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10 March 2011

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* This work is partially supported by Research Grants ECO2009-14457-C04-01 (Ministerio de Ciencia e Innovación) and 10PXIB300141PR (Xunta de Galicia and FEDER).

Abstract. In a scenario with a continuum of asymmetrically informed agents, we analyze how the initial information of a trader may be altered when she becomes a member of a coalition. In contrast to a perfect competition frame, we first show that neither arbitrarily small coalitions nor large coalitions are enough to block an allocation which is not in the core, due to the market failure produced by asymmetric information. However, under mild assumptions, we extend the characterizations of the core provided by Vind and Schmeidler (1972) to economies with asymmetrically informed traders. We then focus on information sharing rules based on the coalitions' size. Assuming the existence of coalitions to which the sharing rule associates an information finer than all the others, we show that the corresponding cores coincide with the one defined by this finest information. Finally, characterizations for the weak fine, the fine and the private core are obtained as particular cases of this equivalence theorem.

JEL Classification: D82, D51, D71, C02.

Keywords: Coalitions, asymmetric information, information sharing, blocking mechanisms, core.

1 Introduction

This paper focuses on the veto mechanism for economies with a continuum of asymmetrically informed agents that trade a finite number of commodities.

Asymmetries in information create market imperfections which are particularly relevant to the rules that regulate the information sharing within coalitions. In this framework the information that each trader initially possesses may vary when she becomes a member of a group. For instance, this variation can occur as a result of an information sharing process among traders belonging to the same coalition or, on the contrary, it could be a consequence of some rules that prevent the use of information other than the common one.

This aspect has been widely acknowledged by the literature and several alternative notions of core have been proposed such as the fine, the coarse, and the private core.

In the fine core, introduced by Wilson (1978), traders in a coalition pool their initial private information whereas in the coarse core, also introduced by Wilson (1978), traders within each coalition only use their common information.

In the private core, defined by Yannelis (1991), the information of an individual is not modified when a coalition is formed, that is, each member maintains her private information independently of the coalition she belongs to.

In this paper we go a step further by focusing on the information changes that may occur when a trader becomes member of a coalition. To this aim, we consider the notion of information sharing rule introduced by Allen (2006) that allows for a great variety of possibilities on trader's information inside a coalition.

Firstly, we look at the measure of blocking coalitions when traders are asymmetrically informed. We analyze the results by Schmeidler (1972) and Vind (1972), which are interpreted as a characteristic of perfect competition. As the asymmetries in the information produce market imperfections, we state examples which show that, in general, these results do not hold in the framework of asymmetric information economies. Then, under mild assumptions on the information sharing rule, we provide extensions of both Schmeidler and Vind's theorems for atomless economies with asymmetrically informed agents.

In the classical core notions which have been addressed in the literature on economies with asymmetric information (private, coarse and fine core), the rule used by a blocking coalition, in regards to information sharing, is fixed a priori and is independent of any characteristic of the coalition itself. To overcome this limitation, we specify information sharing rules such that in each coalition the capacity of its members to share or not share their initial information is regulated on the size of the coalition.

This idea is initially developed in a general frame; we assume that in the market there is a number of relevant sizes for the coalitions and an equivalent number of information sharing rules. When a trader takes part in a coalition, she has access to a specific sharing rule according to the size of the coalition itself. Whenever a class of coalitions is provided with an information which is finer than all the others, we show that the resulting core coincides with the one associated with this finest information sharing rule. We prove this equivalence by using our extension of the Vind's theorem.

We then detail the veto mechanism by analyzing particular and more explicit sharing rules, following the traditional ways of modelling information within coalitions (common, pooled and private information). We consider two mechanisms with a precise economical meaning. In the first one, the process of information sharing may only take place within coalitions with a size smaller than an exogenous threshold. It has a natural interpretation if the transmission of information is costly; larger the size of the coalition more difficult the communication among its members. Thus, only small coalitions pool information. In contrast, in the second mechanism, traders pool information only in coalitions whose measure is larger than another exogenous threshold. In this case, the intuition is that members within big coalitions presume that their own information is irrelevant (the probability of finding members with the same information is increasing with the size of the group) and, consequently, they spontaneously share out their information. Under the conditions which guarantee Vind's result in an asymmetric information framework, our main result states that the core solutions associated with these mechanisms are equivalent and, depending on the informational requirement for an allocation to be feasible, they coincide with either the weak fine core or the fine core.

