Social Status, Human Capital Formation and the Long-run Effects of Money

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ABSTRACT

This study examines the effects of monetary policy in a two-sector cash-in-advance economy of human capital accumulation. Agents concern about their social status represented by the relative physical capital and relative human capital. We find that if the desire for social status depends only on relative physical capital, money is superneutral in the growth-rate sense. However, if the desire for social status depends on relative human capital, the money growth rate will have a positive effect on the long-run economic growth rate. Furthermore, an increase in the desire to pursue human capital will raise the long-run growth rate, but an increase in the desire to pursue physical capital will lower it.

Keywords: Cash-in-advance economy; Endogenous growth; Social status.

JEL Classification: C62, E52, O42.

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1. INTRODUCTION

Social status has long interested economists in a variety of research fields; see, among others, Cole et al. (1992), Bakshi and Chen (1996), Fisher and Hof (2005) and Chang et al. (2008). The concept of social status can be traced back to the "spirit of capitalism" of Weber (1958) and the "wealth effects" of Kurz (1968). In contemporary research, the desire for social status is usually represented by wealth-induced preference in economic models. By motivating the accumulation of physical capital, the presence of wealth-enhanced social status will affect households' consumption and savings decisions. This will in turn affect economic growth (Zou, 1994; Wirl, 1994).

There is a growing literature concerning the role of social status in a monetary economy and its impact on the effectiveness of monetary policy. Tobin (1965) demonstrates that in a descriptive aggregate model, a permanent increase in the growth rate of nominal money raises the inflation rate and lowers the real interest rate. This leads to portfolio substitution from real balances to capital (Tobin's effect).[1] Stockman (1981) develops a cash-in-advance (CIA) model where both consumption and investment are liquidity constrained and shows that an increase in the money growth rate will lower the steady-state value of physical capital.[2] Social status is introduced into a CIA model by Gong and Zou (2001), Chang et al. (2000, 2003) and Chen and Guo (2009) in order to study how wealth-enhanced social status affects the impact of monetary policy on economic growth.[3] Gong and Zou (2001) and Chang et al. (2000, 2003) demonstrate that with the presence of social status, the money growth

1 However, Sidrauski (1967) argues that in an infinite-horizon, representative-agent model, money growth does not affect the steady-state value of physical capital.

2 Based on a CIA model, Abel (1985) shows that monetary policies will not affect the steady-state value of physical capital if the cash-in-advance constraint is applied solely to consumption. Note that endogenous growth is not exhibited in Stockman (1981) and Abel (1985). The effectiveness of monetary policy in an endogenous-growth CIA model is examined by Chang et al. (2000), Suen and Yip (2005) and Chen and Guo (2011).

3 Note that Chang et al. (2000) and Chen and Guo (2009) display endogenous growth while Gong and Zou (2001) and Chang et al. (2003) do not.

rate will positively affect the level or growth rate of output if consumption is liquidity constrained. However, Chen and Guo (2009) argue that if the cash-in-advance constraint is applied to both consumption and investment, an increase in the growth rate of money will decrease the economic growth rate even under wealth-induced preferences. The related literature is summarized in Table 1.[4]

<Table 1 is inserted about here>

The focus of previous studies examining the role of social status in a monetary economy has been on the role of physical capital. These studies have largely ignored the role of human capital. In this paper, we investigate how social status affects the effectiveness of monetary policy in a two-sector endogenous-growth CIA model with Lucas-type human capital accumulation.[5] Furthermore, the incorporation of human capital into the model allows us to consider the impact of social-status formation.[6] We assume that higher social status is bestowed on relative physical capital and relative human capital. To the best of our knowledge, this is the first paper to study the impact of monetary policy in a monetary economy with social status represented by relative human capital. Feathermand and Stevens (1982) demonstrate that human capital appears to be an important determinant of social status. Besides a monetary reward, professionals often earn rewards in the form of social esteem. Based on this idea, Fershtman and Weiss (1993) and Fershtman et al. (1996) develop a general equilibrium model with social status represented by occupation which depends on human capital to study the interplay between occupation and economic growth.

Monetary policies can affect the accumulation of physical capital through two

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[4] Table 1 is not intended to be an exhaustive literature review and many important contributions may not be included.

[5] The superneutrality of money in a two-sector CIA model with wealth-induced social status is studied by Chen (2011a, 2011b).

[6] In the literature, social status is represented by a variety of formations. For example, Corneo and Jeanne (1997b) and Clemens (2004) use relative wealth to represent social status. Rauscher (1997) and Corneo and Jeanne (1997a) use conspicuous consumption to represent social status.

channels. Tobin's effect demonstrates that a higher money growth rate increases investment due to the portfolio substitution effect. However, an increase in the inflation rate also reduces the real money balance which in turn will affect investment through the cash-in-advance constraint. There are two types of cash-in-advance constraints considered in this paper. We first assume that consumption is liquidity constrained. In this case, an increase in the money growth rate reduces future consumption through the cash-in-advance constraint. This induces the agent to use current consumption to substitute for future consumption and reduces investment. Second, we assume that both consumption and investment are liquidity constrained. In this case, an increase in the money growth rate induces a lower real balance which restricts the agent's ability to both consume and invest.

In both cases, we show that a two-sector endogenous-growth CIA model with social status can be represented by a four-dimensional dynamic system, and we also examine the existence and uniqueness of the equilibrium. We find that the superneutrality of money in the growth-rate sense depends on the nature of the formation of social status. When the social status depends only on relative physical capital, changes in monetary policy will not affect economic growth and money is growth-rate superneutral. This is because although the existence of wealth-induced preferences increases physical capital accumulation, it lowers the incentive for human capital accumulation. However, when the desire for social status depends on relative human capital, the money growth rate will positively affect the economic growth. The presence of human-capital-induced preference reinforces the incentive for human capital accumulation (direct effect). Since human capital and physical capital are complements in the process of production, an increase in the human capital accumulation will increase the return of investment in physical capital (indirect effect). Both effects encourage economic growth.

When the desire to pursue physical capital increases, the incentive of physical capital accumulation becomes higher and causes higher investment in physical capital. This induces a larger fraction of human capital to be involved in output production and the long-run growth rate will decrease since a smaller fraction of human capital is devoted to human capital accumulation. On the other hand, when the desire to pursue human capital increases, it strengthens the motivation of human capital accumulation. The long-run economic growth will increase because of a higher fraction of human capital devoted to human capital accumulation.

The remainder of this paper is organized as follows. In the next section we develop a two-sector endogenous growth model where consumption is liquidity constrained and examine the growth-rate superneutrality of money. In Sections 3, we study the effects of money growth rate on the long-run growth in a monetary economy where both consumption and investment are liquidity constrained. The final section concludes.

