Determination of Profit and Loss Sharing Ratios in Interest-Free Business Finance

Hasan, Zubair

International Islamic University of Malaysia

1985

Online at https://mpra.ub.uni-muenchen.de/3013/
MPRA Paper No. 3013, posted 03 May 2007 UTC
Determination of Profit and Loss Sharing Ratios in Interest-free Business Finance

ZUBAIR HASAN
Associate Professor of Economics,
Zakir Husain College, University of Delhi, India

ABSTRACT. Current models of profit-and-loss-sharing (PLS) financing assume that the ratio in which profits are shared may be different from the ratio the financier's capital bears to the total capital of the firm, which is the ratio in which losses must be shared. This can be better justified as a growth promoting policy than as an instrument for ensuring distributive justice.

In a system where interest and PLS financing co-exist, the aggregate profit-sharing ratio is a function of the overall rate of return on investment, rate of interest, degree of leverage, and risk premium. PLS financing is more profitable to financiers in the long run than interest financing.

Introduction

Islam prohibits interests. Participation in the financing of a business is possible only on a profit and loss sharing (PLS) basis. The ramifications of this change, at the theoretical and applied levels, are in a process of continuous exploration in Islamic economics. Discussions on how to structure and operate a sharing system have not been infrequent in the literature. Yet surprisingly, the basic question of the determination of profit and loss sharing ratios and some of the related issues remain by and large unresolved. Cursory explanations have been many, but a formal integrated analysis of the subject has rarely been attempted.

The present paper makes a preliminary effort at filling this gap in the context of an economy where finance is available both on the profit sharing basis and on the basis of interest. Its main objectives are:

(i) To analyze the process of ratios determination at macro and micro levels, showing how demand and supply of investment funds play their role, and changes in different variables may affect the equilibrium ratio, and
(ii) To indicate some of the implications of the sharing arrangement, in particular the examination of its profitability for the financiers as compared to lending on interest.

Broadly speaking, the ratio question has three main aspects: juridical, economic, and accounting. Each is important and intimately related to the others. However, we shall primarily be concerned with the first two aspects.

There has been some controversy on the issue of profit and loss sharing ratios in Islamic jurisprudence.\(^1\) We shall briefly present below the different viewpoints on the subject without going into the finer juristic points. We shall explain the position contemporary Islamic economists have taken as a matter of near agreement, and which we have also maintained, for constructing a sharing model. This position allows, a 'divergence' between the profit and loss sharing ratios of a financier. This is justified on the ground that the financier supplies only capital whereas the owner of an enterprise, who has also invested his own capital in the enterprise, supplies both capital and work. But a problem arises when this principle is applied to a modern joint stock company. The owners, in this case the share holders, supply only capital, work being done by hired labour including managers. Should those supplying finance to such a company be allowed to share its profits in a ratio different from the one in which they share its losses?

**Current Position**

Profit and loss sharing - in short PLS-mode is, have been designed, especially in the area of interest-free banking, on a principle which in fact is characteristic of *mudarabah*. Here one assigns his resources to the productive sector through an entrepreneur in exchange for a share in the return on that investment, financial losses, if any, falling entirely on the former (Ariff, p.6). Jurists generally agree that in *mudarabah* the PLS ratios of the financier are to be different and capital can be contributed in the form of money or money's worth. Furthermore, the financier has no right to participate in the business decision making (Ghazali, pp.84-85).

However, modern business situations are in most cases different from pure *mudarabah* in the sense that owners of the firm - the entrepreneurs - contribute a substantial portion of the capital \(K\) employed in its production activity. Because of this invariably 'mixed' character of real world cases, a straightforward application of the juridical principles of *shirkah* or *mudarabah* becomes difficult.

One comes across, in a broad way, two types of opinion about *shirkah* arrangements among the jurists. The Hanafis and Hanbalis with some supporters among the Shafiis maintain that while the suppliers of capital will always bear the loss in proportion to their financial contribution, they are free to negotiate among themselves for their profit sharing ratios. Evidently the implication is that their profit and loss sharing ratios could be different. This we may term as the 'ratio divergence' approach. It makes no distinction between *shirkah* and *mudarabah* so far as the profit and loss sharing principle is concerned. In contrast, the Malikis and most of the Shafiis are of the view that, in *shirkah*, profit and loss are both to be shared in the same ratio \(\sigma = \frac{L}{K}\), \(L\) being the financier's contribution to the total capital \(K\). This one may call as the 'ratio identity' approach.
Islamic economists have almost always preferred the 'ratio divergence' approach for erecting their PLS models, and for two reasons. First, they claim, at least by implication, that the bulk of juridical opinion seems to support this approach. One need not dispute the contention. But since the other viewpoint - the ratio identity approach - has some important adherents among Islamic jurists the exercise of discretion in favour of ratio divergence must be supported by more convincing economic arguments than has so far been done.

