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## **A Remark on Intensive Differential Rent and the Labour Theory of Value in Ricardo**

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### **Abstract**

One of the foundations of the labour theory of value used by Ricardo in the *Principles* is that rent does not enter into commodity prices. In response to objections raised by Malthus and Say, Ricardo defended this idea by arguing that even where all cultivated land pays rent, the last dose of capital employed on the land does not and no rent is therefore involved in the price of the product of this capital. We will show that this claim, which has convinced several generations of economists, is based on a misleading argument.

In particular, we will show that the intensive differential rent paid on land of the worst quality under cultivation enters into the agricultural product price and so, even in the most favourable case, commodities are no longer exchanged at a ratio corresponding to the relative quantities of labour they embody.

**Keywords:** differential rent, labour theory of value, methods of production, Ricardo, Sraffa, Smith.

**JEL Classification:** B12, D46.

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## 1. Introduction

The primary result of Ricardo's analysis is undoubtedly the presence of an inverse relationship between the wage rate and the rate of profit. With the aid of the labour theory of value, Ricardo succeeded where Adam Smith had failed and pinpointed the link between profits and wages.

Although many of Ricardo's conclusions have been shown to be valid in general, his arguments are strongly based on the hypothesis that commodities are exchanged with one other at a ratio determined by the quantities of labour they embody.<sup>1</sup> He was therefore obliged first to deny Smith's claim that prices are not determined by the labour they embody in a developed country and second to refute the objections raised by Say and Malthus.

As is well known, Ricardo's labour theory of value is essentially grounded on two assertions: i) that the employment of capital in the production of each commodity is proportional to the employment of labour; ii) that rent is not a component part of the prices of commodities. The first was intended as an approximation, in the sense that exceptions were possible and their relevance should be judged case by case. The second is instead a result that Ricardo believed he had established through his theory of differential rent. We shall focus here solely on the latter, as the former has already received a great deal of attention.

If extensive differential rent were the only possible form, then Ricardo's result would hold, as the price of agricultural products would be determined in this case by the cost of production on the least fertile land under cultivation, and land of this quality pays no extensive differential rent at all. Ricardo therefore appears to have found a reply to Smith's idea of rent as a component part of commodity prices:

that corn which is produced by the greatest quantity of labour is the regulator of the price of corn; and rent does not and cannot enter in the last degree as a component part of its price. Adam Smith, therefore, cannot be correct in supposing that the original rule which regulated the exchangeable value of commodities, namely, the comparative quantity of labour by which they were produced, can be altered by the appropriation of land and payment of rent. Raw material enters into the composition of most commodities, but the value of that raw material, as well as corn, is regulated by the productiveness of the portion of capital last employed on land, and paying no rent; and therefore rent is not a component part of the price of commodities. (Ricardo 1951-73: vol. 1, p. 46)

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<sup>1</sup> "Ricardo's basic theorem on distribution—'a rise of wages ... would invariably lower profits'—is thus strictly dependent on his measure of value" (Stigler 1952, p. 203).

Although differences in the quality of land always involve extensive differential rent, however, it can hardly be maintained that the rents of landowners consist exclusively of this element. Where every piece of land is owned, there is clearly no land that is cultivated without the payment of rent to the owner. This is the objection Say raised against Ricardo's theory.

In a letter to Say dated 11th January 1820, Ricardo replies to this point as follows:

You appear to me to have mistaken also an opinion of mine on which you comment in a note of the translation of my book. My argument respecting rent, profit and taxes, is founded on a supposition that there is land in every country which pays no rent, or that there is capital employed on land before in cultivation for which no rent is paid. You answer the first proposition, but take no notice of the second. The admission of either will answer my purpose. (Ricardo 1951-73: vol. 8, pp. 149, 150)

In the first place, Ricardo expressly states that the presence of rent-free land is a "supposition", something admittedly not proven. In the second, he does not reject Say's argument that there is no cultivated land yielding no rent to its owner, which is also summed up in a note<sup>2</sup> to Chapter XXXII of the *Principles*. He confines himself to defending his theory on the basis of intensive differential rent,<sup>3</sup> arguing that even if Say were right to believe that all the cultivated land received rent, the last dose of capital employed on this land would still pay no rent and his theory would thus be valid in any case.

We also find this argument stated in an earlier letter of Ricardo to James Mill, dated December 22nd 1818, in which he reports on a discussion with Malthus:

He [Malthus] has altered his opinion you know about there being land in every country which pays no rent, and appears like M. Say to think that when that is proved, my doctrine of rent not entering into price is overthrown – they neither of them advert to the other principle which cannot be touched, of capital being employed on land, already in cultivation, which pays no rent. (Ricardo 1951-73: vol. 7, p. 372)

The ultimate foundation of Ricardo's idea that rent does not enter into the price of commodities therefore appears to be the theory of intensive differential rent.

