Rational Interacting Agents and Volatility Clustering: A New Approach

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ABSTRACT

Here, we show that agents who are *ex ante* rational, if allowed to interact locally, may generate clustering of volatility. Hence, there is no need to reject the notion of rationality in agent based models.

KEYWORDS: Volatility Clustering, Rationality, Local Interactions

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Rational Interacting Agents and Volatility Clustering: A New Approach

An intriguing statistical property of financial market data is clustering of volatility where large absolute price changes tend to follow large absolute price changes and small absolute price changes tend to follow small absolute price changes. Mandelbrot (1963) first discovered this phenomenon in commodity prices. However, it is the pioneering work of Engle (1982) and Bollerslev (1986) on autoregressive conditional heteroskedastic (ARCH) models and their generalization GARCH models that brought this phenomenon to the forefront of financial market research.

In empirical work, volatility clustering is usually modeled by a statistical model such as GARCH or one of its extensions. As noted by Engle (2001), these models are only statistical descriptions of the data and they do not provide any structural explanation as to why the phenomenon arises. Rather, the statistical models postulate that volatility clustering has an exogenous source and is for example caused by the clustered arrival of random news about the economic fundamentals.

Representative agent finance models cannot explain this phenomenon (see Kurz (1997)). Consequently, researchers have been looking at models with heterogeneous interacting agents for clues regarding this phenomenon. In this spirit, several studies have considered modeling financial markets in analogy with ecological systems where various trading strategies co-exist and evolve via a “natural selection” mechanism (Arifovic et al (2000), Arthur et al (1997), Lebaron et al (1999), Lebaron (2001)). The idea behind these models, the prototype of which is the Santa Fe artificial stock market, is that a financial market can be viewed as a population of agents that differ in the decision rules they use. A decision rule is defined as a mapping from an agent’s information set to the set of actions. These decision rules evolve according to profitability with the most profitable decision rules favored by “natural selection”, usually via genetic algorithms. Notably, Lux and Marchesi (2000) study an agent-based model in which volatility clustering arises from behavioral switching of market participants between fundamentalist and chartist behavior. Fundamentalists expect that the price follows the fundamental value in the long run. Noise traders try to identify price trends, which results in a tendency to herding.

Agents are allowed to switch between these two behaviors according to the performance of the various strategies.

These types of models typically have two features in common. Firstly, they do not allow agents to interact with each other directly. All interactions are mediated through the price system. Specifically, prices are realized through agents' actions resulting in different profits to agents using different decision rules. Agents evaluate their decision rules and corresponding switch from one rule to another, giving rise to a new set of prices, and the process repeats. Agents are not allowed *talk to* each other directly; hence, social influences operating via direct communication are ignored. Secondly, all agents are not *ex ante* rational. Some agents are not doing the best they can given the information that they have. Noise traders are a notable example. Here, it is important to distinguish between *ex post* and *ex ante* rationality. *Ex post* rationality requires that given the realized outcomes, a person must have done the best that could be done. It is rationality in hindsight. That is, in order to be *ex post* rational, a person's beliefs must have correctly predicted the realized outcomes (expectations thereof). Ex *ante* rationality requires that a person must be doing the best he or she can do given the information he or she has at the time of making his or her decision. A person can be *ex ante* rational and *ex post* irrational. For example, if new information arrives between the time of making the decision and realization of outcomes then a person can easily be *ex ante* rational and *ex post* irrational. Agent based models not only reject *ex post* rationality but also *ex ante* rationality by allowing noise traders, as one example. It is reasonable to reject *ex post* rationality, however, rejecting *ex ante* rationality may be considered a stretch.

In this paper, I put forward an agent based model of asset pricing in which all agents are *ex ante* rational and also directly interact with other agents in their circle of influence. I show that such a model can generate volatility clustering, hence, in agent based modeling one does not have to reject the reasonable notion of *ex ante* rationality. Rational expectations finance cannot explain volatility clustering because it maintains the assumption of *ex post* rationality. Agent based finance explains volatility clustering but it goes too far in rejecting rationality and rejects not only *ex post* but also *ex ante* rationality. This paper seeks to make a contribution to agent based finance by demonstrating that a system of interacting agents that are *ex ante* rational and also *talk* to each other may generate volatility clustering.

