Power Spot Price Models with negative Prices

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Abstract

Negative prices for electricity are a novelty in European power markets. At the German EEX spot market negative hourly prices have since occurred frequently, down to values as extreme as minus several hundred €/MWh. However, in some non-European markets as USA, Australia and Canada, negative prices are a characteristic for a longer period already. Negative prices are in fact natural for electricity spot trading: plant flexibility is limited and costly, thus, incurring a negative price for an hour can nevertheless be economically optimal overall. Negative prices pose a basic problem to stochastic price modelling: going from prices to log-prices is not possible. So far, this has been dealt with by "workarounds". However, here a thorough approach is advocated, based on the area hyperbolic sine transformation. The transformation is applied to spot modelling of the German EEX, the ERCOT West Texas market and the exemplary valuation of an option. It is concluded that the area hyperbolic sine transform is well and naturally suited as a starting point for modelling negative power prices. It can be integrated in common stochastic price models without adding much complexity. Moreover, this transformation might be in general more appropriate for power prices than the log transformation, considering fundamentals of power price formation. Eventually, a thorough treatment of negative prices is indispensable since they significantly affect business.

Inhalt

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[1] This article is based on a talk delivered at the 2nd International Ruhr Energy Conference 2010 (INREC 2010). The article has been submitted to "The Journal of Energy Markets" and is currently subject to review.

Introduction

The German power exchange EEX permitted negative price outcomes for the spot auction in autumn 2008. Since then, negative prices have occurred frequently, down to values as low as -500 €/MWh, see *Fig. 1*. This has triggered considerable attention and debates, not only in the energy trading business but even in the media. Paying money instead of demanding when selling a commodity to a counterparty appears counterintuitive. However, this is a natural consequence of the properties of the commodity power. Generally, this is because generation of power is of limited flexibility due to technical and regulatory constraints. When it is not possible for a generation facility to follow a demand slump closely, its generation is sold off with discount, even down to negative prices.
For the modeller negative prices pose a basic problem: the usual initial transition from prices to log prices is not possible. Surprisingly, (to the author's best knowledge) no solution dealing realistically with this issue has been proposed so far. Instead, the existing approaches are workarounds. The simplest being just the exclusion of all negative occurrences from analysis, or, introducing a shifted price with zero level at the observed minimum price (eg, -500 €/MWh), see Sewalt and De Jong (2003) and Knittel and Roberts (2001). The justifications given are that the workaround solution is supposed to be sufficient or that negative prices do not really have an influence because they are relatively rare. However, energy traders and business practitioners do not feel comfortable with this, see Sprenger and Laege (2009). Quite the contrary, they notice a significant impact of negative

prices on the value of their position, eg, when holding a structured Off-Peak position[2]. Looking again at the EEX price history of 2009 (*Fig. 1*), it indeed appears justified to consider negative or downward jumps as having replaced the upwards spikes in the times before economic downturn.
In this article a simple but effective way to deal realistically with negative (as well as positive) spot prices is developed. Namely, replacing the log transformation by the area hyperbolic sine transformation. An appealing feature of this approach is that it complicates stochastic price modelling not seriously. Basic cases can still be expressed by closed-form expressions. Moreover, indications are found that the log transformation is in general not well suited for power prices, but the area hyperbolic sine transformation is, independently of the occurrence of negative prices.

Negative power prices

In this section, a brief account of the economical and technical background of negative power spot prices is given. For detailed overviews on this subjects see, eg, Dettmer and Jacob (2009).
The German EEX spot exchange was the first market in Europe permitting negative prices. The Scandinavian Nordpool spot followed in the end of 2009. However, outside Europe, power markets permitting negative price are an established institution. The possibility of negative prices was built in their design from the beginning on. Two striking examples are ERCOT West Texas and AEMO, region South Australia. They exhibit very frequent, often strongly negative prices.
Negative prices at EEX occur when very low demand coincides with high supply. Low demand situations are constituted, eg, by public holidays and Sunday nights, amplified by slumps in industrial activity due to the economic crisis. A typical high supply situation is constituted by high wind power infeed. The grid operator, who has to take the wind production with priority, then bids this power at EEX for a negative price to achieve market clearing. Also, production from conventional power plants can be bid into the market for a negative price. This is, eg, when the plant produces above the marginal generation costs for matching the total demand in some hour. It can then nevertheless be economically optimal to leave the plant online for that hour when the loss from the negative price is smaller than the costs from a modified production schedule. An alternative production schedule would have to take into account a number of factors incurring costs, eg, ramping costs, start-up costs, costs for procuring energy from alternative sources during a mandatory downtime of the plant. Another negative price situation is constituted by grid transmission bottlenecks, eg, high wind production in Denmark cannot be sufficiently transported to hydro pump storages in northern Scandinavia.
Summarizing, negative prices occur because production of power has limited flexibility for economical, technical and regulatory reasons. When it is not possible for a power generation facility to follow a demand slump closely, its generation is sold off with discount. This is why it was consequential to allow for negative prices at EEX[3]. It was shown that this is

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[2] A profile with hourly varying loads in the low demand periods.

[3] Before, spot auctions at EEX frequently attained no intersection of bid and offer curves, necessitating partial market clearing only.

economically rational in order to optimize market clearing, see Viehmann and Sämisch (2009).
It remains to be seen if other markets follow the model of EEX and Nordpool. Especially, when considering that European power markets are getting more and more integrated. Interestingly, for another commodity, natural gas (delivered to NBP), there were some negative prices in the past. The situation was analogous to power: a production facility (a gas field) provided excess production in a low demand period which could not be sufficiently adapted for technical reasons.

Modelling concept

The area hyperbolic sine transformation

Stochastic modelling of power prices usually starts with log transforming $x = \ln p$ of the original prices p. Instead, I propose to replace the log transformation by the area hyperbolic sine transformation:

$$x = \sinh^{-1}\left(\frac{p-\xi}{\lambda}\right) \qquad (1)$$

, where ξ is an offset and λ a scale parameter. The behaviour of this function compared to the natural logarithm is depicted in *Fig. 2*. The most important property is the asymptotic log behaviour

$$\sinh^{-1}(p) = \ln\left(p + \sqrt{p^2+1}\right) \approx \operatorname{sign}(p) \cdot \ln(2\,|\,p\,|)$$

for $|\,p\,| \to \infty$ [4]. The log function is a good approximation for small $|\,p\,| > 2$ already. The positive and negative log-like parts are connected by an approximately linear part at $|\,p\,| \approx 0$. The transformation appears to be a natural choice because it preserves the log behaviour which is a proven method. However, it is now presupposed that the properties of prices $p < 0$ are a "mirror image" of $p > 0$. We recall a basic rationale for the log transformation: the volatility/variability of prices increases with the absolute price level: $dp \sim p$. This is why returns $\frac{dp}{p}(t)$ or log returns $\ln p(t) - \ln p(t-1)$ are studied. It appears to be plausible that this is also the case for negative power prices: $|\,dp\,| \sim |\,p\,|$ for $p < 0$. This can be substantiated by studying bid / ask curves of EEX spot auctions, see *Fig. 3*. As it is well known the curves are getting steeper when going to high prices and thus the spot price gets more variable/ spiky when the auction outcome (intersection of curves) moves into that price region. We see that the same holds for the negative price region. Note: this finding already rules out the „shift zero price level down to a negative lowest level" approach to the problem.

[4] The offset and scale parameter are temporarily omitted. The discussed properties do not qualitatively depend on these, of course.

Basic stochastic differential equation and price distribution

As a start, it is necessary to understand basic stochastic and statistical features of modelling built on the area hyperbolic sine function[5]:

$$x = \sinh^{-1}(p)$$

To this end, the same standard route as for the log transformation can be taken: analysing a basic model for the spot price process, the Ornstein-Uhlenbeck (OU) process:

$$dx = k \cdot (m - x)dt + \sigma \cdot dW \quad (2)$$

, with k and m the mean reversion rate and level, W the standard Wiener process and volatility σ.