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<sup>2</sup> "M. Say, in his notes to the French translation of this work, has endeavoured to show that there is not at any time land in cultivation which does not pay a rent, and having satisfied himself on this point, he concludes that he has overturned all the conclusions which result from that doctrine." (Ricardo 1951-73: vol. 1, p. 413)

<sup>3</sup> Samuelson (1959, p. 9, footnote 2) proposed a reformulation of the Ricardian theory of rent in terms of differential calculus and marginal productivity. In this case, assuming a continuum of different qualities of land – as in von Thunen theory for example – the extensive differential rent would be compatible with a situation in which almost every cultivated land yields a rent to its proprietor. But clearly this was not the case considered by Ricardo, since he explicitly referred to the intensive rent.

Ricardo ultimately succeeded in defending his idea and convincing several generations of economists<sup>4</sup> that even though all cultivated land receives rent, the last dose of capital employed on land pays no rent and therefore rent does not enter into the price of commodities.

With the aid of Sraffa's rigorous restatement of differential rent theory<sup>5</sup>, we intend to show here that Ricardo's argument about intensive rent is misleading and the widely accepted conclusion that intensive rent does not enter into the price of agricultural commodities is incorrect.

In particular, after restating Ricardo's argument, we shall analyse intensive differential rent from a different point of view and, using the same data as Ricardo's example, show that it enters into the (relative) price of corn. As a result, when there are intensive rents, commodities cannot be exchanged at prices determined by the embodied quantities of labour.

## 2. Ricardo's argument

Ricardo presents his theory of intensive differential rent by means of an example in the second chapter of the *Principles*. He imagines that a capital of £1,000 – which can be regarded as wages anticipated at the beginning of the year – applied on an acre of the most fertile land gives an output of 100 quarters of corn, whereas the same capital applied on an acre of less fertile land would give 80 quarters. He then observes that if the application of a second £1,000 of capital on an acre of the most fertile land gave rise to an increase in product of 85 quarters, this use of capital would be clearly more advantageous, where possible, than the cultivation of less fertile land.

In this case, rent arises even without different qualities of land being cultivated, in that Ricardo views the difference between the 100 quarters obtained by the first dose of capital and the 85 quarters obtained with the second – i.e. 15 quarters, or the value of 15 quarters – as representing the landowner's rent for an acre. The amount of (gross) profit on a capital of £1,000 is thus the equivalent of 85 quarters of corn for the first investment of capital as well as the second. Ricardo concludes that:

the capital last employed pays no rent. For the greater productive powers of the first £1000, fifteen quarters is paid for rent, for the employment of the second £1,000 no rent whatever is paid. (Ricardo 1951-73: vol. 1, p. 72)

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<sup>4</sup> Authoritative examples include Marshall (1893, p. 86) and S. Hollander (1979, p. 203).

<sup>5</sup> We refer in particular to the analysis described by Sraffa (1960: 75, 76 - §§ 87, 88) and put into formal terms and developed in greater depth by various economists, including Montani (1975), Kurz (1978), Guichard (1982) and D'Agata (1983). Cf. also: Kurz and Salvadori (1995, p. 289).

Though capable of showing that more intense cultivation of the most fertile land can be more advantageous than cultivating inferior land, Ricardo's argument in no way proves that rent does not enter into the price of corn.

The arbitrary division of capital into doses of £1,000 each is in fact misleading and induces the reader to believe that the last dose of capital employed alone gives an output of 85 quarters of corn. In other words, these 85 quarters seems to be the product of £1,000 capital, without any (further) employment of land. This impression is incorrect, however, because corn cannot be produced without land.

The application of a second dose of capital on the most fertile land derives, as Sraffa (1960) shows, from the possibility of cultivating land of this quality by two alternative methods. With the first, method  $\alpha$ , 100 quarters of corn are obtained by employing a capital of £1,000 on an acre of land, which means that one quarter of corn is obtained with a capital of £10 and 0.01 acres of land. With the second,  $\beta$ , 185 quarters are produced by employing a capital of £2,000 on an acre of land, and so the production of one quarter of corn requires a capital of £10.81 and 0.0054 acres of land. Corn is produced by means of capital and land both with method  $\alpha$  and with method  $\beta$ .

As a result, when the rent rate is equal to the value of 15 quarters of corn, every single unit of capital employed pays a rent. In particular, every £1 of capital employed pays a rent corresponding to the value of 0.015 quarters of corn with method  $\alpha$  and 0.0075 quarters with method  $\beta$ .

With his doses of capital, Ricardo instead seeks to present method  $\beta$  as a combination of method  $\alpha$  and a third method making it possible produce corn with no land. The argument is misleading, however, since Ricardo then uses this imaginary third method to prove that rent does not enter into the price of corn.

