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February 1985

Online at <https://mpra.ub.uni-muenchen.de/29910/>  
MPRA Paper No. 29910, posted 21 Apr 2011 20:47 UTC

# Economic Development and Loglinearity in the Rank-Size Distribution of Urban Systems<sup>\*</sup>

by

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This paper is a revised and extended version of an invited presentation made at the P.E.C.U.S. (Annual Permanent Conference on Ukrainian Studies) at Harvard University, May 29-31, 1981.

A shorter version has been accepted for publication in a forthcoming issue of the Atlantic Economic Journal.

I would like to express my thanks to Chauncey Harris, University of Chicago, Lev Dobriansky, Georgetown University and Igor Stebelsky, University of Windsor, for their encouragements and their helpful comments.

Mai 1985



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**INTRODUCTION**

Several researchers have attempted to correlate economies development with the degree of urbanization of a given region or country (BECKMANN, 1958; BERRY, 1961; EL-SHAKS, 1972; BRUNET, 1980). While different approaches are proposed to explain the regularity in the distribution of cities in terms of competing forces interacting at an aggregate level, less attention seems to have been devoted to study economies where these forces would have been willingly and constantly modified, such as in a state planned economy.

HARRIS (1970) has addressed this question in his extensive work on the U.S.S.R. Since his study covered only the 1959 census year, nothing can be said on the dynamics of economic development with respect to a changing urban structure. Furthermore, it would seem interesting to observe how the regional changes occurring at the level of the component republics are reflected on the evolving superstructure.

The purpose of this paper is to examine, empirically, the dynamics of the soviet urban System in light of the rank-size distribution hypothesis and to expand the analysis initiated by HARRIS on the U.S.S.R. The first part of this paper will present a brief recall of the theoretical aspects dealing with the measurement of city distribution, with reference to the various points of view on the topic. In the second part, the rank-size hypothesis will be tested for each census year during the 1897-1979 period for the U.S.S.R. to observe changes in the slope values of the distribution, expressed in logarithmic form, of several classes of cities. In order to assess how the changes in city growth of one of the major republics in the System might be related to the trends identified at the superstructure, a new set of analysis will be carried out for the corresponding period, involving also that region. Finally, a general interpretation, based on the summary of the findings, will be proposed for further discussion.

## **I. CONCEPTS OF CITY SIZE DISTRIBUTION**

### **a) Principles of measurement**

The location and the distribution of economic activities in a given economic system have been observed by geographers, demographers, urbanists and economists

in order to detect some predictable patterns or regularities over place and time (BRUNET, 1976). Most analysis are mainly descriptive, but some attempts have been made to derive some generalized models, even though no universal and well accepted theory has emerged from the various propositions.

The most direct approach to express the importance of the urban structure in an economy is thru the measurement of the aggregate level of urbanization, such as the percentage of urban population in the country. Even at this point, comparisons are sometimes risky, given the arbitrary definitions of urbanized areas and the changes in the political regions or city boundaries thru time. One particular problem is the determination of the minimum size of the communities taken under consideration. The availability and the reliability of the census data may further complicate the analysis (DUNCAN, 1957; PARR, 1976).

When it comes to the distribution of the urban System, there is no unanimity as to the rationale of the form or the significance of its representation, but some explanations on the regularity of the distribution of cities have been proposed. Starting with the early works, see the bibliography in BRUNET (1976), the-concepts of concentration indexes have evolved from the notion of rank-size rule, where the product of the city by its rank would be equal to the value of the largest city in the System, to the Pareto-curve and the lognormal distributions. Most studies could be even classified as "empirical curiosities" rather than formal doctrines, and few efforts were directed at recognizing and assessing, with proper mathematical techniques, the deviations from the expected overall pattern (DUNCAN, 1957). The analysis of the growth process and of its stability thru time, of different economic Systems or of specific political regional components within an overall structure were also mostly neglected, at least from the theoretical point of view.

