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Abstract

This paper studies party discipline in a congress within a political agency framework with retrospective voting. Party discipline serves as an incentive device to induce office-motivated congress members to perform in line with the party leadership’s objective of controlling both the executive and the legislative branches of government. I show first that the same party is more likely to control both branches of government (i.e., unified government) the stronger the party discipline in the congress is. Second, the leader of the governing party imposes more party discipline under unified government than does the opposition leader under divided government. Moreover, the incumbents’ aggregate performance increases with party discipline, so a representative voter becomes better off.

JEL classification: D72.

Keywords: Party discipline; Political agency; Retrospective voting; Office-motivated politicians.

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1. Introduction

In modern democracies, party discipline is defined as the ability of the leadership of a party to control its members in the legislature. Party discipline usually refers to the ability of a party leader to get her party’s congress members to support the party line rather than to follow the special interests of their home districts. Party discipline has been a topic of frequent study in the empirical literature (see Heller and Mershon 2008, Krehbiel 2000, McCarty et al. 2001, Rohde 1991, and Snyder and Groseclose 2000, among many others).

A number of authors have produced formal models of party discipline. For example, some researchers have elaborated on informational arguments, pointing out that strong party discipline informs voters about the future policy of a candidate who, once elected, cannot deviate from the party’s official platform (Ashworth and Bueno de Mesquita 2004, Castanheira and Crutzen 2010, Cox and McCubbins 1993, Snyder and Ting 2002). In a similar vein, Grossman and Helpman (2008) defined party discipline as a party’s ability to induce ex-post adherence to a preannounced position. In other words, the level of party discipline determines the “extent of commitment to party platforms.” In some other studies, party discipline has been modeled as the ability of the party leadership to control its members in the legislature such that they vote in line with the party’s ideological position (Colomer 2005, Eguia 2011, Iaryczower 2008, McGillivray 1997, Patty 2008, Volden and Bergman 2006). In these models, the party leaders’ objective is to discipline party members who might have different ideological preferences. Diermeier and Feddersen (1998a, 1998b) provided an institutional explanation for cohesive voting of legislators in parliamentary systems. They showed that the vote-of-confidence procedure common in parliamentary democracies creates an incentive for cohesion in voting.

This paper complements the aforementioned literature by analyzing party discipline under the assumption of office-motivated party leaders who want their party to control both the executive and the legislative branches of government. In the framework used here, party leaders impose party discipline on their congress members in order to motivate the latter to perform in line with the party’s objective of controlling the two branches of government rather than to only seek reelection in their home districts. The paper therefore emphasizes the role of party discipline as an incentive device that motivates congress members to perform in the interests of their party leadership.

In this paper, I build a political agency model of interaction between office-motivated politicians (an executive, an opposition leader, and a congress member) and their constituency
in the presence of a moral hazard problem. A representative voter cares about the politicians’ performances, which are observable but not contractible. The executive, as the leader of the governing party, cares about her party controlling the executive and legislative branches of government. This means that the executive seeks to be reelected herself and also wants her partisan ally to win in the congressional election. The leader of the opposition also wants her party to win the executive and legislative elections, and so wants the incumbent executive to be thrown out of office (which would lead the opposition to win the presidential election) and wants the congress to be controlled by her partisan ally.

Consider first a benchmark case with no party discipline in the congress. The incumbent congress member is office-motivated and cares only about his own reelection chances. The representative voter applies optimal retrospective voting rules to reward the incumbents. The executive is reelected if her performance exceeds an optimal threshold. The reelection of the congress member is conditioned on the congress member’s own performance and also on the executive’s performance in order to incentivize the executive, who cares about the reelection prospects of the congress member. Moreover, the voter will use different retrospective voting rules in the cases of unified and divided government. Under unified government, the executive wants her party’s congress member to be reelected in the coming election. The incumbents’ preferences are therefore positively aligned, which implies positively correlated optimal retrospective voting rules. In the case of divided government, the executive prefers the congress member to be thrown out of office so that the executive’s partisan ally wins the congressional election. The incumbents’ preferences are thus negatively aligned, and so are the optimal retrospective voting rules.

Assume now that the party leaders (i.e., the executive and the opposition leader) can impose costly party discipline on their party’s congress member. The executive will impose party discipline on her party’s congress member in the case of unified government, whereas the opposition leader will do so in the case of divided government (when the congress is controlled by the opposition leader’s partisan ally). Party discipline means that the congress member supports the goals of the party leadership. In the framework used here, this implies that the congress member will share the party leader’s objective of controlling the two branches of government. Strong party discipline thus leads to further alignment of the incumbents’ preferences. In the case of unified government, a disciplined congress member wants the executive to be reelected in the coming elections, and so the incumbents’ preferences become even more positively aligned. Under divided government, in contrast, the congress member

\footnote{For the sake of tractability, it is assumed that there is a single national district and that the congress therefore consists of one congress member.}
prefers the executive to be thrown out of office, which makes the incumbents’ preferences even more negatively aligned. The voter thus adopts more correlated (positively under unified government or negatively under divided government) retrospective voting rules to motivate the congress member to perform better.

The party leaders choose to impose party discipline in order to “delegate”, in some sense, the party’s goal of controlling the two branches of government to their party’s congress members. Strong party discipline means more aligned incumbents’ preferences. In turn, the latter implies more correlated retrospective voting rules, which motivate the congress member to exert higher effort. The executive can therefore can “free-ride” on the congress member’s performance, which leads to lower effort by the executive. Nonetheless, stronger party discipline increases the incumbents’ aggregate performance, making the voter better off. Moreover, if party discipline were cheap to impose, the party leaders would choose to discipline their party’s congress member as much as possible, since the probability of controlling the two branches of government increases with party discipline. Under unified government, the reelection outcomes of the incumbents are positively correlated. Stronger party discipline increases this correlation further. Therefore, the incumbents are more likely to be reelected together. In the case of divided government, the reelection outcomes are negatively correlated, and they become even more negatively correlated the stronger the party discipline is. It is more likely then that the congress member is reelected while the executive is thrown out of office.

My results indicate that the leader of the governing party, i.e., the executive, imposes more party discipline under unified government than the opposition leader does under divided government. The reason is that stronger party discipline allows the executive to free-ride on the congress member’s performance and reduces the executive’s effort (and the associated cost). Thus, the executive takes this effect into account when choosing the level of party discipline in the congress, which results in stronger party discipline under unified government than under divided government.