2. A TWO-SECTOR CIA MODEL

We begin our analysis by considering a two-sector CIA model of human capital accumulation, with the economy comprising of a representative, infinitely-lived agent. Following the idea proposed by Corneo and Jeanne (1997), we assume that social status is represented by the *relative* physical (human) capital. The representative agent cares about both consumption (c_t) and social status, which is composed by the relative physical capital and relative human capital. The discounted lifetime utility is:

$$\int_0^\infty \left[\log(c_t) + \beta v(k_t / K_t) + \sigma v(h_t / H_t)\right]\exp(-\rho t)dt,$$

where $\rho \in (0,1)$ is the discount factor, k_t and K_t respectively denote individual and aggregate physical capital and h_t and H_t respectively represent individual and aggregate human capital. The parameters $\beta \geq 0$ and $\sigma \geq 0$ respectively denote the desire for social status represented by physical and human capital. Let $v'(q_t / Q_t)$

represent the derivative of function $v(q_t/Q_t)$ with respect to (q_t/Q_t). The periodic utility function $v(q_t/Q_t)$ has the properties that $v'(1)=1$, $v_{q_t}=\partial v/\partial q_t = v'(q_t/Q_t)/q_t > 0$ and $v_{q_t q_t} = \partial^2 v/\partial q_t^2 < 0$ where $q_t = k_t$, h_t and $Q_t = K_t$, H_t.

We consider an endogenous growth monetary economy comprising of two sectors, one of which produces output used for consumption and investment, while the other produces human capital. Let u_t and $(1-u_t)$ respectively represent the proportion of human capital devoted to the production of output and the accumulation of human capital. Labor supply, which is inelastic, is normalized to unity. There is a constant-returns-to-scale production technology for output (y_t) which is produced by using both physical capital and effective labor ($u_t h_t$):

$$y_t = Ak_t^{\alpha}(u_t h_t)^{1-\alpha}, \tag{1}$$

where $A > 0$ is total factor productivity and $\alpha \in (0,1)$ denotes the capital share of output.

Following Lucas (1988), we assume that human capital accumulation evolves based on the following equation:

$$\dot{h}_t = B(1-u_t)h_t, \tag{2}$$

where $B > \rho$ represents the technology parameter for human capital accumulation.[7]

In period t, the government injects money into the economy by giving nominal lump-sum transfers to the representative agent. We assume that the nominal money supply grows at the rate of $\mu > 0$. Let p_t and m_t denote the common price and individual real money balance in period t. The representative agent uses the real money balance in period t, determined in the previous period $t-1$, to buy goods for consumption and investment. Thus, the cash-in-advance constraint for the

[7] We assume $B > \rho$ so that the growth rate at equilibrium is positive when social status is not present.

representative agent is:

$$c_t + \phi i_t \le m_t . \tag{3}$$

When $\phi = 0$, the cash-in-advance constraint is only applied to consumption. When $\phi = 1$, both consumption and investment are subject to the cash-in-advance constraint.

The budget constraint for the representative agent is:

$$c_t + i_t + \dot{m}_t = r_t k_t + w_t u_t h_t - \pi_t m_t + \tau_t , \tag{4}$$

where i_t denotes investment, r_t is the real rental rate of physical capital, w_t is the real wage, $\pi_t = \dot{p}_t / p_t$ represents the inflation rate and $\tau_t = \mu m_t$ is the real lump-sum transfers that households receive from the monetary authority.

The law of motion of capital follows:

$$\dot{k}_t = i_t - \delta k_t , \tag{5}$$

where $\delta \in [0,1]$ is the depreciation rate of physical capital. The real interest rate is therefore represented by ($r_t - \delta$).

Let λ_{ht}, λ_{mt} and λ_{kt} respectively represent the co-state variables of Eqs. (2), (4) and (5) and ξ_t represents the multiplier of Eq. (3). The first-order necessary conditions for the representative agent's optimization problem are:

$$\frac{1}{c_t} = \lambda_{mt} + \xi_t , \tag{6}$$

$$\lambda_{mt} + \phi \xi_t = \lambda_{kt} , \tag{7}$$

$$\lambda_{mt} w_t = \lambda_{ht} B , \tag{8}$$

$$\dot{\lambda}_{kt} = (\rho + \delta)\lambda_{kt} - \beta v'\left(\frac{k_t}{K_t}\right)\frac{1}{K_t} - \lambda_{mt} r_t , \tag{9}$$

$$\dot{\lambda}_{mt} = (\rho + \pi_t)\lambda_{mt} - \xi_t , \tag{10}$$

$$\dot{\lambda}_{ht} = \rho\gamma_t - \sigma v'\left(\frac{h_t}{H_t}\right)\frac{1}{H_t} - \lambda_{mt} w_t u_t - \lambda_{ht} B(1-u_t), \tag{11}$$

plus the transversality conditions:

$$\lim_{t\to\infty} e^{-\rho t}\lambda_{kt} k_t = 0, \qquad \lim_{t\to\infty} e^{-\rho t}\lambda_{mt} m_t = 0, \qquad \lim_{t\to\infty} e^{-\rho t}\lambda_{ht} h_t = 0.$$

Eq. (6) demonstrates that the marginal benefit equals the marginal cost of consumption. Eqs. (7) and (9) govern the evolution of physical capital, where the marginal utility benefit from agents' status-seeking physical capital accumulation is captured by the term $\beta v'\left(\frac{k_t}{K_t}\right)\frac{1}{K_t}$. Eq. (10) demonstrates that the marginal values of real money holdings equals the marginal costs. Eq. (8) states that the marginal returns of human capital from final production and human capital accumulation should be equal and it determines the allocation of human capital. Eqs. (8) and (11) together govern the evolution of human capital over time, where $\sigma v'\left(\frac{h_t}{H_t}\right)\frac{1}{H_t}$ reflects agents' status seeking of human capital accumulation.