This brings us to the proponents' second reason. They argue that profit is the result of the combined effort of capital and enterprise. Hence, keeping the profit and loss sharing ratios equal for the financier would be unfair to the entrepreneur. It will discriminate in favour of the dormant supplier of funds. This is straight logic. Yet it has far-reaching implications.

The argument requires that $\sigma$ must always be less than $\frac{L}{K}$. This sets a constraint on the bargaining power of the financiers and may sometimes be welcome to that extent. But if the gap $g = \frac{L}{K} - \sigma$ is to provide for part of the compensation for the entrepreneurial endeavour, one may face serious difficulties.

To begin with, can we ensure that the magnitude of $g$ remains only fair? It is difficult to rely on market arbitration or design state intervention for the purpose. Again, can the beneficiaries of the ratios’ gap earnings or at least their majority always be shown to deserve the advantage? Presumably this may be possible, in a broad way, in case of proprietary businesses or closely-held small companies. But it is extremely difficult to identify the entrepreneurial services, more so the person performing them, in a large multiproduct corporation. The ratio gap earnings will go to the stockholders but their vast majority takes no more active part in business than the outside financiers. Should we then discriminate between the two?

Furthermore, in large corporations there is no work done by any of the functionaries for which remuneration is not prefixed and deducted from gross revenues before distributable profits are arrived at. In the event, will not ratio divergence lead to duplication of payments?

Last, but by no means the least, not a few consider $\sigma$ as a policy variable. For example, it is suggested that $\sigma$ may be manipulated by the central bank of a country for controlling credit (Uzair, pp. 48-50). Efficacy of this proposal as a policy instrument apart, it clearly comes into conflict with the juridical position that $\sigma$ is open to negotiations among the parties (Ariff, p. 14). Furthermore, will not a sectoral administration of $\sigma$ e.g. in banking alone be discriminatory and a global one almost impossible to enforce?

The questions we have raised above are difficult to answer. But these are the questions that have to be answered satisfactorily before the ratio divergence approach can be applied without discretion.
To us the case for ratio divergence seems rather weak on grounds of social justice in
the present day business circumstances. Perhaps one can support it better as an
expedient promotive of growth. The proposition \( \frac{\sigma}{L} < \frac{K}{L} \) may work as an incentive to
produce and enhance the scope for accumulation, thus helping the process of economic
development in the Islamic countries short in enterprise and capital.

However, the success of the policy depends, among other things, on how far the
surplus of business earnings - \( g \times x \) profits is retained for capital formation. In case it
leaks out largely into the income and consumption streams, ratio divergence may tend to
(i) increase distributional inequalities, (ii) distort demand and therefore investment
patterns, and (iii) encourage, like interest finance, the destabilizing forces in the
economy. Policies aimed at minimizing such leakages must be devised. Appropriate
fiscal measures may help.

But at present, given the postulate of divergence between the profit and loss sharing
ratios in interest-free finance, let us examine briefly the process of their determination in
the economy.

**Macro-Level Analysis**

Normally business as a whole is not expected to suffer losses, though individual
firms may not always earn profits. At the macro-level then the problem is one of
determining the profit sharing ratio with the constraint \( \frac{\sigma}{L} < \frac{K}{L} \).

In Islamic economics one often comes across the observation that the profit sharing
ratio is determined by the forces of demand and supply of investible funds in the
economy (Siddiqi 1983, p.125). But this has remained largely in the nature of a cause
and effect statement. There has seldom been an effort either to identify the relevant
variables of the process or to demonstrate precisely how the apparatus would work to
establish the equilibrium ratio.

Again, there is a tendency, even if veiled, to strike a parallel with the process of
interest rate determination.\(^3\) This must be resisted. For \( \sigma \) is a ratio for sharing, not a
price like the rate of interest (Ariff, p.22). Furthermore, unlike the case of interest, the
element of risk and uncertainty plays a dominant role in the determination of \( \sigma \). For
these and other reasons one cannot apply here the demand and supply tools in a
straightforward mechanistic way.

