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1 February 2011

Online at <https://mpra.ub.uni-muenchen.de/29859/>  
MPRA Paper No. 29859, posted 30 Mar 2011 19:14 UTC

# Asset Prices, Nominal Rigidities, and Monetary Policy: Role of Price Indexation\*

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February 1, 2011

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\*I would like to thank Charles Carlstrom, Timothy Fuerst, Masaru Inaba, and Keiichiro Kobayashi for their helpful comments and suggestions. Of course, all remaining errors are mine. The author acknowledges financial support from the Canon Institute for Global Studies.

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## **Abstract**

Carlstrom and Fuerst (2007) [“Asset prices, nominal rigidities, and monetary policy,” *Review of Economic Dynamics* 10, 256–275] find that monetary policy response to share prices is a source of equilibrium indeterminacy because an increase in inflation implies a high real marginal cost and low share prices in a sticky-price economy. We find that if the New Keynesian Phillips curve has a lagged inflation term caused by price indexation, this effect is weakened. Moreover, equilibrium indeterminacy caused by monetary policy response to share prices never arises if all the firms that cannot re-optimize their prices follow price indexation.

**Keywords:** asset prices; monetary policy; equilibrium determinacy; price indexation

**JEL classification:** E32; E44; E52

# 1 Introduction

One of the classical topics in monetary policy is on the stance of a central bank on asset price fluctuations. Japan's experience of the boom during the late 1980s and the long stagnation during the 1990s and the recent boom and bust experience of the U.S. economy seem to imply that a central bank should respond to asset price fluctuations.

Many researchers have investigated this topic. Bernanke and Gertler (2001) and Gilchrist and Leahy (2002) find the unimportance of responding to asset prices. Iacoviello (2005) shows that monetary policy response to asset prices generates welfare gain. Faia and Monacelli (2007) find that monetary policy should negatively respond to asset prices.

A recent paper by Carlstrom and Fuerst (2007) provides a negative answer to this question. They show that equilibrium indeterminacy arises if monetary policy responds to share prices in a standard sticky-price economy. An increase in inflation reduces firm's profits, and the share prices decline since they reflect the firm's profits. Then, monetary policy response to share prices implicitly weakens overall reactions to inflation. This is a source of equilibrium indeterminacy.

In this paper, we extend the model of Carlstrom and Fuerst (2007) by introducing price indexation and show that equilibrium determinacy is likely to arise. Under price indexation, the New Keynesian Philips curve is hybrid and has a lagged inflation term. It is shown that the effect of an increase in inflation on real marginal costs is weakened through the hybrid Phillips curve. Moreover, equilibrium indeterminacy caused by monetary policy response to share prices never arises if all the firms that cannot re-optimize their prices follow price indexation.

An increase in inflation increases the real marginal cost under the sticky-price setting without price indexation, since a fraction of firms cannot change their prices. This increase in the real marginal cost implies low share prices. Then,

monetary policy response to share prices implicitly weakens overall reactions to inflation. Contrary to this, firms following price indexation can keep their real marginal cost constant in the long run since the past inflation reflects this increase in inflation.

Fuhrer and Moore (1995) emphasize the inflation persistence by empirical analyses, and Gali and Gertler (1999) develop a model with the hybrid New Keynesian Phillips curve. Many state-of-the-art DSGE models à la Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) employ price indexation. Therefore, it is important to consider the type of New Keynesian Phillips curve used when we investigate the relationship between monetary policy and share prices.

The rest of this paper is organized as follows. Section 2 introduces our model. Section 3 presents the main results and their interpretation. Finally, Section 4 presents our concluding remarks.

## **2 The model**

We employ a standard sticky-price model with shares, like Carlstrom and Fuerst (2007). The difference between our model and theirs is that we introduce price indexation in sticky prices.

### **2.1 Households**

The household begins period  $t$  with  $M_t$  cash balances,  $B_t$  one-period nominal bonds that pay  $R_{t-1}$  gross risk-free interest rate,  $S_t$  shares of stock that sell at price  $Q_t$ .