As is common in the literature, we assume that the cash-in-advance constraint is strictly binding ($\xi_t > 0$). In a symmetric equilibrium all agents consume and invest the same amount of goods, own the same amount of capital and real money balance and devote the same amount of human capital in production. That is, $c_t = C_t$, $i_t = I_t$, $k_t = K_t$, $h_t = H_t$, $m_t = M_t$ and $u_t = U_t$ where C_t, I_t, M_t and U_t respectively represent the aggregate consumption, investment, real money balance and the fraction of human capital devoted to the production of output. In the equilibrium, the factor prices are:

$$r_t = A\alpha K_t^{\alpha-1}(U_t H_t)^{1-\alpha}, \tag{12}$$

$$w_t = A(1-\alpha)K_t^{\alpha}(U_t H_t)^{-\alpha}. \tag{13}$$

The law of motion of the real money balance is:

$$\dot{M}_t = (\mu - \pi_t)M_t. \tag{14}$$

The goods market clearing condition implies:

$$C_t + \dot{K}_t + \delta K_t = Y_t, \tag{15}$$

where Y_t is the aggregate output. We define two stationary variables as $z_t = \frac{C_t}{K_t}$ and $x_t = \frac{H_t}{K_t}$. Combining Eqs. (2) and (15) gives us the evolution of x_t:

$$\frac{\dot{x}_t}{x_t} = \frac{\dot{H}_t}{H_t} - \frac{\dot{K}_t}{K_t} = \delta + B(1-U_t) - A(U_t x_t)^{1-\alpha} + z_t. \tag{16}$$

Substituting Eqs. (8) and (13) into Eq. (11), we can derive:

$$\frac{\dot{\lambda}_{ht}}{\lambda_{ht}} = \rho - B - \frac{\sigma}{\lambda_{ht}H_t}. \tag{17}$$

2.1 Cash for Consumption Only

We first consider the case where the cash-in-advance constraint is applied only to consumption (that is, $\phi = 0$). We define another stationary variable as $\psi_t = \frac{1}{K_t \lambda_{kt}}$.

The inflation rate is determined by Eqs. (6), (7), (9) and (10):

$$\pi_t = -1 + \delta - A\alpha(U_t x_t)^{1-\alpha} - \beta\psi_t + \frac{\psi_t}{z_t}. \tag{18}$$

Combining Eqs. (7), (9) and (15), the evolution of ψ_t is governed by:

$$\frac{\dot{\psi}_t}{\psi_t} = -\rho - A(1-\alpha)(U_t x_t)^{1-\alpha} + \beta\psi_t + z_t. \tag{19}$$

Using Eqs. (3), (14), (15) and (18), the dynamics of z_t is based on:

$$\frac{\dot{z}_t}{z_t} = 1 + \mu + \beta\psi_t - A(1-\alpha)(U_t x_t)^{1-\alpha} + z_t - \frac{\psi_t}{z_t}. \tag{20}$$

Combining Eqs. (8) and (17) gives us:

$$\frac{\dot{\lambda}_{ht}}{\lambda_{ht}} = \rho - B - \frac{\sigma B \psi_t}{A(1-\alpha)x_t^{1-\alpha}U_t^{-\alpha}}. \tag{21}$$

Combining Eqs. (7)-(9), (16) and (21), the dynamics of U_t is governed by:

$$\frac{\dot{U}_t}{U_t} = \frac{(1-\alpha)\delta}{\alpha} + \frac{B}{\alpha}\left[1 + \frac{\sigma\psi_t}{A(1-\alpha)x_t^{1-\alpha}U_t^{-\alpha}}\right] - B(1-U_t) - z_t - \frac{\beta}{\alpha}\psi_t. \tag{22}$$

Thus, the dynamic behavior of the economy is represented by a four- dimensional dynamic system of Eqs. (16), (19), (20) and (22) in x_t, ψ_t, z_t and U_t.

2.2 The BGP Equilibrium

Along the balanced growth path (BGP) equilibrium, C_t, Y_t, K_t, H_t and M_t grow at a common growth rate (g^*), while the common growth rate for λ_{kt} and λ_{mt} is ($-g^*$). Thus, along the BGP equilibrium, the stationery variables ψ_t, z_t, x_t, and U_t do not grow. Let ψ^*, z^*, x^* and U^* represent the steady-state values of ψ_t, z_t, x_t and U_t. From Eqs. (16), (19), (20) and (22), the steady-state conditions are:

$$z^* = \frac{(1-\alpha)[B(1-U^*)+\delta]+\rho}{\alpha+\beta(1+\mu+\rho)}, \tag{23a}$$

$$\psi^* = (1+\mu+\rho)z^*, \tag{23b}$$

$$A(1-\alpha)(U^*x^*)^{1-\alpha} = [1+\beta(1+\mu+\rho)]z^* - \rho, \tag{23c}$$

$$B\left[1 + \frac{\sigma U^*(1+\mu+\rho)z^*}{-\rho+[1+\beta(1+\mu+\rho)]z^*}\right] = B(1-U^*) + \rho. \tag{23d}$$

From Eqs. (23a), (23d), it follows that U^* is implicitly determined by:

$$f(U^*, \mu, \beta, \sigma) = 0, \tag{24a}$$

where

$$f(U,\mu,\beta,\sigma) = B(1-U) + \rho - B\left\{1 + \frac{\sigma U(1+\mu+\rho)z(U,\mu,\beta)}{-\rho+[1+\beta(1+\mu+\rho)]z(U,\mu,\beta,\sigma)}\right\} \tag{24b}$$

and

$$z(U,\mu,\beta)=\frac{(1-\alpha)[B(1-U)+\delta]+\rho}{\alpha+\beta(1+\mu+\rho)}. \tag{24c}$$

Eq. (24c) implies that $\partial z(U,\mu,\beta)/\partial U<0$ .To examine the existence and uniqueness of the steady state, we take the first order derivatives of $f(U,\mu,\beta,\sigma)$ and obtain:

$$\frac{\partial f(U,\mu,\beta,\sigma)}{\partial U}$$
$$=-B\left\{1+\sigma(1+\mu+\rho)\frac{z(U,\mu,\beta)\{-\rho+[1+\beta(1+\mu+\rho)]z(U,\mu,\beta)\}-\rho U\dfrac{\partial z(U,\mu,\beta)}{\partial U}}{\{-\rho+[1+\beta(1+\mu+\rho)]z(U,\mu,\beta)\}^2}\right\}<0$$

Thus, $f(U,\mu,\beta,\sigma)$ is a monotonically decreasing function in U . Since $U\in[0,1]$, the boundary values of $f(U)$ are:

$$f_0=f(0)=\rho>0,$$
$$f_1=f(1)=\rho-B\left[1+\frac{\sigma(1+\mu+\rho)[(1-\alpha)\delta+\rho]}{(1-\alpha)\{\rho+[1+\beta(1+\mu+\rho)]\delta\}}\right].$$

Therefore, a sufficient condition for the existence and uniqueness of the steady state is that $f_1<0$. That is,

$$B>B_1=\frac{\rho(1-\alpha)\{\rho+[1+\beta(1+\mu+\rho)]\delta\}}{(1-\alpha)\delta[1+(\sigma+\beta)(1+\mu+\rho)]+[1-\alpha+\sigma(1+\mu+\rho)]\rho}.$$

Function $f(U,\mu,\beta,\sigma)$ is illustrated in Figure 1. We summarize our results in the following proposition.

Proposition 1. When the desire for the social status is present and there is a cash-in-advance constraint applied to consumption in a two-sector CIA model with human capital accumulation, there exists a unique BGP equilibrium if $B>B_1$.