Rather, we have to connect \( \sigma \) with the overall rate of profit in the economy and
integrate the notion of a risk premium into the analysis frame to see how the demand
and supply forces may operate in this case. For the purpose of this exercise, elementary
in nature, we assume:

1. That the PLS system operates in competition with the customary institutions
   providing funds on interest.\(^4\)
2. That both firms and financiers attempt maximizing their profits, subject of course
to the ethical constraints of Islam.
3. That there are no taxes or transaction cost.
4. That all productive investments are equally risky, hence the risk premium is the same in all cases.\(^5\)
5. That profit expectations of the owners and the financiers concerning a given investment \(K\) as well as their risk estimates coincide.
6. That there are no market imperfections to impede the processes of adjustment.

Now, let us list the symbols - including those already introduced - that we shall use for facilitating the discussion in this and the following sections.

- \(K\): total capital employed in a firm's business
- \(K_0\): owner's contribution in \(K\) (\(K_0 = \text{equity}\))
- \(L\): financier's share in \(K\) (\(K = K_0 + L\))
- \(p\): profit measured as gross revenue minus all contractual payments and capital consumption but including interest.
- \(\sigma^*\): financiers' ratio for sharing profit attributable to \(L\)
- \(\lambda\): financial leverage (\(\lambda = \frac{L}{K}\))
- \(\sigma\): proportion of profit going to the financiers (\(\sigma = \sigma^* \lambda\))
- \(r\): rate of profit on total investment (\(r = \frac{p}{K}\))
- \(r_i\): rate of interest
- \(r_{i0}\): rate of profit on \(K_0\) under PLS finance
- \(r_{iL}\): rate of profit on \(L\) under PLS finance
- \(\alpha\): risk premium fraction
- \(x = r_{i0} - r\)
- \(y = r_0 - r\)

All the rates and ratios are per unit. The absolute values of a variable (\(K, K_0, L, \) or \(P\)) may be added for an aggregative analysis.

In financing systems with and without interest operating side by side in the economy, the main criterion for the firms in choosing their source of external finance is assumed to be the rate of profit they expect to earn on the owner's capital (\(K_0\)) in each case. However, there is a flaw in this viewpoint.

The firms are aware that under the PLS system the risk of enterprise would be transferred from them to the financiers in the same ratio as \(L/K\). In contrast, they have to bear the entire risk in the case of interest finance. In fact, it tends to increase with borrowings because of rising fixed interest commitments. In view of this vital difference between the alternatives the firms may normally be willing to allow the PLS financiers a profit sharing ratio that would leave with them (firms) sufficient earnings for the owners to have a rate of return on their investment \(K_0\) comparable with the one that customary finance would permit after deducting from profits not only interest but an appropriate risk premium as well.\(^6\) For the firms the minimal requirement for preferring the PLS finance would thus be:
\[(1 - \sigma) P \geq P - r_i \lambda K - \alpha \lambda K\]

Simplifying we get,

\[\sigma \leq \frac{\lambda}{r} (r_i + \alpha)\]  \hspace{1cm} (1)

On similar reasoning, the financiers may be willing to take a share of profits which is more than, or at least equal to, the interest income plus reasonable compensation for risk, i.e.

\[\sigma P \geq P - r_i \lambda K + \alpha \lambda K\]

which reduces to

\[\sigma \geq \frac{\lambda}{r} (r_i + \alpha)\]  \hspace{1cm} (2)

Given our assumptions, in equilibrium

\[\sigma = \frac{\lambda}{r} (r_i + \alpha)\]  \hspace{1cm} (3)

This is a situation where the firms will not tend to prefer one source of finance to another except because of religious motivation. It also shows that the profit sharing ratio in the PLS system of our construct is primarily a function of four variables.

\[\sigma = f(r, r_i, a, K)\]  \hspace{1cm} (4)

The relationships between \(r, r_i\) and \(\alpha\) on the one hand, and their interconnections with the forces of demand and supply concerning investible funds on the other are of the same sort as we have them in secular economics. Also, the processes of their determination or modification due to variations in business and social environment operate along the customary lines. But demand and supply presumably neither determine nor are determined by the profit sharing ratio \(\sigma\) in the strict sense. Rather, decisions on \(K\) are to be such that \(\sigma\) remains in harmony with the other variables of the system.