The retrospective-voting approach in the political agency framework used here started with the seminal work of Barro (1973), to be followed by the work of Ferejohn (1986), Persson et al. (1997), Austen-Smith and Banks (1989), Banks and Sundaram (1993, 1996), and others. In addition to having a sound theoretical framework, this approach has received considerable empirical support (see, for example, Peltzman 1992 and Besley and Case 1995a, 1995b, 2003). Besley (2006) “emphasizes the empirical potential of these models in explaining real world
policy choices.” In a recent article in the New York Times, Glaeser pointed out that the “president . . . is both our leader and our employee. We (the voters) chose him, our taxes pay his salary, and we can fire him in four years.” The political agency approach may therefore be appropriate for modeling political interactions between politicians and voters. Even so, elected politicians can only be offered implicit incentive schemes; it is difficult to reward public policies with explicit contracts.

The rest of the paper is organized as follows. Section 2 outlines a model. Section 3 presents the formal analysis. Finally, Section 4 concludes the paper.

2. Model

Consider a single national district where an executive $E$ and a congress member $C$ implement policies on behalf of a representative voter. There is no ideological heterogeneity in the politicians’ and voters’ policy preferences. Each politician $i \in \{E, C\}$ performs a policy task determined by her unobservable effort $e_i \geq 0$ and her random ability $a_i \sim N(0, \sigma^2)$. The cost of the effort of politician $i$ is given by $e_i^2$. Ability and effort are additive. Politician $i$’s performance

$$p_i = e_i + a_i$$

(by not its composition in terms of effort and ability) is observed by the representative voter.\footnote{Besley (2006), p. 3.} \footnote{Edward L. Glaeser “Lower (and more realistic) presidential expectations," January 20, 2009. Available online at http://economix.blogs.nytimes.com/2009/01/20/lower-and-more-realistic-presidential-expectations/ (accessed December 1, 2010).} \footnote{One district is assumed in order to keep the analysis clean and the results tractable. The analysis of a model where there are several districts and where a representative voter from each district elects a congress member to the national legislature is left for future research.} \footnote{Since there is no heterogeneity in policy preferences, I can, without loss of generality, assume a representative voter.} \footnote{Assuming a nonzero average ability of the politicians leads to more complicated algebra but similar results. An analysis of this case is available upon request.} \footnote{A simple cost specification $e_i^2$ allows a closed-form solution in this framework. The results would be qualitatively the same for a strictly convex and increasing cost function.} \footnote{An extended version of the model is available upon request, where the cost of effort for an executive and a congress member under unified government is different from that under divided government (e.g., because of synergy). The results of this extended model are qualitatively the same.} \footnote{The representative voter is assumed to observe the politicians’ performances separately. One can think, for example, of pork barrel spending as a measure of the congress member’s performance, and of the nation’s overall performance as a measure of the executive’s performance.}
I assume that there are two political parties and that each politician is affiliated with one of these parties. The state variable $\Gamma \in \{U, D\}$ is introduced, where $\Gamma = U$ corresponds to the case of unified government (i.e., the same party controls the two branches of government) and $\Gamma = D$ corresponds to the case of divided government (i.e., one party controls the presidential (executive) office and the other party controls the congress).

The presidential and congressional elections are held simultaneously. In each election, the candidates (an incumbent and a challenger) are affiliated with opposite parties. The incumbents and the challengers are identical in all respects except party label.

**Politicians**

Consider first a contest for the presidential office. It is assumed that the incumbent executive is the leader of one political party and the challenger is the leader of the other political party. As party leaders, they care about their party’s chances of controlling the two branches of government.$^{11}$ Their goal is to maximize the probability of their party winning in the presidential and congressional elections. So the net objective function of the incumbent executive $E$, denoted by $\Psi^E$, in state $\Gamma$, is given by

$$
\Psi^U_E (e_E, e_C) = Pr (E \text{ is reelected and } C \text{ is reelected}) - \frac{e_E^2}{2},
$$

$$
\Psi^D_E (e_E, e_C) = Pr (E \text{ is reelected and } C \text{ is not reelected}) - \frac{e_E^2}{2}.
$$

First, in each of the two states, the executive $E$ wants to be reelected. Second, in the case of unified government, $\Gamma = U$, the executive $E$ prefers the congress member $C$ to be reelected too. In the case of divided government, $\Gamma = D$, $E$ wants the incumbent congress member to be thrown out of office. This implies that the challenger (from $E$’s party) will be elected in the congressional election.

The leader of the opposition, denoted by $O$, is a challenger for the presidential office and has the same objective (in reverse). $O$ wants both incumbents to be dismissed in the case of unified government, $\Gamma = U$, which would imply $O$’s own appointment to the executive office and her partisan ally’s election to the congress. In the case of divided government,

$^{11}$Several authors have made similar assumptions about politicians’ partisan preferences. Fréchette et al. (2008) assumed that the party leader’s objective is to maximize the reelection chances of the party’s incumbent politicians. In turn, Brollo and Nannicini (2010) assumed that an executive wants to maximize “the political capital represented by aligned mayors” by increasing the likelihood that a municipality is run by a mayor aligned with the central government. Zudenkova (2011) considered politicians with aligned preferences who care about their party’s overall representation in the executive and legislative branches of government, and not just their own reelection prospects.
\(\Gamma = D\), the opposition leader wants the congress member \(C\) to be reelected and the executive \(E\) to be thrown out of office, which would lead to \(O\)’s party controlling the two branches of government. \(O\)’s objective function, \(\Psi^O\), is

\[
\begin{align*}
\Psi^O_U (e_E, e_C) &= Pr (E \text{ is not reelected and } C \text{ is not reelected}), \\
\Psi^O_D (e_E, e_C) &= Pr (E \text{ is not reelected and } C \text{ is reelected}).
\end{align*}
\]