The utility function is

$$U\left(C_t, H_t, \frac{M_{t+1}}{P_t}\right) = \frac{C_t^{1-\sigma}}{1-\sigma} - \phi \frac{H_t^{1+\gamma}}{1+\gamma} + V\left(\frac{M_{t+1}}{P_t}\right), \quad (1)$$

where  $\sigma > 0$ ,  $\phi > 0$ ,  $\gamma > 0$ ,  $V(\cdot)$  is increasing and concave,  $C_t$  denotes consumption,  $H_t$  denotes labor supply,  $P_t$  denotes aggregate price level, and  $M_{t+1}/P_t$  denotes real cash balances at the end of period  $t$ .

The budget constraint of household is

$$\begin{aligned} P_t C_t + M_{t+1} + B_{t+1} + P_t Q_t S_{t+1} \\ \leq P_t W_t H_t + M_t + R_{t-1} B_t + P_t (Q_t + D_t) S_t + X_t, \end{aligned} \quad (2)$$

where  $W_t$  denotes wage rate,  $D_t$  denotes dividends of share, and  $X_t$  denotes monetary injection.

The first order conditions of households are

$$\phi C_t^\sigma H_t^\gamma = W_t, \quad (3)$$

$$C_t^{-\sigma} = \beta C_{t+1}^{-\sigma} \cdot \frac{R_t}{\Pi_{t+1}}, \quad (4)$$

$$C_t^{-\sigma} Q_t = \beta C_{t+1}^{-\sigma} [Q_{t+1} + D_{t+1}], \quad (5)$$

where  $\Pi_{t+1} \equiv P_{t+1}/P_t$  denotes gross inflation. Equation (3) is the intratemporal optimization condition, equation (4) is the Euler equation for consumption, and equation (5) is the Euler equation for share.

Equation (5) can be rewritten as familiar asset prices equations:

$$Q_t = \left[ Q_{t+1} + D_{t+1} \right] \frac{\Pi_{t+1}}{R_t}. \quad (6)$$

## 2.2 Firms

There are monopolistically competitive intermediate-goods firms and competitive final-goods firms.

The production technology of final-goods firms is

$$Y_t = \left( \int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \quad (7)$$

where  $\theta$  denotes the elasticity of substitution and  $Y_t(i)$  denotes outputs of intermediate-goods indexed by  $i$ . The profit maximization of final-goods firms implies the demand curve for  $Y_t(i)$  as

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\theta} Y_t, \quad (8)$$

where  $P_t(i)$  denotes the price level of intermediate-goods indexed by  $i$ . Combining equations (7) and (8) yields the following price index for intermediate goods:

$$P_t = \left( \int_0^1 P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}. \quad (9)$$

The intermediate-goods firms are monopolistically competitive, and they produce intermediate-goods  $Y_t(i)$  employing labor  $H_t(i)$  from households. The production function of intermediate-goods firm is

$$Y_t(i) = H_t(i). \quad (10)$$

The cost minimization problem implies

$$W_t = Z_t, \quad (11)$$

where  $Z_t$  denotes the Lagrange multiplier of the cost minimization problem, and it can be interpreted as the real marginal cost.

Intermediate goods firms set their prices subject to Calvo-type price staggeredness with price indexation. The price can be re-optimized at period  $t$  only with probability  $1 - \kappa$ . Among  $\kappa$  firms that cannot re-optimize their prices, a fraction  $\eta$  firms index their prices to the past inflation  $\pi_{t-1}$ .

As in Smets and Wouters (2007), under this setting, we obtain the hybrid New Keynesian Phillips curve,

$$\pi_t = \frac{\beta}{1 + \eta\beta} \pi_{t+1} + \frac{\eta}{1 + \eta\beta} \pi_{t-1} + \lambda z_t, \quad (12)$$

where

$$\lambda \equiv \frac{(1 - \kappa)(1 - \kappa\beta)}{\kappa(1 + \eta\beta)}, \text{ and}$$

$\beta$ ,  $\pi_t$  and  $z_t$  denote the discount factor, the log deviations from a steady state of inflation and the real marginal cost, respectively.