If there were a method of producing corn without land, it would be a mere truism to assert that rent does not enter into the cost of corn. No such method exists, however, because the investment of the second dose of capital cannot take place independently of the first. The fact that the second dose of capital is not employed with a third independent method becomes clear when we observe that, according to Ricardo's reasoning, a change in the quantity of corn obtained from £1,000 with method  $\alpha$  – from the first dose of capital – would entail a change in the amount of product obtained from the last dose of capital.<sup>6</sup>

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<sup>6</sup> For example, if capital became more productive with method  $\alpha$ , so that £1,000 capital produced 110 quarters of corn per acre, the quantity of corn obtained with the second dose of capital would be reduced to 75 quarters because the quantity produced by method  $\beta$  (185 quarters per acre), being a well-defined method of production, is independent of any improvement in method  $\alpha$ . As a result, the second dose of capital cannot be regarded as employed with a third independent method.

Whenever rent is paid for the use of land of a certain quality, every single unit of capital invested in that land pays a rent. As a result, if an intensive differential rent is due for the last piece of land under cultivation, then rent enters into the price of corn. This is proved in the next section by means of a simple example based on the same data used here.

### 3. A different argument

Let us consider an example with only two commodities: an agricultural product, say corn, and an industrial product, say steel.

Let us assume that capital in each sector consists exclusively of wages paid in advance for the period of production, which is one year in both sectors. As a result, capital and profits can clearly be seen to be proportional to the employment of labour in each sector. If rents do not enter into the price of commodities, as Ricardo claims, they should therefore be exchanged at a relative price equal to the ratio of the amounts of labour they embody.

The total quantities to be produced are fixed and equal to 740 quarters of corn and 25 tons of steel. The surface area of uniform land available is also fixed and equal to 6 acres.

Steel is produced by just one method, employing 2 units of labour per ton of output. Corn can instead be obtained by two methods,  $\alpha$  and  $\beta$ . With method  $\alpha$ , 100 quarters of corn are produced per acre of land by the employment of 10 units of labour, whereas 185 quarters of corn per acre of land are obtained by employing 20 units of labour with method  $\beta$ .<sup>7</sup>

If the rate of rent were zero, method  $\alpha$  would be cheaper than  $\beta$  because less labour is employed per quarter of corn. Due to the scarcity of land, however, the use of this method alone would allow the production of only 600 quarters of corn as against the 740 required by the market. Methods  $\alpha$  and  $\beta$  must therefore necessarily coexist and the rate of rent must rise in order to make the unit costs identical within the two methods,<sup>8</sup> a task that it could not accomplish if rent did not enter into the costs (cf. also Appendix 2).

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<sup>7</sup> It should be noted that if  $w = £100$ , method  $\alpha$  makes it possible to produce 100 quarters per acre with a capital of £1000 and method  $\beta$  to produce 85 quarters more per acre through a further investment of £1000, as in Ricardo's own example.

<sup>8</sup> Until 740 quarters of corn are produced, the price of corn is above its cost of production with method  $\alpha$  and agricultural producers therefore obtain extra profits. Since the six acres of land are entirely cultivated, the classical mechanism of capitalistic competition allows landowners to appropriate the extra profits in the form of rents. The rising rents tend to increase the unit cost of production with method  $\alpha$  with respect to the unit cost with method  $\beta$ . This process goes on until the unit cost of production for corn is the same with both methods.

When the rent reaches this level, the 6 acres of land will be entirely cultivated: 4.35 acres (4 acres, 1 rood and 16 perches) with method  $\alpha$  and 1.65 acres (1 acre, 2 roods and 24 perches) with method  $\beta$ .

In this case, given a wage rate  $w$ , the profit rate  $r$ , the prices  $p_c$  and  $p_s$  and the rent rate  $\rho$  must be such that<sup>9</sup>:

$$20 w (1 + r) + \rho = 185 p_c \quad (1)$$

$$10 w (1 + r) + \rho = 100 p_c \quad (2)$$

$$2 w (1 + r) = p_s \quad (3)$$

By solving system (1)-(3), we obtain the result that the rent rate  $\rho$  is equal to the value of 15 quarters of corn, as in Ricardo's example, and the relative price of corn in terms of steel is 1/17.

We must now determine the amount of labour embodied in corn relative to the amount embodied in steel so as to compare this value with the relative price. If Ricardo's argument about intensive rent is right, they must be equal. If instead they differ, this is due to the fact that rent enters into the price.

The usual determination of the labour time embodied in the commodities does not apply in the case considered here, which involves the more general problem of calculating the amounts of labour embodied in cases where more than one method of production is in use in the same industry. This issue has already received some attention, e.g. in Morishima (1973), Flashel (1983) and Toker (1984).

