South Sudan Referundum: A Macroeconomic Analysis of Post-Secession Scenario

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27 October 2010

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Abstracts

The purpose of this paper to analyse financial stability in small open economy, with dual foreign exchange markets, enduring political uncertainty and facing the likelihood of perminant adverse export shock. The finding in the paper indicate, given capital outflow is maintained at minimal level, there exist stable equilibrium exchange rates, despite the adverse export shock. However, for the foreign exchange market to adjust more quickly towards a new steady state equilibrium the central bank need to build sufficient foreign exchange reserves. If the reserve level remains at low levels the recovery process from the adverse shock will take longer time, as periodic devaluation of the official rate remain the only available tool for the central bank. When expanding fiscal deficit and declining official reserves force the government adopting a floating exchange rate system, our model predict depreciation of foreign exchange rate is identical to domestic money growth.

Keywords: parallel rate, official rate, Stability, Steady-state.
JEL: C10, C50, G10

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1. Introduction

The Comprhensive Peace Agreement (CPA) signed in 2005 between Southern rebel group and the government of Sudan ended a 21-year old civil war, by granting Southern Sudan semi-autonomy, leading to a referendum in early 2011 to choose full independence or remain within united Sudan. Under the CPA, during the transitional period, before the referendum, the two sides divide equally the revenue from oil produced in the South, which constitutes currently about 75 percent of foreign currency source for Sudan government in the North. Most analyst agree on likely vote will be for seperation, rather than for a unity. If that happens then the financial system in the remaining North-Sudan expected to undergo substantial changes, as the government has not taken so far any serious measures accommodating the big shock. What may make the matter even more worse for the North is that, currently Sudan is under US sanctions imposed since 1997, it targets 31 government owned companies and limits their access to US financial system and bars their use of US dollar in international financial transactions. Due to big drop in the official foreign currency reserves, caused mainly by the crude oil price fall in 2009, the government imposed in recent months quantitative restrictions on selling foreign currencies to individuals and business entities and requires heavy documentations to legitimize transfer of foreign currencies. Currently oil revenue accounts about 75 percent of

foreign currency revenue for the central government, and the rest comes from export of primary agriculture products and foreign investment flows. Since the beginning of the current year, speculation has been rising by investors and currency traders that the government reserve level is deteriorating, adding more presure on foreign currency demand in the parallel market. To curb speculative effects on foreign exchange markets, the highest financial authority (the Central bank) decided to tighten control on exchange bureaux by enforcing additional restrictions on foreign exchange sales and cracking down on black marketeers for foreign currencies. As a result, the salient feature of Sudan foreign exchange market, at this point of time is dual foreign exchange markets, operating side by side, an official exchange rate, which is manged and determined by the central bank, and freely floating parallel market, which is sensitive to speculations and rumors about the economic and the political situation in the country[2].

Since economic stability at the post-secession era depends on political stability and resolving contentious issues related to border identification, we exclude the senario of war conflict between the two sides even though the political situation in the country remains foggy at the current time, but still there is high likelihhod of the two sides reaching an agreement in favor of a limited economic cooperation at post-secession era, as the South crude oil needs to flow through the pipe lines and refineries in the North, the only outlet for the South oil export at the current time[3].

The remaining parts of the paper structured as follows. In what follows, we will present an open economy macroeconomic model that combines

[2] The official exchange rate system currently is a managed float system, in which the central bank calculates an indicative rate based on previous day transactions and intervenes in the market if quotes break away from a plus/minus 3 percent margin around that rate.

[3] Infact, the impact of war breakout between the two sides can be infered from the impact of massive capital flight from the country, which is included in our analysis.

various decision making by economic agents. Section three includes proporties of steady state and stability conditions.The final section summerize the findings of the paper.