Indeed, representing the complexity of economic activities should require more variables than just the size and the rank of the cities comprising a System. Distance between cities, for one thing, may greatly influence the distribution of the private and governmental functions and, therefore, the density of the occupations and of the population in a given area. Other

factors, such as the physical constraints to the movement of people and merchandises, historical traditions, cultural preferences and political and economic policies inevitably influence urban settlements, specially when they have an impact on fertility, mortality or migrations (DUNCAN, 1957; EL-SHAKS, 1972; Von BOVENTER, 1973).

## **b) Empirical regularities and deviations**

HILL (1974) has provided a formal derivation of the probabilistic model of the rank-size distribution, based on the Bose-Einstein form of entropy, an approach often adopted by the population physicists.

In notation form, the rank-size distribution can be represented as a special case of the lognormal distribution:

$$P_R = P_1 / R^q$$

$$\text{or} \quad \log P_R = \log P_1 - q' \log R \quad (1)$$

where  $P_1$  = population of the first city

$P_R$  = population of the city of rank R

$q$  = parameter (assumed equal to -1)

When  $P_1$  is not chosen from the sample of cities, it represents the theoretical size of the largest city in the System when the parameter  $q$  representing the slope of the distribution is equal to -1.

Equation (1) can be interpreted as the discrete version of the continuous Pareto-curve distribution, using transposed axes.

The regularity of the rank-size distribution providing for the linearity would result from the presence of competing forces such as proposed by the Yule-Simon model (PARR, 1976). This approach is based on 2 crucial assumptions: the so-called Law of Proportionate Effect, where the probability of a given growth rate

is the same for each size class of cities and, secondly, that the proportion of growth from new cities (those entering the System, given a predetermined threshold level for the truncated lognormal distribution) would be constant over time (PARR and SUZUKI, 1973). However, both assumptions have been severely contested with empirical tests (PARR, 1976),

Throughout the empirical studies, general patterns of regularity have been observed for many urban Systems at many different times (ZIPF, 1949; ALLEN, 1954; BERRY, 1961; EL-SHAKS, 1972; BRUNET, 1976, 1980). However, what divides the scholars in their acceptance of the rank-size distribution is the recognition and the interpretation of departures from the "normal" shape. Furthermore, when logarithmic scales suggest only minor changes from the distribution, they should be tested with more rigorous statistical methods, such as the Chi-square test of goodness of fit, rather than a casual acceptance of high values for  $R^2$  (DUNCAN, 1957).

In principle, three types of deviations could occur. Concavity may indicate that the lower part of the distribution is sharply dropping. This becomes particularly common when all the settlements have been included in the sample. Practically, it means that the very smallest population units tend to regroup themselves at some minimum size of functional and economic efficiency, rather than remain as a large number of isolated entities (DUNCAN, 1957; PARR, 1976). In fact, PARR (1976) suggests that the rank-size distribution, when it conforms to the lognormal form, can be found only above a certain minimum city size and is referred to as the truncated distribution. ZIPF (1949) has observed that convexity to the origin could be found for some countries and at given periods of time, indicating the primacy pattern of the system. This primacy concern has been examined in detail by EL-SHAKS (1972) in relation with the take-off phase of economic development. He proposed the measurement of the degree of primacy of a city or of a system, but his mathematical representation suffers from notation errors with his use of the subscripts all throughout his 4 main equations. Eventually, the system could display both convexity and concavity, showing an S-shaped curve for the distribution (STEWART, 1958).

Acceptance of the rank-size hypothesis requires, inevitably, plausible

explanations for the type of deviation observed in a system. Since the urban structure, with all its particularities, evolves thru time, attempts to explain the changing deviations in light of a wide range of possible disturbance factors will remain a formidable challenge to the adventurous analyst' (ZIPF, 1949; DUNCAN, 1957; PARR, 1976; BRUNET, 1980).