The party leaders can impose a certain type of control, known as party discipline, on their congress members. In modern democracies, party discipline usually refers to the ability of a party leader to get her party’s congress members to support the party’s goals rather than to follow the special interests of their home districts. In the absence of party discipline, a congress member just wants to satisfy the wishes of a representative voter in his home district in order to be reelected in the coming elections. In the context of the present model, no party discipline would mean that congress member \(C\)’s objective would be simply to maximize his reelection probability \(Pr_C (e_C)\). Assume now that the party leaders can ensure party discipline by imposing the party’s objective (i.e., their own objective) on their party’s congress members. In particular, a party leader \(j \in \{E, O\}\) can impose a level of party discipline \(\vartheta_j \in [0, 1]\) to get her party’s congress member to support the party’s official goal of controlling the two branches of government. I assume that this level of party discipline determines the strength with which congress member \(C\) shares the preferences of his party leadership. Party discipline is costly to enforce; the cost is \(k \vartheta_j^2\), with \(k > 0\), and can be interpreted as the cost of party whips, whose primary task is to ensure party discipline in the legislature (usually by offering rewards to party members or threatening them with punishment). The opposition leader \(O\) cannot impose party discipline in the case of unified government, since the congress member \(C\) belongs to the governing party. \(O\) can, however, ensure party discipline in the case of divided government by making her party’s congress member \(C\) support the party’s objective of holding both branches of government. In turn, the executive \(E\) can control her party’s congress member only in the case of unified government. Under divided government, \(E\) has no party member in the congress. Thus, it follows that the congress member’s net objective function, denoted by \(\Psi^\Gamma_C\), becomes

\[
\begin{align*}
\Psi^U_C (e_E, e_C) &= \vartheta_E Pr (E \text{ is reelected and } C \text{ is reelected}) + Pr_C (e_C) - \frac{e_C^2}{2}, \\
\Psi^D_C (e_E, e_C) &= \vartheta_O Pr (E \text{ is not reelected and } C \text{ is reelected}) + Pr_C (e_C) - \frac{e_C^2}{2}.
\end{align*}
\]

The reasonable assumption here is that even with party discipline, the congress member still values his own reelection more than his party leadership’s goal of controlling both branches.
of government. That is why the congress member’s own reelection probability is included in the objective function.

Note that the politicians’ incentives are aligned, as the politicians share their party’s common goal of controlling both branches of government. As a party leader, the executive (in addition to her own reelection) cares also about her party’s chances of winning in the congressional election. Moreover, party discipline can be enforced in the congress such that the congress member performs not only in his own self-interest but also in the interests of his party leadership. In the case of unified government, the executive can impose party discipline on her party’s congress member. Then the incumbents’ preferences are positively aligned, as each incumbent (in addition to her own reelection) also wants her counterpart to be reelected. In the case of divided government, the opposition leader can enforce the party’s objective on her party’s congress member. The incumbents’ preferences are then negatively aligned, as each incumbent wants to be reelected herself and wants her counterpart to be thrown out of office (which implies the reelection of her partisan ally).

Representative Voter

The voter cares about policy outcomes according to a linear utility function

\[ p_E + p_C. \]

The politicians are held accountable for their performance at the moment of election. I assume that the voter uses retrospective voting to reappoint the incumbents, i.e., bases the reelection decision on the politicians’ performances \( p_E \) and \( p_C \) to incentivize their efforts.

The incumbents care not only about their own reelection prospects but also about their parties’ chances of controlling the two branches of government. This gives the voter an additional tool to increase the politicians’ accountability. The voter will reward politician \( i \) for that politician’s own performance \( p_i \) in order to give her an incentive to perform well. Moreover, since executive \( E \) cares about the reelection chances of congress member \( C \), the voter will condition the reelection of the latter on the executive’s performance \( p_E \). This will provide an extra incentive for the executive, who wants her partisan ally to win in the congressional election. In the same vein, since congress member \( C \) might also share the party leader’s objective of controlling the two branches of government, the executive’s reelection will be conditioned on the congress member’s performance \( p_C \), to incentivize congress member \( C \). Therefore, owing to the alignment of the incumbents’ preferences, the optimal retrospective voting rule for incumbent \( i \)’s reelection might depend on both incumbents’ performances, \( p_E \) and \( p_C \). It is assumed here that the voter applies linear retrospective rules determined by
scalars $\lambda_E$ (for $E$’s reelection) and $\lambda_C$ (for $C$’s reelection), $\lambda_E, \lambda_C \in \mathbb{R}$. In particular, the voter conditions the reelection of executive $E$ on the joint performance of the two incumbents given by a linear combination $p_E + \lambda_{EPC}$. By analogy, the reelection of congress member $C$ depends on $p_C + \lambda_{CPE}$. To prevent bizarre outcomes (such as the possibility that incumbents with poor performances might be reelected while ones with better performances are not), the restriction $\lambda_E\lambda_C < 1$ is imposed.

The voter knows that the only alternative to reappointing incumbents is to elect challengers of average ability who will exert equilibrium efforts $e'_E$ and $e'_C$ (where $e'_i$ denotes the voter’s perception of $e_i$). Thus, the voter compares the incumbents’ performances with their challengers’ expected performances and votes accordingly. The executive $E$ will be reelected if $p_E + \lambda_{EPC} \geq e'_E + \lambda_{E}e'_C$. In turn, congress member $C$ will be reappointed if $p_C + \lambda_{CPE} \geq e'_C + \lambda_{C}e'_E$.

Intuition suggests that the optimal retrospective voting rules will differ between the two states. In the case of unified government, the positively aligned incumbents’ preferences imply that the executive has an extra incentive to perform well if the congress member’s reelection chances increase with the executive’s performance. By analogy, if the congress member’s success raises the executive’s reelection prospects, then the congress member is more eager to perform well. However, in the case of divided government, the negatively aligned incumbents’ preferences lead to different optimal reelection rules. In this case, each incumbent will perform better if her success decreases the reelection chances of her incumbent counterpart, affiliated with the rival political party.

**Timing**

This is a sequential game between the politicians and the representative voter. The timing of events is as follows. First, the incumbents are drawn randomly, and a state $\Gamma \in \{U, D\}$ is realized. Second, in the case of unified government, $\Gamma = U$, the executive $E$ imposes a level of party discipline $\theta_E$ on the congress. In the case of divided government, $\Gamma = D$, it is the leader of the opposition $O$ who imposes a level of party discipline $\theta_O$ on the congress. Next, the voter commits to retrospective voting rules determined by scalars $\lambda_E$ (for $E$’s reelection) and $\lambda_C$ (for $C$’s reelection). The incumbents then exert efforts $e_E$ and $e_C$. Finally, the politicians’ abilities $a_E$ and $a_C$ are realized, and policy outcomes $p_E$ and $p_C$ are observed. The presidential and congressional elections are held simultaneously, and the voter applies the chosen retrospective voting rules to reward (reelect) or punish (dismiss) the incumbents.