2. The macroeconomic model

The macroeconomic model made up of the optimal rules of the various decision-making agents in the economy. Of these agents, private sector producers choose output and input levels for both home goods and export goods. Inputs include both labour and imported producer goods. Firms also set how much of any export revenue to divert to the foreign exchange markets by under-invoicing export sales and over-invoicing import bills. Households choose how much of their total financial wealth to hold in domestic currency, and how much to hold in foreign currency. Finally, the government determines its fiscal stance and financing, and the rules for pricing and rationing in the official foreign exchange market. We will deal with each in turn, and then assemble the various decision rules into steady-state equilibrium conditions for the two parts of foreign exchange markets.

i. Domestic producers' decisions

Domestic firms can produce for home consumption or export. Exports are exogenously determined, as they depend mainly on international terms of trade. Home goods (y) have a Cobb-Douglas production function with imported producer goods I_p and domestic labour L_y as the only inputs:

$$Y \leq I_p^{(1-\alpha)} L_y^{\alpha}, \qquad 0 \leq \alpha \leq 1 \qquad (1)$$

It is assumed that there is a quasi-fixed exchange rate denoted as e, applicable to some official transactions, and a freely floating nominal exchange rate, denoted as b, mainly applicable to parallel market transactions[4]. This latter rate takes whatever level is required to achieve asset market equilibrium. In the remaining parts of the paper we will refer to the quasi-fixed rate as the official rate, and the free rate by the parallel rate.

The domestic currency purchase price of imported producer goods is the foreign currency price P^*_m multiplied by the free exchange rate relevant for competitive purchases. Competitive purchases of imported producer goods will always be made at the parallel rate b, since the government impose restrictions for purchases at the official rate.

The government stipulate that private firms convert a portion, $(0 < \phi < 1)$ of their export proceeds at the parallel rate b, and the remaining part at the official rate, e which is always lower than b.

Despite foreign exchange regulations firms can divert additional export proceeds illegally at the parallel rate b, by under-invoicing export procceds. As a result, the decision of how much of export proceeds to surrender at the official rate versus the parallel rate depends on the size of ϕ, which will determines the amount of export proceeds to evade foreign exchange regulations.

The income of domestic firms consist of revenue from goods produced for domestic consumption and revenue from export. The expenditure side includes imported capital goods and labour cost for both export and domestic consumption goods. It is also included in the expenditure side

[4] It is important to realize that the parallel market in this sense is not a black market because it is officially recognized market by the government, and basic reason for this is that the government want to deter emergence of black market for foreign exchange.

the cost of under-invoicing export revenue, which is assumed linear function of ϕ, or more specifically $(\phi/2)$. Thus, firms' decision rules for all the choices above are found by maximising their profit function:

$$Max[p_y Y + \{\phi b(1-(\phi/2)+(1-\phi)e\}X - bP^*_m I_p - W(L_x + L_y)] \qquad (2)$$

With respect to L_y, I_p and Φ, subject to production technology constraint and to the usual non-negativity restrictions. The first order conditions for I_p, L_y and Φ are:

$$P_y(1-\alpha)I_p^{-\alpha}L_y^{\alpha} = P^*_m\ b \qquad (3)$$

$$P_y\ \alpha Y/L_y = W \qquad (4)$$

$$\Phi = (1 - 1/\pi) \qquad (5)$$

Where $\pi = b/e$, refers to the paralle rate premium, and P^*_m is the dollar value of import price. Concavity of equation (5) implies that, while rising premium induce diversion of foreign currency from the official market to the parallel market by under-invoicing export revenue, but in a decreasing rate due to higher penalty cost when the size of under-invoicing increases.

After a few manipulations and substitutions of equations (3) and (4) we get:

$$P_y = P^*_m\ b(I_p/L_y)^{\alpha} + w(I_p/L_y)^{\alpha-1} \qquad (6)$$

When the ratio of capital and labour inputs (I_p/L_y) combined in fixed proportions, equation (6) reduces to:

$$P_y = \beta_1 P_m^* b + \beta_2 w \qquad (7)$$

$$where \quad \beta_1 = (I_p / L_y)^{\alpha}, \quad \beta_2 = (I_p / L_y)^{\alpha-1}$$

Thus, domestic inflation can be expressed as a function of imported inflation and domestic wage cost and the paralle rate:

$$\dot{P}_y = \beta_1 [P_m^* \dot{b} + b\dot{P}_m^*] + \beta_2 \dot{w} \qquad (8)$$
$$= \beta_2 \dot{w} + \beta_3 \dot{b} + \beta_4 \dot{P}_m^*$$
$$for \quad \beta_2 > 0, \beta_3 > 0, \beta_4 > 0$$

Where a dot over a variable denotes change over time.