According to a universally accepted basic definition, the amount of labour embodied in a commodity is the labour time employed directly in its production plus the labour embodied in all the commodities that are its means of production. It follows that if some commodities are produced by the simultaneous use of different methods, there will be different amounts of labour embodied in each commodity. This is perfectly consistent, as we shall see, both with Marx's theory and with Ricardo's.

Let us start with Marx, who asserts that a commodity can have various "individual values" corresponding to the different methods of production in use. The differences between the individual values of the same commodities can arise from various circumstances, including the cultivation of

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<sup>9</sup> As regards the equations determining intensive differential rent simultaneously to prices, we consider the analysis put forward by Sraffa (1960, pp. 75, 76 - §§ 87, 88) and later put into formal terms by various authors, in particular, Montani (1975, pp. 80 - 83), Kurz (1978, pp. 26 - 34) and Kurz and Salvadori (1995, p. 289). Equations (3)-(5) are the same as though found there but written for the simple example addressed. It is also worth observing that although Sraffa and the scholars mentioned above have a correct theory of intensive differential rent, they fail to note that it can be used to refute Ricardo's conclusions about rent and the labour theory of value (for a discussion of the differences between Sraffa's and Ricardo's theory of rent see also Fratini, 2008).



different kinds of land<sup>10</sup>, differences in the technical knowledge and, more simply, the fact that two or more methods are equally profitable, as at a switch point between two techniques or in the case of intensive rent addressed here. Each commodity also has a “market value”<sup>11</sup> (or “social value”), however. In cases where the price of a commodity is regulated by the amount of labour it embodies, competition entails “the establishment of an equal market-value and market-price by averaging the various individual values of the commodities” (Marx 1909, p. 212).

In Ricardo’s analysis the distinction between individual and market value is less explicit but still present, at least when differential rent is considered. In fact, by saying that the “corn which is produced by the greatest quantity of labour is the regulator of the price of corn” (Ricardo 1951-73: vol. 1, p. 46), Ricardo is implicitly arguing that while corn can embody different amounts of labour, it has just one price, which is equal to the greatest of these.<sup>12</sup> Therefore, in the case under consideration, Ricardo’s view differs from Marx’s precisely because the market value is the highest individual value in the former and the average of the individual values in the latter.<sup>13</sup>

Let us now return to our example. Due to the absence of capital goods, the calculation is very simple. The individual amounts of labour embodied in a quarter of corn with methods  $\alpha$  and  $\beta$  are respectively  $1/10$  and  $4/37$  ( $= 20/185$ ), and the labour embodied in a ton of steel is 2. The market

<sup>10</sup> “The difference in the fertility of the soil brings it about that the same quantities of labor and capital, hence the same value, express themselves in different quantities of agricultural products, so that these products have different individual values.” Marx 1909, p. 948.

<sup>11</sup> “[A] market value [...] must be distinguished from the individual value of the commodities produced by the various producers. [...] The individual value of some of these commodities will be below the market-value, that is to say, they require less labor-time for their production than is expressed in the market-value, while that of others will be above the market-value. We shall have to regard the market-value on one side as the average value of the commodities produced in a certain sphere, and on the other side as the individual value of commodities produced under the average conditions of their respective sphere of production and constituting the bulk of the products of that sphere.” Marx 1909, p. 210.

<sup>12</sup> On the contrary, according to Marx: “[i]t is only extraordinary combinations of circumstances under which commodities produced under the least or most favorable conditions regulate the market-value.” Marx 1909, p. 210.

<sup>13</sup> According to the analyses of Flashel (1983) and Toker (1984), this point can be stated in formal terms as follows. Let us assume there are  $n$  commodities – labelled  $1, 2, \dots, n$  – and each one of them can be produced by many methods. Let us denote by  $b_{jk}$  the amount of commodity  $j$  (with  $j = 1, 2, \dots, n$ ) produced with method  $k$ , and by  $a_{ijk}$  and  $\ell_{jk}$  respectively the amounts of commodity  $i$  and labour employed in that activity. If  $s_j$  is the number of activities simultaneously in use for the production of commodity  $j$ , then the individual values  $v_{jk}$  (with  $k = 1, 2, \dots, s_j$ ) and the social or market value  $v_j$  must satisfy the following equations:

$$v_{jk} \cdot b_{jk} = \sum_{i=1}^n v_i \cdot a_{ijk} + \ell_{jk} \quad \forall j = 1, 2, \dots, n, \forall k = 1, 2, \dots, s_j.$$