## **II. RANK-SIZE HYPOTHESIS AND THE SOVIET URBAN SYSTEM**

### **a) Importance of previous soviet urban studies**

As to the involved effort of soviet scholars on the subject, HARRIS (1970) reported that there was more than one thousand research works published on soviet cities. He also indicated that about 400 specialists were covering many aspects of soviet urban and population geography. While little details are mentioned as to their specified works, Harris emphasized that:

"Soviet geographers and planners have devoted much attention to the question related to size of cities. The soviet literature on optimum size of cities and on the need for limiting the size of the great metropolises is particularly extensive ... showing that cities in the size range of 50 to 200 thousands are most efficient in terms of the urban economy ...." (p. 46).

Actually, while some critical efficiency aspects, such as economies of scale in the production and moreover in the distribution of public goods and services, are certainly preoccupying soviet authorities, one could suppose that other factors, like ethno-cultural and political considerations do not play a minor role in their planning policies. This point is clearly substantiated by another quotation from Harris:

"The 22nd and 23rd congresses of the Communist Party of the Soviet Union in 1961 and in 1966 adopted programs of fostering the growth of small and middle-sized cities. New industrial establishments are to be built primarily in middle-sized and smaller cities. A large number of

monographs and articles have recently been devoted to the possibilities and problems of locating industries in small and medium-sized towns, particularly of the western parts of the U.S.S.R. ' (p. 47).

As to the specific works on the distribution of cities in the soviet urban network, Harris referred in particular to V.G. Davidovich and O.A. Konstantinov and to their use of "... ingenious graphs, statistical indicators and projections (to throw) light on the regularities that exist in the settlement patterns of the country" (p. 49-50).

## **b) Testing the rank-size hypothesis for the U.S.S.R.**

### **i) Data description**

City population data were extracted from the Statistical Supplement compiled by Chauncey Harris for the 1897, 1926, 1939 and 1959 census years. The observations refer to actual political boundaries, while in fact these had been modified after each war period. The minimum city size of 10 000 inhabitants did apply only to the 1959 census, while all data available was used for the previous years, rounding the figures to the nearest thousand.

More recent information for 1970 and 1979, not available in the Supplement, came from official sources, the Nark'hoz yearbooks. The minimum size of cities available was 50 000 for 1970 and 100 000 for 1979, providing us with a more limited but still substantial number of observations.

A special remark which should be introduced at this point concerns the problem of city boundaries. Since we deal with politically defined limits, not with urban agglomerations which would be more representative of the concentration of economic activities, any change in the boundaries from one census to another for any given leading city may indeed obscure the analysis of the dynamics of the System. For example, data obtained for 1959 raised the population of Moscow from 5 046 000 to 6 009 000 inhabitants, a hefty 19% increase, simply by redefining the city boundaries of the capital, while the

other major cities remained unchanged.

Using data from each census year, the city distribution slope values were calculated with a regression test for each of the following cases: first, the whole sample of observations, then the 5 largest cities, the middle-sized group (rank 6 thru 50) and finally the smallest cities group for the remaining cities (51 to the end). No particular claim is made for the choice of the subdivisions and other cutting points could have been adopted. Furthermore, while there is an obvious interest to isolate the very largest cities in a distinct group, limiting their number to 5 could certainly be disputed on the basis of statistical relevance. The fact that numerous studies on the subject have selected the same basis allows for a direct comparison of the results, albeit recognizing the previous warning.

## **ii) Results for the U.S.S.R.**



Figure 1. City rank-size distribution slopes: U.S.S.R. (1897-1979).

Table 1 provides the summary of the results for the individual tests performed for each group of cities in each given census year. The number of cities varies from each sample, depending on the availability of the data from the source documents, as well as the cutting point at the minimum size.