The paper proceeds in the following order. In Section 2, we present the economic model. In Section 3, we extend the core characterization by Schmeidler and Vind to the framework of asymmetric information. In Section 4, the process of information sharing within a coalition is regulated according to its measure. We then obtain an equivalence result for the veto mechanisms depending on the information that coalitions of a certain measure may acquire. In Section 5, we analyze information sharing rules that use common, private or pooled information. We obtain the coincidence of the associated cores as particular cases of the previous equivalence result.

2 General set-up

2.1 Preliminary notations

Given a set Ω, a partition of Ω is a collection of nonempty, pairwise disjoint subsets, often called blocks, whose union is Ω.
A binary relation $\succeq$ can be defined on the set $\mathcal{P}$ of all partitions of Ω as follows.
Given $P, Q \in \mathcal{P}$, P is finer than Q (or, equivalently, Q is coarser than P), denoted $P \succeq Q$, if for every $A \in P$ there is $B \in Q$ such that $A \subseteq B$.
The binary relation $\succeq$ is easily shown to be reflexive, transitive and antisymmetric, so that $(\mathcal{P}, \succeq)$ is a partially ordered set.
Moreover, $(\mathcal{P}, \succeq)$ is a lattice where, for every $P, Q \in \mathcal{P}$, the supremum $P \vee Q$ and the infimum $P \wedge Q$ are defined as follows:

$$\begin{array}{ll} P \vee Q & \text{is the coarsest partition finer than both } P \text{ and } Q \\ P \wedge Q & \text{is the finest partition coarser than both } P \text{ and } Q. \end{array}$$

By denoting $A(\omega)$ the block in the partition A of Ω which contains ω, it holds that:

$$(P \vee Q)(\omega) = P(\omega) \cap Q(\omega)$$

$$(P \wedge Q)(\omega) = \{z \in \Omega : z \in R(z_1), z_1 \in R(z_2), ..., z_k \in R(\omega)\},$$

where $z_1, \ldots, z_k \in \Omega$ and $R(\omega)$ is either $P(\omega)$ or $Q(\omega)$.

The definitions above can be easily extended to a finite number of partitions.

2.2 The economic model

Let $\mathcal{E}$ be an exchange economy with asymmetric information and a continuum of traders modelled by the finite measure space $(I, \mathcal{A}, \mu)$, where I denotes the set of agents, $\mathcal{A}$ is the Lebesgue σ-algebra and μ is the Lebesgue measure.

The economy extends over two time periods $\tau = 0, 1$ and consumption takes place at $\tau = 1$. At $\tau = 0$ there is uncertainty over the states of nature and agents make contracts that may be contingent on the realized state of nature at period $\tau = 1$, that is, trading is characterized by ex ante contract arrangements. The exogenous uncertainty is described by a measurable space $(\Omega, \mathcal{F})$, where Ω is a finite set of k states of nature and $\mathcal{F}$ denotes the algebra of all events.

Agents are partially and asymmetrically informed with respect to states of nature. Specifically, the initial information of trader $t \in I$ is described by a measurable partition

Π_t of Ω. The interpretation is as follows: trader t is not able to distinguish the states of nature which are in the same block of Π_t and then if state $\bar{\omega}$ occurs, she only observes the block of the partition Π_t that contains such a state.

There is a finite number ℓ of commodities in each state and therefore $(\mathbb{R}^\ell_+)^k$ is the commodity space. Each agent t is characterized by her endowments $e(t,\omega) \in \mathbb{R}^\ell_+$ for each $\omega \in \Omega$ and has a preference relation over the consumption set represented by the utility function $U_t : (\mathbb{R}^\ell_+)^k \to \mathbb{R}_+$. The utility function U_t is said to be monotone if for every $x, y \in (\mathbb{R}^\ell_+)^k$ such that $x \geq y$, it holds that $U_t(x) \geq U_t(y)$.