The above system has  $\sum_j s_j$  equations and  $\sum_j s_j + n$  unknowns. Therefore, in order to close the system, we need  $n$  more equations. In the case of Marx’s conception of market value, these are (cf. Flashel 1983, p. 442, and Toker 1984, p. 152):

$$v_j \sum_{k=1}^{s_j} b_{jk} = \sum_{k=1}^{s_j} v_{jk} \cdot b_{jk} \quad \forall j = 1, 2, \dots, n.$$

In the case of Ricardo, they can instead be written as follows:

$$v_j = \max\{v_{j1}, v_{j2}, \dots, v_{js_j}\} \quad \forall j = 1, 2, \dots, n.$$

or social labour value of corn relative to that of steel is  $2/37$  and therefore smaller than the relative price ( $2/34$ ).<sup>14</sup> Rent does enter into the price of corn and causes an increase with respect to the level corresponding to the relative quantities of labour embodied.

#### 4. Conclusion

Contrary to what Ricardo claims and many economists have since believed, if the last piece of land under cultivation receives intensive differential rent, it enters into the price of corn or agricultural products in general.

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<sup>14</sup> It is worth observing that, as Sraffa points out (1960, p. 75), the relative price of corn in terms of steel can be determined by a system in which land does not appear. This possibility has been studied in general and formal terms by Guichard (1982). Since this way of framing the equations might generate a sort of optical illusion, some clarification will be necessary in order to avoid possible misunderstanding.

Sraffa's price equations – as well as similar equations found in Walras's theory (see Fratini & Levrero 2011) – derive from the idea that in a normal position, every sector of the economy must earn zero extra-profits and commodity prices must therefore equal their average costs of production (including the ordinary profit on the capital per unit of product). In other words, it is the equality between average costs and prices that makes it possible to determine the latter. Once this economic principle is used to write the equations, however, the system can then be addressed with every tool made available by mathematics.

As regards the case in hand, equations (1) – (3) are written according to the principle of the equality between average costs and prices: this is the economically meaningful system of equations. It is nevertheless mathematically sound to use equations (1) and (2) in order to obtain:

$$10 w (1 + r) = 85 p_c. \quad (4)$$

In doing this, we have used an equation in order to eliminate a variable, namely the rate of rent, as is usual when solving a system “by substitution”. Moreover, equations (3) and (4) form a system whose solution – for a given wage rate – makes it possible to obtain the relative price of the two commodities and the rate of profit.

This procedure is certainly correct in formal terms and, contrary to what one might think, makes no difference to our argument. The relative price of corn determined by solving equations (3) and (4) must be  $2/34$  here too, and the ratio between the amounts of labour embodied in the two commodities is still  $2/37$ . There is thus no contradiction between our result and the possibility of eliminating land from the system referred to by Sraffa and Guichard.

This way of solving the equations may, however, generate the optical illusion mentioned above. The fact that the rate of rent does not appear in equations (3) and (4) might seem to suggest that the relative price is not affected by it. This is not true, as the relative price depends on the relative average cost of the two commodities, and the average cost of corn *is* affected by the rate of rent. More precisely, if the rate of rent did not make the average cost of corn production with method  $\alpha$  equal to that with method  $\beta$ , equation (4) could not exist, as is clear from the way in which it is obtained. The price determined by equation (4) therefore includes the rate of rent even if it does not appear.

Moreover, great care must be taken not to infer from equation (4) that 10 units of labour are embodied in 85 quarters of corn. This would be true if 85 quarters were the gross product obtained by the employment of 10 workers with the least favourable method, i.e. if equation (4) had exactly the same economic meaning as equation (1). But this is not so. The 85 quarters of corn in equation (4) are not the gross product of 10 workers (which is instead 100 quarters with method  $\alpha$  and 92.5 with method  $\beta$ ) but a physical amount of gross profits. Equation (4) tells us only that a capital of  $10w$  employed in agriculture must earn the value of 85 quarters of corn as gross profits regardless of the method used. This fact is also proved by observing that while equations (1) and (2) can similarly be used in order to obtain

$$p = 15 p_c. \quad (5)$$

we cannot infer from this either that wages and profits do not affect the price of corn or that one quarter of corn “embodies”  $1/15$  acres of land. Equation (5) is instead what allows Ricardo to claim that the rent for an acre is the value of 15 quarters of corn.

This has been proved by means of Ricardo's own example. In particular, even assuming that capital in each sector consists exclusively of wages anticipated for one year, we have shown that in the presence of intensive differential rent, the relative price of commodities cannot be equal to the ratio of the quantity of labour they embody.

Malthus and Say were therefore right to argue that Ricardo's idea of rent not entering into the price of commodities is based on the presence of cultivated lands which pay no rent, a situation to be found only in very particular cases, such as a newly colonised country. But where all cultivated land pays a rent, which seems to be the general case, rent is an element of the cost and the price of agricultural commodities.