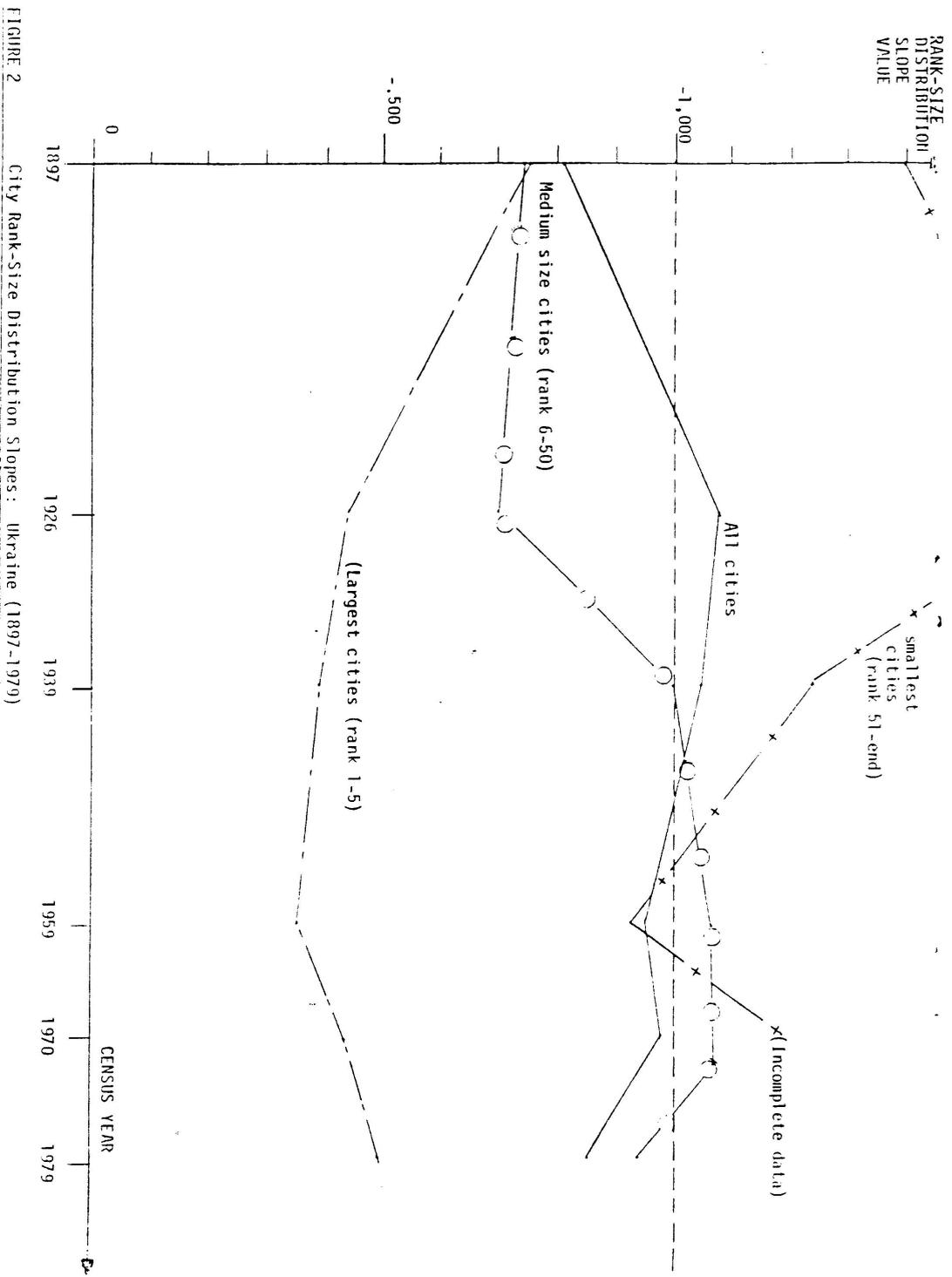
However, the truncated distribution does not affect the comparisons for the two largest groups and, given a sufficiently large number of remaining observations, should have probably only a minor impact on the third group as well as on the whole sample, since they are computed in logarithmic form.