Equation (3) can be rearranged to solve for imports of producer goods. Substituting for P_y from equation (7) as well, the optimal level of imported producer goods is stated as:

$$I_p = \sigma(1-\alpha)Y \qquad (9)$$

$$where \quad \sigma = \beta_1 + \beta_2 (w/P_m)$$

where (w/p_m) is the ratio of marginal productivity of labour and imported capital inputs used for home goods production.

Since change in real exchange rate reflect change in the relative price of tradables to non-tradables, or $r = (eP^*_X / P_y)$ then change in the parallel exchange rate affect the real exchange through its effect on domestic inflation (equation 8). In this definition of real exchange rate the relation between real exchange rate and export defined as: $(\partial X / \partial r) > 0.$, so that appreciation in real exchange rate induce non-oil commodity exports.

ii. Household portfolio allocation:

Households choose between domestic and foreign assets, a portfolio allocation decision. Households' nominal financial asset

portfolio H is assumed to consist only of domestic money holdings M, and foreign money holdings F[5]. Since households buy foreign currency F only in the parallel market, and therefore value it at the parallel exchange rate b, the domestic currency value of households' nominal wealth H can be expressed as:

$$H = M + bF \qquad (10)$$

Let λ be the fraction of financial wealth H that households want to hold in foreign currency. Both foreign currency F and domestic currency M earn zero interest, but F will provide a return whenever the parallel rate b changes[6]. The fraction λ will therefore rise with the actual rate of increase in the parallel rate[7].

In equilibrium, desired holdings of foreign money λH must equal the actual stock bF of foreign money being held, so we can solve for $H = bF/\lambda$. Replacing H in equation (10) and rearranging to solve for M, and dividing both sides by, e, to convert to foreign currency values, valued at the official rate, so that:

$$m = [(1 - \lambda)/\lambda)]\, \pi F \qquad (11)$$

where m = M/ e. The fraction λ is a function of the rate of increase in parallel rate, but this can be broken down into appreciation of the official rate, $\hat{e}$, and of the parallel market premium $\hat{\pi}$. Under fixed official rate, parallel market rate change equal to change in the parallel premium rate. Thus letting $\Lambda(\hat{\pi})$ stand for the relationship of $(1 - \lambda)/\lambda$, and solving for $\hat{\pi}$ equation (11) can be expressed as:

[5] As the stock market is underdeveloped, its impact on domestic savings is negligable.

[6] A principal motive of holding domestic money despite its zero interest earning, because of zero risk of holding it.

[7] Lizondo (1987), and Dornbush (1983) use perfect foresight assumption as they assume that λ rise with the expected increase of the free rate.

$$\hat{\pi} = \Psi(m/\pi F) \quad \Psi' < 0 \qquad (12)$$

Equations (11) and (12) represents the portfolio-balance or the asset market equilibrium condition. Equation (11) indicates that the higher the expected rate of increase of the parallell rate (depreciation), the lower is the ratio of domestic money to foreign currency holdings.

iii. Government decisions

The government determines much of the context for decisions of other agents in the economy, and also acts as a separate agent. For instance, the government decrees and administers a set of foreign exchange controls which regulate entry into the official exchange market. In this market the government buys foreign currencies from households at the official rate *e*, and allocates it to pay for government imports (*G*). The government can buy from only one source: private sector export revenue *X*.

We assume that government spending *G* is entirely on imports, including payment of interest on foreign debt, and that no new foreign debt is incurred. Further, we assume that any of *G* that is not financed by taxes must be financed by borrowing from the Central Bank.

The change in the stock of domestic money *M*, is equal to the change in central bank domestic credit, $\dot{D}$, plus the change in (domestic currency value of) foreign currency reserves held by the government, $\dot{eR}$. The change in domestic credit reflects government borrowing from the central bank to finance its deficit, $G - T$ or (in foreign currency) $g - t$ (where t=T/e, and g=G/e) that is,

$$\begin{aligned} \dot{m} &= \dot{D} + \dot{R} \qquad (13) \\ &= (g - t) + \dot{R} \end{aligned}$$

Since $(M/e) = m$

The value of $\dot{R}$ is determined in the official exchange market, to which we now turn.

iv. Foreign exchange markets

(a) The official reserves:

The current account balance in the official market (for brevity we will call it official resrves)[8], $\dot{R}$, is determined as a fraction of export revenue channelled through the official exchange market, $((1-\phi)eX)$, less government imports (G) all valued in foreign currency. Algebraically, the official reserves can be expressed as:

$$\dot{R} = [(1-\Phi)X(r)] - g \qquad (14)$$

Where r is the real exchange rate. Substituting for Φ (equal to $(1-1/\pi)$) from equation (5) yield the final form of the official reserves:

$$\dot{R} = (X(r)/\pi) - g \qquad (15)$$

Thus, the status of the official reserves is affected by the real exchange rate and the parallel rate premium. For example, as implied by equation (5) rising parallel rate premium affect official reserve balance inversely through the smaller proportion of export proceeds allocated to the official foreign exchange market. It can also be verified from equation (15), as adverse shock to export raise the parallel premium, balancing official cuurent account, $(\dot{R})$, requires fiscal deficit reduction, (g).

[8] There are no capital account transactions in the official market.

(b) The parallel FX market:

After determining the official current account, the current account in the parallel market, $\dot{F}$, is determined by subtracting total imports (private sector and government), capital flight, and official reserves, from the total inflow of foreign currency to the economy:

$$\dot{F} = X(r) - (I_p + g) - C - \dot{R} \qquad (16)$$

where I_p is imports of producer goods, and C is the capital flight measured in foreign currency units. Since there is no commonly accepted definition of capital flight in the economic literature (Kant, 2002), in this paper we define capital flight in more broader terms as the outflow of capital from a country in response to risk and uncertainty in the economic policies of that country. As a result, given declining official reserves and increasing parallel rate premium invoke more stringent foreign exchange regulations and abalance of payment crisis posibility then change in capital flight depends on the deviation of parallel rate premium from a benchmark level that balance official current account. More formally, from equation (15) the premium level that balance official reserves $(\dot{R} = 0)$ can be determined as:

$\tilde{\pi}]_{\dot{R}=0} = (X/g)$

Given change in the ratio of capital flight to total private wealth, (a=C/H), depends on the location of the premium from its benchmark level, $(\tilde{\pi})$, then:

$$a = f(\pi - \tilde{\pi}) \quad for\ f' > 0 \qquad (17)$$

Equation (17) imply change in the ratio of capital flight, a, depends on the official reserve status, which depends on whether the premium is

above or below the benchmark level $(\tilde{\pi})$. Denoting $\theta = (\pi - \tilde{\pi})$ then from (17) the following relations hold[8]:

$$\frac{da}{d\theta} > 0 \quad if \quad \theta > 0 \Rightarrow \dot{R} < 0$$

$$\frac{da}{d\theta} < 0 \quad if \quad \theta < 0 \Rightarrow \dot{R} > 0$$

$$\frac{da}{d\theta} = 0 \quad if \quad \theta = 0 \Rightarrow \dot{R} = 0$$

Taking into account total private financial wealth, (H= m + πF), and Substituting for $\dot{R}$, from equation (15) and I_p from equation (8) we get the final form of the current account balance in the parallel exchange market:

$$\dot{F} = (1 - 1/\pi)X(r) - \sigma(1-\alpha)Y - a(\theta)(m + \pi F) \qquad (18)$$

Equation (18) completes the model, which consist of the differential equations (12), (13), (15) and (18) in $\dot{m}, \dot{F}, \dot{R}, and\ \hat{\pi}$. To simplify, we reduce the differential equation system to three equations by substituting for $\dot{R}$ into equation (13) giving

$$\dot{m} = (X(r)/\pi) - t \qquad (19)$$

The third differential equation is (12), is repeated here as:

$$\hat{\pi} = \Psi(m/\pi F) \quad \Psi' < 0 \qquad (12)$$

The dynamic system of the model now represented by the equations (12), (18), and (19).

In the next section we illustrate the adjusment process of the economy when speculative effects and adverse shock hit the economy under the case of insufficient reserve level. But before that we need to

[8] since capital flight in our context is defined as outflow of foreign currencies due to continous deterioration of official reserves, it serve as a proxy variable for political uncertainty.

investigate existance of steady state solution for the dynamic system of the model.