Setting,

\[\lambda (r_i + \alpha) = \beta\]  \hspace{1cm} a constant, in equation \hspace{1cm} (3)

We get

\[\sigma r = \beta\]  \hspace{1cm} (5)

The curve is a rectangular hyperbola with centre \((0, 0)\) and is convex from below. It shows that \(\sigma\) is inversely related to \(r\). By varying the value of the parameter \(\beta\) we have a system of curves for explaining the nature and implications of relationships underlying the function. Figure 1 presents a set of such curves.
Some of the conclusions that follow from this simple diagram are:

1. Other things being equal marginal reduction in \( \sigma \) declines for equal increases in \( r \). The implication is that the resistance of the financiers to downward adjustments in their profit sharing ratio tends to increase even when profit expectations may be rising. On the other hand, they are likely to insist on a more than proportionate rise in \( \sigma \) when profit expectations are generally declining. This is clear from the movement along any of the curves.

\[
\left( \frac{\partial \sigma}{\partial r} = \frac{\beta}{r^2} \right)
\]

2. For any given value of \( B = B_0 \) and \( r = r' \) a point like \( A \) above the relevant curve indicates a \( \sigma \) more favourable to financiers and a point such as \( A' \) below the curve, a \( \sigma \) more advantageous to the firms. Both depict states of incompatibility and forces will tend to set in motion to bring \( \sigma \) back on to the curve. At point \( A \) the financiers would be willing to supply more of funds under PLS relative to the firms' demand. Competition among the financiers will eventually restore \( \sigma \) to the equilibrium level on the curve. Opposite is likely to happen at point \( A' \). Supply of funds will tend to be less than their demand and firms will compete with each other until \( \sigma \) is forced up appropriately. In this way one can see demand and supply play a role in keeping \( \sigma \) around the equilibrium level.

3. As \( r \) on the one hand and \( \lambda \), \( r_i \), and \( \alpha \) together on the other, have opposite effects on \( \sigma \) they may, in a dynamic situation, neutralise each other's influence such that \( \sigma \) remains constant along a line \( \bar{\sigma} \bar{\sigma} \) even when profit expectations fluctuate.

4. For given profit expectations, \( \sigma \) may vary directly with the net change in \( r_i \), \( \alpha \), and \( \lambda \) along a line such as \( \bar{r} \bar{r} \).
5. Given $\lambda$, $\sigma$ changes directly with variations in $(r_i + \alpha)$ relative to $r$.
6. With $(r_i + \alpha)$ constant, $\sigma$ varies directly with fluctuations in $\lambda$ relative to $r$.
7. $\lambda$, $r$, and $\alpha$ remaining unchanged $\sigma$ varies directly with the rate of interest $r_i$.

With these conclusions, let us turn to the analysis of the problem of profit sharing ratio at the micro-level.

**Firm's Equilibrium**

Given profit expectation, the firm will be in equilibrium when it maximizes the rate of return $r_0$ on the owners’ part of its investment. Here, the problem is of finding the relevant combination of $\sigma$ and $\lambda$. Putting in equation (3)

$$\frac{r_i + \alpha}{r} = b \quad \text{a constant}$$

we have

$$\sigma = b \lambda \ (0 < b < 1) \quad (6)$$

This shows that $\sigma$ varies directly with $\lambda$ in a linear relationship and a 'family' of straight lines passing through the origin can be obtained by varying the slope $b$. Any line of the set such as the one with $b = b_0$ in Fig. 2.2 below may be viewed for the firm as the supply schedule of $L$ expressed in terms of $\lambda(=\frac{L}{K})$.  

![Fig. 2](image-url)
Notice that the combinations of $\sigma$ and $\lambda$ along the line are only such that keep $n_1$ unchanged. This clearly follows from equation (2). Thus, for given $n_1$, the supply of L for an individual firm is perfectly elastic despite - indeed because of - $\sigma$ varying with $\lambda$.

However, in PLS finance, as in that based on interest, the firm may not push up $\lambda$ infinitely. Even though the rate of profit $r_0$ on the owners' investment tends to rise as $\lambda$ increases the firm may not like to reduce the proportion of $K_0$ in $K$ beyond a limit where its own employable financial resources may tend to become idle. This sets a minimum level for absolute earnings, say $M$, that must be available for the owners. Liquidity and income retention considerations may also influence the requirement for $M$. Under the circumstances the firm may seek a combination of $\sigma$ and $\lambda$ that would be compatible with $M$. Symbolically we have:

$$ (1 - \sigma) P \geq M $$

or

$$ \sigma \leq 1 - \frac{M}{P} $$

(7)