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12 Linear retrospective rules are considered for reasons of tractability, as they allow a closed-form solution.
The game is analyzed backwards to solve for a subgame perfect equilibrium. The incumbents’ efforts $e_E^\Gamma$ and $e_C^\Gamma$ in each state $\Gamma \in \{U, D\}$ under linear rules $\lambda_E$ and $\lambda_C$ are found first. Next, I solve for the scalars $\lambda_E^\Gamma$ and $\lambda_C^\Gamma$ that determine the voter’s retrospective voting rules in each state $\Gamma$. Finally, I examine the executive’s choice of $\vartheta_E$ (if $\Gamma = U$) and the opposition leader’s choice of $\vartheta_O$ (if $\Gamma = D$) for imposing party discipline on their party’s congress members.

**Intuition**

Intuitively, the party leaders’ objectives of controlling the two branches of government imply the alignment of the incumbents’ preferences. The voter conditions the reelection of the congress member on the performances of the two incumbents in order to provide the correct incentives to the executive. Imposing party discipline on the congress member leads to even further alignment of the incumbents’ preferences, which is used by the voter to increase accountability. In the presence of party discipline, the voter will condition the reelection of each incumbent on the performances of both of them. In other words, party discipline in the congress serves as an extra incentive device for the congress member. The party leaders will choose to enforce a certain level of party discipline that allows them to “delegate” to their party’s congress member the implementation of the task of controlling the two branches of government.

**3. Analysis**

Consider the incumbents’ decisions about the efforts $e_E^\Gamma$ and $e_C^\Gamma$ under linear rules $\lambda_E$ and $\lambda_C$ when the executive has chosen $\vartheta_E$ in the case of unified government and the opposition leader has chosen $\vartheta_O$ in the case of divided government. The executive’s net objective function is given by

$$
\Psi_E(\vartheta_E, \vartheta_C) = Pr \left( \left\{ p_E + \lambda_E p_C \geq e'_E + \lambda_E e'_C \right\} \cap \left\{ p_C + \lambda_C p_E \geq e'_C + \lambda_C e'_E \right\} \right) - \frac{\vartheta_E^2}{2} - \frac{\vartheta_O^2}{2},
$$

$$
\Psi_C(\vartheta_E, \vartheta_C) = Pr \left( \left\{ p_E + \lambda_E p_C \geq e'_E + \lambda_E e'_C \right\} \cap \left\{ p_C + \lambda_C p_E < e'_C + \lambda_C e'_E \right\} \right) - \frac{\vartheta_C^2}{2},
$$

where $p_E = e_E + a_E$ and $p_C = e_C + a_C$. The congress member’s net objective function depends on the level of party discipline and is given by

$$
\Psi_C(\vartheta_E, \vartheta_C) = \vartheta_E Pr \left( \left\{ p_E + \lambda_E p_C \geq e'_E + \lambda_E e'_C \right\} \cap \left\{ p_C + \lambda_C p_E \geq e'_C + \lambda_C e'_E \right\} \right) + Pr \left( p_C + \lambda_C p_E \geq e'_C + \lambda_C e'_E \right) - \frac{\vartheta_C^2}{2},
$$

10
\[
\Psi^D_C(e_E, e_C) = \vartheta_O \Pr \left( \{ p_E + \lambda_E p_C < e'_E + \lambda_E e'_C \} \cap \{ p_C + \lambda_C p_E \geq e'_C + \lambda_C e'_E \} \right) \\
+ \Pr \left( p_C + \lambda_C p_E \geq e'_C + \lambda_C e'_E \right) - \frac{e'_C}{2}.
\]

The incumbents make efforts \( e_E \) and \( e_C \) before knowing the realizations of their abilities \( a_E \) and \( a_C \), and take the voter’s expectations \( e'_E \) and \( e'_C \) as given. The following proposition establishes results for the incumbents’ efforts \( e'_E \) and \( e'_C \) under linear rules \( \lambda_E \) and \( \lambda_C \). (A proof can be found in the Appendix.)

**Proposition 1.** Under linear retrospective voting rules \( \lambda_E \) and \( \lambda_C \), \( \lambda_E \lambda_C < 1 \), the incumbents exert efforts \( e'_E \) and \( e'_C \) equal to

\[
e'_E = \frac{1}{2 \sqrt{2\pi} \sigma} \left( \frac{1}{\sqrt{1 + \lambda_E^2}} + \frac{\lambda_C}{\sqrt{1 + \lambda_C^2}} \right),
\]

\[
e'_C = \frac{1}{2 \sqrt{2\pi} \sigma} \left( \frac{1}{\sqrt{1 + \lambda_C^2}} + \frac{\lambda_E}{\sqrt{1 + \lambda_E^2}} \right) + \frac{1}{\sqrt{2\pi} \sigma \sqrt{1 + \lambda_C^2}}
\]

in the case of unified government, \( \Gamma = U \), and

\[
e'_E = \frac{1}{2 \sqrt{2\pi} \sigma} \left( \frac{1}{\sqrt{1 + \lambda_E^2}} - \frac{\lambda_C}{\sqrt{1 + \lambda_C^2}} \right),
\]

\[
e'_C = \frac{1}{2 \sqrt{2\pi} \sigma} \left( \frac{1}{\sqrt{1 + \lambda_C^2}} - \frac{\lambda_E}{\sqrt{1 + \lambda_E^2}} \right) + \frac{1}{\sqrt{2\pi} \sigma \sqrt{1 + \lambda_C^2}}
\]

in the case of divided government, \( \Gamma = D \).

I turn now to the voter’s choice of linear retrospective voting rules determined by the scalars \( \lambda_E \) and \( \lambda_C \). Maximizing \( E (p_E + p_C) = e'_E + e'_C \) with respect to \( \lambda_E \) and \( \lambda_C \) yields an equilibrium. The results are summarized in the following proposition (the proof is straightforward).