### 2.3 Monetary policy

We assume that monetary authority follows a Taylor rule:

$$r_t = \tau\pi_t + \tau_q q_t, \quad (13)$$

where  $r_t$  and  $q_t$  denote the log-deviations from a steady state of  $R_t$  and  $Q_t$ , respectively.

If  $\tau_q > 0$ , a central bank responds to asset price fluctuations.

### 2.4 Equilibrium

The market clearing conditions are

$$H_t = \int_0^1 H_t(i) di, \quad (14)$$

$$S_t = 1, \quad (15)$$

$$B_t = 0. \quad (16)$$

The resource constraint is

$$C_t = Y_t \quad (17)$$



and the aggregate production function is

$$Y_t = \frac{1}{\Delta_t} H_t, \quad (18)$$

where  $\Delta_t$  is a measure of resource cost of price dispersion:

$$\Delta_t \equiv \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\theta} di. \quad (19)$$

In this paper, we ignore effects from the price dispersion for simplicity.

We focus on a equilibrium where all monopolistic competitive firms are symmetric in this paper. The firm's profits are paid out as dividends to the shareholders. For simplicity, we assume that the measure of firms is equal to the measure of households. The dividend of intermediate-goods firms is given by

$$D_t = Y_t - W_t H_t. \quad (20)$$

By equation (11), the dividend is written by

$$D_t = (1 - Z_t) Y_t. \quad (21)$$

## 2.5 Linearized system

The linearized equilibrium system is given as follows:

$$(\sigma + \gamma)c_t = w_t, \quad (22)$$

$$\sigma(c_{t+1} - c_t) = r_t - \pi_{t+1}, \quad (23)$$

$$q_t = \beta q_{t+1} + (1 - \beta)d_{t+1} + (\pi_{t+1} - r_t), \quad (24)$$

$$d_t = c_t - \frac{z}{1 - z} z_t, \quad (25)$$

$$w_t = z_t, \quad (26)$$

$$\pi_t = \frac{\beta}{1 + \eta\beta} \pi_{t+1} + \frac{\eta}{1 + \eta\beta} \pi_{t-1} + \lambda z_t, \quad (27)$$

$$r_t = \tau \pi_t + \tau_q q_t, \quad (28)$$

where the lower letters denote the log-deviations from a steady state.

As shown by Carlstrom and Fuerst (2007), the dividend is given by

$$d_t = -Az_t, \quad (29)$$

where

$$A \equiv \frac{z(1 + \sigma + \gamma) - 1}{(1 - z)[\sigma + \gamma]}.$$

We employ an assumption on  $A$  following Carlstrom and Fuerst (2007).

**Assumption 1.**  $A > 0$ .

Under this assumption, an increase in the real marginal cost decreases the dividend.

The equilibrium system is reduced to the following matrix form:

$$\begin{bmatrix} 1 & 0 & \chi & 0 \\ \frac{\beta}{1+\eta\beta} & 0 & 0 & 0 \\ 1 & 0 & -(1-\beta)A & \beta \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_{t+1} \\ \pi_t \\ z_{t+1} \\ q_{t+1} \end{bmatrix} = \begin{bmatrix} \tau & 0 & \chi & \tau_q \\ 1 & -\frac{\eta}{1+\eta\beta} & -\lambda & 0 \\ \tau & 0 & 0 & 1 + \tau_q \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ \pi_{t-1} \\ z_t \\ q_t \end{bmatrix},$$

where

$$\chi \equiv \frac{\sigma}{\sigma + \gamma} > 0.$$

The first equation is the consumption Euler equation (23); the second, the New Keynesian Phillips curve (27); and the third, the Euler equation for share (24).

In this paper, we impose the following restriction.

**Assumption 2.**  $1 < \tau \leq 2$ .

We make this assumption to easily prove Proposition 1. However, according to our numerical robustness check, the result in this paper is robust even if  $\tau > 2$ .

### 3 Main results

#### 3.1 Results

The main results of this paper are as follows.