<Figure 1 is inserted about here>

Eq. (24a) indicates that the solution of U^* is:

$$U^* = U^*(\mu, \beta, \sigma) . \tag{24d}$$

The BGP growth rate can be calculated by using Eq. (2):

$$g^*(\mu, \beta, \sigma) = B(1 - U^*(\mu, \beta, \sigma)) . \tag{25}$$

We now turn to examine the effects of monetary policy. The first-order partial derivate of $f(U, \mu, \beta, \sigma)$ with respect to μ is:

$$\frac{\partial f(U, \mu, \beta, \sigma)}{\partial \mu} = \frac{-\sigma B U z(U, \mu, \beta)(1-\alpha)[B(1-U) + \delta + \rho]}{\{-\rho + [1 + \beta(1 + \mu + \rho)] z(U, \mu, \beta)\}^2 [\alpha + \beta(1 + \mu + \rho)]} . \tag{26}$$

Regarding social status, there are four possible situations. First, we consider the case where the desire for social status is not present ($\beta = \sigma = 0$). In this case, Eq. (26) indicates that $\partial f(U, \mu, \beta, \sigma) / \partial \mu = 0$. That is, changes in the money growth rate will not affect the fraction of human capital used for human capital accumulation and the long-run economic growth rate. This confirms the results found by Marquis and Reffett (1991). Furthermore, using Eqs. (23a)-(23d) and (25), one can derive a constant BGP growth rate:

$$g^* = B - \rho . \tag{27a}$$

A permanent increase in μ reduces the real money balance, which depresses future consumption through the cash-in-advance constraint. Thus, the agent tends to use current consumption to substitute for future consumption, thereby reducing investment to the detriment of physical capital accumulation. On the other hand, an increase in the inflation rate raises the cost of money holdings. This causes the representative household to substitute out of real balances and into physical capital. These two effects cancel each other out so that an increase in μ does not affect the long-run economic growth rate.

Note that the BGP growth rate in the CIA model with the standard Barro-Rebelo "*AK*" type production is:

$$g^* = A - \rho - \delta . \tag{27b}$$

Eqs. (27a) and (27b) indicate that when the desire for social status is not present, money is superneutral in the growth-rate sense, independent of the consideration of human capital accumulation. However, the long-run growth rate is driven by the production function in the "*AK*" model while it is driven by the human capital formation in an endogenous model with human capital accumulation.

Next, we consider the case where the desire for social status is represented by the level of physical capital ($\sigma = 0$). In this case, Eq. (26) indicates that $\partial f(U, \mu, \beta, \sigma) / \partial \mu = 0$ and money is superneutral in the growth-rate sense. The long-run growth rate is the same as the one given by Eq. (27b). This finding of the growth-rate superneutrality of money in a two-sector endogenous-growth economy with the wealth-induced preference is in line with the results in Chen (2011a) where the social status is represented by the absolute physical capital.

Note that this result is very different from the result found in the CIA model without human capital accumulation. Chang, Hsieh and Lai (2000) show that the BGP growth rate in the CIA model with "*AK*" type production function and status seeking is:

$$g^* = A - \frac{\rho}{1 + \beta(1 + \mu + \rho)} - \delta . \tag{27c}$$

Eq. (27c) implies that money growth rate positively affects the economic growth rate in the long run when human capital accumulation is not taken into consideration. Our finding is different from theirs because the presence of social-status seeking will strengthen the motivation of physical capital accumulation but lead to a relative reduction in the incentive for human capital accumulation. The former is beneficial to economic growth while the latter hurts the economic growth. Overall, the economic growth rate is not affected by the money growth rate.

Third, when social status is represented by the level of human capital ($\beta = 0$,

$\sigma > 0$), Eq. (26) indicates that under this case, $\partial f(U,\mu,\beta,\sigma)/\partial\mu < 0$. Then an increase μ will shift the curve of $f(U,\mu,\beta,\sigma)$ in Figure 1 downward. This will reduce U^* and increase ($1-U^*$). Therefore, an increase in μ will raise g^*. The presence of social status strengthens the motivation of human capital accumulation. Moreover, since physical capital and human capital are complements in the production function, an increase in the rate of human capital accumulation will also raise the returns to physical capital and investment in physical capital will increase. Both increases in human capital and physical capital accumulation encourage the economic growth rate.

The last case we consider is that in which social status is represented by the levels of physical capital and human capital. In this case, $\partial f(U,\mu,\beta,\sigma)/\partial\mu < 0$. and an increase in the money growth rate will raise the long-run economic growth rate. From our analysis in the previous two cases, we know that the non-superneutrality of money is caused by the presence of social status represented by relative human capital. We summarize our findings in Table 1 and in the following proposition:

Proposition 2: *When a cash-in-advance constraint is applied to consumption, the growth-rate superneutrality of money is conditional on the specification of social status formation. Money is superneutral in the growth-rate sense if there is no desire for social status or if social status depends only on relative physical capital. However, if social status depends on relative human capital, an increase in the money growth rate will raise the BGP growth rate.*

To study the impact of social status, we first calculate the first-order partial derivate of $f(U,\mu,\beta,\sigma)$ with respect to β:

$$\frac{\partial f(U,\mu,\beta,\sigma)}{\partial \beta}=\frac{\sigma BU(1+\mu+\rho)^2(1-\alpha)z(U,\mu,\beta)[B(1-U)+\delta]}{\{-\rho+[1+\beta(1+\mu+\rho)]z(U,\mu,\beta)\}^2[\alpha+\beta(1+\mu+\rho)]}>0. \quad (28)$$

Eq. (28) implies that an increase in β will shift $f(U^*)$ upward. Therefore, an increase in β raises U^* and reduces $(1-U^*)$, so the long-run economic growth rate will decrease. This is because an increase in β induces higher investment in physical capital and also causes a larger fraction of human capital to be involved in output production. With the decrease in the fraction of human capital devoted to human capital accumulation, the BGP growth rate decreases.

Then we calculate the first-order partial derivative of $f(U,\mu,\beta,\sigma)$ with respect to σ:

$$\frac{\partial f(U,\mu,\beta,\sigma)}{\partial \sigma}=\frac{-BU(1+\mu+\rho)z(U,\mu,\beta)}{-\rho+[1+\beta(1+\mu+\rho)]z(U,\mu,\beta)}<0. \quad (29)$$

Eq. (29) implies that an increase in σ will shift $f(U^*)$ downward .Therefore, an increase in σ reduces U^* and thus raises g^*. Since an increase in σ reinforces the motivation to accumulate human capital, it causes a higher fraction of human capital to be devoted to human capital accumulation. Therefore, the long-run economic growth increases.