**Proposition 2.** The optimal linear retrospective voting rules that the voter uses for the incumbents’ reelection are determined by scalars \( \lambda_E^U \) and \( \lambda_C^U \) such that

\[
\lambda^U_E = \vartheta_E \text{ and } \lambda^U_C = \frac{1}{2 + \vartheta_E}
\]

in the case of unified government, \( \Gamma = U \), and

\[
\lambda^D_E = -\vartheta_O \text{ and } \lambda^D_C = -\frac{1}{2 + \vartheta_O}
\]
in the case of divided government, $\Gamma = D$. Under these optimal rules, the politicians’ efforts are equal to

\[
    e^U_E = \frac{1}{2\sqrt{2\pi}\sigma} \left( \frac{1}{\sqrt{1 + \vartheta_E^2}} + \frac{1}{\sqrt{5 + 4\vartheta_E + \vartheta_E^2}} \right),
\]

\[
    e^U_C = \frac{1}{2\sqrt{2\pi}\sigma} \left( \frac{\vartheta_E^2}{1 + \vartheta_E^2} + \frac{(2 + \vartheta_E)^2}{\sqrt{5 + 4\vartheta_E + \vartheta_E^2}} \right),
\]

in the case of unified government, $\Gamma = U$, and

\[
    e^D_E = \frac{1}{2\sqrt{2\pi}\sigma} \left( \frac{1}{\sqrt{1 + \vartheta_O^2}} + \frac{1}{\sqrt{5 + 4\vartheta_O + \vartheta_O^2}} \right),
\]

\[
    e^D_C = \frac{1}{2\sqrt{2\pi}\sigma} \left( \frac{\vartheta_O^2}{1 + \vartheta_O^2} + \frac{(2 + \vartheta_O)^2}{\sqrt{5 + 4\vartheta_O + \vartheta_O^2}} \right)
\]

in the case of divided government, $\Gamma = D$.

As expected, in the case of unified government, the reelection of one incumbent is positively correlated with the performance of the other incumbent. Thus, the success of one incumbent promotes the reelection of the other incumbent. So the positively aligned incumbents’ preferences imply positively correlated reelection outcomes under unified government.

In the case of divided government, the reelection of one incumbent is negatively correlated with the performance of the other incumbent. Therefore, the success of one incumbent hinders the reelection of the other incumbent. Thus, under divided government, the negatively aligned incumbents’ preferences lead to negatively correlated reelection outcomes. In fact, two-sided coattail effects arise. On the one hand, the executive’s performance affects the congress member’s reelection, which implies a presidential coattail effect. On the other hand, the executive’s reelection depends on the congress member’s performance, which results in a reverse coattail effect.\(^\text{14}\)

Moreover, in the absence of party discipline, $\vartheta_j = 0$, $j \in \{E, O\}$, the voter uses a joint retrospective voting rule only to reward the congress member $C$. The optimal rule for reappointing executive $E$ is a simple cutoff rule such that $E$ is reelected if her performance $p_E$ exceeds the equilibrium level of effort $e^*_E$ (where $e^*_E$ denotes the voter’s perception of $e_E$).

Intuitively, in the absence of party discipline, the congress member $C$ cares only about his

\(^{13}\)Note that the equilibrium values of $\lambda^U_E$ and $\lambda^U_C$ satisfy the condition $\lambda^U_E \lambda^U_C < 1$.

\(^{14}\)See Zudenkova (2011) for a formal model of coattail voting.
own reelection; for that reason, there is no way to incentivize the congress member by conditioning the executive’s reelection on C’s performance. If party discipline is enforced in the congress, \( \vartheta_j \neq 0 \), the congress member will share his party leader’s goal of controlling both branches of government. So the congress member will care about the executive’s reelection prospects, and the voter can incentivize the congress member by conditioning \( E \)’s reelection on the congress member’s performance. There are thus two forces at work to motivate the congress member to perform well. First, the party leader imposes party discipline to align the congress member’s preferences with those of the party leadership. Second, given this alignment of preferences, the voter conditions the executive’s reelection on the congress member’s performance. These two forces jointly serve as an incentive device to encourage the congress member to exert higher effort.

The stronger the party discipline (represented by \( \vartheta_j \)) in the congress, the more correlated (positively if \( \Gamma = U \) and negatively if \( \Gamma = D \)) \( E \)’s reelection is with \( C \)’s performance. Indeed, the more the congress member shares the party leadership’s objective of controlling the two branches of government, the more incentive the congress member has to perform better. The voter provides the congress member with an optimal incentive scheme by making the executive’s reelection more dependent (positively if \( \Gamma = U \) and negatively if \( \Gamma = D \)) on the congress member’s performance. However, the stronger the party discipline in the congress, the less correlated (positively if \( \Gamma = U \) and negatively if \( \Gamma = D \)) is \( C \)’s reelection with \( E \)’s performance. The reason is that stronger party discipline implies more incentive for the congress member but at the same time less incentive for the executive, who can now partly “delegate” the goal of controlling the two branches of government to the congress member and “free-ride” on \( C \)’s effort. The voter thus adopts an optimal voting rule for \( C \)’s reelection that is less dependent on \( E \)’s performance the stronger the party discipline is.

Consider now the equilibrium levels of the efforts \( e^\Gamma_E \) and \( e^\Gamma_C \), which have the same functional form for the two states. The only difference is that under unified government, \( \Gamma = U \), they depend on the level of party discipline \( \vartheta_E \) that the executive enforces in the congress. Under divided government, \( \Gamma = D \), they depend on the level of party discipline \( \vartheta_O \) that the opposition leader enforces in the congress. The reason for this is that the politicians’ preferences are symmetric between the two states, which implies symmetry of the optimal retrospective voting rules. Note, moreover, that the executive’s effort \( e^\Gamma_E \) decreases and the congress member’s effort \( e^\Gamma_C \) increases with the level \( \vartheta_j \) of party discipline in the congress: \( \frac{de^\Gamma_E}{d\vartheta_j} < 0 \) and \( \frac{de^\Gamma_C}{d\vartheta_j} > 0 \). As mentioned above, stronger party discipline leads to extra incentive for the congress member and less incentive for the executive. Under the optimal retrospective voting rules, the congress member will be incentivized to exert higher effort to implement
the party leadership’s goal of controlling both branches of government. The executive, meanwhile, will exert less effort, as she can free-ride on the performance of the disciplined congress member. It is important to stress that the sum of the incumbents’ efforts $e^E + e^C$ increases with the level $\vartheta_j$ of party discipline in the congress: $\frac{d(e^E + e^C)}{d\vartheta_j} > 0$. So the stronger the party discipline, the better off the representative voter is. Note, moreover, that the equilibrium levels of the efforts $e^E$ and $e^C$ decrease with variance $\sigma^2$ of the politicians’ ability (since more randomness in the incumbents’ performances makes the reelection probabilities less sensitive to effort and thus reduces the incumbents’ incentives).