For given $P$, the equation $\sigma = 1 - \frac{M}{P}$ gives the demand schedule of the firm for L which, like the supply schedule, can be interpreted in terms of $\lambda$. As $M$ increases L decreases, other things being equal. Thus, $\lambda$ varies inversely with $M$. This is clear from the equality of the demand and supply schedules. In a state of equilibrium, combining (6) and (7), we have

$$ \lambda = \left[ 1 - \frac{M}{P} \right] \frac{1}{b} $$

(8)

$\lambda$ and $M$ being the variables, the inverse function is:

$$ M = P(1 - b\lambda) \text{ and } \frac{\partial M}{\partial \lambda} = -pb $$

It follows from (7) that the relationship between $\sigma$ and $M$ is linear with a negative slope $-\frac{1}{P}$. The slope varies directly with $P$ and a 'set' of lines passes through the point $\sigma = 1$. One such line is shown in Fig. 2.1.

It is interesting to see at this stage how the inequality $\sigma < \lambda$ is built into the micro aspect of a PLS system. Islamic economists invariably maintain that profit must be, in the first instance, apportioned to L in accordance with the value of $\lambda$ i.e. profit attributable to L is $\lambda P$. It is from $\lambda P$ that the financiers get an agreed proportion $\sigma$ (Siddiqi 1978, pp. 36-37). The procedure sets $\sigma P = \sigma^* \lambda P$ or $\sigma = \sigma^* \lambda$. Comparing this with (5) we have $\sigma^* = b$ as well. Notice that $\sigma^*$ and $\lambda$ being fractions $\sigma < \lambda$ and $\sigma < \sigma^*$ but $\sigma^* \geq \lambda$. 
For a given $\sigma$, $\sigma'$ varies inversely with $\lambda$. In fact it is the same thing as inverse of the function in (6) now taking $b (= \sigma')$ as variable. We may have a set of equi-$\sigma'$ curves for different values of the parameter $\sigma$ as shown in Fig. 2.3. It follows that there can be circumstances where a firm may expect a reduction in $\sigma'$ as it seeks more funds under PLS arrangement though such a change is of little consequence unless $\sigma$ is reduced. Fig. 2.3 also shows that for any $\lambda$, $\sigma'$ varies directly with $\sigma$. Again, $\sigma'$ may remain constant if changes in $\sigma$ and $\lambda$ balance each other's influence.

Fig. 2.4 translates the relationship shown in Fig. 2.1 in terms of $\sigma'$. We find

$$\sigma' = \frac{1}{\lambda} \left( 1 - \frac{M}{P} \right)$$

which gives a linear relationship between $\sigma'$ and $M$ with a negative slope $-\frac{1}{\lambda P}$. Since $P > \lambda P$ the slope of the demand curve in Fig. 2.1 is kept greater than that in Fig. 2.4.

The equilibrium of the firm is shown by the continuous line circuit starting from $M_0$ in Fig. 2.1. Passing through the points E, F, G and H it gives interconnected values of the variables as $\sigma_0$, $\lambda_0$, $\sigma_0$ and back to $M_0$, all the four held in mutual balance as balls placed in a bowl. A change in $M$, $\sigma$, or $\lambda$ may start a chain reaction until a similar equilibrium circuit is reestablished. In a simple illustration, if $\lambda$ changes from $\lambda_0$ to $\lambda_1$ in Fig. 2.2 showing a reduction in $\frac{L}{K}$, the firm will have larger $M (= M_1)$, the profit sharing ratio of the financier falling to $\sigma_1$, but $\sigma'$ remaining unchanged, the equilibrium point in Fig. 2.3 shifts on to a lower equi-$\sigma$ curve. The demand curve in Fig. 2.4 shifts to the right because its slope involving $\lambda$ has changed such that $\sigma'$ remains the same. The new equilibrium circuit is shown by the broken line; its portion between Fig. 2.3 and Fig. 2.4 runs over the old circuit.

Attention may be drawn to a matter of significance at the micro level in arranging PLS finance for business. It is a consequence of the changes in $\frac{L}{K}$.

In customary finance the rate of interest cuts off the financier with a periodic payment for his contribution $L$ to the total investment in a firm's business. It leaves him unconcerned about (i) the initial volume of $K$, and (ii) any subsequent changes in $K$ unless the security of his contribution $L$ is threatened.