***Proposition**:*

A sufficient condition for existance of steady state solution for the system of equations (12), (18), and (19) is that the size of capital flight should be as small as negligable, so that $a \to 0$ *in equation (18).*

A prove of existence of a unique solution for the steady state values is given in the appendix of this paper. Analysis of the characteristic polynomial of the linear approximation of the equations (12), (18), and (19), shows that the dynamic model has one positive and two negative roots if the ratio of capital flight is small as negligable. That is, the steady state discussed above is a saddle-point solution, therefore the economy can (re-)converge to the steady state from a distance away. If the condition of $a \to 0$ is not satisfied there is no guarantee for existance of sustainable unique solution for the state variables, $\pi, F, and\ m$.

Assuming the parallel market always adjust to steady state, $\dot{F} = 0$, the adjustment process of the economy is illustrated using phase diagrams of the equations (12) and (19), under two different cases: (a) At the pre-secession period when speculation about official resrve depletion is rising; and, (b) At the post-secession era, represented by adverse export shock to the economy.

3-Dynamic Analysis

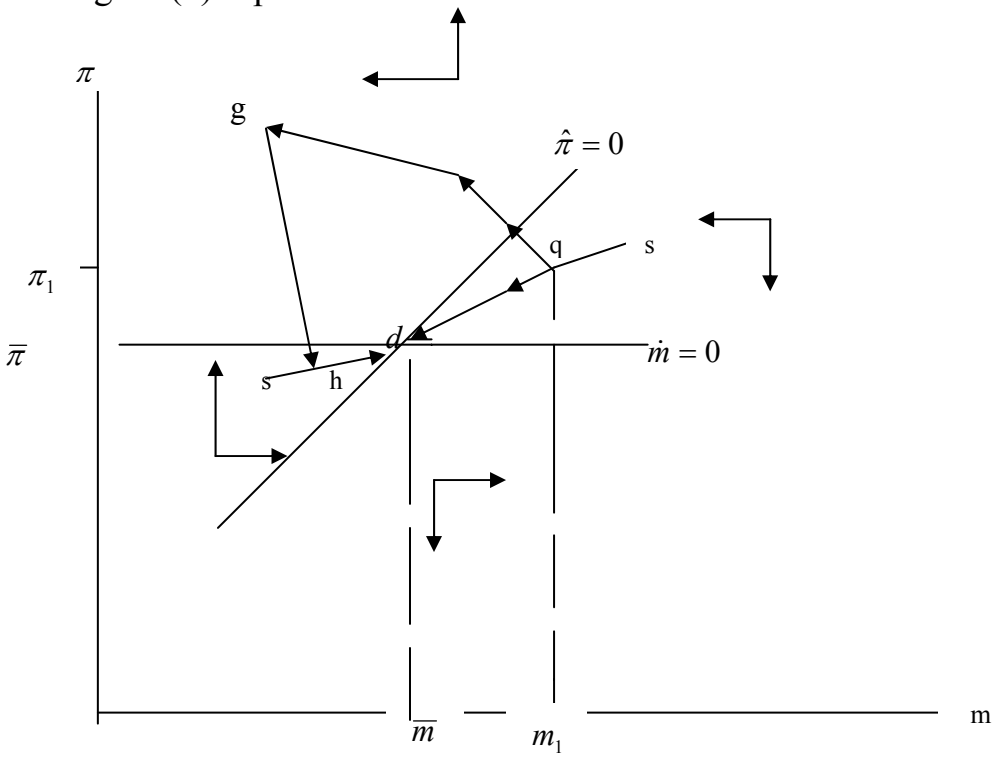
It should be noted that political uncertainty in our context is due to the likelihood of the country split into two independent parts, resulting in a permanent adverse export shock to the economy in the North, which will lose substantial amount of oil resource. However, at the pre-secession period speculations are rising with regard to official resrves depletion and sustainabilty of the official exchange market. In the following we analyse the dynamics of the financial system under each of the two cases.

(a) Speculative Effects:

Currently, as the date of the refrendum to decide upon unity or seperation of the country is approaching, foreign reserves with the central bank hit low levels, as the public anticipation of future currency depreciation rises[6]. The adjustment process in the financial system due to the rising demand on foreign currency in the parallel market is explained in figure (2). As indicated in the proposition, steady-state solution exist only if capital outflow is maintained at low levels. Given this condition is satisfied, the initial steady state equilibrium represented at pont d in figure (2). Rising demand for foreign currencies in the parallel market represented by upward shift in the parallel rate premium from point d to q. At point q expanding spread between the official rate and the parallel rate aggrevates reserve loss and induces further anticipation of official rate devaluation, which will push the parallel rate premium at the point, g. When official reserves reach a minimum level at point g, the government resort to a devaluation of the official rate that brings the system to the point h. During the adjustment process from h to d, along the saddle path

some of the reserves previously lost is replenished. It should be realized that, the recovery process from points g to h and then to d, depends on the level of the intial official reserves, as the adjustment process is expected to take longer time, the lower is the official reserves level. In other words, when official reserve level is low, the distance between points g and h, becomes wider and that requires bigger devaluation of the official exchange rate to locate the system along the sadle path at point h. Thus, the adjusment process when official resrve level is low, or insufficient, is more costly in terms of the social cost pertaining to the impact of official rate devaluation on income distribution and resource re-allocation.

Figure (2): Speculative effects and insufficient reserves



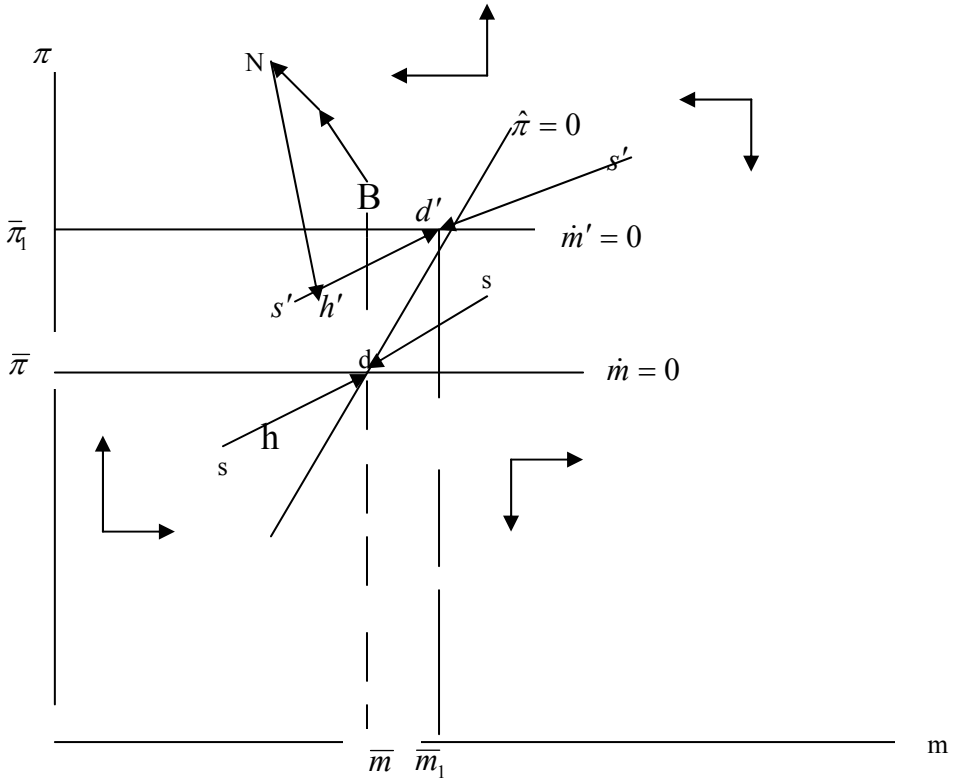
[6] According to the IMF country report of 2009, Sudan foreign currency resrves declined from about

(b) Perminant Export Shock:

In case the referundum results comes in favor of seperation of the South, the economy in the North expected to undergo substantial changes as it loses large part of crude oil export. Given this situation hold, the adjusment process in figure (3) illustrates the impact of a perminant adverse shock in export which cause an upward shift in the $\dot{m} = 0$ schedule to $\dot{m}' = 0$. On impact, the depreciation of the parallel rate raise the premium level to the point B. The adjustment from B to N will be affected by reduction in the demand for domestic money in private assets portfolio and the government withdrawal from its reserves to sustain the official rate. However, by the time official resrves reach its benchmark minimum, the government resort to discrete devaluation of the official rate so that the system jumps from N to h, and then converge along the saddle path towards the new equilibrium at point d' with international reserves partially restored, along the new saddle path s's'. It should be realized that for the adjustment process towards equilibrium level to be feasible it is required that initial reserve level must exceed the losses that take place during the transition, from points d to N; otherwise, a balance of payment crisis will take place. It is also to be noted that the new steady state solution at point d' is maintained at higher parallel premium level, reflecting a wider spread between the two exchange rates at the steady state solution. Thus, the steady state premium level depends on size of the adverse export shock. If the shock is too big, in such away that it induces intolerable spread between the two exchange rates, the government is left with the only option of unifying the exchange rates by adopting free floating rate, to which we turn in the following section.