I. The Model

Our model has two sets of assumptions regarding 1) the data generating process and 2) the belief formation of agents. Manski (2003) argues that in many economic situations, the underlying model or the data generating process is only partially identifiable. It is impossible to correctly identify the date generating process because of the inherent structural instability in the system. The true model can only be partially identified. Econometricians agree about the part which has been identified but disagree about what has not been identified. They complete the model by using their own subjective judgments as they need to make a decision. True model is only partially known and they know that the true model is only partially known. However, at all times, they are very clear about what is objectively known and what has been assumed subjectively. Any two econometricians in this situation would disagree if they have different subjective judgments. However, none of them can be proven to be wrong *ex ante* as long as their predictions remain consistent with the objectively known part of the model. Hence, econometricians making forecasts can disagree and still be *ex ante* rational. In the spirit of Manski (2003), we assume the following:

All agents have partial knowledge about the true model in the following way. There is a commonly known part about which every one agrees and there is an unknown part about which agents disagree. Each period all agents receive the same information. Let θ_{tp} be the interpretation of this information according to the commonly known part of the model. The magnitude of θ_{tp} indicates the expectation about the next period's price level according to the commonly known part of the model. That is, $E_t[p_{t+1}] = \theta_{tp}$.

Each agent's private belief (private expectation about the next period's price level), x_{it} is some perturbation of θ_{tp} :

$$x_{it} = \theta_{tp} + v_{it} = E_{it}[p_{t+1}] \tag{1}$$

Where v_{it} has a mean of zero and a uniform distribution with a range fixed by the partially known true model.

Each agent interacts with people in his social circle. These interactions influence his belief. This is captured by considering a 2-dimensional lattice and assigning a cell to each agent with neighboring cells as his neighbors. Each agent's belief is affected by his interactions with his neighbors. Let z_{it} represent the belief of agent i after interacting with his neighbors:

$$z_{it} = f\left(neighbors' beliefs, x_{it}\right) = E_{it}[p_{t+1} \,|\, Social\ Interactions] \tag{2}$$

where f is some function describing how neighbors' beliefs influence an agent's belief.

Each agent is a mean-variance maximizer. Each agent optimizes given his belief, z_{it} . The standard optimization exercise with one risk-free and one risky asset produces a demand curve for the risky asset of each agent. Assuming that the number of shares outstanding is constant and by equating them with aggregate demand, we can solve for the equilibrium price:[2]

$$p_t = \frac{1}{(1+r)N}\left\{\sum_i^N E_{it}[p_{t+1} \,|\, Social\ Interactions] + \sum_i^N E_{it}[d_{t+1}]\right\} \tag{3}$$

where r is the one period risk free net return, N is the total number of agents and d_{t+1} is the intervening dividend. Similarly, in the next period, the whole process repeats, new information arrives $\left(\theta_{(t+1)p}\right)$, new private beliefs $\left(x_{i(t+1)}\right)$ are formed, new beliefs after social interactions are formed, and the new equilibrium price p_{t+1} is determined.

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[2] See the Appendix for a derivation of this equation. Brock and Hommes (1998) derive this equation as an extension of asset pricing model to the case of heterogeneous expectations. Some authors such as Arthur et al. (1997) derive this equation from no-arbitrage arguments without any explicit optimization exercise. Also see Chiarella and He (2001, 2002, 2003), Farmer and Joshi (2002), Lebaron (2000), Lebaron et al. (1999), Lux and Marchesi (1999), and Lesourne (1992).

Once p_{t+1} is known, each agent compares the expectation error of his private belief, $\left| p_{t+1} - E_{it}[p_{t+1}] \right|$ with the expectation errors of his neighbors. If his expectation error is greater than the expectation error of his neighbors, he assigns a greater weight to their opinion in the next period. If his error is smaller, he assigns a lower weight to the opinion of his neighbors in the next period. The exact functional form of this updating is immaterial to the existence of volatility clustering.[3]

Note that in this model, agents influence each others' beliefs directly. All agents have beliefs consistent with existing information. Each agent is still optimizing given his or her belief and these optimal actions, in aggregate, determine the price of the risky asset.

II. Simulation Results[4]

In what follows, subscripts ij denote the location of an agent in a 2-dimensional plane divided into cells (lattice).