<Table 2 is inserted about here>

Because the model is represented by a four-dimensional dynamical system, it is difficult to examine the stability around the equilibrium analytically. Thus, we resort to numerical methods and study the local property of the dynamic behavior around the equilibrium by assigning reasonable parameter values. We set $\alpha=1/3$, $\phi=0.2$, $\rho=0.025$, $A=0.1$, $B=0.02$ and $\delta=0.05$. Table 2 presents values of U^*, z^*, x^*, and g^* with varying μ, β and σ. Rows 2, 3 and 4 display the effects with an increase in μ from 0.05 to 1, rows 2, 5 and 6 present the impacts of an increase in β from 0.8 to 1.2 and rows 2, 7 and 8 show the impacts with an increase in σ from 0.8 to 1.2. Our numerical results show that an increase in μ or σ will lower U^*, but will raise g^*. On the other hand, an increase in β will increase U^*, but will decrease g^*. To study the stability around the equilibrium, we compute the Jacobian matrix of the dynamical system of Eqs. (16), (19), (20) and (22) evaluated at the steady state and calculate its eigenvalues. In all cases shown in Table 2, there are three eigenvalues with positive real parts and one eigenvalue with negative real part. Since there are three jump variables (ψ_t, z_t and U_t) and one non-jump variable (x_t) in the model, this indicates that the BGP equilibrium exhibits saddle-path stability in all cases.

3. CASH FOR CONSUMPTION AND INVESTMENT

We now turn to consider the case where $\phi = 1$. That is, the cash-in-advance constraint applied to both consumption and investment. In order to simplify the derivation, instead of using p_t, we define a new stationary variable $\theta_t = \frac{\lambda_{kt}}{\lambda_{mt}}$. Combining Eqs. (6)-(8) and (17), we have:

$$\frac{\dot{\lambda}_{ht}}{\lambda_{ht}} = \rho - B - \frac{\sigma B z_t \theta_t}{A(1-\alpha) x_t^{1-\alpha} U_t^{-\alpha}}. \tag{30}$$

Combining the binding cash-in-advance constraint and the resource constraint, we can derive:

$$M_t = A_t K_t^{\alpha} (U_t H_t)^{1-\alpha}. \tag{31}$$

Taking logarithms on both sides of Eq. (31) and differentiating with respect to time, then combining with Eq. (14), the inflation rate is endogenously determined by:

$$\pi_t = \mu - \alpha \frac{\dot{K}_t}{K_t} - (1-\alpha)\left(\frac{\dot{U}_t}{U_t} + \frac{\dot{H}_t}{H_t} \right). \tag{32}$$

Combining Eqs. (6), (7), (9), (10) and (32), we have:

$$\frac{\dot{\theta}_t}{\theta_t} = (1-\alpha)\delta - (\alpha+\beta)z_t + \frac{(\theta_t - 1)A\alpha(U_t x_t)^{1-\alpha}}{\theta_t} + (1-\alpha)\left[\frac{\dot{U}_t}{U_t} + B(1-U_t)\right] - 1 - \mu + \theta_t \,. \tag{33}$$

Combining Eqs. (6), (7), (9) and (15) gives us:

$$\frac{\dot{z}_t}{z_t} = -\rho + (1+\beta)z_t + \left(\frac{\alpha}{\theta_t} - 1\right)A(U_t x_t)^{1-\alpha} \,. \tag{34}$$

Using Eqs. (8), (10), (16), (30) and (32), the dynamics of U_t can be expressed as:

$$\frac{\dot{U}_t}{U_t} = 1 + \mu + BU_t - \theta_t + \frac{\sigma B \theta_t z_t}{A(1-\alpha)x_t^{1-\alpha}U_t^{-\alpha}} \,. \tag{35}$$

Eqs. (16) and (33)-(35) constitute a four-dimensional dynamic system of equations represented by x_t, θ_t, z_t and U_t.

3.1 The BGP Equilibrium

Along the BGP equilibrium, the stationery variables x_t, θ_t, z_t and U_t do not grow. From Eqs. (16) and (33)-(35), the steady state values x^*, θ^*, z^* and U^* are determined by the following equations:

$$\theta^* = 1 + \mu + \rho \,, \tag{36a}$$

$$z^* = \frac{(1+\mu+\rho-\alpha)[B(1-U^*)+\delta] + \rho(1+\mu+\rho)}{\alpha + \beta(1+\mu+\rho)} \,, \tag{36b}$$

$$A(U^* x^*)^{1-\alpha} = z^* + B(1-U^*) + \delta \,, \tag{36c}$$

$$(1-\alpha)(\rho - BU^*)[z^* + B(1-U^*) + \delta] = (1+\mu+\rho)\sigma B U^* z^* \,. \tag{36d}$$

From Eqs. (36b) and (36d), it follows that U^* is implicitly determined by:

$$\Lambda(U^*, \mu, \beta, \sigma) = 0 \,, \tag{37a}$$

where

$$\begin{aligned} &\Lambda(U, \mu, \beta, \sigma) \\ &= (1-\alpha)(\rho - BU)[z(U,\mu,\beta) + B(1-U) + \delta] - (1+\mu+\rho)\sigma B U z(U,\mu,\beta) \end{aligned} \tag{37b}$$

and

$$z(U,\mu,\beta)=\frac{(1+\mu+\rho-\alpha)[B(1-U)+\delta]+\rho(1+\mu+\rho)}{\alpha+\beta(1+\mu+\rho)}. \tag{37c}$$

Eq. (37a) indicates that the solution of U^* is:

$$U^*=U^*(\mu,\beta,\sigma). \tag{37d}$$

Once U^* is determined, z^* and x^* can be determined uniquely by Eqs. (36b)-(36c). Eq. (37b) implies that to guarantee the existence of the solution of $U^*\in(0,1)$, the parameters are subject to the constraint $\rho>BU^*$.