US$ 1.7 billion in 2006 to about US$ 390 million in 2009.

Figure (3): Export shock



(c) Unification of Exchange Markets:

Let us assume that due to expanding fiscal deficit and deteriorating official reserves the government decided to abandon the dual foreign exchange policy and liberalize its foreign exchange market by adopting a single exchange rate floating freely and applicable to all transactions. In order to investigate the behavior of the post-unification floating exchange rate we introduce some changes into the basic model. Under a floating exchange rate system, the government eases the foreign exchange rate restrictions by allowing the private sector to buy and sell foreign currency determined by supply and demand conditions. Given the exchange rate is now flexible, there is a unified exchange rate which will be denoted by

μ. Since the central bank no longer intervenes in the foreign exchange market the stock of official reserves is constant $(\dot{R} = 0)$. Under the new foreign exchange system the change in domestic money supply is reflected only by change in domestic credit: $\dot{M} = \dot{D}$, therefore

$$\dot{m} = \left[\frac{(g-t)}{m} - \hat{\mu}\right]m \qquad (20)$$

where $(g-t) = (G-T)/\mu, \ and \quad m = M/\mu$.

The condition for portfolio equilibrium under the floating exchange rate system is given by:

$$m = \Lambda(\hat{\mu})F \qquad (21)$$

Given the parallel market equilibrium condition still hold $(\dot{F} = 0)$, the dynamic system of the model is now represented by the equations (20) and (21). In order to reduce the system into a single equation we invert equation (21) and substitute it into equation (20) to get:

$$\dot{m} = \left[\frac{(g-t)}{m} - \Psi(m/F)\right]m \qquad (22)$$

where $\Psi(m/F) = \hat{\mu}, \quad \Psi = \Lambda^{-1}, \Psi' < 0$

Since $[(g-t)/m] = \dot{M}/M$, we denote the growth in domestic nominal money (or domestic credit expansion) by $\eta(t)$ so that equation (22) can be expressed as:

$$\dot{m} = [\eta(t) - \Psi(m/F)]m \qquad (23)$$

When domestic credit expands at a constant rate[7] $\eta(t) = \eta.$

Equation (23) represents the evolution of domestic money growth under floating exchange rate system, and it indicates that in the steady-state $(\dot{m} = 0)$ the rate of exchange rate depreciation is identical to domestic credit expansion[8]:

[7] The condition of constant rate of domestic credit expansion is crucial for existance of stable equilibrium values of the state variables, M, F, and π.

[8] The size of the public deficit (g-t) or domestic credit expansion does not change between the two regimes of foreign exchanges because there is no effective open market operations that allows the

$$\hat{\mu}^* = \eta \qquad (24)$$

equation (24) predict that if fiscal deficit persist after the liberalization of foreign exchange system, depreciation of foreign exchange rate will be identical to domestic credit expansion.

4. Concluding remarks:

We set up a dynamic model capturing salient features of small open economy enduring political uncertainty and operating under dual exchange rates, a free exchange rate applicable to a wide range of private capital transactions and quasi-fixed exchange rate applicable to officially sanctioned transactions. The objective in the paper to assess financial stability in the economy, taking into account diminishing official foreign currency reserves, and political uncertainty that entails the likelihood of seperation of the country into two independent parts, invoking adverse perminant export shock. The finding in the paper indicate steady state equilibrium exchange rate system can be attained given that capital flight remains at minimal level. The size of export shock determines the size of equilibrium exchange rate premuim (divergence of the free rate from the official rate), and the level of the initial official reserves determine the length of time for the process to adjust towards a new steady state equilibrium. The lower initial official resrve level is, the longer time is needed to recever from adverse shock and adjust towards a new equilibrium steady state. When fiscal deficit and declining official reserves force the government to abandon the dual exchange system in favor of floating single exchange rate system, our model predict depreciation of foreign exchange rate is identical to domestic money growth.