**Proposition 1.** *Under Assumptions 1 and 2, a necessary and sufficient condition for equilibrium determinacy is*

$$\tau_q < \tau_q^{max} \equiv \frac{\lambda(\tau - 1)(1 + \eta\beta)}{A(1 - \eta)(1 - \beta)}.$$

If  $\tau_q > \tau_q^{max}$ , there is equilibrium indeterminacy or no stationary equilibrium.

*Proof.* See Appendix. □

**Proposition 2.** *Under Assumptions 1 and 2, there is equilibrium indeterminacy or no stationary equilibrium if  $\tau_q > \tau_q^{max}$ . A sufficient condition for equilibrium indeterminacy is*

$$\tau_q^{max} < \tau_q < \frac{2(1 + \eta\beta)[\chi + \lambda(2 - \tau)] + \chi(4\beta + \eta)}{(2 - \eta)[\chi + 2A(1 - \beta)]}.$$

*Proof.* See Appendix. □

At a limit of  $\eta = 0$ , the threshold  $\tau_q^{max}$  is the same as the threshold in Proposition 1 of Carlstrom and Fuerst (2007).

The threshold  $\tau_q^{max}$  depends on the fraction of price indexation firms,  $\eta$ .

**Proposition 3.**  $\tau_q^{max}$  is increasing in  $\eta$ .

*Proof.* Since  $\lambda = \frac{(1-\kappa)(1-\kappa\beta)}{\kappa(1+\eta\beta)}$ , we obtain

$$\tau_q^{max} = \frac{(1 - \kappa)(1 - \kappa\beta)(\tau - 1)}{\kappa A(1 - \eta)(1 - \beta)}.$$

□

Then, in the case where the fraction of price indexation is large, equilibrium determinacy is likely to arise even if monetary policy responds to share prices. Especially, in the case where  $\eta = 1$ , equilibrium indeterminacy never arises even if monetary policy responds to share prices.

**Proposition 4.** *Under Assumptions 1 and 2, if all the firms follow price indexation,  $\eta = 1$ , then equilibrium indeterminacy never arise.*

*Proof.*  $\lim_{\eta \rightarrow 1} \tau_q^{max} = \infty$ . □

### 3.2 A Taylor principle interpretation

The Taylor principle establishes that a permanent increase in the inflation rate leads to a more-than-proportionate increase in the nominal interest rate. Following Bullard and Mitra (2002) and Carlstrom and Fuerst (2007), we interpret our results according to this principle.

A one percentage point permanent increase in the inflation rate causes the marginal cost to increase by  $\frac{(1-\eta)(1-\beta)}{\lambda(1+\eta\beta)}$  percentage point through the New Keynesian Phillips curve. Since  $\lambda = \frac{(1-\kappa)(1-\kappa\beta)}{\kappa(1+\eta\beta)}$ , this can be rewritten as

$$\frac{\kappa(1-\eta)(1-\beta)}{(1-\kappa)(1-\kappa\beta)}. \quad (30)$$

This decreases dividends and share prices by  $\frac{A\kappa(1-\eta)(1-\beta)}{(1-\kappa)(1-\kappa\beta)}$ . The total effect on the nominal rate is given by

$$\tau - \tau_q \frac{A\kappa(1-\eta)(1-\beta)}{(1-\kappa)(1-\kappa\beta)}. \quad (31)$$

If this total response is greater than unity, the rule satisfies the Taylor principle.

If  $0 \leq \eta < 1$ , then  $\frac{A\kappa(1-\eta)(1-\beta)}{(1-\kappa)(1-\kappa\beta)} > 0$ . Thus, monetary policy response to share prices weakens the total response to inflation and is a source of equilibrium indeterminacy. However, this effect is decreasing in  $\eta$  since  $\frac{A\kappa(1-\eta)(1-\beta)}{(1-\kappa)(1-\kappa\beta)} > 0$  is decreasing in  $\eta$ . This is because the effect of an increase in inflation on the real marginal

cost, through the hybrid Phillips curve, is weakened. In particular, if  $\eta = 1$ , then  $\frac{A\kappa(1-\eta)(1-\beta)}{(1-\kappa)(1-\kappa\beta)} = 0$  and the total effect on the nominal interest rate of inflation is  $\tau$ . Therefore, monetary policy response to share prices is not a source of equilibrium indeterminacy in this case.