From Eq. (37c), we can easily derive that $\partial z(U,\mu,\beta)/\partial U<0$. Using Eq. (37a)-(37c), we have:

$$\frac{\partial\Lambda(U,\mu,\beta,\sigma)}{\partial U}=\frac{\partial g_1(U,\mu,\beta)}{\partial U}-\frac{\partial g_2(U,\mu,\beta,\sigma)}{\partial U}<0, \tag{38}$$

where

$$g_1(U,\mu,\beta)=(1-\alpha)\left\{-B[z(U,\mu,\beta)+B(1-U)+\delta]+(\rho-BU)\left(\frac{\partial z(U,\mu,\beta)}{\partial U}-B\right)\right\}<0$$

and

$$g_2(U,\mu,\beta,\sigma)=\sigma B(1+\mu+\rho)\frac{(1+\mu+\rho-\alpha)[B(1-2U)+\delta]+(1+\mu+\rho)\rho}{\alpha+\beta(1+\mu+\rho)}>0.$$

Eq. (38) indicates that $\Lambda(U,\mu,\beta,\sigma)$ is a monotonically decreasing function in U. Since $U\in[0,1]$, the boundary values of $\Lambda(U,\mu,\beta,\sigma)$ are:

$$\Lambda_0=\frac{(1-\alpha)\rho(1+\mu+\rho)[(B+\delta)(1+\beta)+\rho]}{\alpha+\beta(1+\mu+\rho)}>0,$$

$$\Lambda_1=\frac{(1+\mu+\rho)\{(1-\alpha)(\rho-B)[\rho+(1+\beta)\delta]-B\sigma[(1+\mu+\rho)(\delta+\rho)-\alpha\delta]\}}{\alpha+\beta(1+\mu+\rho)},$$

where Λ_0 and Λ_1 are boundary values evaluated at $U=0$ and $U=1$, respectively.

Therefore, a sufficient condition for the uniqueness of the steady state is that $\Lambda_1<0$. That is,

$$B > B_2 = \frac{\rho(1-\alpha)[\rho+(1+\beta)\delta]}{\sigma[(1+\mu+\rho)(\delta+\rho)-\alpha\delta]+(1-\alpha)[\rho+(1+\beta)\delta]}.$$

The BGP growth rate is given in Eq. (26).

To examine how the money growth rate affects the long-run growth rate, we first consider case where the desire for social status is not present ($\beta = \sigma = 0$) or is represented solely by physical capital ($\sigma = 0$). Eq. (37a) indicates that under these two cases, the fraction of human capital devoted to production is constant and equals $U^* = \rho / B$. Thus, money is superneutral in the growth-rate sense since changes in the money growth rate do not affect the fraction of human capital devoted to human capital accumulation and the long-run economic growth rate. These results are consistent with the findings in Marquis and Reffett (1991) and Chen (2011b). In both cases, the long-run economic growth rate is: $g^* = B - \rho$.

Our findings of the growth-rate superneutrality of money when social status is not present or when it is represented solely by physical capital are very different from previous findings in the "*AK*" model without human capital accumulation. Chen and Guo (2000) show that in the "*AK*" model with cash constraint applied on both consumption and investment, the BGP growth rate is driven by the production function:

$$g^* = A - z^*_{AK} - \delta,$$

where $z^*_{AK} = [\rho(1+\mu+\rho) + A(\mu+\rho)]/[(1+\beta)(1+\mu+\rho)]$. This implies that an increase money is not growth-rate superneutral because an increase in the money growth rate will reduce the long-run growth rate ($\partial g^* / \partial \mu < 0$), independent of the presence of status seeking.

To understand why our result is different from those obtained in the "*AK*" model, note that BGP growth rate in our model is governed by the human capital formation. From the economy's resource constraint and Eq. (34), we have:

$$\frac{\dot{C}_t}{C_t} = \frac{A\alpha(U_t x_t)^{1-\alpha}}{\theta_t} - \delta + \beta z_t - \rho = \frac{r_t}{\theta_t} - \delta + \beta z_t - \rho \,. \tag{39}$$

Eq. (39) can be interpreted as an Euler equation in a model with relative wealth preferences and CIA constraint. The modified real rate of return $R_t = (r_t / \theta_t - \delta + \beta z_t)$ is composed by the effective real rate of return $(r_t / \theta_t - \delta)$ and the status related component βz_t. Using the steady-state conditions, we can derive that under these two conditions:

$$\frac{\partial z^*}{\partial \mu} = \frac{\alpha\{(1+\beta)[B(1-U^*)+\delta]+\rho\}}{[\alpha+\beta(1+\mu+\rho)]^2} > 0 \,, \tag{40a}$$

$$\frac{\partial \dfrac{A\alpha(U^* x^*)^{1-\alpha}}{\theta^*}}{\partial \mu} = -\frac{\beta\alpha[(1+\beta)(B+\delta)-\beta\rho]}{[\alpha+\beta(1+\mu+\rho)]^2} = -\beta\frac{\partial z^*}{\partial \mu} \le 0 \,. \tag{40b}$$

An increase in the money growth rate raises the inflation rate and the cost of money holdings. This will encourage investment in physical capital because the representative household substitutes out of real balances and into physical capital. But a higher inflation rate reduces the real money holdings and lowers consumption and investment through the cash-in-advance constraint. When the desire for social status is not present ($\beta = \sigma = 0$), the two effects cancel each other out and the effective real rate of return remains unaffected (Eq. (40b)). As a result, investment of physical capital is not affected and the allocation of human capital between production function and human capital accumulation remains the same. Thus, BGP growth rate is not affected by changes in the rate of money growth.

When the desire for social-status of physical capital is present ($\sigma = 0$), it lowers the effective real rate of return (Eq. (40b)). However, the presence of social status of physical capital raises the status related component of the modified real rate of return (Eq.(40a)). These two effects will cancel each other out and the modified real rate of return remains unaffected by the growth rate of money ($\partial R^* / \partial \mu = 0$). Then the allocation of human capital between production function and human capital accumulation remains unchanged and BGP growth rate is still not affected by the monetary policy and money is still growth-rate superneutral.

When social status depends on the relative human capital ($\sigma > 0$), we compute the first-order partial derivate of $U^* = U^*(\mu, \beta, \sigma)$ with respect to μ and get:[8]

$$\frac{\partial U^*(\mu, \beta, \sigma)}{\partial \mu} < 0.$$

This indicates that an increase μ reduces U^* and raises $(1 - U^*)$. Therefore, an increase in μ will increase g^* and money is not growth-rate superneutral. This is because the presence of social status of human capital strengthens the motivation to accumulate human capital and this will in turn raise the economic growth rate.

Our result of a positive output-growth effect of money in a CIA model where consumption and investment are liquidity constrained is very different from those found in the literature. As shown in the third column of Table 1, previous studies demonstrate that if both consumption and investment are liquidity constrained, an increase in the money growth rate will negatively affect the steady-state output or the BGP growth rate in a one-sector CIA model, regardless of the presence of social status. Marquis and Reffett (1991) and Chen (2011b) find that the positive output-growth effect of money no longer exists when introducing human capital accumulation into a traditional CIA model. In a two-sector CIA model money is superneutral in the growth-rate sense when there is no social-status seeking or when the desire for social status is represented by physical capital. In this paper, we show that an increase in the money growth rate can positively affect the BGP growth rate if the social status formation depends on human capital. Our results are summarized in the following proposition:

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[8] See Appendix for the first-order partial derivatives of $U^* = U^*(\mu, \beta, \sigma)$ with respect to μ, β and σ.