By means of the Ito formula and standard relations for hyperbolic functions we get the stochastic differential equation

$$\frac{dp}{\sqrt{1+p^2}} = \left[k \cdot (m - \sinh^{-1} p) + 0.5 \cdot \sigma^2 \frac{p}{\sqrt{1+p^2}} \right] dt + \sigma \cdot dW \quad (3)$$

This is relatively similar to the SDE for the log case

$$\frac{dp}{p} = \left[k \cdot (m - \ln p) + 0.5 \cdot \sigma^2 \right] dt + \sigma \cdot dW$$

Both SDE generate the same dynamics for large p. Additionally, *(3)* exhibits a linear form $dp \approx L(p)dt + \sigma \cdot dW$ for small $|p|$. There is an explicit probability distribution solution to *(3).* The stationary distribution to the OU process for x:

$$x \sim N(m, \frac{\sigma^2}{2k}) \equiv N(m, \sigma_{OU}^2)$$

reads

$$f(p) = \frac{1}{\sigma_{OU}\sqrt{2\pi}\sqrt{1+p^2}} \exp\left(-\frac{(\sinh^{-1} p - m)^2}{2\sigma_{OU}^2} \right) \quad (4)$$

for p. f is known as the Johnson SU distribution (Johnson et al 1994). Summarizing, it has to be pointed out that switching from the log to the hyperbolic sine transformation has not qualitatively complicated the analysis. We still get a closed-form solution, the Johnson distribution replacing the log-normal distribution.

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[5] Stochastic processes involving the hypebolic sine have been employed before to model volatility smiles. However, prices in that context are strictly positive and described by ordinary returns, see Brigo et al (2003) and Carr et al (1999).

Fig. 4 depicts typical examples of the Johnson distribution for different means and variances, overlaid with log-normal distributions with the same mean and variance with respect to p (not feasible for negative means). Additionally, empirical histograms of EEX spot prices are displayed. For large (positive) mean and large variance, both the Johnson and log-normal distribution cannot be distinguished, as expected. Empirically, this case corresponds to the histogram of typical high price hour types (working day, early evening). A right-skewed, upward spiking (fat tail) behaviour is observed. Going to typical lowest price hour types (Sundays and holidays, deep night), the opposite is observed: left-skewed and downward spiking. This is (qualitatively) matched by a specification of the Johnson distribution, but of course not by a log-normal type distribution.

Spot price modelling

It now needs to be analysed how the hyperbolic sine transformation integrates with state-of-the-art spot price models and if the simulation results are satisfactorily able to reproduce the spot price histories with negative prices.
As the state-of-the-art spot price model the "independent spike model" is taken, see De Jong (2006) and De Jong and Schneider (2009). The model produces daily spot prices by means of a three-regime switching process, regimes for normal prices and upward and downward spikes. This is the model formulated as a discrete time process:

Mean-reverting regime M: $dx_t^M = \alpha(\mu - x_{t-1}^M) + \sigma \cdot \varepsilon_t$

Spike regimes: $x_t^S = \mu + \sum_{i=1}^{n_t+1} Z_{t,i}$

High spike regime: $Z_{t,i} \sim N(\mu^H, \sigma^H), \quad n_t \sim POI(\lambda^H), \quad \mu^H > 0$

Low spike regime: $Z_{t,i} \sim N(\mu^L, \sigma^L), \quad n_t \sim POI(\lambda^L), \quad \mu^L < 0$

Markov transition matrix: $\Pi = \begin{bmatrix} 1-\pi^{MH}-\pi^{ML} & \pi^{MH} & \pi^{ML} \\ \pi^{HM} & 1-\pi^{HM} & 0 \\ \pi^{LM} & 0 & 1-\pi^{LM} \end{bmatrix}$

The normal price process is an OU process with mean-reversion level and rate μ and α. The (log) prices in the spike regimes are directly drawn from fat-tailed distributions (Poisson compounded Gaussians). For further explanations, especially on the parameter estimation method, see the articles.
Besides applying the area hyperbolic sine transformation it is necessary to remove the deterministic component from the (daily) prices:

$$s_t + x_t = x'_t = \sinh^{-1}(p_t)$$

The day type specific components are captured by dummy variables $1_D(t)$, D={Mon, .., Fri, Sat/Bridge days, Sun/holidays} being the day type set. Seasons and trends are captured by a moving average $MA(t) = x'(t') * G(t-t')$, $G(t-t')$ being a Gaussian convolution kernel with standard deviation of 20 days. This choice of the standard deviation is appropriate to capture winter-summer effects as well slowly moving fundamentals. Additionally, for the moving average calculation spikes from the price series are removed beforehand, applying a standard 3 standard deviation iterative method. This is because the $MA(t)$ is meant to refer to the mean-reverting level of the normal price process. The coefficients of the components $b \cdot MA(t)$, $d_i \cdot 1_D(t)$ are finally estimated by linear regression.

Case EEX

As a first case study, the EEX hourly price history from Oct 2008 – the point in time from which on negative prices have been possible – till Jan 2010 are modelled. The modelling consists of two stages. First, the independent spike model is employed to produce scenarios of daily prices. Secondly, the daily prices are endowed with hourly profiles by means of a historical price sampling and re-scaling method. *Fig. 5* shows the historical daily as well as hourly prices of the period. The two day which visibly stick out from the daily price series have been removed (Oct and Dec 2009). These days, the only ones so far at EEX with a negative price of the full day, are exceptionally strong downward spikes. It does not yet appear to be meaningful to define an own regime or distributions for this kind of extreme event at this point in time, based on two examples only so far. Nevertheless, there are still many days in the truncated history with quite negative hourly prices.
The parameter estimation is accomplished as described above and the determined parameters are shown in *Tab. 1* . Up spiking is marginal, but down spiking is pronounced (see the values of the normal-to-spike regime jumping probability π^{MX} and the Poisson parameters λ). This is, of course, plausible, since as stated above, downward jumps have replaced the upwards spikes in the times before economic downturn.

Scaling of hourly profiles

A simulated daily price is converted into 24 hourly prices by drawing an appropriately similar day from history and matching it to the day's price by a scaling transformation. Beforehand, the EEX historical days are tagged by month, day type D and spike regime. The historical spike regime characterisation is provided by the parameter estimation algorithm. It outputs a triple of probabilities[6] for each day, indicating this day's price being in one of the regimes. For simulated day with price p_t^{sim} the spike regime is definite and, so, among all historical days with matching month and day type a historical regime probability weighted random choice is carried out. The historical hourly profile S_h^{hist} then is transformed to $S_{t,h}^{sim}$ such that average price equals the day's price, $\overline{S_{t,h}^{sim}} \equiv p_t^{sim}$. In the past, with positive prices only, there was a simple way to transform: multiplying each hour by a factor $C = \frac{p_t^{sim}}{\overline{S_h^{hist}}}$ [7]. Thereby, in terms of hourly price differences, Offpeak hour prices are shifted a bit, Peak hours prices strongly, which is realistic. However, when some hours of S_h^{hist} are negative, the transformation yields an implausible result. Imagine the case of a "stronger market" in simulation than in history, $p_t^{sim} > p_t^{hist}$. The factor transformation, the C being >1, would

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[6] The estimator's output are likelihoods which are converted into probabilities

[7] A similar sampling re-scaling method has been employed in Culot et al (2006) for power spot market modeling (no negative power prices in Europe at that time).