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government to adjust its stock of money supply according to its reserve level. In otherwords, the government is always a buyer of foreign currency, not a seller of foreign currency.

Appendix

Prove of the proposition

To establish existance of steady state solution fo r the system of equations:

$$\hat{\pi} = \Psi(m/\pi F) \qquad \psi' < 0 \qquad (12)$$

$$\dot{F} = (1 - 1/\pi)X - \sigma(1-\alpha)Y - a(m + \pi F) \qquad (18)$$

$$\dot{m} = (X/\pi) - t \qquad (19)$$

Linearizing equations (12), (18) and (19) around the steady state values of $\overline{F}, \overline{m}, and\ \overline{\pi}$, to give the following matrix equation:

$$\begin{bmatrix} \hat{\pi} \\ \dot{F} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} \hat{\pi}_\pi & \hat{\pi}_F & \hat{\pi}_m \\ \dot{F}_\pi & \dot{F}_F & \dot{F}_m \\ \dot{m}_\pi & \dot{m}_F & \dot{m}_m \end{bmatrix} \begin{bmatrix} \pi - \overline{\pi} \\ F - \overline{F} \\ m - \overline{m} \end{bmatrix}$$

the values of the partial derivatives in the Jaccobian matrix can be determined as follows:

$$\frac{\partial \hat{\pi}}{\partial \pi} = \frac{-\Psi' m}{\pi^2 F} > 0, \quad \frac{\partial \hat{\pi}}{\partial m} = \frac{\Psi'}{\pi F} < 0, \frac{\partial \hat{\pi}}{\partial F} = -\frac{\Psi' m}{\pi F^2} > 0$$

$$\frac{\partial \dot{m}}{\partial F} = 0, \frac{\partial \dot{m}}{\partial \pi} = -\frac{X}{\pi^2} < 0, \frac{\partial \dot{m}}{\partial m} = 0$$

$$\frac{\partial \dot{F}}{\partial m} = -a < 0, \frac{\partial \dot{F}}{\partial \pi} = \left(\frac{X}{\pi^2} - aF \right) = ?, \frac{\partial \dot{F}}{\partial F} = -a\pi < 0$$

A necessary condition for a saddle-point solution is that the determinant of the Jaccobian matrix be positive:

$$|J| = \hat{\pi}_\pi \left[\dot{F}_F \dot{m}_m - \dot{F}_m \dot{m}_F \right] - \hat{\pi}_F \left[\dot{F}_\pi \dot{m}_m - \dot{m}_\pi \dot{F}_m \right] + \hat{\pi}_m \left[\dot{F}_\pi \dot{m}_F - \dot{F}_F \dot{m}_\pi \right] > 0$$

Since the determinant of the Jaccobial matrix is positive, either there are two roots that are negative and the third is positive, or all the three roots are positive. In the latter case the steady state is unstable. To rule out instability we analyse the characteristic polynomial P(r) of the Jaccobian matrix:

$$P(r) = \begin{bmatrix} \hat{\pi}_\pi - r & \hat{\pi}_F & \hat{\pi}_m \\ \dot{F}_\pi & \dot{F}_F - r & \dot{F}_m \\ \dot{m}_\pi & 0 & \dot{m}_m - r \end{bmatrix}$$

Where r is a characteristic root of the equation system. The polynomial properties include:

$$p(r) = \nu_0 + \nu_1 r + \nu_2 r^2 + \nu_3 r^3$$
$$\nu_0 = r_1 r_2 r_3 > 0, \qquad (\det | J |)$$
$$\nu_1 = r_1 r_2 + r_1 r_3 + r_2 r_3$$
$$\nu_2 = r_1 + r_2 + r_3 \qquad (trace)$$

If the sum of the cross products $(\nu_1 < 0)$ then at least one of the three roots is negative, and the case of instability is ruled out. Using the equation for ν_1 and the determinant equation, we can show that:

$$\nu_1 = \left[-\dot{F}_\pi \hat{\pi}_F + \hat{\pi}_\pi \dot{m}_m - \dot{m}_\pi \hat{\pi}_m \right] < 0 \quad if \ \dot{F}_\pi > 0$$

the condition of $\dot{F}_\pi > 0$, is satisfied only if $a \to 0$, which is the case that rules out the possibility of an unstable solution, and leaves only the saddle point solution.

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