### Party Discipline

Consider now the problem of a party leader $j \in \{E, O\}$ who has to decide on the level of party discipline $\vartheta_j$ to impose on her party’s congress member. The leader of the governing party, i.e., the executive $E$, can control the legislature only in the case of unified government, since under divided government the congress member is affiliated with the opposite party. In contrast, the leader of the opposition $O$ can impose party discipline only under divided government. The party leaders’ net objective functions can be found by substituting $e^E$, $e^C$, $\lambda^E$, and $\lambda^C$ into $\Psi^U_E(e^E, e^C)$ and $\Psi^D_O(e^E, e^C)$ (a detailed derivation can be found in the Appendix). This yields

$$\Psi^U_E(\vartheta_E) = \frac{1}{4} + \frac{1}{2\pi} \arctan \left( \frac{1 + \vartheta_E}{2} \right)^2 - k\vartheta_E^2 - \frac{1}{16\pi\sigma^2} \left( \frac{1}{1 + \vartheta_E^2} + \frac{1}{\sqrt{5 + 4\vartheta_E + \vartheta_E^2}} \right)^2,$$

$$\Psi^D_O(\vartheta_O) = \frac{1}{4} + \frac{1}{2\pi} \arctan \left( \frac{1 + \vartheta_O}{2} \right)^2 - k\vartheta_O^2,$$

where $\arctan(\cdot)$ is the arctangent function. Note that stronger party discipline (higher $\vartheta_j$) increases the probability of the party controlling both branches of government. In the case of unified government, $\Gamma = U$, stronger party discipline implies more positively aligned incumbents’ preferences, so that the voter uses more positively correlated retrospective voting rules. Under these rules, the incumbents are more likely to be reelected together than they are to receive opposite rewards. Thus, the probability that both incumbents will be reelected increases with the level of party discipline $\vartheta_E$ that the executive imposes under unified government. In the case of divided government, $\Gamma = D$, stronger party discipline leads to more negatively aligned incumbents’ preferences. The voter then applies more negatively correlated

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15It is important to stress that variance $\sigma^2$ of the politicians’ ability is required to be quite different from zero in order to guarantee that the politicians’ individual rationality (i.e., participation) constraints are satisfied. In particular, the results hold for values of $\sigma \geq 1$. 

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voting rules. Thus, it is more likely that one incumbent will be dismissed while the other is reelected. So the probability that the congress member is reelected while the executive is thrown out of office increases with the level of party discipline \( \vartheta_O \) that the opposition leader imposes under divided government.

Owing to the symmetry of the party leaders’ preferences, the voter uses linear retrospective voting rules that are symmetric between the states. So the probability of controlling both branches of government has the same functional form for the two states. The only difference is that under unified government, \( \Gamma = U \), this probability depends on the level of party discipline \( \vartheta_E \) that the leader of the governing party (i.e., the executive) enforces in the congress. Under divided government, \( \Gamma = D \), it depends on the level of party discipline \( \vartheta_O \) that the opposition leader enforces on her party’s congress member. However, the net objective functions of the party leaders differ between the states such that \( \Psi_E^U (\vartheta_E) < \Psi_D^O (\vartheta_O) \) for any \( \vartheta_E = \vartheta_O \in [0, 1] \). The reason is that under unified government, the executive takes into account the effect of imposing party discipline on her own performance in office. \( E \)'s choice of the level of party discipline \( \vartheta_E \) modifies the amount of effort \( e_E \) that \( E \) puts into policy implementation, and the cost of this effort, \( \frac{e_E^2}{2} \), which is included in \( E \)'s objective function \( \Psi_E^U \). The following proposition specifies the party leaders’ optimal choices of the levels of party discipline in the congress. (The proof is straightforward.\(^{16}\))

**Proposition 3.** Under unified government, \( \Gamma = U \), the executive imposes a level of party discipline \( \vartheta_E^* \), which is a decreasing function of the cost parameter \( k \), defined implicitly by the first-order condition

\[
\frac{d}{d \vartheta_E} \Psi_E^U (\vartheta_E^*) = 0 \text{ if } k \geq \frac{7 + 2\sqrt{5} + 40\sigma^2}{200\pi\sigma^2}
\]

and

\[ \vartheta_E^* = 1 \text{ if } k < \frac{7 + 2\sqrt{5} + 40\sigma^2}{200\pi\sigma^2}. \]

Under divided government, \( \Gamma = D \), the opposition leader imposes a level of party discipline \( \vartheta_O^* \), which is a decreasing function of the cost parameter \( k \), defined implicitly by the first-order condition

\[
\frac{d}{d \vartheta_O} \Psi_D^O (\vartheta_O^*) = 0 \text{ if } k \geq \frac{1}{5\pi}
\]

and

\[ \vartheta_O^* = 1 \text{ if } k < \frac{1}{5\pi}. \]

Moreover, the executive always enforces stronger party discipline in the congress than the opposition leader does, i.e., \( \vartheta_E^* \geq \vartheta_O^* \) for any cost parameter \( k > 0 \).

\(^{16}\)One can check that the second-order conditions hold.
The reason why the executive $E$ imposes more party discipline than does the opposition leader $O$ is that the amount of effort that $E$ puts into policy implementation, $e_E$, decreases with $\theta_E$, and so does the corresponding effort cost $\frac{c_E^2}{2}$. The executive, therefore, is more eager than the opposition leader to impose party discipline, since that leads to less policy work for the executive. Indeed, stronger party discipline implies a higher amount of effort $e_C$ by the congress member towards policy implementation and allows the executive to “free-ride” on her party’s congress member’s performance.

The level of party discipline that the executive imposes under unified government approaches that of the opposition leader under divided government as the variance $\sigma^2$ of the politicians’ ability increases: $\lim_{\sigma \to \infty} \vartheta^*_E(\sigma^2) = \vartheta^*_O$. A larger variance $\sigma^2$ implies more randomness in the politicians’ performances and makes the reelection probabilities less sensitive to effort. The executive therefore exerts less effort $e_E$, and the executive’s incentive to impose party discipline approaches that for the opposition leader.