This result also rehabilitates an assertion made by Adam Smith and criticised by Ricardo. In countries where all the land is owned and all cultivated land pays a rent, the "original rule" is in fact altered in that the price of commodities is no longer determined by the relative quantity of labour embodied. In conclusion, therefore:

As soon as the land of any country has all become private property, the landlords, like all other men, love to reap where they never sowed, and demand a rent even for its natural produce. The wood of the forest, the grass of the field, and all the natural fruits of the earth, which, when land was in common, cost the labourer only the trouble of gathering them, come, even to him, to have an additional price fixed upon them. He must then pay for the licence to gather them; and must give up to the landlord a portion of what his labour either collects or produces. This portion, or, what comes to the same thing, the price of this portion, constitutes the rent of land, and in the price of the greater part of commodities makes a third component part. (Smith 1976: vol. 2, p. 67 – I.vi.8)

### **Appendix 1. Different definitions of embodied labour**

The basic definition of the labour embodied in a commodity is given in section 3 as the labour time employed directly in its production plus the labour embodied in all the commodities that are its means of production. As is known, this value can be determined in the usual case, where the number of methods in use is equal to that of commodities and there is no joint production, either by adding the labour embodied at every stage or by means of indirect ways, and primarily by determining relative prices in cases where wages are the only form of income.

If we apply this procedure to equations (1)–(3) on page 7 and posit that  $r = \rho = 0$ , it is clear that the system is over-determined. Once steel is adopted as the numéraire commodity, the system has three independent equations and only two unknowns, namely the wage rate  $w$  and the price of corn in terms of steel. This problem can be avoided, however, by the expedient of allowing the

prices (in terms of steel) of the corn produced with methods  $\alpha$  and  $\beta$  to differ. In this case, the system becomes:

$$20 w = 185 p_c^\beta \quad (\text{A.1})$$

$$10 w = 100 p_c^\alpha \quad (\text{A.2})$$

$$2 w = 1 \quad (\text{A.3})$$

and its solution is  $w = 1/2$ ,  $p_c^\alpha = 1/20$  and  $p_c^\beta = 2/37$ .  $p_c^\alpha$  and  $p_c^\beta$  are thus nothing other than the relative individual values of corn in terms of steel we have already determined in sec. 3.

Another indirect way to determine the labour embodied in commodities is the one Sraffa used with reference to the case of joint production.<sup>15</sup> As he writes in chapter IX of *Production of Commodities*:

if we wish to increase by a given amount the quantity in which a commodity enters the net product of the system, while leaving all the other components of the net product unchanged, we normally must increase the total labour employed by society. It is therefore natural to conclude that the quantity by which labour has be increased for this purpose goes in its entirety, whether directly or indirectly, to produce the additional quantity of the commodity in question. (Sraffa 1960, p. 57)

The use of this procedure to calculate the labour embodied by the commodities in the case of joint production was severely criticized by Steedman (1976 and 1977) and Toker (1984) because, as Sraffa himself admitted, it can imply negative quantities of labour embodied for some commodity.

Let us nevertheless try to apply this definition to the example considered here. In order to do this, we shall compare the amounts of labour used to produce 740 and 739 quarters of corn. According to Sraffa's argument, the difference between them will be equal to the labour embodied in the last quarter of corn.

As can be easily verified,<sup>16</sup> if 740 quarters of corn are to be produced on 6 acres of land, then 435.29 quarters have to be obtained by means of method  $\alpha$  and 304.71 by means of method  $\beta$ . It follows that the amount of labour employed in the corn sector is  $435.29(10/100) + 304.71(20/185) = 76.471$ . If 739 quarters are to be produced, then the quantities of corn obtained

<sup>15</sup> I thank Neri Salvadori for bringing Sraffa's particular conception of embodied labour to my attention.

<sup>16</sup> If  $C$  is the quantity of corn to be produced,  $C_\alpha$  and  $C_\beta$  are the quantities produced respectively by means of methods  $\alpha$  and  $\beta$ , and 6 acres is the available surface of land, this necessarily gives us:

$$\begin{aligned} C_\alpha + C_\beta &= C \\ C_\alpha/100 + C_\beta/185 &= 6. \end{aligned}$$

with methods  $\alpha$  and  $\beta$  are respectively 436.47 and 302.53, and the labour employed is  $436.47(10/100) + 302.53(20/185) = 76.353$ .

The resulting difference between the quantities of labour employed is thus 0.118 or  $4/34$ . Given that 2 units of labour are embodied in a ton of steel, it therefore follows that the ratio of the quantities of labour embodied, which is  $2/34$ , is equal to the relative price. This result calls, however, for closer examination.