shift positive hour upwards which is correct. However, negative hours would take on even more negative prices which is of course the opposite of what is observed on a real market. The search for a plausible new transformation has been guided by the following considerations. The factor transformation preserves log normal distributions, which is the basic price distribution for the positive prices only world, assuming a log transformation on the prices and a basic OU model for the price process. So, if all hours of a historical day are distributed log-normally, $\ln S_h^{hist} \sim N(m_h^{hist}, \sigma_h^{hist})$, scaling with the factor C yields new log normal distributions $\ln S_h^{sim} \sim N(m_h^{hist} + \ln C, \sigma_h^{hist})$ with equally shifted means. This principle is transferred to the hyperbolic sine transformation and Johnson distribution world. We again consider a sample day. Find a δ such that

$$\sinh^{-1} S_h^{hist} \sim N(m_h^{hist}, \sigma_h^{hist}) \quad \rightarrow \quad \sinh^{-1} S_h^{sim} = \delta + \sinh^{-1} S_h^{hist} \sim N(m_h^{hist} + \delta, \sigma_h^{hist})$$

and $\overline{S_{t,h}^{sim}} \equiv p_t^{sim}$. With $\sinh^{-1} \Delta \equiv \delta$ we get

$$S_h^{sim} = \sinh\left(\sinh^{-1} \Delta + \sinh^{-1} S_h^{hist}\right) = \Delta \cdot \cosh\left(\sinh^{-1} S_h^{hist}\right) + S_h^{hist} \cdot \cosh\left(\sinh^{-1} \Delta\right)$$

by means of an addition theorem. In contrast to the log normal case the sine hyperbolic case cannot explicitly be solved for Δ. Instead, we simplify by noticing $|\sinh x| \cong |\cosh x|$, an approximation which is very good already for small $|x|$, eg, $|x| \geq 2$. Thus, we can approximate:

$$S_h^{sim} = \Delta \cdot \cosh\left(\sinh^{-1} S_h^{hist}\right) + S_h \cosh\left(\sinh^{-1} \Delta\right) \approx \begin{cases} \Delta \cdot S_h^{hist} + S_h^{hist} \cosh\left(\sinh^{-1} \Delta\right) & \text{for } S_h^{hist} > 0 \\ \Delta \cdot (-S_h^{hist}) + S_h^{hist} \cosh\left(\sinh^{-1} \Delta\right) & \text{for } S_h^{hist} < 0 \\ 0 \quad \text{for } S_h^{hist} \approx 0 \end{cases}$$

The approximation does in fact not need to be very exact because for the purpose of solving for Δ we only need price means:

$$p^{sim} = \overline{S_h^{sim}} \approx \frac{1}{24} \sum_{S_h^{hist} > 0} S_h^{hist} \cdot (\Delta + \cosh \sinh^{-1} \Delta) + \frac{1}{24} \sum_{S_h^{hist} < 0} S_h^{hist} \cdot (-\Delta + \cosh \sinh^{-1} \Delta)$$

$$= \Delta \cdot \frac{1}{24} \sum_h |S_h^{hist}| + \cosh \sinh^{-1} \Delta \cdot \frac{1}{24} \sum_h S_h^{hist}$$

resulting in

$$\Rightarrow \overline{S_h^{sim}} \approx \Delta \cdot \overline{|S_h^{hist}|} + \cosh \sinh^{-1} \Delta \cdot \overline{S_h^{hist}}$$

, which can easily be solved numerically for Δ.

Fig 6 shows two examples of applying the transformation. For an hourly profile with positive prices only, the new transformation and the old factor transformation yield basically the same result (left). For the profile containing negative prices the new transformation only yields a plausibly looking result (right).

Simulation results

The simulated trajectories, for daily as well as for hourly prices, resemble the historical trajectories very well, see *Fig 7*. The goodness is confirmed by comparing price statistics and their moments, *Fig. 8* and *Tab 2*. Only the kurtosis of the historical hourly prices is not fully captured which is, however, a typical issue affecting virtually all power spot price models.

Case ERCOT West

ERCOT West is a power grid zone in Western Texas. This market region is specific insofar as it has quite a high share of wind power production, in conjunction with low population density and restricted transmission to other regions. The price plots, *Fig 9*, show extremely volatile and spiky quarter-hourly prices, strong spikes occurring for positive as well as negative prices. Even the daily prices are remarkably volatile and spiky, with a lot of days exhibiting completely negative average price.
Here, the aim is to reproduce the daily prices from Jan 2008 – Feb 2010 (the history is chosen to be roughly the same time period as for EEX). Modelling is restricted to daily prices because the quarter-hourly structure is too irregular to be captured with the means applied in this work. Further, it can be seen from the price plot that the time series is structurally changing, being significantly more spiky and negative in the first half of the period. This will be subjected to a simplification, see below, since the overarching aim of this work is to show how to generally integrate negative prices into stochastic spot price modelling, not yet dealing with very specific features.

Extended area hyperbolic sine transformation

When applying the hyperbolic sine transformation $x_t^{'} = \sinh^{-1}(p_t)$ as in EEX case above, we get an unsatisfying result, see *Fig 10*. Compared to the original time series the downward spikes are strongly amplified whereas the upward spikes suppressed, comparatively. The transformation does not preserve the characteristics of the data[8]. This is because the $\sinh^{-1}(p)$ transformation effectively, compared to range of price levels p, behaves step-like, see *Fig 11*. The main body of prices are around a mean level at $p > 0$ therefore gets compressed, the downward spikes crossing the axis gets torn away. So, one should look for a less disruptive transformation around $p = 0$. To this end the original form of the transformation *(1)* is reconsidered:

$$x' = \sinh^{-1}\left(\frac{p - \xi}{\lambda}\right)$$

where offset ξ and scale λ can be chosen arbitrarily. For $\xi \neq 0$ and $\lambda \neq 1$ the stochastic process characteristics stay the same. The Johnson distribution is still the basic solution to the OU process. Here, $\xi = 20$ and $\lambda = 30$ are taken, see *Fig 11*. This shifts the turning point to the mean level of prices and widens the linear part of the transformation. Now, the transformed time series $x_t^{'}$ looks much more "natural", see *Fig 10*. The choice of values for

[8] Nevertheless, a parameter estimation for the spot price model was carried out on this data. This is, however, only successful when the model's spike regimes are endowed with a more extremer and fat-tailed distribution.

ξ and λ had been ad hoc, guided by the appearance of the data[9]. In a naive fashion, the parameterization of the transformation here is somewhat similar to the idea of the Box-Cox transformation, providing an appropriate local transformation for every price level p. Interestingly, Weron (2008) has already remarked – though for a set-up with positive power prices only – that the log transformation does not appear appropriate for power prices. It produces artificial downward spikes for $p \approx 0$, although those prices are not judged as extraordinarily jumping from an expert's view. He then employs no transformation to the data at all (thus, equivalent to the trivial linear transformation). This, however, produces the drawback of leaving spikes undamped and requires therefore special distributions for the spike regime.

The transformation elaborated in this work combines the advantages of both the log and the linear approach. The choice of ξ and λ produces a linear regime of the transformation encompassing the main, non-spiking range of prices. In the positive and negative spike price range, however, the transformation behaves again log like, allowing for the "conventional" modelling.

Why is the modified area hyperbolic sine transformation working well? It is posited that this is because the transformation function reflects fundamental economics of power prices. The linear range of the function coincidences with the middle part of the generation stack (merit order curve). There, we find the variable power generation cost function of mid-load producing plants (hard coal, CCGT). This function increases with little curvature, approximately linear with demand, and is not sensitive to changes in demand or supply. Contrasting, the upper generation stack, consisting of plants with high costs (eg, gas turbines), is strongly curved upwards and sensitive to load changes. This can cause upward spikes. The lower end of the stack contains generation with variable costs close to 0, amongst others, base load plants (lignite, nuclear) and wind generation. There is no curvature, but the inflexible, must-run characteristics of these generation types nevertheless makes them sensitive to load changes and can lead to spot auction offers far below production costs (as shown above).