An allocation $f : I \times \Omega \longrightarrow \mathbb{R}^\ell_+$ is physically feasible if $\int_I f(t,\omega) d\mu(t) \leq \int_I e(t,\omega) d\mu(t)$ for every $\omega \in \Omega$. In order to define the informational feasibility condition of an allocation, let us associate to each consumer t a partition $\mathcal{P}_t$ of Ω which may differ from the initial information Π_t. The interpretation is that during the trading process the information that each agent initially possesses may vary; as a consequence, the constraint that information imposes over allocations is expressed with reference to the partition $\mathcal{P}_t$ rather than Π_t.

The allocation f is feasible if it is physically feasible and $f(t,\cdot)$ is $\mathcal{P}_t$-measurable for almost all $t \in I$.

For the sequel, it is also worth emphasizing the following points on how uncertainty and information are modelled.

- As a consequence of Ω being finite, only a finite number, say n, of different partitions of Ω exist. Let $\hat{\Pi}_1, \hat{\Pi}_2, \ldots, \hat{\Pi}_n$ denote these partitions.

- Given a coalition $S \subseteq I$, $\bigvee_{t\in S} \Pi_t$ denotes the maximum information that coalition S can dispose of, whenever each member of S opts for sharing her initial information with everybody else within the coalition. Formally, this is the coarsest refinement of each partition Π_t, that is:

$$\bigvee_{t\in S} \Pi_t = \left\{ \bigcap_{t\in S} \Pi_t(\omega) : \omega \in \Omega \right\}$$

  where, following the notation stated before, $\Pi_t(\omega)$ is the block in the partition Π_t which contains ω.

- $\bigwedge_{t\in S} \Pi_t$ is the common information partition associated to the coalition S. Thus, $A \in \bigwedge_{t\in S} \Pi_t$ if and only if A is the smallest subset of Ω such that for every $\omega \in A$, $\Pi_t(\omega) \subseteq A$, for every $t \in S$.

The first point is of particular interest for our purposes. In fact, regarding information, we can identify only a finite number n of agents' types, which are defined by:

$$I_i = \{t \in I \text{ such that } \Pi_t = \hat{\Pi}_i\}, \qquad i \in \{1, 2, \ldots, n\}.$$

We assume that, for every $i \in \{1, \ldots, n\}$, the set I_i is measurable and $\mu(I_i) > 0$.

For each coalition $S \subseteq I$, we can focus on the information types which are actually present within such a coalition; they are represented by the following set:

$$I_S = \{i \in \{1, 2, \ldots, n\} : \mu(S \cap I_i) > 0\}$$

Note that for every coalition S we have $S = \bigcup_{i \in I_S} S_i$, where S_i is the set of individuals in S who, regarding information, are of type i, i.e., $S_i = \{t \in S \text{ such that } \Pi_t = \hat{\Pi}_i\}$.

Using this notation, we have that $\hat{\Pi}_i(\omega) \in \left(\bigwedge_{t \in S} \Pi_t\right) \bigcap \left(\bigvee_{t \in S} \Pi_t\right)$ for some $i \in I_S$ if and only if $\hat{\Pi}_i(\omega) = \Pi_t(\omega)$ for every $t \in S$.

2.3 Information and coalitions

In this Section, we deal with the information that agents can dispose of when they become members of a coalition and the associated blocking mechanism. In order to model how the initial information of a trader changes, we use the notion of the information sharing rule as introduced by Allen (2006).

Given a coalition S, an information sharing rule for S is a function $\Upsilon(S)$ which associates a partition $\Upsilon_t(S)$ to each member $t \in S$; the partition $\Upsilon_t(S)$ is intended as the information that agent t can dispose of once the coalition S has been formed. Thus, an information sharing rule Υ for the economy $\mathcal{E}$ is a collection $(\Upsilon(S))_{S \in \mathcal{A}}$.