It is important to mention that ensuring party discipline in the congress improves not only the gross utility of the party leaders but also the utility of the representative voter. In fact, if party discipline were cheap to impose, there would be no conflict of interest between the voter and the party leaders, who would prefer the highest possible level of party discipline in the congress. Since party discipline is costly to impose, however, the party leaders choose a moderate level of party discipline that maximizes their net utility. But imposing party discipline has an ambiguous effect on the congress member’s utility. For large values of the variance $\sigma^2$ of the politicians’ ability, the congress member’s utility increases with stronger party discipline. A lower variance $\sigma^2$ increases the congress member’s effort such that at some point his net utility begins to change nonmonotonically with the level of party discipline.

**4. Conclusions**

This paper has studied party discipline under the assumption of office-motivated politicians. In a political agency model with moral hazard, party discipline serves as an incentive device to motivate legislators to perform in line with the party leaders’ objective of controlling both the executive and the legislative branches of government.

The party leaders choose to impose party discipline to “delegate,” in some sense, the party line of controlling the two branches of government to their party’s congress member. Stronger party discipline implies more aligned incumbents’ preferences. As a result, the voter adopts more correlated retrospective voting rules, conditioned on the government being unified or divided. In the case of unified government, the reelection outcomes for the incumbents
are positively correlated to incentivize the congress member, who wants the executive to be reelected. Under divided government, the reelection outcomes are negatively correlated to incentivize the congress member, who wants the executive to be thrown out of office and the congress member’s partisan ally to win the presidential election. The congress member therefore performs better, for the sake of his party as well as for himself. So the executive can free-ride on the congress member’s performance, which decreases the executive’s effort. However, the aggregate performance of the incumbents increases with stronger party discipline, so the representative voter becomes better off.

The probability of controlling both branches of government is strictly increasing in the level of party discipline. In the case of unified government, the reelection outcomes are positively correlated, so the executive and the congress member are more likely to be reelected together. Under divided government, the reelection outcomes are negatively correlated, and it is more likely that the congress member is reelected while the executive is thrown out of office. The results show that the leader of the governing party (i.e., the executive) imposes stronger party discipline under unified government than does the opposition leader under divided government. The reason is that the executive takes into account the impact of party discipline on her own policy effort and on the associated cost of this effort, which are strictly decreasing in the level of party discipline.

Even though the model is very stylized, it yields a number of empirically testable predictions. First, according to my results, stronger party discipline in the congress is expected to increase the probability that the same party will control both branches of government, i.e., the probability of unified government. Second, the predictions indicate that the leader of the governing party imposes stronger party discipline on her party’s congress members under unified government than does the opposition leader under divided government. Finally, stronger party discipline is expected to increase the aggregate performance of the incumbents. Testing these predictions implies identifying and measuring party discipline. This might be a hard but feasible task, and has been addressed to some extent by a number of authors (see the references to empirical studies of party discipline in the Introduction).

Appendix

A. Proof of Proposition 1

The executive $E$ is reelected if $p_E + \lambda_E p_C \geq e'_E + \lambda_E e'_C$ or $a_E + \lambda_E a_C \geq e'_E - e_E + \lambda_E (e'_C - e_C)$, where $a_E + \lambda_E a_C \sim N \left(0, \left(1 + \lambda_E^2\right) \sigma^2\right)$. The congress member $C$ is reelected if $p_C + \lambda_C p_E \geq$
\( \varepsilon'_{c} + \lambda_{c} \varepsilon'_{e} \) or \( a_{c} + \lambda_{c} a_{E} \geq \varepsilon'_{c} - \varepsilon_{c} + \lambda_{c} (\varepsilon'_{E} - \varepsilon_{E}) \), where \( a_{c} + \lambda_{c} a_{E} \sim N \left( 0, (1 + \lambda_{c}^{2}) \sigma^{2} \right) \).

The density function of a bivariate normal distribution of random variables \( a_{E} + \lambda_{E} a_{c} \) and \( a_{c} + \lambda_{c} a_{E} \), denoted by \( f_{a_{E} + \lambda_{E} a_{c}, a_{c} + \lambda_{c} a_{E}} (x, y) \), is

\[
f_{a_{E} + \lambda_{E} a_{c}, a_{c} + \lambda_{c} a_{E}} (x, y) = \frac{1}{2\pi \sigma^{2} (1 - \lambda_{E} \lambda_{E})} \exp \left\{ -\frac{(x - \lambda_{E} y)^{2} + (y - \lambda_{c} x)^{2}}{2\sigma^{2} (1 - \lambda_{E} \lambda_{E})} \right\}.
\]

Note that the condition \( \lambda_{E} \lambda_{c} < 1 \) guarantees that this density function is well defined. The executive’s net objective is equal to

\[
\Psi_{E}^{U} (e_{E}, e_{c}) = \int_{e_{E} - \varepsilon_{E} + \lambda_{E} (e'_{c} - e_{c})}^{+\infty} \int_{e'_{c} - \varepsilon_{c} + \lambda_{c} (e'_{E} - e_{E})}^{+\infty} f_{a_{E} + \lambda_{E} a_{c}, a_{c} + \lambda_{c} a_{E}} (x, y) \ dy \ dx - \frac{e_{E}^{2}}{2} - \frac{k \theta_{E}^{2}}{2},
\]

\[
\Psi_{E}^{D} (e_{E}, e_{c}) = \int_{e_{E} - \varepsilon_{E} + \lambda_{E} (e'_{c} - e_{c})}^{+\infty} \int_{-\infty}^{e'_{c} - \varepsilon_{c} + \lambda_{c} (e'_{E} - e_{E})} f_{a_{E} + \lambda_{E} a_{c}, a_{c} + \lambda_{c} a_{E}} (x, y) \ dy \ dx - \frac{e_{E}^{2}}{2}.
\]

One should carefully take the first-order condition with respect to actual effort \( e_{E} \), taking \( e'_{E} \) as given. After imposing the equilibrium requirements \( e'_{E} = e_{E} \) and \( e'_{c} = e_{c} \), one obtains the executive’s equilibrium effort \( e'_{E} \) under linear retrospective voting rules \( \lambda_{E} \) and \( \lambda_{c} \), \( \lambda_{E} \lambda_{c} < 1 \):

\[
e'_{E}^{U} = \frac{1}{2\sqrt{2\pi} \sigma} \left( \frac{1}{\sqrt{1 + \lambda_{E}^{2}}} + \frac{\lambda_{c}}{\sqrt{1 + \lambda_{C}^{2}}} \right),
\]

\[
e'_{E}^{D} = \frac{1}{2\sqrt{2\pi} \sigma} \left( \frac{1}{\sqrt{1 + \lambda_{E}^{2}}} - \frac{\lambda_{c}}{\sqrt{1 + \lambda_{C}^{2}}} \right).
\]

It is straightforward to check that the second-order condition holds.