Estimation and simulation results

Concerning the initially described irregularities in the ERCOT price history some modifications and simplifications have been made in parameter estimation and simulation, compared to the standard method for the EEX case.

The change of spike intensity over time is not accounted for (by, eg, parameters changing over time). Instead, a constant parameter set is estimated from a "homogenized" history. This means, only the distribution of historical prices is fitted, discarding the original day-to-day estimation based on a Bayesian scheme. Since we know that the model price distribution is a mixture of a normal (the mean-reverting regime) and two compound normal distributions (the spike regimes), most spot price model parameters can be estimated by maximum likelihood estimation on the price distributions. One further step is needed. We need the day-to-day regime switching probabilities for the Markov matrix, but only know the total probability of the process being in the spike regime from the mixture-of-distributions weights. To this end the average time the process stays in the spike regime is estimated. All

[9] Clearly, in the future there should be a mathematical method in place for determining the parameter values, see Conclusions.

days' prices are labelled as spikes if their normalized likelihoods of belonging to the spike price distributions is >0.5. The parameters are shown in *Tab. 3*. One spike regime was discarded by the estimation. Both upwards and downwards spikes are captured by a single, wide distribution, which seems a more parsimonious modelling of the strongly volatile price dynamics than two distinct spike modes. The pronounced spikiness can be seen from the high value of about 20% for the probability of jumping from the normal into the spike regime.

Another model modification had to be applied to the simulation regarding the significant inhomogenity of the deterministic price level (which, as for the EEX case, is estimated and then re-added to the simulation stochastic process). If in the simulation a price level combines with a spike in way that does not correspond to a historical situation (as described above: all stochastic parameters including spikes are made uniform over the complete period), some spikes with unrealistically large amplitudes are produced ($\approx 0.5\%$ of all prices). These are capped and floored to the historically observed absolute maximum and minimum. Otherwise, two or three of those events strongly distort the moments of the simulated price distribution.

The simulated trajectories, see *Fig 12*, reveal that the "average dynamics" is matched. The effect of artificial parameter homogeneity can be observed: some spikes occur at (historically) wrong times (other trajectories match better). The moments of simulated and historical price distributions match satisfyingly, see *Tab 4* (although less well than for the EEX case).

An application example: a simple option on the spot price

Knowing that the power market can exhibit very low or even negative prices in the future a market participant seeks to protect himself against the risk. Eg, a power generator can purchase a strip of, eg, daily put options on the spot price to secure minimum revenues for his production. The payoff function for the option for day T is the expected value

$$V = \mathrm{E}\left[\max\left[(K - p(T)),0\right]\right]$$

, given a strike K and maturity time T[10]. Here, it is shown how this put option value can be calculated in the hyperbolic sine framework and how values differ from a conventional log approach.

In order to make the central effect clear the initially introduced simple OU process[11] is again employed, omitting a specific spike regime treatment. As shown above the stationary price distribution $p(T)$ is the Johnson distribution ($p \in \Re$):

$$f_{SU}(p) = \frac{1}{\lambda\sigma_{SU}\sqrt{2\pi}\sqrt{1+\left(\frac{p-\xi}{\lambda}\right)^2}}\exp\left(-\frac{\left(\sinh^{-1}\left(\frac{p-\xi}{\lambda}\right)-m_{SU}\right)^2}{2\sigma_{SU}^2}\right)$$

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[10] For reasons of simplicity, an interest rate of 0 is assumed.

[11] A deterministic component is omitted.

, here formulated for the full area hyperbolic sine transformation $x = \sinh^{-1}\left(\frac{p-\xi}{\lambda}\right)$. When starting out with the log transformation $x = \ln p$, ($p > 0$), the usual log-normal distribution

$$f_{LN}(p) = \frac{1}{\sigma_{LN}\sqrt{2\pi}\,p}\exp\left(-\frac{(\ln p - m_{LN})^2}{2\sigma_{LN}^2}\right)$$

results. For V there are explicit formulas for both the log and the hyperbolic case. For the log case it is the usual expression:

$$V_{LN} = \Phi(d_{LN})$$

, with Φ being the cumulative standard normal distribution function and $d_{LN} = \frac{-m_{SU} + \ln K}{\sigma_{LN}}$. For the hyperbolic case we get:

$$V_{SU} = \ldots$$
$$-0.5\cdot\lambda\cdot\exp\left(0.5\cdot\sigma_{SU}^2\right)\cdot\left[\exp(m_{SU})\cdot\Phi(d_{SU}-\sigma_{SU}) - \exp(-m_{SU})\cdot\Phi(d_{SU}+\sigma_{SU})\right] - \xi\cdot\Phi(d_{SU}) + K\cdot\Phi(d_{SU})$$

with $d_{SU} = \frac{-m_{SU} + \sinh^{-1}\left(\frac{K-\xi}{\lambda}\right)}{\sigma_{SU}}$.

We assume that we know the maturity spot price $p(T)$ distribution's expected value $\mathrm{m} = \mathrm{E}[p(T)]$ and standard deviation σ [12]. We then can fit both the log-normal and the Johnson distribution, comparing the option values based on either assumption. We have

$$\sigma_{LN} = \sqrt{\ln\left(\frac{\sigma^2}{m^2}+1\right)} \quad , \quad m_{LN} = \ln m - 0.5\cdot\sigma_{LN}^2$$

And

$$\sigma_{SU} = \sqrt{\ln\left(\sqrt{2\frac{\sigma^2}{\lambda^2} + 2\frac{(m-\xi)^2}{\lambda^2} + 1 + \frac{(m-\xi)^4}{\lambda^4}} - \frac{(m-\xi)^2}{\lambda^2}\right)} \quad ,$$

$$m_{SU} = \sinh^{-1}\left(\frac{m-\xi}{\lambda}\cdot\exp\left(-0.5\cdot\sigma_{SU}^2\right)\right)$$

An exemplary calculation is carried out with $\mathrm{E}[p(T)] = 40$, $K = 10$, $\sigma \in (5,10,\ldots,50)$ and $\xi = 20$, $\lambda = 30$. The results are shown in *Fig 13*. To begin with, V_{SU} is much bigger than

[12] From the forward curve and a market volatility indicator.

V_{LN} for large σ. This is trivial because $f_{SU}(p)$ then comprises negative prices as opposed to $f_{LN}(p)$. However, $V_{SU} > V_{LN}$ holds for all σ, both valuations do not coincidence for the case of a positive prices only distribution. The explanation comes from the form of the price distribution. The Johnson distribution attributes a bigger mass to small prices $p \approx 0$. This result indicates that the option valuation based on the hyperbolic sine framework should always be considered to avoid undervaluation, being aware of the fact that every power spot market exhibits a substantial amount of prices $p \approx 0$, even if negative prices are not permitted.

Concluding remarks

It is increasingly recognized that negative prices are an inherent feature of the commodity power. Constraints on the supply side limit the flexibility of a generation facility and forces to sell off production with discount in case of demand slumps. This is why permitting negative bid, offers and auction results in the power spot market is economically reasonable.

Several non-European power markets pioneered, the European markets EEX and Nordpool have introduced negative prices recently. It is an exciting question if with the ongoing European market integration the concept is going to spread to other regional markets. Eg, a formal spot market coupling is on the way to be realized between Germany/Scandinavia and France/Belgium/Netherlands.

It is obvious that under these circumstances the so far prevailing tendency to deal with the "problem" by "workarounds" or exclusion/negligence is to be abandoned and sound integration into the various stochastic power price modelling frameworks needs to be achieved.

The solution proposed in this article is to replace the usual initial log transformation of prices by the area hyperbolic sine transformation. Several arguments were provided to support this approach. Firstly, the choice is natural, leaving the transformation for positive prices almost unchanged and mirroring the logarithm feature to the negative price axis. This choice is equivalent with the finding that the price dynamics in negative price region is analogous to the one in the positive region, volatility basically depending on the absolute price level. Secondly, combining the hyperbolic sine transformation with stochastic models does not significantly increase the difficulty of treatment compared to the log case. Interestingly, an analogue to the log-normal distribution as a theoretical "basic distribution" is found: the Johnson distribution. This is consistent with the difficulty argument since it is also a closed form expression. This convenient characteristics has been exploited for the valuation of an option in the last section.