We use the following parameter values for simulation, assuming a uniform distribution with the lower and upper limits given by:

$$\theta_{tp} \in [\textit{current price} - 0.05 \times \textit{current price}, \textit{current price} + 0.05 \times \textit{current price}]$$

$$v_{ijt} \in [-0.05 \times \textit{current price}, 0.05 \times \textit{current price}]$$

$$r = 0.05,\ N = 40000,\ E_{ijt}[d_{t+1}] = 1 \forall i, j, t$$

θ_{tp} represents the interpretation of new information according to the commonly known part of the model. Each period, θ_{tp} takes a random value from a uniform distribution with the range defined above. The role of θ_{tp} in the model is to ensure a steady arrival of new information each period. The results presented here are robust to the range of values

[3] As long as agents are updating in the right direction, the model generates clustering of volatility.
[4] Simulations were carried out in Netlogo.

θ_{tp} can take.[5] v_{ijt} is the idiosyncratic element in each agent's belief. Each period, for each agent, a random value is drawn from a uniform distribution with the range defined above. Together, these two parameters ensure that different agents have different interpretations. We use the simplest linear form for the function z_{ijt} (belief after social interactions, that is, $z_{ijt} = E_{ijt}[p_{t+1} \mid Social\ Interactions]$):

$$z_{ijt} = a \times b_{ijt} + c \times x_{ijt} \tag{4}$$

Where a and c are positive parameters, b_{ijt} is the average belief in the neighborhood of agent ij , that is, $b_{ijt} = \dfrac{x_{(i+1)j} + x_{(i-1)j} + x_{i(j+1)} + x_{i(j-1)}}{4}$ where $x_{(i+1)j}, x_{(i-1)j}, x_{i(j+1)},$ and $x_{i(j-1)}$ denote the beliefs of neighbors immediately to the right, left, above, and below the agent, respectively. We assume that each agent has 4 neighbors.[6] x_{ijt} is the own belief of agent ij . Equation (4) states that the belief after social interactions depends on the average belief in one's social circle as well as on one's own initial predisposition. Parameters a and c control the relative importance that an agent attaches to others' opinion in his social circle. We will refer to a as intensity of social influence and c as own confidence.

For updating confidence, if the expectations error of an agent is greater than the expectation error of average neighborhood belief, that is,

If $|p_{t+1} - x_{ijt}]| > |p_{t+1} - b_{ijt}|$

then a goes up by an amount g which is randomly drawn from a uniform distribution: $g \in [0.10 \times a, 0.50 \times a]$.

If the expectation error of an agent is less than the expectation error of average neighborhood belief then a goes down by g .

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[5] Of course, negative values are not allowed since price is a strictly non-negative variable.

[6] Face-to-face interactions with people with whom one has strong social ties are likely to have the strongest influence on one's judgment. Typically, the number of such people is small. Results are similar for either 4 or 8 neighbors. Due to the sampling issue, it is clear that results will get weaker for larger neighborhoods with our model becoming equivalent to modern asset pricing model for very large neighborhoods (social influence will cancel out).

The rationale behind this assumption is as follows: if a person's social circle outperforms him then plausibly he will assign a greater weight to their opinion in the next future. How much greater? That depends on his state of mind at the moment of decision, which depends on a lot of environmental factors. These environmental factors are essentially random.

A number of representative simulations are run:

1. Simulation without social influences. This simulation is run to establish the benchmark.
2. Simulation with social interactions and the following parameter values: a=0.80, c=0.20
3. Simulation with social interactions and the following parameter values: a=0.60, c=0.20
4. Simulation with social interactions and the following parameter values: a=0.40, c=0.20
5. Simulation with social interactions and the following parameter values: a=0.20, c=0.20

If there is no social influence, our model reduces to modern asset pricing model (substitute a=0, and c=1 in equation 4). See Figure 1. Unsurprisingly, Figure 1 is similar to output from a typical modern asset pricing model. Returns are measured as changes in log-price. Figures 2, 3, 4, and 5 show the returns generated by our model corresponding to simulations 2, 3, 4, and 5 as described above. Tables 1, 2, 3, and 4 show the results from a statistical test for volatility clustering corresponding to simulations 2, 3, 4, and 5. As can be seen, there is clear evidence of volatility clustering.