Proposition 3: *When a cash-in-advance constraint is applied to consumption and investment, money is superneutral in the* growth-rate *sense if the desire for social status does not depend on relative human capital. However, if the desire for social status depends on relative human capital, an increase in the money growth rate will raise the BGP growth rate.*

To study the impact of social status, we calculate the first-order partial derivates of $U^*(\mu,\beta,\sigma)$ with respect to β and σ and obtain:

$$\frac{\partial U^*(\mu,\beta,\sigma)}{\partial \beta} > 0 \quad \text{and} \quad \frac{\partial U^*(\mu,\beta,\sigma)}{\partial \sigma} < 0 .$$

Therefore, similar to the economy where the cash-in-advance constraint is solely applied to consumption, an increase in β will lower the BGP growth rate while an increase in σ will raise the BGP growth rate.

<Table 3 is inserted about here>

Using the same parameterization as in Table 2, Table 3 presents values of U^*, z^*, x^*, and g^* with varying μ, β and σ when both consumption and investments are liquidity constrained. The numerical results show that U^* will decrease and g^* will increase with an increase in μ or σ. But an increase in β will increase U^* lower g^*. In all cases, there are three eigenvalues with positive real parts and one eigenvalue with negative real part for the Jacobain matrix of the dynamical system of Eqs. (16) and (33)-(35) evaluated at the steady state. This implies that the BGP equilibrium exhibits saddle-path stability since there are three jump variables (θ_t, z_t and U_t) and one non-jump variable (x_t) in the model.

4. CONCLUSIONS

This paper examines the growth-rate superneutrality of money in a two-sector endogenous-growth CIA model with desire for social status. We consider two scenarios: the cash-in-advance constraint applied solely to consumption and the cash-in-advance constraint applied to consumption and investment. In both scenarios, we find that money is growth-rate superneutral when the desire for social status depends only on relative physical capital. However, when the formation for social status depends on relative human capital, an increase in the growth rate of money will positively affect the long-run growth rate. This positive output-growth effect of money overturns the traditional consensus that an increase in the money growth rate will lower the long-run growth rate when both consumption and investment are liquidity constrained.

We conclude this paper with the suggestion that this study can easily be extended and applied to a variety of studies, pointing out three possible directions. First, we can extend the model by assuming that human capital accumulation also depends on the input of physical capital and examine how changes in the human capital accumulation function affects the effectiveness of monetary policies. Second, we consider a non-separable utility function. Third, we can consider a cash-in-advance constraint applied to consumption, physical capital investment, and human capital investment and examine the impacts of these constraints on economic growth.

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Table 1   Related literature of the effectiveness of monetary policy

	$c_t \leq m_t$	$c_t + i_t \leq m_t$
Panel A (one-sector model): $y = Ak^{\alpha}$		
No social status	Abel (1985): $dk^* / d\mu = 0$	Stockman (1981): $dk^* / d\mu < 0$
With social status	Gong and Zou (2001) and Chang et al. (2000): $dk^* / d\mu > 0$	Gong and Zou (2001) and Chang et al. (2003): $dk^* / d\mu < 0$
Panel B (one-sector *AK* model): $y = Ak$		
No social status	Chang et al. (2000): $dg^* / d\mu = 0$	Suen and Yip (2005): $dg^* / d\mu < 0$
With social status	Chang et al. (2000): $dg^* / d\mu > 0$	Chen and Guo (2009): $dg^* / d\mu < 0$
Panel C (two-sector model): Lucas-type human capital formation		
No social status	Marquis and Reffett (1991): $dg^* / d\mu = 0$	Marquis and Reffett (1991): $dg^* / d\mu = 0$
With social status represented by k	Chen (2011a): $dg^* / d\mu = 0$	Chen (2011b): $dg^* / d\mu = 0$
With social status represented by h	**This paper:** $dg^* / d\mu > 0$	**This paper:** $dg^* / d\mu > 0$
With social status represented by k and h	**This paper:** $dg^* / d\mu > 0$	**This paper:** $dg^* / d\mu > 0$

Note: Results of Suen and Yip (2005) shown in Table 1 correspond to the case where the elasticity of intertemporal substitution is less than or equal to 1.

Table 2 The effects of μ, β and σ when $c_t \le m_t$

μ	β	σ	U^*	z^*	x^*	g^*	Roots
0.05	0.8	0.8	0.7689	0.0515	1.4212	0.0046	– + + +
0.07	0.8	0.8	0.7655	0.0508	1.4158	0.0047	– + + +
0.10	0.8	0.8	0.7606	0.0499	1.4079	0.0048	– + + +
0.05	1.0	0.8	0.7942	0.0434	1.2119	0.0041	– + + +
0.05	1.2	0.8	0.8169	0.0374	1.0644	0.0037	– + + +
0.05	0.8	1.0	0.7029	0.0522	1.6000	0.0059	– + + +
0.05	0.8	1.2	0.6476	0.0528	1.7784	0.0070	– + + +

Table 3 The effects of μ, β and σ when $c_t + i_t \le m_t$

μ	β	σ	U^*	z^*	x^*	g^*	Roots
0.05	0.8	0.8	0.6936	0.0574	1.2787	0.0061	– + + +
0.07	0.8	0.8	0.6882	0.0581	1.3126	0.0062	– + + +
0.10	0.8	0.8	0.6802	0.0590	1.3633	0.0064	– + + +
0.05	1.0	0.8	0.7120	0.0484	1.0565	0.0058	– + + +
0.05	1.2	0.8	0.7283	0.0419	0.8994	0.0054	– + + +
0.05	0.8	1.0	0.6409	0.0581	1.5254	0.0072	– + + +
0.05	0.8	1.2	0.5955	0.0586	1.7609	0.0081	– + + +

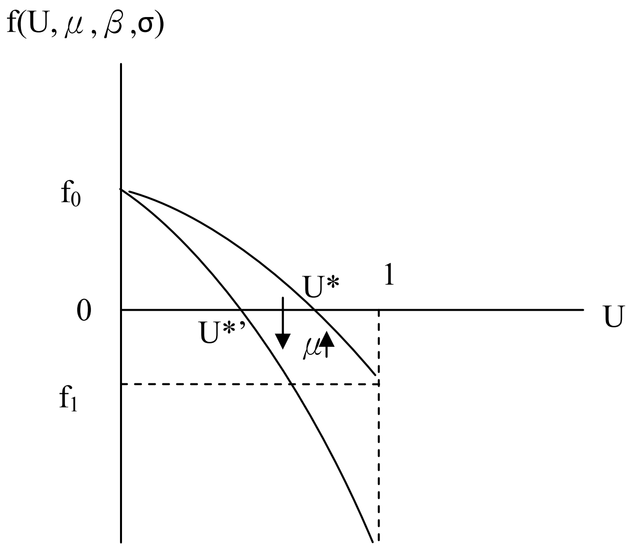

Figure 1 The determination of U^*

Appendix

First, we study the effects of money growth rate by computing the first-order partial derivate of $U^* = U^*(\mu, \beta, \sigma)$ with respect to μ. From Eq. (37b), we can derive that:

$$\frac{\partial \Lambda(U,\mu,\beta,\sigma)}{\partial \mu} = [(1-\alpha)(\rho - BU) - \sigma BU(1+\mu+\rho)]\frac{\partial z(U,\mu,\beta)}{\partial \mu} - \sigma BUz(U,\mu,\beta), \tag{A1}$$

where

$$\frac{\partial z(U,\mu,\beta)}{\partial \mu} = \frac{\alpha\{(1+\beta)[B(1-U)+\delta]+\rho\}}{[\alpha+\beta(1+\mu+\rho)]^2} > 0.$$