Thirdly, the area hyperbolic sine transformation exhibits a connection to fundamentals of power prices, the generation stack and its production cost function. This is, eg, constituted by a linear regime of transformation for the moderate (normal) price regime. It has been noted before (Weron, 2008) that the log transformation does not work well in that price regime, producing artificial distortions. Here, it was found that the area hyperbolic sine transformation as an effective combination of linear and log transformation performs well, preserving the characteristics of the data.

Summing up, it is posited that the introduction of the area hyperbolic sine transformation is the natural step for power price modelling as response to the permit of negative prices at power spot exchanges. This is also supported by the fact that the transformation can be applied to power markets with positive prices only equally well.

There is one important issue for negative power prices to be dealt with in future work. The power markets are not yet in a “steady state” regarding the handling of negative price occurrences. The changing nature of these occurrences was an issue in this work for the case of ERCOT West. It is also obvious when comparing EEX 2010 data with 2009. The frequency and severity of negative occurrences is strongly reduced in 2010 compared to 2009. This is on the hand attributed to a learning effect of energy traders, eg, changed power plant production schedules. On the other hand, this is due to changing regulations concerning renewable energy (eg, wind) marketing. So, it is difficult to anticipate currently how a steady market state in the future could look like and the energy trading business is reliant on close-to-the-market modelling activities on this subject.

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Figures and tables

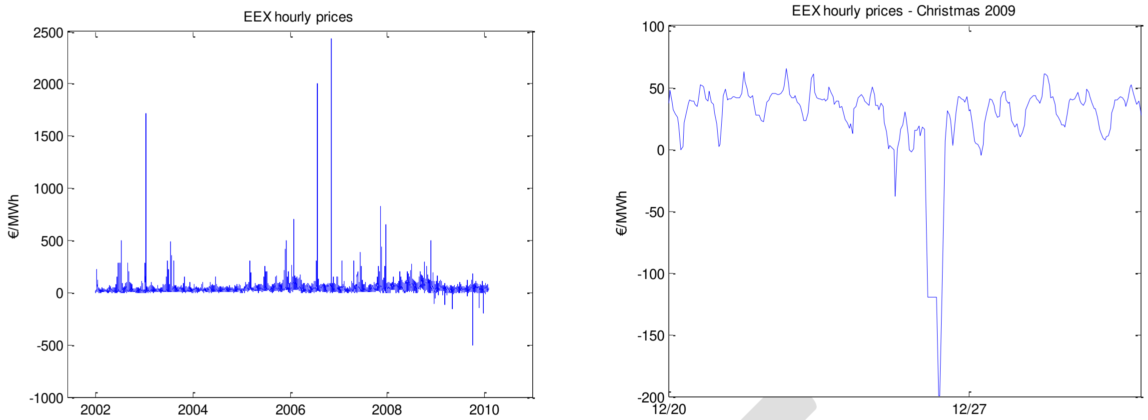

Fig. 1: The history of EEX hourly prices.

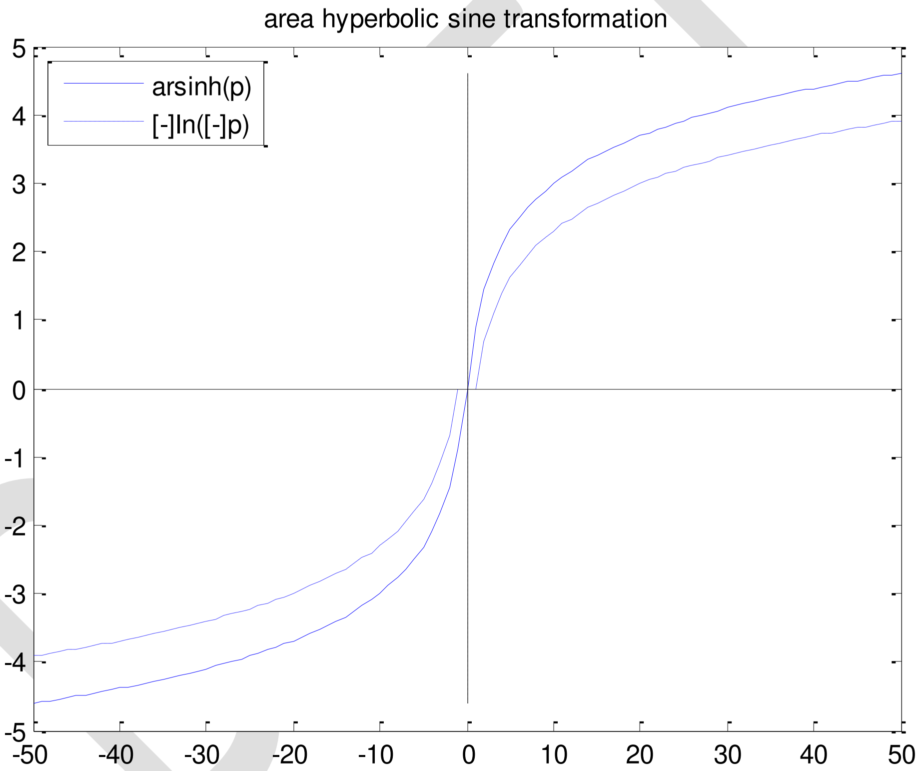

Fig. 2: Price transformation $\sinh^{-1}(p)$ along with $\ln(p)$ and $-\ln(-p)$ for comparison.

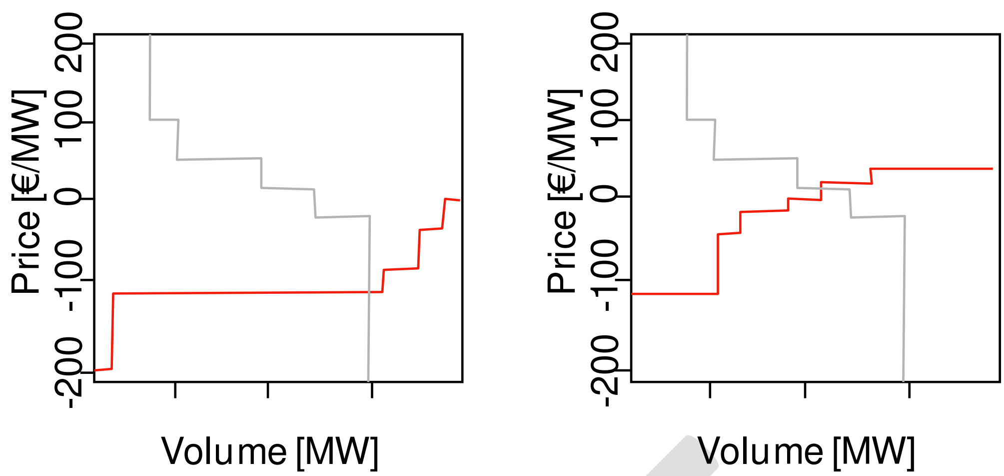

Fig. 3: Bid/demand (grey) and ask/supply (red) curves of hours 1 (left) and 14 (right) for the EEX auctions of Dec 26th, 2009 (schematically re-drawn).