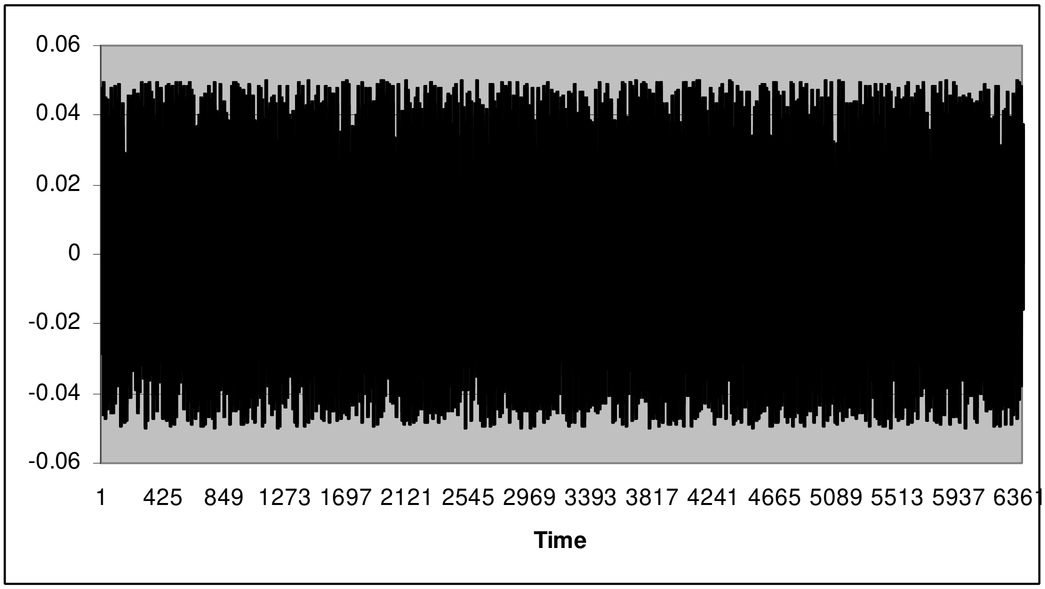

Figure 1: Returns

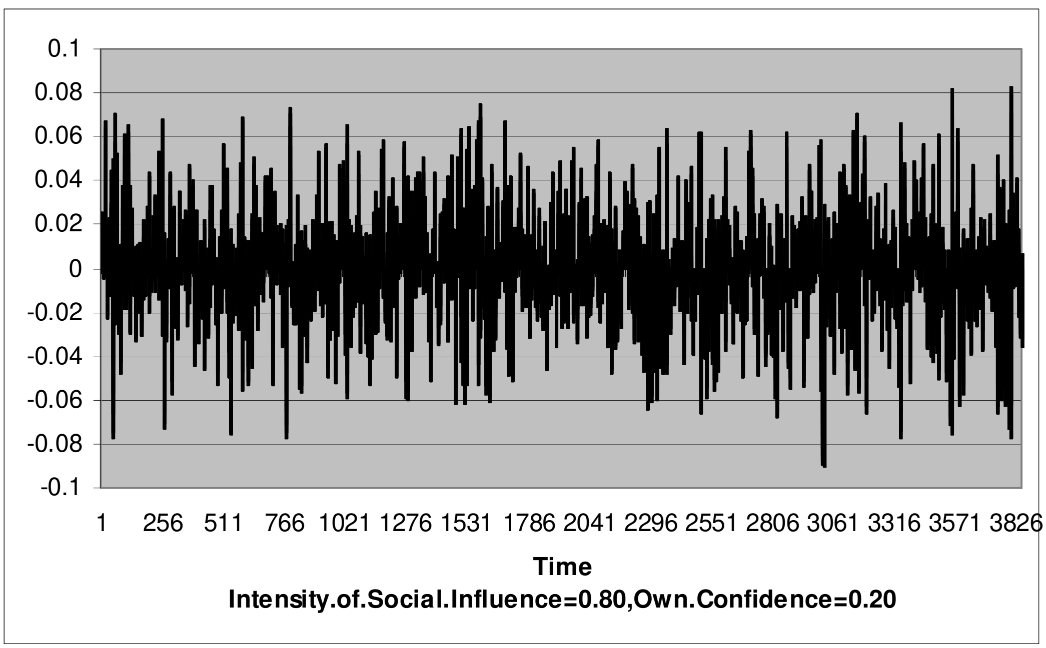

Figure 2: Returns

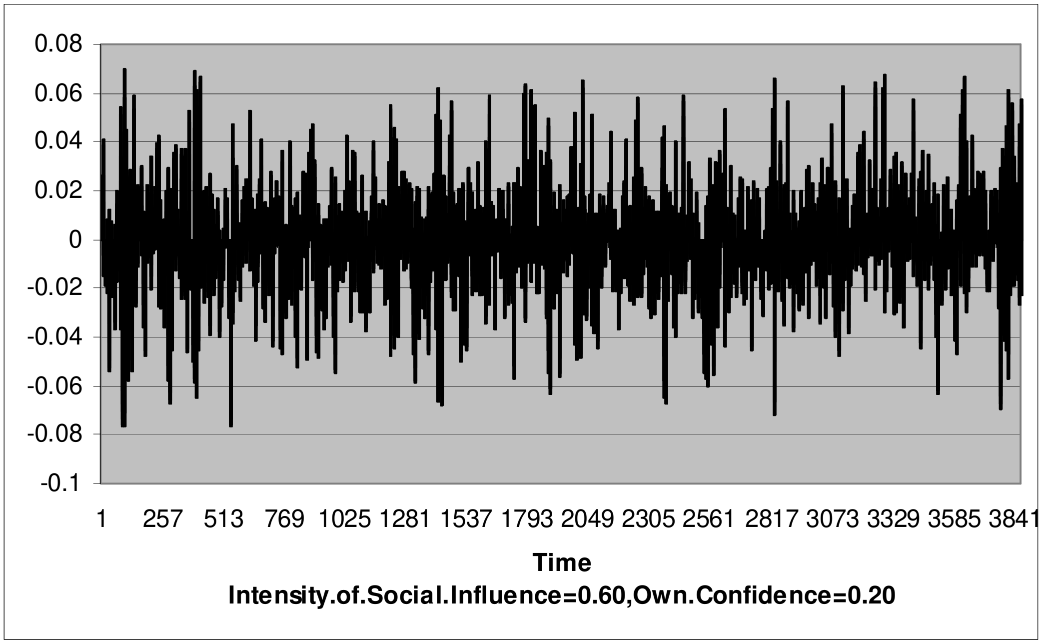

Figure 3: Returns

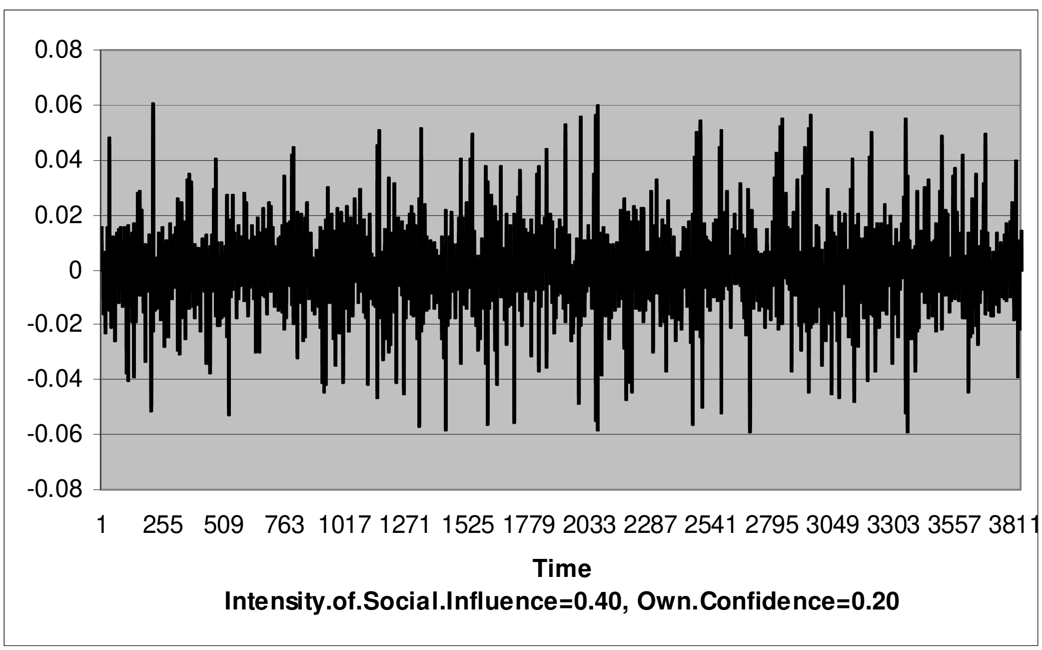

Figure 4: Returns

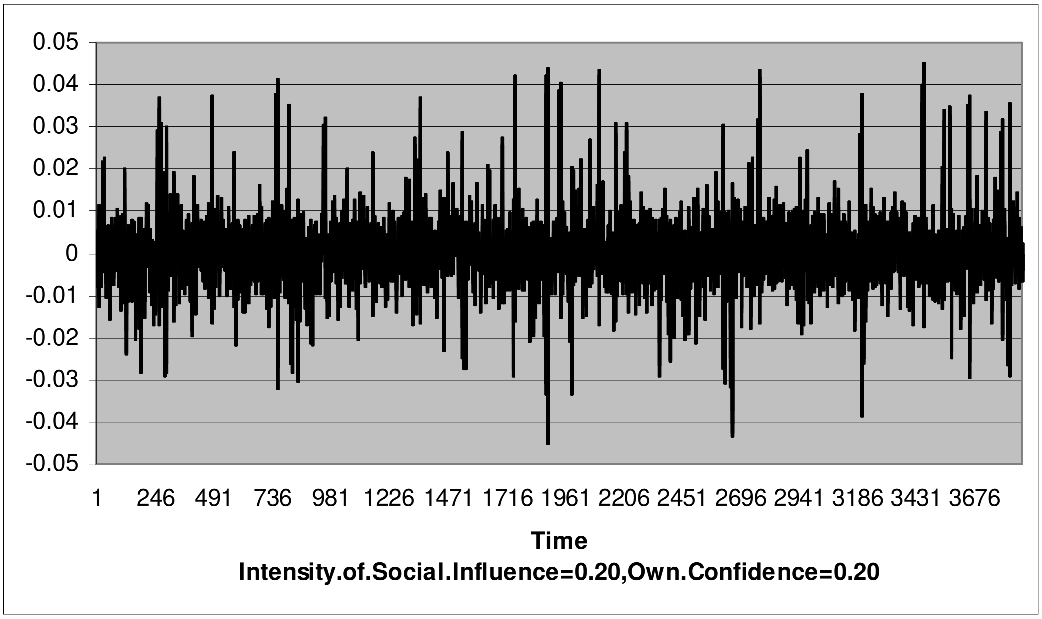

Figure 5: Returns

Table 1: ARCH Test

ARCH(1) REGRESSION: TEST FOR VOLATILITY CLUSTERING					
	Estimate	**Error**	**t-Value**	**p-Value**	**ARCH EFFECT**
ARCH0	0.000093	1.17E-05	7.94	<.0001	Yes
ARCH1	0.9133	0.0689	13.26	<.0001	Yes

Table 2: ARCH Test