Overall, the city rank-size distribution was very close to "normal" in 1897, under the Tsarist régime, with a slope of  $-0.979$ . This could be seen as a surprising result, given the state of the economy of this period and the lower degree of urbanization. Since then, the Soviet urban System is characterized by a constant decline, from an initial rise to  $-1.100$  in 1926, to a significant low value of  $-0.788$  in 1979. This pattern seems very typical with the behavior suggested by EL-SHAKS (1972) for an economy starting first from a level of underdevelopment with a distribution similar to the rank-size type, then reaching the primate pattern during its early stage of development, to eventually return to the linear form. The disturbing fact in this case comes from the "degrading" of the distribution from 1959. Many important events have not been properly captured in the evolution of the System, like the 29 years period comprising the change of régime in 1917 and the subsequent internal revolutionary conflicts, the shorter 13 years segment covering the Great Famine of 1933 in Ukraine, and the 20 years including the devastating effects of World War II.

The primate pattern of the System becomes evident when examining the largest cities group for any given year, including 1897 ( $-1.123$ ), suggesting a permanence of leadership throughout the two political régimes. A similar continuity is found in the middle-sized group, where the very low range of values from  $-0.773$  to  $-0.592$  does indicate a lack of a solid urban basis at the intermediate level. However, two different patterns could describe the changes in the smallest cities group. Until 1939, the trend is upwards, from  $-1.203$  in

1897 up to -1.340. The slope has then dropped much lower, varying more closely around the target value of -1.0 with a range of values from -1.099 to -0.987 (see Figure 1).

At this point, two remarks are worthwhile mentioning. Firstly, the slope values in 1979 are lower but still close to those in the pre-soviet era of 1897. Secondly, the slope for each city group test shows a declining trend since 1959. In fact, all values are inferior to -1.0 in 1979, except for the largest cities group (-1.086), a reminder of the undisputed dominance of the system by the two leading Russian cities, Moscow and Leningrad. Eventually, if the indicated trend persists, the major cities group could also see its slope value pass below the -1.0 mark in the future. A possible interpretation to be given for the previous behavior of the system is that the urban structure of the U.S.S.R. has evolved to a state somewhat similar to the one of the pre-revolutionary years, giving rise to secondary influence centers, with the emergence of regional capitals at the intermediate level. The increasingly stronger basis of smallest cities, combined with the permanent presence of highly decentralized and competing economic centers at the intermediate level may only increase the tension between leading cities and the lower urban levels for further power sharing.



### **iii) Results for the Republic of Ukraine**

In order to assess the changes within the U.S.S.R. and to parallel them with the overall modifications, a complete set of tests has been performed on Ukraine, the largest non-russian federated republic. Similar conditions are used in the selection and the processing of data as those applied to the U.S.S.R. and summary of the results are provided in the lower part of Table 1 with graphic representation on Figure 2.

Overall, the city rank-size distribution for Ukraine shows great similarity with the lognormal distribution, except for two years, 1897 and 1979. Detailed analysis indicates that the leadership of the System, such as represented here by the 5 largest cities, has been seriously atrophiated after 1897, as a result of a change of political régime. As a word of caution, it should be mentioned that data represent population within city limits, not urban agglomeration which would modified the slope values and allow for a more meaningful interpretation of the apparent lack of strong leadership in the urban network of Ukraine, as seen in the previous remark on redefining city limits, on page 5. The middle-sized cities conform more to linearity, since 1939, while the smallest cities group remains generally with high slope values, suggesting a sharp drop of the curve at that extremity, except in 1959 (-0.918). Insufficient number of observations for 1970 and 1979, due to an increase of the minimum city size, should warn against hasty interpretation for that group.

### **c) Testing urban network shifts within the U.S.S.R.**

Given the different patterns associated with each system examined previously it should be enlightening to further analyze any possible relationship existing between them. In other words, do they develop independently of one another or do the interactions manifest themselves only at certain levels and for some given period? Such hypothesis could be formally tested using appropriate mathematical techniques to determine their validity. The preliminary nature of this investigation will require only a graphical representation of the possible relationships.