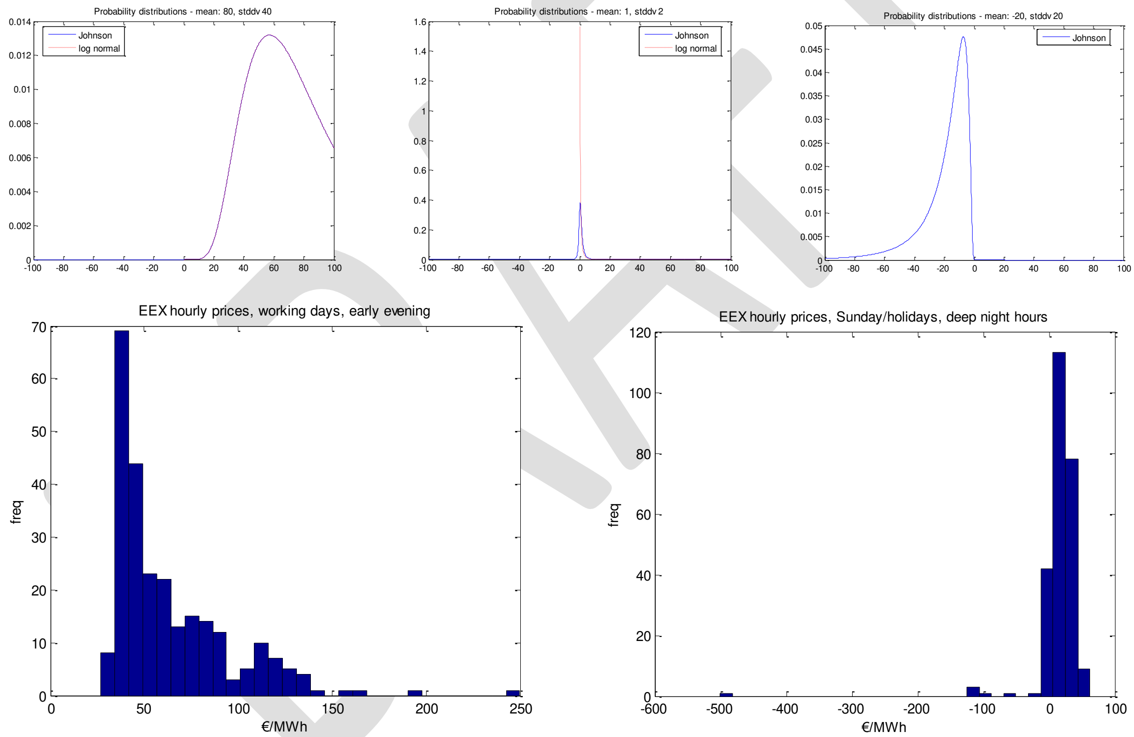

Fig. 4: Qualitative comparison of empirical spot price distributions and theoretical distributions. Theoretical distributions: Johnson and log-normal with same mean and standard deviation (except for the negative mean case, right).

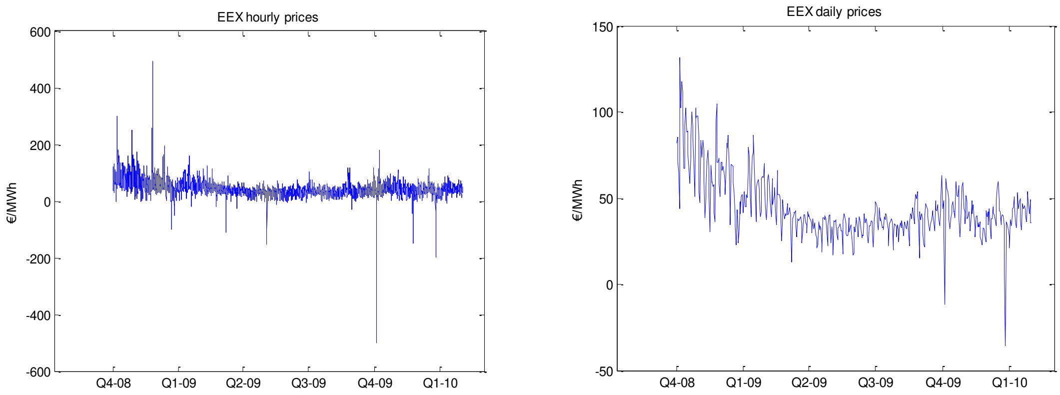

Fig. 5: EEX hourly and daily price history from Oct 2008 till Jan 2010, time period as taken for modelling.

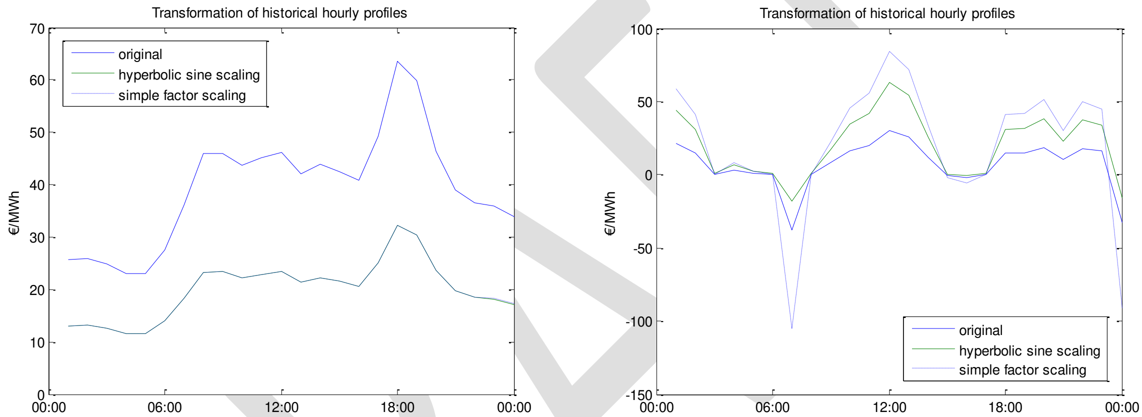

Fig 6: Scaling transformations of historical hourly price profiles EEX to new daily average prices (20 €/MWh for both cases). Left: simple factor scaled profile hidden by practically identical hyperbolic sine scaled profile.

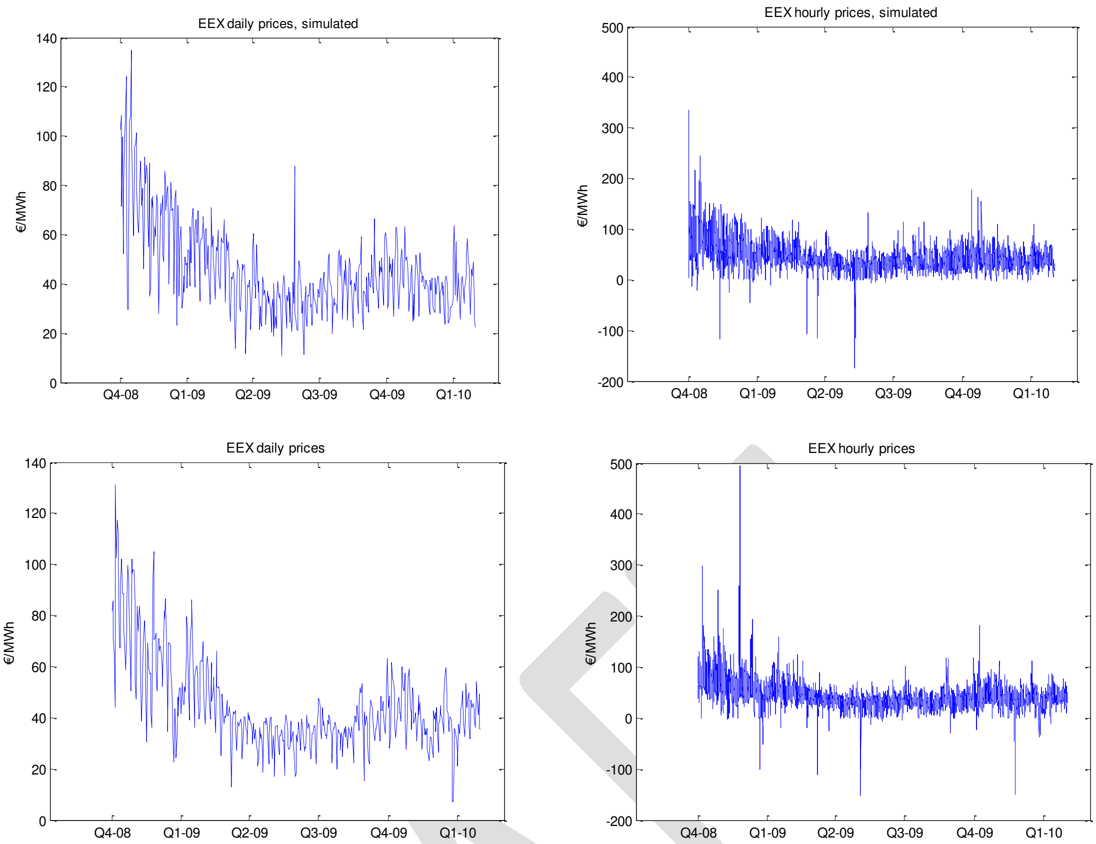

Fig 7: Exemplary simulated and historical trajectories.

