of the expected income in the urban sector relative to the expected income in the rural sector. The specification will deviate somewhat from Harris-Todaro. First, relative wages will be used as a proxy for relative incomes. Second, actual wages will determine migration rather than expected wages in the absence of unemployment. The basic migration equation has the form given in equation (L-1), where *MIGR* represents the level of migration from rural to urban sectors.

$$MIGR_l = \chi_l^m \left(\frac{AWAGE_{Urb,l}}{AWAGE_{Rur,l}} \right)^{\omega_l^m} \qquad \text{(L-1)}$$

$$AWAGE_{g,l} = \frac{\sum_{i \in g} \left(\frac{W_{i,l}}{1+\tau_{i,l}^{fl}} \right) L_{i,l}^d}{\sum_{i \in g} L_{i,l}^d} \qquad \text{(L-2)}$$

The variable *AWAGE* is the average wage in the respective sectors. Letting the index *g* represent the geographic index, the average wage formula is given by equation (L-2). Note that the average wage is calculated based on the net-of-tax wage rate, the rate which matters to the worker deciding to migrate or not.

Labor market equilibrium conditions are now based on two separate labor markets rather than the integrated market of the standard model. Thus with segmented markets, equations (F-1) through (F-3) are dropped from the model specification and are replaced by equations (L-3) and (L-4).

$$L_{g,l}^s = \sum_{i \in g} L_{i,l}^d \qquad \text{(L-3)}$$

$$W_{i,l} = \phi_{i,l}^l W_{g,l}^e \quad \text{for} \quad i \in g \qquad \text{(L-4)}$$

Equation (L-3) determines the equilibrium wage rate by sector—i.e. rural and urban. It sets the aggregate geographic labor supply equal to the demand for labor in the same geographic zone, i.e. it determines the variable W^e which is now indexed by both geographic zone as well as labor type. Equation (L-4) is equivalent to (F-3), but the relative wages are evaluated with respect to the zone-specific equilibrium wage.

The remaining loose end is the definition of labor supply and this is given by equations (L-5) and (L-6). It is assumed that labor supply net of migration is given in any given period. In the case of comparative static simulations geographic labor supply is simply exogenous and set to its base level, L_0. In the case of dynamic simulations, labor supply in each zone grows at some exogenous rate, g^L and migration is subtracted from this amount in the rural zone, equation (L-5), and is added to labor supply in the urban zone, equation (L-6). Equation (L-7) determines the total economy-wide labor supply for each labor type.

$$L^s_{Rur,l} = \left(1 + g^L_{Rur,l}\right) L^s_{Rur,l,-1} - MIGR_l \qquad \text{(L-5)}$$

$$L^s_{Urb,l} = \left(1 + g^L_{Urb,l}\right) L^s_{Urb,l,-1} + MIGR_l \qquad \text{(L-6)}$$

$$L^s_{Tot,l} = L^s_{Rur,l} + L^s_{Urb,l} \qquad \text{(L-7)}$$

Figures

Figure 1: Nested structure of production

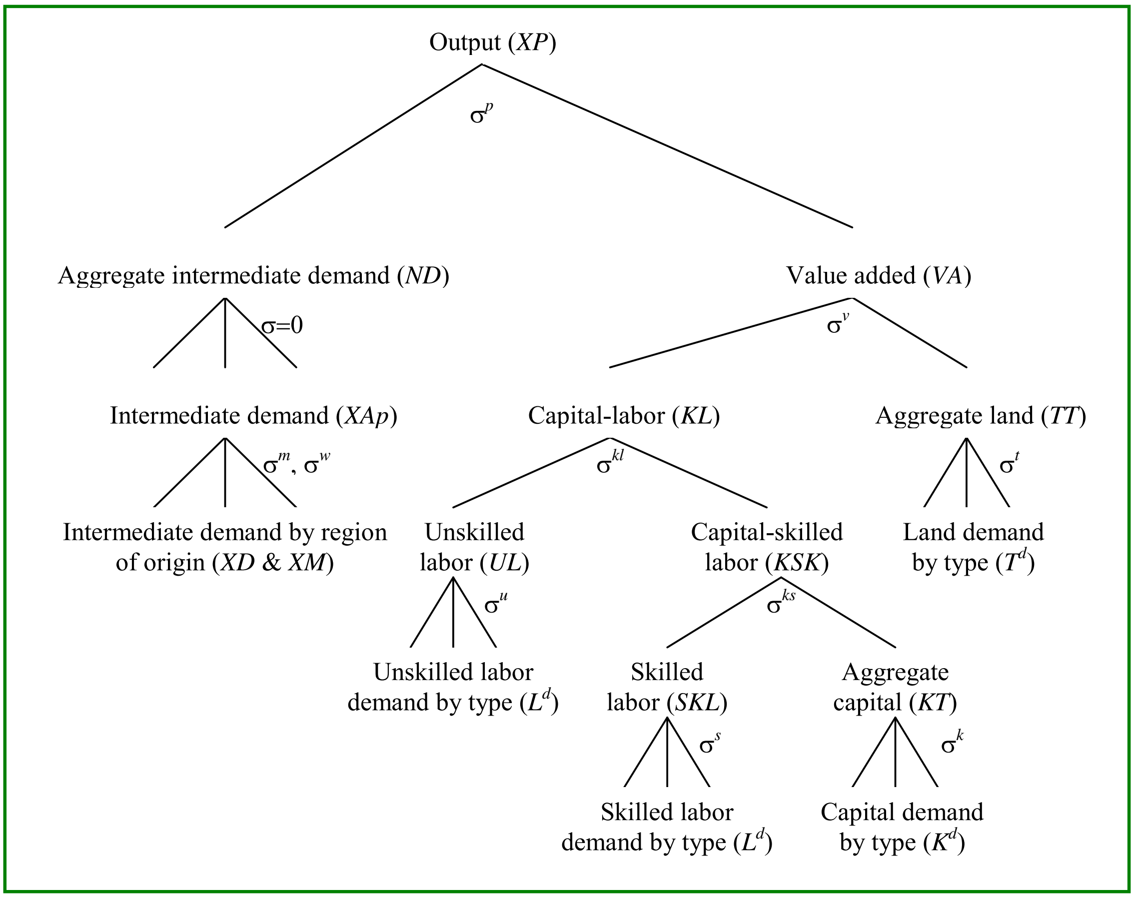

Figure 2: Nested structure of consumer demand