However, in a PLS system both these matters assume importance for the non-owner financier. It is always necessary, though seldom easy, that the financiers and the owners agree at the time of making a PLS contract on the estimate of $K$ employed or to be employed in the firm's business or required for a project. This is essential to give the financier a precise idea of his loss sharing ratio $\frac{L}{K} = \lambda$, and negotiate, with the knowledge of risk involved, for a profit sharing ratio. This brings into focus the importance of the accounting valuations in PLS financing, especially in case of a going concern.
Even when the initial volume of $K$ is settled, and the financiers' contribution to it decided, their reaction to subsequent changes in $K$ relative to $L$ remains uncertain.

For any $\sigma^*$ agreed upon, if later variations in $K$ leave the expected rate of profit $r$ unchanged the same has no effect on $r_L$. However, if a reduction in $K$, given $L$ takes place that would raise $\lambda$ the loss sharing ratio. So the financier may oppose the change. On the other hand, he may welcome an increase in $K$ under the circumstances, for that would reduce $\lambda$ without decreasing $r_L$.

But the situation becomes complicated as profit expectations change with variations in $K$. If a reduction in $K$ promises to raise $r$, as may for instance be the case under schemes of rationalization, the financier may not resist the change provided he considers the expected increase in $r_L$ adequate enough to compensate him for a simultaneous increase in $\lambda$. The reverse is true if $r$ is expected to fall.

Likewise, if increase in $K$ is expected to go with a reduction in $r$, the financier may or may not oppose the change depending on how he evaluates the resultant reduction in $\lambda$ - $L$ remaining fixed - compared to the likely fall in $r_L$.

**Relative Profitability**

Doubts have recently been raised regarding the success of the PLS institutions operating in competition with those working on the basis of interest. It is suspected that the former are likely to be much less profitable and may tend to vanish in the long run unless efforts are made to sustain them through strategies of non-financial sort of supplementary business (Nienhaus, 1983).

We shall argue that the logic of a PLS system negates such a presumption. The incorporation of the risk premium factor in the process of arriving at the equilibrium profit sharing ratio $\sigma$ through equations (1) to (3) is clearly indicative of superior profitability expectation for providing funds on a PLS basis. The proposition further unfolds itself in another connection i.e. the leverage effects discussed below.

One distinctive feature of interest finance is that leverage the use of term loans as part of business finance - normally tends to raise the return $r_o$ on the owners' investment, relative to the overall rate of profit $r$. In other words, $r_o$ tends to be greater than $r$.

Assuming that profit expectations are realized, we may set

$$X = \frac{P - r\lambda K}{(1 - \lambda)K}$$

which reduces to

$$X = \frac{\lambda}{1 - \lambda} (r - r_o), (r > r_o) \tag{9}$$

Given ($r - r_o$), $x$ varies directly with $\lambda$ and at an increasing rate

$$\frac{\partial X}{\partial \lambda} = \frac{r - r_o}{(1 - \lambda)^2}, \text{and } \frac{\partial^2 X}{\partial \lambda^2} = \frac{2(r - r_o)}{(1 - \lambda)^3}.$$
But as $\lambda$ increases, the risk of the owners also increases (Baumol, p.461) and presumably at a faster rate, after a point, than the rise in $r$. Therefore, a stage is likely to be reached soon when additional borrowing on interest may become more risky than profitable for the owners. Leverage cannot be pushed beyond the limits of safety. This is a well known fact in the interest based business finance.

The ratio divergence approach by keeping $\sigma < \lambda$ provides for a leverage effect to the benefit of the owners in a PLS system as well. To formalize matters, we may write:

$$Y = \frac{(1-\sigma)K - P}{(1-\lambda)K}$$

Simplifying we get

$$Y = \frac{r(\lambda - \sigma)}{(1-\lambda)}$$

(10)

Substituting in this the value of $\sigma$ from (3) we have:

$$Y = \frac{\lambda}{(1-\lambda)}(r - r - \alpha), (r > r + \alpha)$$

(11)

A comparison of equations (9) and (11) shows that the leverage gains for the owners are likely to be smaller under PLS finance, the excess of $x$ over $y$ being measured by the ratio $\frac{\lambda \alpha}{(1-\lambda)}$. Also, the rate of increase in $y$ is smaller

$$\frac{\partial^2 X}{\partial \lambda^2} > \frac{\partial^2 y}{\partial \lambda^2} = \frac{2(r - r - \alpha)}{(1-\lambda)^3}.$$ However, the owners are compensated for the short fall in profit by the transfer of risk to the financiers.