ARCH(1) REGRESSION: TEST FOR VOLATILITY CLUSTERING					
	Estimate	Error	t-Value	p-Value	ARCH EFFECT
ARCH0	0.000072	8.80E-06	8.18	<.0001	Yes
ARCH1	0.8818	0.0674	13.07	<.0001	Yes

Table 3: ARCH Test

ARCH(1) REGRESSION: TEST FOR VOLATILITY CLUSTERING					
	Estimate	**Error**	**t-Value**	**p-Value**	**ARCH EFFECT**
ARCH0	4.38E-05	5.27E-06	8.31	<.0001	Yes
ARCH1	0.8147	0.0634	12.85	<.0001	Yes

Table 4: ARCH Test

ARCH(1) REGRESSION: TEST FOR VOLATILITY CLUSTERING					
	Estimate	**Error**	**t-Value**	**p-Value**	**ARCH EFFECT**
ARCH0	2.18E-05	2.65E-06	8.23	<.0001	Yes
ARCH1	0.6922	0.057	12.15	<.0001	Yes

III. Conclusions

This paper shows that one does not need to reject the notion of *ex ante* rationality in financial markets as agent based models normally do. Clustering of volatility can be explained within the framework of agent based modeling without rejecting *ex ante* rationality if agents are allowed to talk to each other. It remains to be seen whether other market anomalies can be explained with this modified approach; a subject of future research.