The congress member’s net objective is equal to

\[
\Psi_{C}^{U} (e_{E}, e_{c}) = \vartheta_{E} \int_{e_{E} - \varepsilon_{E} + \lambda_{E} (e'_{c} - e_{c})}^{+\infty} \int_{e'_{c} - \varepsilon_{c} + \lambda_{c} (e'_{E} - e_{E})}^{+\infty} f_{a_{E} + \lambda_{E} a_{c}, a_{c} + \lambda_{c} a_{E}} (x, y) \ dy \ dx
\]

\[+ (1 - F_{a_{c} + \lambda_{c} a_{E}} (e'_{c} - e_{c} + \lambda_{c} (e'_{E} - e_{E}))) - \frac{e_{E}^{2}}{2}, \]

\[
\Psi_{E}^{D} (e_{E}, e_{c}) = \vartheta_{O} \int_{-\infty}^{e_{E} - \varepsilon_{E} + \lambda_{E} (e'_{c} - e_{c})} \int_{e'_{c} - \varepsilon_{c} + \lambda_{c} (e'_{E} - e_{E})}^{+\infty} f_{a_{E} + \lambda_{E} a_{c}, a_{c} + \lambda_{c} a_{E}} (x, y) \ dy \ dx
\]

\[+ (1 - F_{a_{c} + \lambda_{c} a_{E}} (e'_{c} - e_{c} + \lambda_{c} (e'_{E} - e_{E}))) - \frac{e_{E}^{2}}{2}.
\]
where $F$ denotes the normal distribution function. Take the first-order condition with respect to actual effort $e_C$, taking $e'_C$ as given, and afterwards impose the equilibrium requirements $e'_E = e_E$ and $e'_C = e_C$. This yields the congress member’s equilibrium effort $e'_C$ under linear retrospective voting rules $\lambda_E$ and $\lambda_C$, $\lambda_E \lambda_C < 1$:

$$e'_C = \frac{\vartheta_E}{2\sqrt{2\pi} \sigma} \left( \frac{1}{\sqrt{1 + \lambda_C^2}} + \frac{\lambda_E}{\sqrt{1 + \lambda_E^2}} \right) + \frac{1}{\sqrt{2\pi} \sqrt{1 + \lambda_C^2}},$$

$$e'_C = \frac{\vartheta_O}{2\sqrt{2\pi} \sigma} \left( \frac{1}{\sqrt{1 + \lambda_C^2}} - \frac{\lambda_E}{\sqrt{1 + \lambda_E^2}} \right) + \frac{1}{\sqrt{2\pi} \sqrt{1 + \lambda_C^2}}.$$

The second-order condition for the congress member’s problem holds too.

### B. Derivation of the party leaders’ objective functions $\Psi_E^U (e'_E, e'_C)$ and $\Psi_O^D (e'_E, e'_C)$

Under unified government, $\Gamma = U$, the voter applies the linear retrospective voting rules determined by scalars $\lambda'_E$ and $\lambda'_C$, and the incumbents exert efforts $e'_E$ and $e'_C$. $E$’s objective function is thus equal to

$$\Psi_E^U (e'_E, e'_C) = \int_{e'_E - e'_E + \lambda'_E (e'_C - e'_C)}^{+\infty} \int_{e'_C - e'_C + \lambda'_C (e'_E - e'_E)}^{+\infty} f_{a_E + \lambda'_E a_C, a_C + \lambda'_C a_E} (x, y) \, dy \, dx - \frac{e'_E^2}{2} - \frac{k \vartheta_E^2}{2}.$$

Imposing the equilibrium requirements $e'_E = e'_E$ and $e'_C = e'_C$, and plugging in the equilibrium values of $e'_E$, $e'_C$, $\lambda'_E$ and $\lambda'_C$ yields

$$\Psi_E^U (\vartheta_E) = \int_{0}^{+\infty} \left[ \int_{0}^{+\infty} f_{a_E + \vartheta_E a_C, a_C + \frac{1}{2 + \vartheta_E} a_E} (x, y) \, dy \right] \, dx - \frac{k \vartheta_E^2}{2} =$$

$$\frac{1}{4} + \frac{1}{2\pi} \left( \frac{1 + \vartheta_E}{2} \right)^2 - \frac{k \vartheta_E^2}{2} - \frac{1}{16\pi \sigma^2} \left( \frac{1}{\sqrt{1 + \vartheta_E^2}} + \frac{1}{\sqrt{5 + 4\vartheta_E + \vartheta_E^2}} \right)^2,$$

where $\arctan (\cdot)$ is the arctangent function.

Under divided government, $\Gamma = D$, the voter uses the linear retrospective voting rules determined by scalars $\lambda'_E$ and $\lambda'_C$, and the incumbents exert efforts $e'_E$ and $e'_C$. The objective
function of the opposition leader $O$ is

$$\Psi^D_O(e^D_E, e^D_C) = \int_{-\infty}^{+\infty} \left[ \int_{e^D_C - e^D_E + \lambda^D_E (e^D_E - e^D_C)}^{+\infty} f_{a_E + \lambda^D_E a_C, a_C + \lambda^D_C a_E} (x, y) \, dy \right] dx - k\theta_O^2/2. $$

After imposing the equilibrium requirements $e'_E = e^U_E$ and $e'_C = e^U_C$ and plugging in the equilibrium values of $e^U_E, e^U_C, \lambda^U_E, \lambda^U_C$, $O$'s objective function becomes

$$\Psi^D_O(\theta_O) = \int_{-\infty}^{0} \left[ \int_{0}^{+\infty} f_{a_E - \theta_O a_C, a_C - 1/(2\pi \theta_O) a_E} (x, y) \, dy \right] dx - k\theta_O^2/2 = \frac{1}{4} + \frac{1}{2\pi} \arctan \left( \frac{(1 + \theta_O)^2}{2} \right) - \frac{k\theta_O^2}{2}. $$

References