Our first observation concerns the increase in the amount of labour employed in order to produce one quarter of corn more – i.e.  $4/34$  – which is greater than the amount needed to obtain one quarter of corn both with method  $\alpha$  ( $1/10$ ) and with method  $\beta$  ( $4/37$ ). The explanation of this fact is quite simple. In order to obtain one more unit of corn with method  $\beta$ , we need  $1/185$  acres of land, but since all the land is already being used, this area is made available by reducing the use of the more land-intensive method, namely  $\alpha$ .<sup>17</sup>

Second, it seems difficult to regard the difference between the increase in the amount of labour employed for the last unit of corn ( $4/34$ ) and the amount directly embodied in that unit ( $4/37$ ) as embodied labour. In actual fact, it is not labour either directly or indirectly embodied in corn, since capital consists exclusively in our example of wages paid in advance. How can this difference ( $4/34 - 4/37$ ) enter into the cost of corn?

The solution seems to be quite simple. As noted in our first observation,  $4/34$  is not only the labour embodied in the last quarter of corn but also includes the additional labour embodied in the other 739 quarters. This additional labour enters into the production cost of the last quarter of corn – as well as every other quarter – in the following way. The production of one unit of corn with method  $\beta$  involves  $4/37$  ( $= 20/185$ ) units of direct labour and  $1/185$  acres of land, and since the rent for an acre of land is the value of 15 quarters of corn, then the rent to be paid on  $1/185$  acres of land is the value of  $3/37$  ( $= 15/185$ ) quarters of corn. If these  $3/37$  quarters of corn were considered not as rent, which they are, but as seed, then the labour directly and indirectly employed in one quarter of corn would be the solution of the following equation:

$$4/37 + x \cdot 3/37 = x \quad (\text{A.4})$$

and therefore  $x = 4/34$ .

To conclude, the difference  $4/34 - 4/37$  should not be included in the amount of labour embodied in corn because it is the amount of labour embodied in the corn that pays the rent: an income and not an input.

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<sup>17</sup> The quantity of corn obtained with method  $\alpha$  drops from 436.47 quarters to 435.29, while that obtained with method  $\beta$  increases by more than one unit from 302.53 to 304.71.

## Appendix 2. Another example

As should now be clear, the purpose of this paper is to show that intensive differential rent enters into the price of agricultural products. This result is obtained in the main part of this paper by assuming that capital consists exclusively of wages paid in advance at the beginning of the year and proving that, even under this assumption, commodities are not exchanged at a ratio determined by their embodied quantities of labour when intensive differential rent is paid for the use of land. The same result will be obtained here by means of a different argument. We shall consider a case with a non-basic agricultural commodity and intensive rent, as found for example both in Montani (1975) and in Kurz (1978), and show that the latter enters into the price of the former.

In particular, let us follow Montani<sup>18</sup> and assume that there are two commodities: an industrial and basic commodity “a”, which is also the numéraire, and an agricultural and non-basic commodity “z”.

As regards technology, we have one method for the production of “a” and two different methods for the production of “z”, namely “ $\alpha$ ” and “ $\beta$ ”. In symbols:

$$a_a \text{ units of “a”} \ \& \ \ell_a \text{ units of labour} \ \rightarrow \ 1 \text{ unit of “a”}$$

$$a_z^\alpha \text{ units of “a”} \ \& \ \ell_z^\alpha \text{ units of labour} \ \& \ \lambda^\alpha \text{ units of land} \ \rightarrow \ 1 \text{ unit of “z”}$$

$$a_z^\beta \text{ units of “a”} \ \& \ \ell_z^\beta \text{ units of labour} \ \& \ \lambda^\beta \text{ units of land} \ \rightarrow \ 1 \text{ unit of “z”}$$

with  $\lambda^\alpha > \lambda^\beta$ , i.e. method “ $\beta$ ” gives a greater output per unit of land.

If both methods have to be in use in order to meet the demand for “z” fully, then, given a post-factum wage rate  $w$  (or a profit rate  $r$ ), the profit rate  $r$  (or the wage rate  $w$ ), the rent rate  $\rho$  and the price of “z”  $p_z$  in terms of “a” are determined, according to Sraffa’s theory, by solving the system:

$$a_a(1+r) + \ell_a w = 1 \quad (\text{A.5})$$

$$a_z^\alpha(1+r) + \ell_z^\alpha w + \lambda^\alpha \rho = p_z \quad (\text{A.6})$$

$$a_z^\beta(1+r) + \ell_z^\beta w + \lambda^\beta \rho = p_z \quad (\text{A.7})$$

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<sup>18</sup> A similar case can be found in Kurz (1978), pp. 26-28.