APPENDIX

Brock and Hommes (1998) put forward a simple model of asset pricing with heterogeneous beliefs (BH model). Their model is now a workhorse for literature with heterogeneous expectations. Guanersdorfer et al (2003), Guanersdorfer (2000), Chiarella and He (2003), Chiarella and He (2002), Chiarella and He (2001), Farmer and Joshi (2002), Lebaron (2000), Lebaron et al (1999), Lux and Marchesi (1999), and Lesourne (1992) are a few examples of use of this model.

In Brock and Hommes (1998), there are two types of assets, a risk free asset and a risky asset. Risk free asset pays a net return of r, which is between 0 and 1. That is, for a dollar of investment, the gross return is $(1+r)$ after a unit interval. Let p_t denote the price of risky asset that pays dividends, d. The dynamics of wealth of an agent type 'a' is described by

$$W_{a,t+1} = W_{at}(1+r) + R_{t+1}S_{at} \tag{A.1}$$

where R_{t+1} is the excess return (in dollars) per share of risky asset over risk free asset, that is, $R_{t+1} = p_{t+1} + d_{t+1} - (1+r)p_t$ and S_{at} is the number of shares of risky asset bought by an agent of type 'a'.

Let E_t and V_t denote conditional expectation and conditional variance, and let E_{at} and V_{at} denote the beliefs of investor type 'a' about these conditional expectation and variance.

Assume that investors are mean-variance maximizers.[7] The demand for shares of risky asset by an agent of type 'a' can be obtained as follows.

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[7] Mean-Variance Optimization is a decision making model proposed by Markowitz (1952) as an alternative to Expected Utility decision model. The Expected utility model gives the same results as the Mean-Variance model if the utility function is quadratic or returns are normally distributed. Levy and Markowitz (1979) show that mean-variance analysis can be regarded as a Taylor approximation (second order) of any given utility function (such as power utility) in the Expected Utility model. Rabin (2000) argues that Expected Utility model is absurd as a model of human decision making. The Mean-Variance model is simpler, though less general; however, it does not suffer from serious plausibility issues such as the one raised by Rabin (2000).

$$Maximize\{E_{at}[W_{a,t+1}] - (e/2)V_{at}[W_{a,t+1}]\}$$
$$\Rightarrow S_{at} = \frac{E_{at}[R_{t+1}]}{eV_{at}[R_{t+1}]} \tag{A.2}$$

where e is interpreted as a risk aversion parameter.[8]

Assume a constant supply of outside shares over time, m. Further, assume that all agents agree about the variance and that the market clears:

$$\sum_1^N E_{at}[p_{t+1}] + \sum_1^N E_{at}[d_{t+1}] - \sum_1^N p_t(1+r) = m \times e \times V_t[R_{t+1}] \tag{A.3}$$

$$\Rightarrow \sum_1^N E_{at}[p_{t+1}] + \sum_1^N E_{at}[d_{t+1}] - \sum_1^N p_t(1+r) = \sum_1^N \frac{m}{N} \times e \times V_t[R_{t+1}] \tag{A.4}$$

$$\Rightarrow \sum_1^N E_{at}[p_{t+1}] + \sum_1^N E_{at}[d_{t+1}] - \sum_1^N \frac{m}{N} \times e \times V_t[R_{t+1}] - \sum_1^N p_t(1+r) = 0 \tag{A.5}$$

$$\Rightarrow \sum_1^N E_{at}[p_{t+1}] + \sum_1^N \{E_{at}[d_{t+1}] - \frac{m}{N} \times e \times V_t[R_{t+1}]\} - \sum_1^N p_t(1+r) = 0 \tag{A.6}$$

Define risk adjusted dividend as, $d`_{t+1} = d_{t+1} - \frac{m}{N} \times e \times V_t(R_{t+1})$:

$$\Rightarrow \sum_1^N E_{at}[p_{t+1}] + \sum_1^N \{E_{at}[d`_{t+1}]\} - \sum_1^N p_t(1+r) = 0 \tag{A.7}$$

$$\Rightarrow \sum_1^N E_{at}[p_{t+1}] + \sum_1^N \{E_{at}[d`_{t+1}]\} = N \times p_t(1+r) \tag{A.8}$$

$$\Rightarrow p_t = \frac{1}{(1+r)N} \left\{ \sum_1^N E_{at}[p_{t+1}] + \sum_1^N E_{at}[d`_{t+1}] \right\} \tag{A.9}$$

Equation (A.9) is the same as equation (3).

[8] Not to be confused with the risk aversion parameter in the Expected Utility Model.

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