It comes about that PLS finance has two distinct features. First it grants to the financiers a higher rate of return on their funds than the rate of interest, the difference being equal to a [From (3)]. There is, unlike interest finance, virtually no limit on leverage, save $M$ which operates in any case. This promises more business for the PLS financiers. Thus, higher return than the rate of interest on investment funds $L$ and greater volume of business creates visions of superior profitability of the system for the financiers.

There is another way of looking at the matter, and in a somewhat broader perspective.

In secular economics the equilibrium rate of profit $r$ for the system is supposed to be composed of two main elements (i) rate of interest $r$ and (ii) a risk premium fraction say $\sigma$. Thus, $r = r + \sigma$. In the PLS system we have built, the risk premium ratio is reduced to $\alpha$, ($\alpha < \alpha^*$). This is implicit in our formulation $\sigma = \frac{r + \alpha}{r}$. For $\sigma$ being less than $\lambda$ we have $\frac{r + \alpha}{r} < 1$ or $r + \alpha < r$. 
Putting the difference $\alpha' - \alpha = m$ as a balancing element, we have
\[ r_i + \alpha + m = r \quad (12) \]

If interest were completely abolished from the Islamic world, $r_i$ and $\alpha$ would merge into a single entity, say $\alpha'$, such that in the pure PLS system we would have
\[ r = \alpha' + m \quad (13) \]

The $\alpha'$ fraction of profit would be shared between the financiers and the firms, the latter also getting the $m$ fraction. To illustrate, as total profits $P = rK$, the firms receive
\[ (1 - \sigma)P = [(1 - \lambda)\alpha' + m]K \quad (14) \]
and the financiers get
\[ \sigma P = \alpha' \lambda K \quad (15) \]

The $mK$ portion in the firms' share of profit may be viewed as the reward for entrepreneurial services in *mudarabah* or as the surplus available for other purposes such as research and development, depending on specific situations.

Dividing (14) on both sides by $(1 - \lambda)K$ and solving we get the rate of return on owners' investment $K_0$ as
\[ r_o = r + m \frac{\lambda}{(1 - \lambda)} \quad (16) \]

The leverage effect thus raises $r_o$ over $r$ by the fraction $\frac{m\lambda}{(1 - \lambda)}$ without involving additional risk.

Similarly, from (15) the rate of return $r_L$ on the financiers' investment turns out to be
\[ \frac{\sigma P}{\lambda K} = \alpha' \quad (17) \]

As $\alpha' r_L = \alpha'$ would certainly be greater than $r_i$ if interest were prevalent, we return to our earlier conclusions concerning relative profitability of the two systems from the viewpoint of financiers.

It follows that a PLS system is likely to be more attractive for both the firms and the financiers, more so in the long run when transition difficulties are over, the system attains maturity, and the people have experienced the advantage. It promises leverage benefits to the firms free of risk and a return higher than the rate of interest to the financiers. There is probably an additional advantage. Fluctuations in $r_0$ are likely to be smaller than in $r_i$ as $y < x$ and $\frac{\partial y}{\partial \lambda} < \frac{\partial X}{\partial \lambda}$. For this reason PLS operations may have a smaller destabilizing potential for the economy as a whole compared to financing on interest.

The difficulty with PLS finance is not that its relative usefulness is suspect in principle. Rather, it may lie in the area of administrative alertness required for its successful operation.
Concluding Remarks

As mentioned above, current models in PLS financing usually rest on ratio divergence approach that emanates essentially from the principle of mudarabah. We have attempted to demonstrate how the profit sharing ratio $\sigma$ is likely to be determined always being the financiers' loss sharing ratio. In the process we have touched upon a few other issues. Our main conclusions are:

1. The ratio divergence approach to PLS finance raises a number of important but perplexing questions which have not so far been adequately considered, much less answered. Perhaps one can support this approach better as a growth promoting policy than as an instrument for ensuring distributive justice in view of modern forms of business organization.

2. In a system where interest and PLS financing exist side by side, the determination of profit sharing ratio is primarily a function of the overall rate of profit on investment, the prevailing rate of interest, the degree of leverage and the risk premium estimates. The forces of demand for and supply of investible funds operate through their influence on these variables and keep the profit sharing ratio in harmony with them.