Under the sticky-price setting without price indexation, a fraction of firms cannot change their prices in every period. Then, a permanent increase in inflation implies a low real marginal cost. Under the sticky-price setting with price indexation, a fraction of firms that cannot re-optimize their prices indexes their prices to the past inflation. In the long run, firms following price indexation can keep their real marginal cost constant since the past inflation reflects this increase in inflation. Therefore, a permanent increase in inflation does not change the real marginal cost if all the firms that cannot re-optimize their prices follow price indexation.

### 3.3 Numerical examples

We have the fraction of firms that follow price indexation  $\eta$  affects the threshold of the central bank's stance to the share prices on equilibrium indeterminacy qualitatively. In this subsection, we investigate the quantitative effects of  $\eta$  on  $\tau_q$ .

For this exercise, we set the parameter values of the model as follows. The discount factor of households,  $\beta$ , is 0.99. The relative risk aversion,  $\sigma$ , is two. The Frisch elasticity of labor,  $\gamma$ , is two. The central bank's stance to inflation,  $\tau$ , is 1.1. The steady-state marginal cost,  $z$ , is 0.85, which implies that the steady-state markup is 15%. These values are taken from those employed by Carlstrom and Fuerst (2007). We set the Calvo-pricing price-stickiness parameter,  $\kappa$ , is 0.75 following the literature, which implies that firms can re-optimize their prices about once a year.

Figure 1 shows the determinacy and indeterminacy regions.

[Insert Figure 1]

The vertical axis means the central bank's stance to the share price. The horizontal axis means the fraction of firms that follow price indexation. The equilibrium indeterminacy arises in the upper-left region. The equilibrium determinacy arises in the lower-right region. Then, a stronger stance of the central bank to the share prices induces equilibrium indeterminacy. However, if the fraction of firms that follow price indexation is sufficiently high, equilibrium indeterminacy is not likely to arise if monetary policy responds to share prices.

## 4 Concluding remarks

Carlstrom and Fuerst (2007) found that monetary policy response to share prices is a source of equilibrium indeterminacy in a standard sticky-price model because an increase in inflation implies a high real marginal cost and low share prices.

In this paper, we investigated a sticky-price model in which the New Keynesian Phillips curve has a lagged inflation term caused by price indexation. We found that if firms follow price indexation, the effect of an increase in inflation on real marginal cost is weakened and equilibrium determinacy is likely to arise. Moreover, equilibrium indeterminacy never arises if all the firms that cannot re-optimize their prices follow price indexation.

Empirical results support the significance of a backward inflation term in the New Keynesian Phillips curve. Therefore, when we discuss the relationship between asset prices and monetary policy, we must consider the type of Phillips curve.

## Appendix

**Proposition 1.** *Under Assumptions 1 and 2, a necessary and sufficient condition for equilibrium determinacy is*

$$\tau_q < \tau_q^{max} \equiv \frac{\lambda(\tau - 1)(1 + \eta\beta)}{A(1 - \eta)(1 - \beta)}.$$

*Proof.* For equilibrium determinacy, just one root should be inside the unit circle and others should be outside the unit circle. It is easily shown that one root is  $1/\beta$ .