Starting with the slope values obtained from the regressions, the changes in

a system could be expressed in terms of the other system, such as with the ratio of the slope values. Trends for the component region will vary relatively to the overall changes between periods.

When the curve indicates an upwards trend, the changes have benefited the component region, providing its absolute slope value is inferior to -1.0 as expected in lognormal distribution. Similarly, it could be claimed that the superstructure has lost some of its relative primacy position if its own absolute slope value was originally above -1.0.

Examining Figure 3, it should be noted that the vertical axis represents now an index of positive numbers, the ratios of the slope values. The base value of 100 stands for the neutral case, when both Systems exhibit the same individual slope values, while horizontal trends would indicate that the same rate of change occurred in both samples for the given period.

When considering all cities in the sample, a persistent trend is noticed in favor of Ukraine, despite the fact that both individual values for 1979 are quite low. In other words, the departure from normality has been more serious for the U.S.S.R. Undoubtedly, it will be difficult to ascertain the extent of this proposition, since the samples have a disproportionate number of observations for that given year (269 against 38). However, a steady move towards normality seems evident for Ukraine, corresponding to a certain

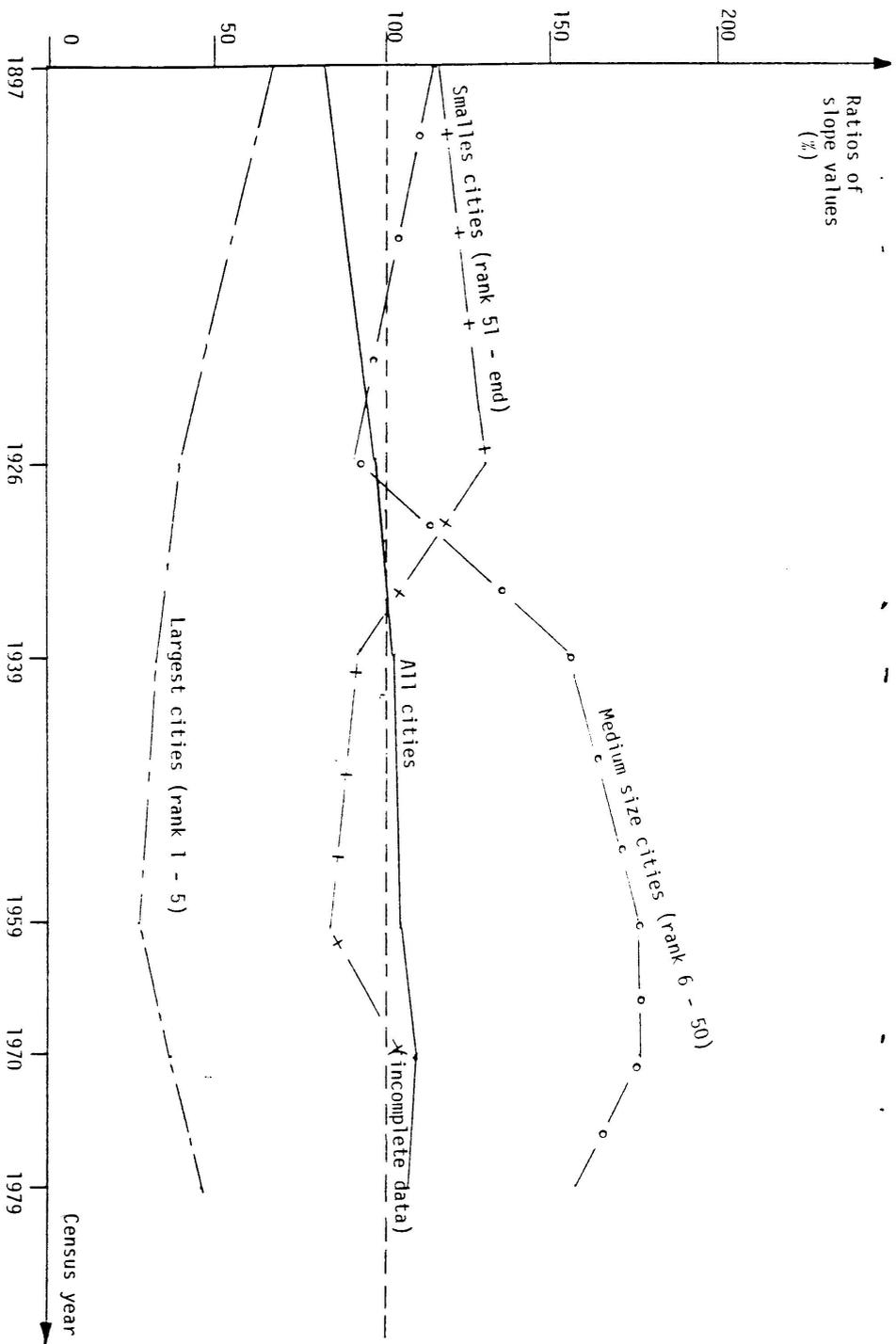


Figure 3 Ratios of city rank-size distribution slopes: Ukraine/U.S.S.R. (1897-1979).

form of decentralization at the level of the U.S.S.R. In fact, dual moves towards normality were present from 1926 to somewhere before 1959, when both Systems developed their urban structure simultaneously, rather than at the expense of each other. Supplementary tests, available upon request from the author, have explored that particular aspect. By developing an extra series of tests with data for the U.S.S.R. without Ukraine, some comparisons were made with results for Ukraine, as if they were totally separated entities. The obvious discrepancy between both systems is with the largest cities group, marked with a slight improvement only from 1959, the period of the Khrushchev era and some measures of decentralization. Symmetrically, the relative absence of developing middle-sized cities in the Soviet Union corresponding to the potential of that economy is a relative advantage to the component republics, such as Ukraine. The