Using Eq. (36d), we can obtain:

$$(1-\alpha)(\rho - BU^*) = \frac{(1+\mu+\rho)\sigma BU^* z^*}{z^* + B(1-U^*)+\delta}. \tag{A2}$$

Substituting Eq. (A2) into Eq. (A1), we have:

$$\frac{\partial \Lambda(U^*,\mu,\beta,\sigma)}{\partial \mu} = -\frac{[B(1-U^*)+\delta](1+\mu+\rho)\sigma BU^*}{z^* + B(1-U^*)+\delta}\frac{\partial z(U^*,\mu,\beta)}{\partial \mu} - \sigma BU^* z(U^*,\mu,\beta) < 0 \cdot \tag{A3}$$

Furthermore, differentiating $\Lambda(U,\mu,\beta,\sigma)$ with respect to U gives us:

$$\begin{aligned}\frac{\partial \Lambda(U,\mu,\beta,\sigma)}{\partial U} &= -(1-\alpha)\left\{B[z(U,\mu,\beta)+B(1-U)+\delta] - (\rho - BU)\left[\frac{\partial z(U,\mu,\beta)}{\partial U} - B\right]\right\} \\ &\quad - (1+\mu+\rho)\sigma B\left[z(U,\mu,\beta) + U\frac{\partial z(U,\mu,\beta)}{\partial U}\right]\end{aligned}. \tag{A4}$$

where

$$\frac{\partial z(U,\mu,\beta)}{\partial U} = -\frac{(1+\mu+\rho-\alpha)B}{\alpha+\beta(1+\mu+\rho)}. \tag{A5}$$

Substituting Eqs. (37c) and (A5) into Eq. (A4) and evaluating this at $U = U^*$, we have:

$$\frac{\partial \Lambda(U^*, \mu, \beta, \sigma)}{\partial U}$$

$$= -(1-\alpha)(1+\mu+\rho)B\frac{(1+\beta)[B(1-U^*)+\delta]+BU^*+(2+\beta)(\rho-BU^*)}{\alpha+\beta(1+\mu+\rho)}$$
$$-(1+\mu+\rho)\sigma B\frac{(1+\mu+\rho-\alpha)[B(1-U^*)+\delta]+\alpha BU^*+(1+\mu+\rho)(\rho-BU^*)}{\alpha+\beta(1+\mu+\rho)} \quad .\ \text{(A6)}$$

Note that from (A2), we have:

$$\rho - BU^* = \frac{(1+\mu+\rho)\sigma BU^* z^*}{(1-\alpha)[z^*+B(1-U^*)+\delta]} > 0\,.$$

It follows that

$$\frac{\partial \Lambda(U^*, \mu, \beta, \sigma)}{\partial U} < 0\,. \quad \text{(A7)}$$

Therefore, the first-order partial derivate of $U^* = U^*(\mu, \beta, \sigma)$ with respect to μ is:

$$\frac{\partial U^*(\mu,\beta,\sigma)}{\partial U} = -\frac{\dfrac{\partial \Lambda(U^*(\mu,\beta,\sigma),\mu,\beta,\sigma)}{\partial \mu}}{\dfrac{\partial \Lambda(U^*(\mu,\beta,\sigma),\mu,\beta,\sigma)}{\partial U}} < 0$$

We now turn to examine the effects of β by computing the first-order partial derivate of $U^* = U^*(\mu, \beta, \sigma)$ with respect to β. From Eq. (37b), we can derive that:

$$\frac{\partial \Lambda(U,\mu,\beta,\sigma)}{\partial \beta} = [(1-\alpha)(\rho - BU) - \sigma BU(1+\mu+\rho)]\frac{\partial z(U,\mu,\beta)}{\partial \beta}, \quad \text{(A8)}$$

where

$$\frac{\partial z(U,\mu,\beta)}{\partial \beta} = -(1+\mu+\rho)\frac{(1+\mu+\rho-\alpha)[B(1-U)+\delta]+\rho(1+\mu+\rho)}{[\alpha+\beta(1+\mu+\rho)]^2} < 0\,.$$

Substituting Eq. (A2) into Eq. (A8) and evaluating this at $U = U^*$, we obtain:

$$\frac{\partial \Lambda(U^*,\mu,\beta,\sigma)}{\partial \beta} = -\frac{(1+\mu+\rho)\sigma BU^*[B(1-U^*)+\delta]}{z^*+B(1-U^*)+\delta}\frac{\partial z(U^*,\mu,\beta)}{\partial \beta} > 0\,. \quad \text{(A9)}$$

Eqs. (A7) and (A9) imply that:

$$\frac{\partial U^*(\mu,\beta,\sigma)}{\partial \beta} = -\frac{\dfrac{\partial \Lambda(U^*(\mu,\beta,\sigma),\mu,\beta,\sigma)}{\partial \beta}}{\dfrac{\partial \Lambda(U^*(\mu,\beta,\sigma),\mu,\beta,\sigma)}{\partial U}} > 0\,.$$

Finally, we study the effects of σ by computing the first-order partial derivate of $U^* = U^*(\mu, \beta, \sigma)$ with respect to σ. From Eq. (37b), we can derive that:

$$\frac{\partial \Lambda(U^*, \mu, \beta, \sigma)}{\partial \sigma} = -B(1+\mu+\rho)U^* z(U^*, \mu, \beta) < 0 . \tag{A10}$$

Eqs. (A7) and (A10) imply that:

$$\frac{\partial U^*(\mu, \beta, \sigma)}{\partial \sigma} = -\frac{\dfrac{\partial \Lambda(U^*(\mu, \beta, \sigma), \mu, \beta, \sigma)}{\partial \sigma}}{\dfrac{\partial \Lambda(U^*(\mu, \beta, \sigma), \mu, \beta, \sigma)}{\partial U}} < 0 .$$