Consider two information sharing rules Υ^1 and Υ^2; we say that Υ^1 is finer than Υ^2, denoted by $\Upsilon^1 \succeq \Upsilon^2$, if for every $S \subseteq I$ we have $\Upsilon^1_t(S) \succeq \Upsilon^2_t(S)$, for every $t \in S$.

Given an information sharing rule Υ, a coalition S Υ–blocks an allocation f via g if:

(i) For every $t \in S$, $g(t, \cdot)$ is $\Upsilon_t(S)$-measurable,

(ii) $\int_S g(t, \omega) d\mu(t) \leq \int_S e(t, \omega) d\mu(t)$ for every $\omega \in \Omega$ and

(iii) $U_t(g(t, \cdot)) > U_t(f(t, \cdot))$ for every $t \in S$.

The core of the economy $\mathcal{E}$ under the information sharing rule Υ, denoted by $C^{\Upsilon}(\mathcal{E})$, is the set of feasible allocations that are not Υ-blocked by any coalition.

It is worth noting that different specifications of both, partitions $\mathcal{P}_t$ which appear in the notion of feasibility, and the information sharing rule Υ, lead to well-known distinct notions of core for asymmetric information economies (see Wilson, 1978, Yannelis, 1991 and Allen, 2006).

3 Asymmetric information and market imperfections

In every economic model where agents engage in cooperation, the formation of coalitions could involve costs depending on the size or measure of the coalition itself. In this respect, it becomes a topic of interest to characterize cooperative solutions in regards to the blocking power of only those coalitions whose formation costs are not too high. In an asymmetric information framework, this aspect requires an additional effort, since trades may not only involve physical resources but also information. Therefore, the cost related to the transmission of information could also be taken into account.

By addressing exchange economies with a finite number of commodities and a continuum of traders with complete information, Schmeidler (1972) shows that it is enough to consider arbitrarily small coalitions in order to obtain the core. In the same issue of Econometrica, Grodal (1972) and Vind (1972) complement the previous result characterizing other families of coalitions which are able to block any allocation that does not belong to the core. In the light of the core-Walras equivalence (see Aumann, 1964), the results by Schmeidler, Grodal and Vind have been interpreted as a characterization of perfect competition. In other words, we can assume that arbitrarily small coalitions (Schmeidler and Grodal) or, symmetrically, arbitrarily big coalitions (Vind) are formed with an arbitrarily low cost. These results heavily rely on Lyapunov's convexity theorem and, therefore, to keep this characterization of perfect competition in a more general framework, as it is for instance an infinite dimensional setting, further assumptions are required (see Hervés-Beloso et al., 2000 and Evren and Hüsseinov, 2008).

Extensions to asymmetric information economies have been firstly obtained by Hervés-Beloso et al. (2005) and also by Evren and Hüsseinov (2008) and Pesce (2010). However, in all these results traders maintain their initial information independently of the coalition they belong to. In this Section our aim is to deepen the analysis when an exogenous mechanism regulates the process of information sharing within coalitions.

In this asymmetric information framework imperfections of the market may arise and, consequently, it should not be surprising that the above mentioned characterizations of perfect competition do not hold. To show this point, we first state two examples which prove that Schmeidler's and Grodal's results and Vind's result, respectively, may fail without extra assumptions on the information sharing rule[1].

Example 1. Consider an economy with asymmetric information, two possible states of nature, a and b, and one commodity in each state. Every agent in $[0,1]$ has an initial endowment given by $(1,1)$, that is, one unit of every commodity in each state. Let x and y denote the consumption in a and b, respectively. Preferences are given by the following utility functions:

$$U_t(x,y) = \begin{cases} x^2 y & \text{if } t \in A = [0,1/2) \\ xy^2 & \text{if } t \in B = [1/2,1] \end{cases}$$

Agents in A have complete information whereas agents in B are not able to distinguish the two states a and b, that is:

$$\Pi_t = \begin{cases} \{\{a\},\{b\}\} & \text{if } t \in A \\ \{a,b\} & \text{if } t \in B \end{cases}$$

We consider the information sharing rule Υ defined as follows:

$$\Upsilon_t(S) = \begin{cases} \bigvee_{t\in S} \Pi_t & \text{if } \mu(S) > \alpha \\ \bigwedge_{t\in S} \Pi_t & \text{if } \mu(S) \leq \alpha \end{cases}$$

Then, the initial endowment allocation is Υ–blocked by every coalition S with measure greater than α such that $\mu(S\cap A) > 0$ and $\mu(S \cap B) > 0$. However, it cannot be Υ–blocked by any coalition S with $\mu(S) \leq \alpha$.