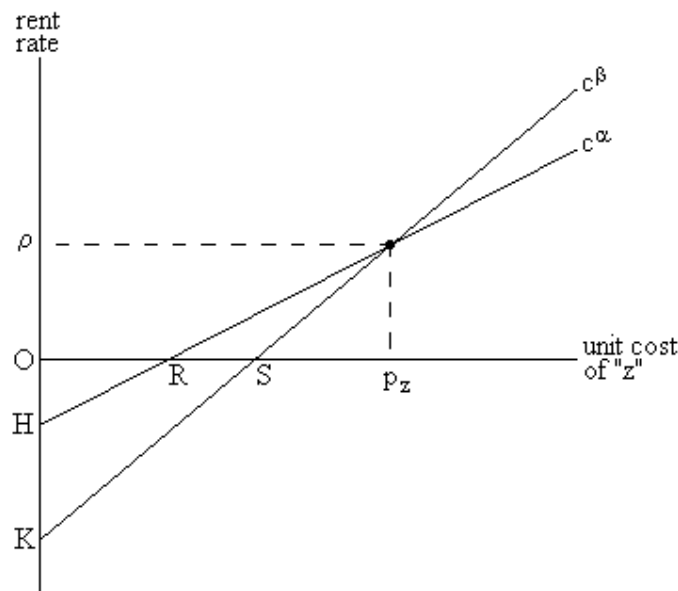
Given the wage rate level  $w^*$ , and having determined the corresponding profit rate  $r^*$  by means of equation (A.5), we can therefore use equations (A.6) and (A.7) – as both Montani and Kurz do – in order to obtain the “price-rent relation” implied by each of the two methods. More precisely, let us denote by  $c^\alpha$  and  $c^\beta$  the unit cost of production<sup>19</sup> of “z” using methods  $\alpha$  and  $\beta$  respectively. From equations (A.6) and (A.7), we thus have:

$$a_z^\alpha(1+r^*) + \ell_z^\alpha w^* + \lambda^\alpha \rho = c^\alpha \quad (\text{A.8})$$

$$a_z^\beta(1+r^*) + \ell_z^\beta w^* + \lambda^\beta \rho = c^\beta \quad (\text{A.9})$$

Leaving aside the details, for which readers are referred to the cited articles of Montani and Kurz, we can state that equations (A.8) and (A.9) entail the functions plotted in fig. 1.

Fig. 1



It is clear from fig. 1 that intensive differential rent enters into the unit cost with both methods,<sup>20</sup> albeit in different amounts, and therefore into the price of “z” in terms of “a”. The unit costs  $c^\alpha$  and  $c^\beta$  are both increasing functions of the rate of rent, but because  $\lambda^\alpha > \lambda^\beta$ ,  $c^\alpha$  grows faster than  $c^\beta$  when this rate increases. There is therefore a positive rate of rent  $\rho$  that allows  $c^\alpha$  to

<sup>19</sup> It is worth noting that Montani (1975, p. 81) – as well as Kurz (1978, p. 27) – does not include the intensive differential rent in the unit cost of production of commodity “z”. He is therefore considering a concept of cost that excludes entrepreneur’s revenues and assuming (implicitly) that landowners organise production, since rents are viewed as a residuum in this case. When the concept of full cost is adopted, however, as it is here, rent must be included in the cost of production of commodity “z” (even when landowners are entrepreneurs).

<sup>20</sup> While our fig. 1 is identical to fig. 4 in Montani (1975, p. 81) and fig. 3 in Kurz (1978, p. 28), they seem to use their figures solely in order to show the existence, under certain conditions, of economically meaningful solutions for equations (A.1), (A.2) and (A.3). The same figure is used here for a different purpose.

catch up with  $c^\beta$  and thus allows the two production methods to be simultaneously in use. Moreover, this rate makes the price  $p_z$  greater than the costs for wages and gross profits both with method  $\alpha$  (i.e. the length of segment OR in fig. 1) and with method  $\beta$  (the length of segment OS).

Therefore, contrary to Ricardo's claim that "no reduction would take place in the price of corn, although landlords should forego the whole of their rent" (1951-1973: vol. I, pp. 74, 75), if  $\rho = 0$  is posited, then both  $c^\alpha$  and  $c^\beta$  will be reduced. In particular, in the case of zero rents, a price equal to OS will allow the producers to pay the workers and recoup the capital with at least the ordinary profit with both the methods.

In conclusion, the intensive differential rent  $\rho$  enters into the price  $p_z$ . This can easily be stated here because "z" is a non-basic commodity and the part of its cost formed by wages and gross profits is therefore not affected by changes in the rate of rent. In this case, rents are clearly in addition to this part. Something similar happens in the Ricardian case discussed above, where capital is assumed to consist solely of wages paid in advance (and no basic commodity therefore exists). In more general cases, the link between intensive differential rent and the price of agricultural products is far more complex.

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