The three remaining roots are the solutions of a characteristic equation:

$$F(x) = x^3 + F_1x^2 + F_2x + F_3,$$

where

$$F_1 \equiv -\frac{1}{\beta\chi} \left\{ \tau_q A(1 - \beta) + \eta\beta(\lambda + \chi) + \chi(1 + \beta + \tau_q) + \lambda \right\} < 0,$$

$$F_2 \equiv -\frac{1}{\beta\chi} \left\{ \eta\tau_q A(1 - \beta) + \lambda\tau(1 + \eta\beta) + \eta\chi(1 + \beta) + \chi[1 + \tau_q(1 + \eta)] \right\} > 0, \text{ and}$$

$$F_3 \equiv -\frac{\eta(1 + \tau_q)}{\beta} < 0.$$

It is shown that  $F(0) = F_3 < 0$ ,  $F'(0) = F_2 > 0$ , and

$$F'(1) = -\frac{1}{\beta\chi} \left\{ \tau_q [(2 - \eta)A(1 - \beta) + \chi(1 - \eta)] + \chi(1 - \beta)(1 - \eta) + \lambda(2 - \tau)(1 + \eta\beta) \right\} < 0$$

since  $\tau \leq 2$ . A necessary condition for equilibrium determinacy is

$$\begin{aligned} F(1) &= 1 + F_1 + F_2 + F_3 \\ &= \frac{1}{\beta\chi} \left\{ \lambda(\tau - 1)(1 + \eta\beta) - \tau_q A(1 - \eta)(1 - \beta) \right\} > 0. \end{aligned}$$

In the case where all roots are real, it is obvious that this condition is also sufficient.

Next, consider the case where two roots are complex. Suppose that  $a \pm bi$  are roots and a norm of  $M \equiv \sqrt{a^2 + b^2}$ . We have

$$\begin{aligned} F(x) &= (x - a + bi)(x - a - bi)(x - r) \\ &= x^3 - (2a + r)x^2 + (M^2 + 2ar)x - M^2r \end{aligned}$$

where  $r$  is a real root in  $(0, 1)$ . For equilibrium determinacy, we will show that  $M^2 > 1$ .

Since  $F'(x) = 3x^2 - 2(2a + r)x + M^2 + 2ar$ ,  $F(x)$  reaches a local minimum at

$$x = x^{Lmin} \equiv \frac{2a + r + \sqrt{r^2 - 2ar + a^2 - 3b^2}}{3}.$$

Since  $F'(0) < 0$  and  $F'(1) > 0$ , it is shown that  $x^{Lmin} > 1$ . It suffices to show that  $x^{Lmin} < a$  for  $M \equiv \sqrt{a^2 + b^2} > 1$ .

The rest of this proof, we show that  $r < a$  at first. Since  $r < x^{Lmin}$ , it is obtained that

$$2(r - a) < \sqrt{r^2 - 2ar + a^2 - 3b^2}.$$

If  $r \geq a$ , we have

$$4(r - a)^2 - (r^2 - 2ar + a^2 - 3b^2) = (r - a)^2 + 3b^3 \geq 0$$

and it is a contradiction. Then,  $r < a$ .

Finally,  $x^{Lmin} < a$  is shown as follows. A necessary and sufficient condition for  $x^{Lmin} < a$  is

$$\sqrt{r^2 - 2ar + a^2 - 3b^2} < a - r.$$

Since  $r < a$ , this condition is reduced to

$$r^2 - 2ar + a^2 - 3b^2 < (a - r)^2,$$

and this is easily shown.

□



**Proposition 2.** *Under Assumptions 1 and 2, there is equilibrium indeterminacy or no stationary equilibrium if  $\tau_q > \tau_q^{max}$ . A sufficient condition for equilibrium indeterminacy is*

$$\tau_q^{max} < \tau_q < \frac{2(1 + \eta\beta)[\chi + \lambda(2 - \tau)] + \chi(4\beta + \eta)}{(2 - \eta)[\chi + 2A(1 - \beta)]}.$$

*Proof.* As in the proof of Proposition 1, it is shown that  $F(0) < 0$ ,  $F'(0) > 0$ ,  $F'(1) < 0$ . A condition  $\tau_q > \tau_q^{max}$  is necessary and sufficient for  $F(1) < 0$ .

In the case where all roots are real, these conditions imply that two roots are inside the unit circle and just one root is outside the unit circle. Then, there is equilibrium indeterminacy. In the case where two roots are complex, there is equilibrium indeterminacy or no stationary equilibrium.