Example 2. Consider the same economy as before except that preferences are given by the following utility functions:

$$U_t(x,y) = \begin{cases} x^2 y & \text{if } t \in [0,1/4) \\ xy^2 & \text{if } t \in [1/4,1/2) \\ xy & \text{if } t \in [1/2,1] \end{cases}$$

[1]The examples we state are in accordance with the work by Serrano et al. (2001) where it is shown that the core convergence to competitive allocations, which is another perfect competition test, may fail when the coarse blocking mechanism is considered.

Then, the initial allocation is coarse blocked by the coalition $[0, 1/2)$ via the allocation which assigns $(4/3, 2/3)$ to every agent in $[0, 1/4)$ and $(2/3, 4/3)$ to agents in $[1/4, 1/2)$.

Note that the common information associated with any coalition $S \subset [0, 1]$ such that $\mu(S \bigcap [1/2, 1]) > 0$ does not distinguish between a and b. Therefore, the initial allocation cannot be blocked by any coalition with measure larger than $1/2$ whenever the sharing information rule is given by the coarse blocking structures.

In order to obtain a general version of both Schmeidler's and Vind's results in the framework of asymmetric information economies, we will state several properties for the information sharing rule Υ which we will use throughout this paper.

(P1) For every $S, S' \in \mathcal{A}$ such that $S' \subseteq S$ and $I_S = I_{S'}$, it holds that:

$$\Upsilon_t(S') = \Upsilon_t(S) \text{ for every } t \in S'$$

(P2) For every $S, S' \in \mathcal{A}$ such that $S' \subseteq S$ and for every $t \in S'$:

$$\Upsilon_t(S) \succeq \Upsilon_t(S')$$

Property (P1) states that when trader t becomes part of two coalitions, one contained in the other, the information she can dispose of in the smaller one is the same as the information she can dispose of in the larger one whenever the information types included in these two coalitions are the same. Property (P2) requires that if we consider an initial coalition and additional members join this group, then, membership in the original coalition cannot become worse off from an informational point of view. The information sharing rules that satisfy (P2) are referred to as nested by Allen (2006) and, under this property, she shows the non-emptiness of the core for NTU games with a finite number of players and asymmetric information.

We remark that properties (P1) and (P2) together can be written as follows:

(P3) For every $S, S' \in \mathcal{A}$ with $S' \subseteq S$ and for every $t \in S'$, it holds that:

$$\begin{aligned} \Upsilon_t(S) \succeq \Upsilon_t(S') &\quad \text{if } I_{S'} \subset I_S; \\ \Upsilon_t(S) = \Upsilon_t(S') &\quad \text{if } I_S = I_{S'} \end{aligned}$$

Note that properties (P1) and (P2) trivially hold for the private information sharing rule Υ^p_t, given by $\Upsilon^p_t(S) = \Pi_t$, for every $S \in \mathcal{A}$ and for every $t \in S$.

Furthermore, (P1) holds for any sharing rule Υ with $\Upsilon(S)$ depending only on the informational types I_S, which are actually present in coalition S. This is particularly the case for the fine information sharing rule, where agents share information within every coalition. It is also the case for the coarse information sharing rule, where agents within a group are restricted to use the common information. In addition, if the information that any agent can dispose of within a coalition does not decrease when the number of types increases[2], then property (P3) is satisfied.

We also consider the next condition which states that the requirements for an allocation f to be informationally feasible imply that, for every t, the bundle $f(t,\cdot)$ is compatible with the information that the individual t disposes of when the large coalition is formed.