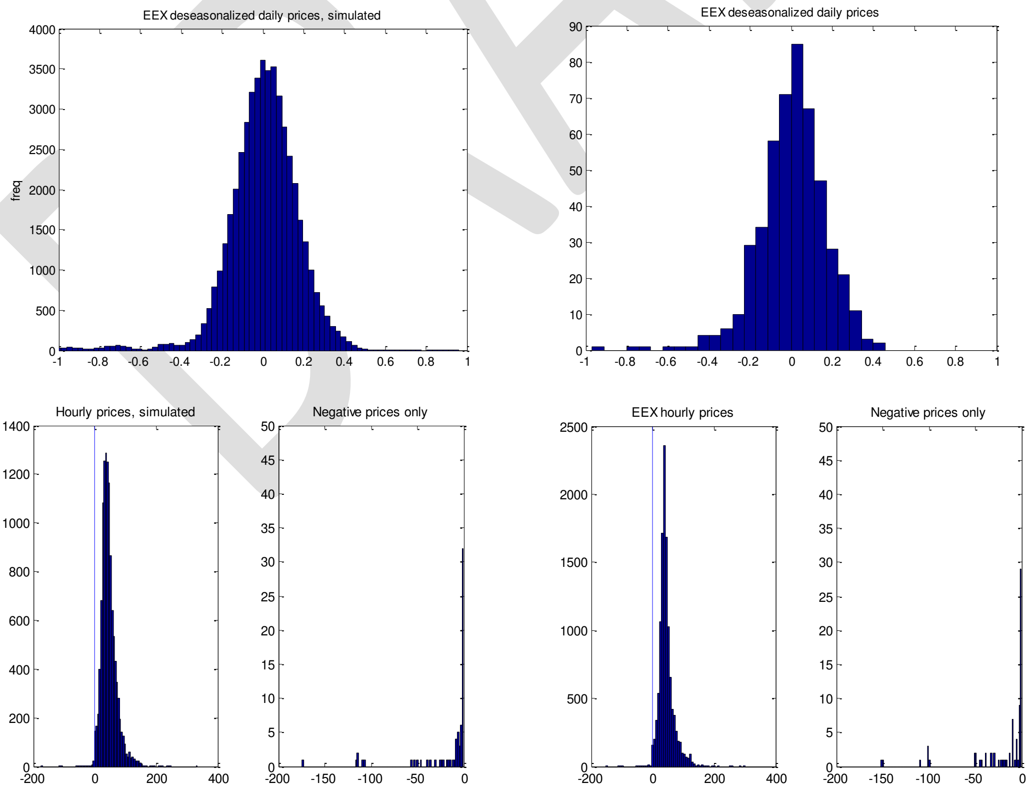

Fig 8: Histograms of daily (x) and hourly prices (the above exemplary simulated scenario only for hourly prices). Left: simulation, right: historical.

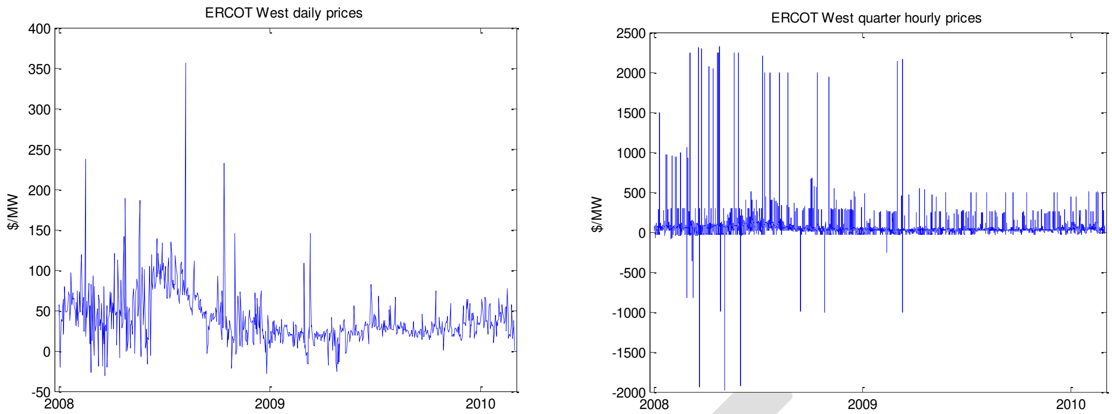

Fig 9: EROCT West market clearing price history Jan 2008 – Feb 2010.

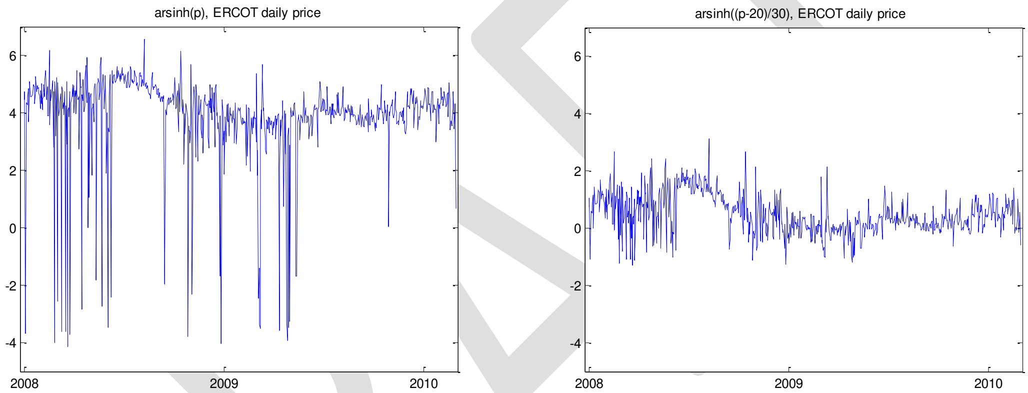

Fig 10: Applying the arsinh transformation $x' = \sinh^{-1}\left(\dfrac{p-\xi}{\lambda}\right)$ to ERCOT daily prices. Left: $\xi = 0, \lambda = 1$. Right: $\xi = 20, \lambda = 30$.

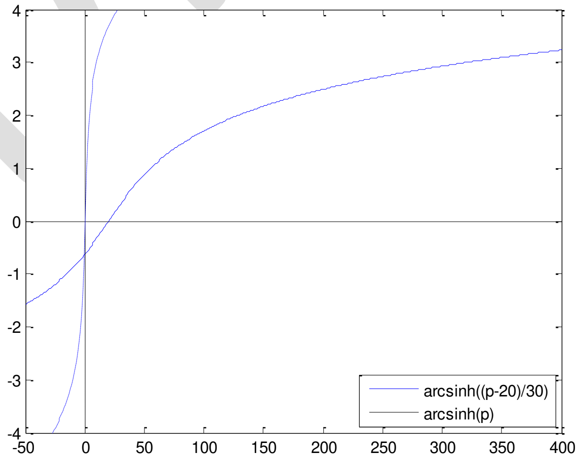

Fig 11: Depiction of arcsinh transformations as applied above.