In the rest of this proof, we show that equilibrium indeterminacy arises if

$$\tau_q < \frac{2(1 + \eta\beta)[\chi + \lambda(2 - \tau)] + \chi(4\beta + \eta)}{(2 - \eta)[\chi + 2A(1 - \beta)]}.$$

This condition is necessary and sufficient for  $F(2) > 0$ . Since  $F(1) < 0$ , there is a real root  $r$  in  $(1, 2)$ .

Suppose that  $a \pm bi$  are roots and a norm of  $M \equiv \sqrt{a^2 + b^2}$ . As in the proof of Proposition 1, we have

$$\begin{aligned} F(x) &= (x - a + bi)(x - a - bi)(x - r) \\ &= x^3 - (2a + r)x^2 + (M^2 + 2ar)x - M^2r. \end{aligned}$$

For equilibrium indeterminacy, we show that  $M^2 < 1$ . To show this, we show that there is a contradiction if  $M^2 > 1$ . We obtain

$$\begin{aligned} F'(1) &= 3 - 2(2a + r) + M^2 + 2ar \\ &> 2(2 - r)(1 - a). \end{aligned}$$

If  $a < 1$ , we have  $F'(1) > 0$  and it is a contradiction since we know  $F'(1) < 0$ . Then,  $M^2 < 1$ .

Finally, we show that  $a < 1$  as follows.  $F(x)$  reached a local maximum at

$$x = x^{Lmax} \equiv \frac{2a + r - \sqrt{r^2 - 2ar + a^2 - 3b^2}}{3}$$

and  $x^{Lmax} < 1$  by  $F'(0) > 0$  and  $F'(1) < 0$ . We have  $x^{Lmax} > a$  since

$$\sqrt{r^2 - 2ar + a^2 - 3b^2} < r - a.$$

Then, it is shown that  $a < 1$ .

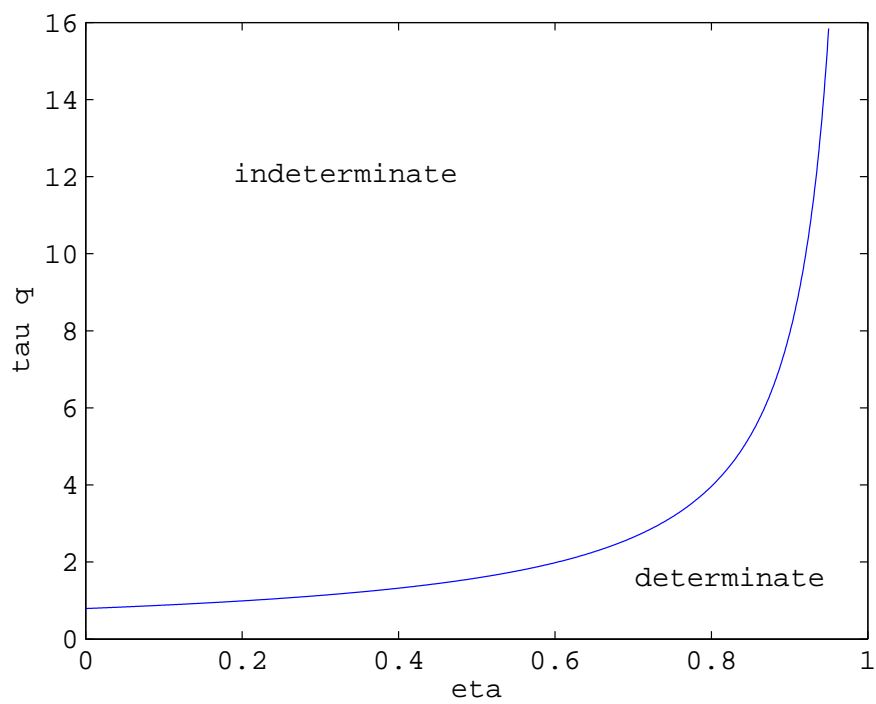
□

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Figure 1: Determinacy and indeterminacy regions



*Notes:* The vertical axis means the central bank's stance to the share price  $\tau_q$ . The horizontal axis means the fraction of firms that follow price indexation  $\eta$ .