(P4) For every $t \in I$, $\Upsilon_t(I) \succeq \mathcal{P}_t$.

Next we state an extension of Schmeidler's result (1972) for asymmetric information economies under an information sharing rule Υ which satisfies the property (P1).

Starting from a blocking coalition S, a crucial point in the proof is that an arbitrarily small blocking coalition can be built in such a way that it contains the same informational types as S. This fact, together with property (P1), makes it possible to overcome the informational constraints over the blocking allocation.

Theorem 3.1 *Consider the asymmetric information economy $\mathcal{E}$ and an information sharing rule Υ with property (P1). Let f be an allocation which is Υ-blocked via g by a coalition S. Then, for any $\varepsilon \in (0,1]$ there exists a coalition $S_\varepsilon \subseteq S$ with $I_{S_\varepsilon} = I_S$ and $\mu(S_\varepsilon) = \varepsilon\mu(S)$ which also Υ-blocks f via the same g.*

Proof. Let S be a coalition that Υ-blocks f under the information sharing rule Υ. That is, there exists an allocation g such that:

(i) $g(t,\cdot)$ is $\Upsilon_t(S)-$ measurable $\mu-$ a.e. in S,

(ii) $\displaystyle\int_S g(t,\cdot)d\mu(t) \leq \int_S e(t,\cdot)d\mu(t)$ and

(iii) $U_t(g(t,\cdot)) > U_t(f(t,\cdot))\ \mu-$ a.e. in S.

[2]This occurs, for example, when agents share their information; on the contrary, it no longer occurs when agents forming a coalition are restricted to use their common information.

For each $S_i = \{t \in S : \Upsilon_t(S) = \hat{\Pi}_i\}$ let us define the vectorial measure m by $m(A) = \left(\int_A (g(t,\cdot) - e(t,\cdot))\, d\mu(t), \mu(A) \right)$ for every $A \subseteq S_i$. Applying Lyapunov's theorem, for any $\varepsilon \in (0,1)$ there exists $S'_i \subseteq S_i$ such that $m(S'_i) = \left(\varepsilon \int_{S_i} (g(t,\cdot) - e(t,\cdot)) d\mu(t), \varepsilon\mu(S_i) \right)$. Let us consider the coalition $S_\varepsilon = \bigcup_{i \in I_S} S'_i$. Since the subcoalitions S'_i are disjoint we have $m(S_\varepsilon) = \sum_{i \in J} m(S'_i)$. Therefore, we can conclude that:

- $\displaystyle\int_{S_\varepsilon} (g(t,\cdot) - e(t,\cdot)) d\mu(t) = \varepsilon \sum_{i \in I_S} \int_{S_i} (g(t,\cdot) - e(t,\cdot)) d\mu(t) \leq 0;$
- $\displaystyle\mu(S_\varepsilon) = \sum_{i \in I_S} \mu(S'_i) = \varepsilon\mu(S);$
- $g(t,\cdot)$ is $\Upsilon_t(S)$ - measurable, by virtue of property (P1).

That is, coalition S_ε Υ-blocks f via g.

Q.E.D.

The previous remarks regarding property (P1) allows us to apply the extension of Schmeidler's result to allocations which do not belong to either private, fine or coarse core.

We now turn to the blocking power of large coalitions by providing an extension of the above mentioned Vind's result to asymmetric information economies.

Theorem 3.2 *Consider the asymmetric information economy $\mathcal{E}$ and an information sharing rule Υ with properties (P3) and (P4). Suppose that utility functions U_t are continuous and monotone. Let f be an allocation which is Υ-blocked via g by a coalition S. Then, for any $\alpha \in (0,1)$ there exists a coalition S_α with $\mu(S_\alpha) = \alpha$ which also Υ-blocks f.*

Proof. Let S be a coalition that blocks f under the information sharing rule Υ. That is, there exists an allocation g such that:

(i) $g(t,\cdot)$ is $\Upsilon_t(S)$ - measurable $\mu -$ a.e. in S,

(ii) $\displaystyle\int_S g(t,\cdot) d\mu(t) \leq \int_S e(t,\cdot) d\mu(t)$ and

(iii) $U_t(g(t,\cdot)) > U_t(f(t,\cdot))\, \mu -$ a.e. in S.