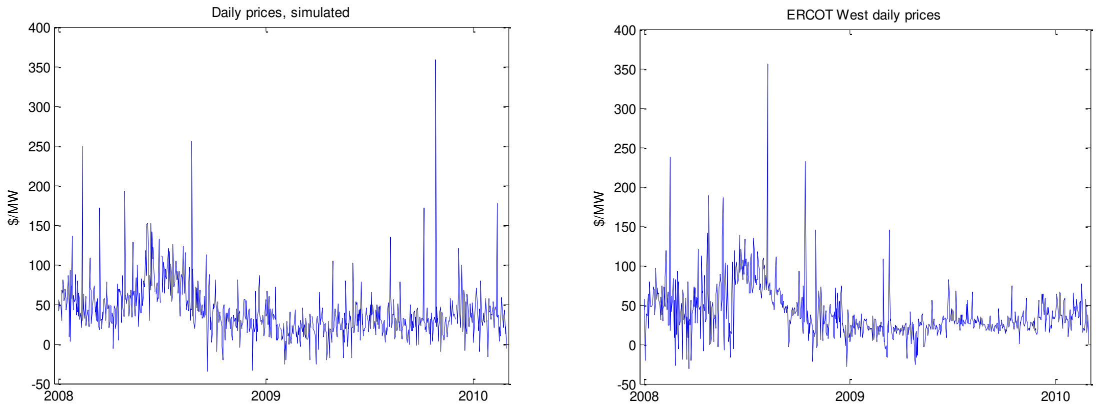

Fig 12: Exemplary simulated and historical trajectory ERCOT.

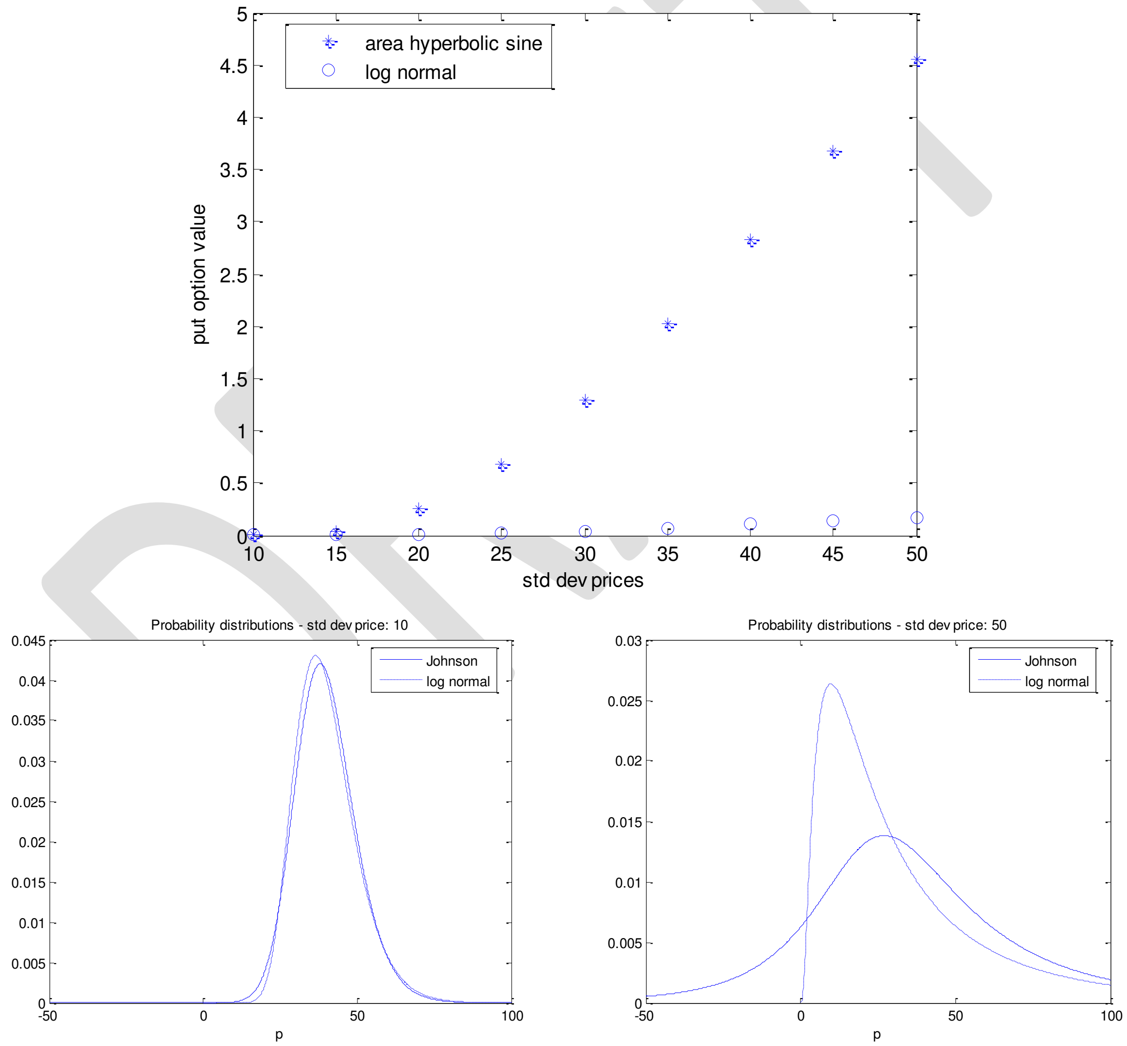

Fig 13: Top: Value of the put option over standard deviation $\sigma(p(T))$ of the spot price at maturity. Bottom: Johnson and log-normal distribution of $p(T)$ for two exemplary $\sigma(p(T))$.

EEX					
Time-series parameters:				Switch probabilities	
Normal regime	α	0,42		from N to H	0,95%
	µ	0,02		from H to N	71,15%
	σ	0,11		from N to L	1,93%
High-spike regi	µH	0,35		from L to N	78,50%
	σH	0,06			
	λH	0,10			
Low-spike regir	µL	0,24			
	σL	0,04			
	λL	2,00			

Tab. 1: Parameter estimates for the (daily) spot price model, case EEX.

EEX		
	historical prices	simulated prices
daily arsinh prices, deseasonalized		
mean	0,00	
standard deviatio	0,18	0,18
skewness	-1,88	-1,56
kurtosis	13,03	11,19
daily prices		
mean	44,74	44,86
standard deviatio	17,69	17,68
skewness	1,51	1,44
kurtosis	5,92	5,93
hourly prices		
mean	44,74	44,83
standard deviatio	24,46	23,94
skewness	1,89	1,57
kurtosis	20,02	10,36

Tab 2: Statistics of historical and simulated prices EEX. Simulation figures aggregated from 100 scenarios.

ERCOT					
Time-series parameters:				Switch probabilities	
Normal regime	α	0,59		from N to S	21,39%
	µ	0,03		from S to N	60,62%
	σ	0,32			
Spike regime	µS	0,00			
	σS	0,74			
	λ	0,50			

Tab 3: Parameter estimates for the (daily) spot price model, case ERCOT.

ERCOT		
	historical prices	simulated prices
daily arsinh prices, deseasonalized		
mean		0,04
standard deviation	0,53	0,52
skewness	-0,43	0,19
kurtosis	6,49	5,51
daily prices		
mean	39,40	40,63
standard deviation	32,40	32,83
skewness	2,50	2,73
kurtosis	18,39	19,78

Tab 4: Statistics of historical and simulated prices ERCOT. Simulation figures aggregated from 100 scenarios.