unbalance between the two Systems at those two levels of urbanization could, in fact, simply represent the expression of the development planning made by the political authorities. To conclude, all the ratios are either rising or are already located in the upper portion of the graph (above the 100% mark) and, given the absolute values observed in Table 1, it appears that the urban structure of the component republic, Ukraine, is either relatively more linearly distributed or improving towards it (or both), at a faster rate than the one in the U.S.S.R. with opposite effects eventually applying to the latter.

### **III. CONCLUDING REMARKS**

The results presented in this paper, while confirming those of HARRIS (1970) for the 1959 census year, replace them in a necessary perspective. In fact, and seen under many different aspects, that period has been marked by several changes in trends. Outlining these tendencies can serve to understand how effectively the state planning practice, in the presence of major events, may result in a particular form of evolution, witness its urban network. Advocates of the rank-size distribution may find rich interpretations for sometimes

opposing thesis and the temptation to engage in "obvious" extrapolations is ever so close. What has been observed, however, was only that the graphic representation of the results seems to be compatible with the thesis of EL-SHAKS (1972) in describing economic development with a departure from and then a return to the rank-size distribution, at least until 1970. The graphs showed also that the leadership of the U.S.S.R. could be done only at the expense of the intermediate urban level, at least when contrasting the changes with one of its major republics, Ukraine. Finally, movements away from some kind of linearity seem to vary with time, despite of interfering forces devoted to control them, although at a much slower pace.

Given the tentative nature of this investigation, further analysis should be undertaken and more tests released for discussion. Urban agglomeration data, control of the minimum city size, variations in the definition of the subgroupings (particularly in the largest cities category), inclusion of other republics and possibly other factors such as those relating to density, ethnic composition and transportation costs should be analyzed. From the theoretical point of view, reassessing the economic and the political viability of federated states and their components could prove both useful and desirable.

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