3. For individual firms, minimum requirement for absolute amount of earnings on the demand side and the proportion of $L$ in $K$ on the supply side tend to settle the equilibrium profit sharing ratio, other things remaining the same. Problems between the owners and non-owner financiers may arise on the issues of initial capitalization of the firm or the project, and the reaction of the financiers to subsequent changes there in are difficult to predict. The accounting aspect of PLS financing assumes significance in these matters.

4. Ratio divergence is likely to generate the same sort of destabilizing forces in the economy - though at a smaller scale - as interest finance creates by magnifying profits (losses) through leverage.

5. PLS financing is likely to be more profitable in the long run from the financiers' viewpoint as compared to interest finance.

These conclusions may not be entirely new, but the way they have been deduced and given form here is probably fresh. Some of the assumptions of the model may not seem realistic. But they are of a simplifying nature. Their removal would complicate the analysis but would hardly affect its broad conclusions. Again, the use of ratios instead of absolute values may have made the discussion at some places a bit less convenient to follow but it has certainly enhanced the range of the argument without consuming more space. For a single relative variation can indicate and summarize more than one absolute change.
Notes

(1) The main sources on the subject are in Arabic. Some literature is also available in Urdu. One reliable and useful discussion has recently been published in English (M.N. Siddiqi, *Partnership and Profit-sharing in Islamic Law*, The Islamic Foundation, Leicester, 1985).

(2) It is interesting to note that while some Islamic economists consider capital and enterprise as a composite production factor (e.g. Uzair, Mohammad, *Interest-free Banking*, Royal Book Company, Karachi, 1978, p.5), the popular ratio divergence approach rests on a separation of the two, at least conceptually.

(3) The argument seems to run as follows: If the interest and profit sharing systems of finance are competitive, in equilibrium PLS financiers must expect a rate of return on their investment $L$ equal to the rate of interest appropriately adjusted for the risk and default factors. Also, for any such rate of return there must be some sharing ratio of profit which gives that rate. Therefore, interest rate and sharing ratio must be determined in an identical manner.

Even if one concedes the argument, the process how $\sigma$ is determined still remains to be explained.

But there is a more fundamental objection. The argument rests on the ratio identity approach. But if $\frac{\sigma}{K} = \frac{L}{K}$, the problem of determining $\sigma$ does not arise. It assumes significance only under ratio divergence where $\frac{\sigma}{K} \neq \frac{L}{K}$. The equality makes the above argument grossly inadequate if not entirely redundant. For $\sigma < \frac{L}{K}$ reduces the profit PLS financiers may expect on $L$ and locks the expectations with $\sigma$ in a circular relationship.

(4) This remains a realistic assumption until interest is completely abolished in Muslim societies. At present the influence of interest rate on the determination of profit sharing ratio cannot be ignored, though we have relaxed this assumption for a moment at a later stage in the discussion to focus on some implications of ratio divergence in a completely interest-free economy.

(5) It is the same as resorting to an averaging process which economists use so often in aggregative analysis.


(8) There must also be an agreement, at least in a broad way, on the accounting principles and procedures for ascertaining the profit to be shared between the owners and the financiers of the firm.

(9) For if expectations fail, the more *expost* profits fall short of *exante* estimates the greater is the burden of interest bearing finance for the owners (leverage given) and more relieving for them are the consequences of PLS finance.
References


العوامل المحددة لنسب المشاركة في الربح والخسارة

أ. لبي حسن
استاذ مشارك - قسم الاقتصاد
كلية فاكر حسن - جامعة دمشق

المتطلب: تفوق النماذج الخالية للمشاركة جواز اختلاف نسبة أقسام الربح، عن نسبة تمويل رأس المال الذي هي نفسها نسبة توزيع الخسائر. وهذا الاختلاف بين النسبتين ملفى الرغبة في تشجيع النمو أكثر مما تقبله اعتبارات العدالة التوزيعية، وخاصة في المنتجات الحديثة التي لا يديرها مالكها.

وفي اقتصاد يقوم فيه التمويل بالمشاركة إلى جانب التمويل بالفائدة، فإن نسبة أقسام الربح على المستوى الكلي للاقتصاد هي دالة للمعدل الكلي للفائدة على الاستثمار، ولعدل الفائدة، ونسبة التمويل إلى رأس المال المشارك، ولعلاقة المحتارة. ويشتهر أن يكون التمويل بالمشاركة في الربح والخسارة أكثر ربحية بالنسبة للمؤسسات الذي يؤخذ على الفائدة.

Determination of Profit and Loss Sharing Rates in Interest-free Business Finance

Page 29