Since (P3) implies (P1), we can apply Theorem 3.1, and therefore consider that S is small enough so that $\mu(S) < \min_i \mu(I_i)$.

Let us define $H(t) = U_t(g(t,\cdot) - U_t(f(t,\cdot)$. Lusin's theorem guarantees the existence of a compact set $K \subset S$, with $\mu(K) > 0$, such that H, f and g are continuous functions on K. Consider the sequence of functions g_n given by $g_n(t,\cdot) = \frac{n}{n+1} g(t,\cdot)$ and let $H_n(t) = U_t(g_n(t,\cdot)) - U_t(f(t,\cdot))$. Note that $g_n(t,\cdot) \leq g_{n+1}(t,\cdot)$ for all n and t. Then by monotonicity of preferences we have that H_n is a monotone increasing sequence of continuous functions defined on K with pointwise limit H. Applying Dini's theorem, we know that there exists $\overline{n}$ such that $U_t(g_n(t,\cdot)) > U_t(f(t,\cdot))$ for every $n \geq \overline{n}$ and every $t \in K$. Take any $N > \overline{n}$ and consider the allocation $\widehat{g}$ given by

$$\widehat{g}(t,\cdot) = \begin{cases} \frac{N}{N+1}\, g(t,\cdot) \text{ if } t \in K \\ g(t,\cdot) \text{ if } t \in S \setminus K \end{cases}$$

By construction we have $\displaystyle\int_S \widehat{g}(t,\cdot) d\mu(t) + \delta \leq \int_S e(t,\cdot) d\mu(t)$, for some $\delta \gg 0$.
Let $\varepsilon \in (0,1)$. Applying Lyapunov Theorem we obtain that there exists $S_\varepsilon \subset S$ such that $\displaystyle\int_{S_\varepsilon} \widehat{g}(t,\cdot) d\mu(t) = \varepsilon \int_S \widehat{g}(t,\cdot) d\mu(t)$ and $\displaystyle\int_{S_\varepsilon} f(t,\cdot) d\mu(t) = \varepsilon \int_S f(t,\cdot) d\mu(t)$. Let h be the allocation given by

$$h(t,\cdot) = \begin{cases} \widehat{g}(t,\cdot) \text{ if } \ t \in S_\varepsilon \\ f(t,\cdot) + \frac{\varepsilon\delta}{2\mu(S \setminus S_\varepsilon)} \text{ if } \ t \in S \setminus S_\varepsilon \end{cases}$$

Note that $U_t(h(t,\cdot)) > U_t(f(t,\cdot))$ for every $t \in S$. Moreover:

$$\int_S h(t,\cdot) d\mu(t) = \int_S \left(\varepsilon \widehat{g}(t,\cdot) + (1-\varepsilon) f(t,\cdot)\right) d\mu(t) + \frac{\varepsilon\delta}{2}$$

By applying Lyapunov's Theorem once more, there exists $A \subset I \setminus S$ such that $\mu(A) = (1-\varepsilon)\mu(I \setminus S)$ and $\displaystyle\int_A \left(f(t,\cdot) - e(t,\cdot)\right) d\mu(t) = (1-\varepsilon) \int_{I \setminus S} \left(f(t,\cdot) - e(t,\cdot)\right) d\mu(t)$.

Now, let ϵ be small enough so that $\mu(S \bigcup A) > \mu(I) - \min_i \mu(I_i)$. This guarantees that the coalition $B = S \bigcup A$ verifies that $\mu(B \bigcap I_i) = \mu(B_i) > 0$ for every $i = 1, \ldots, n$ which implies $I_B = \{1, \ldots, n\} = I_I$. Then, since (P1) is implied by (P3), we have that $\Upsilon_t(B) = \Upsilon_t(I)$ for every $t \in B$.

Consider the allocation z defined by

$$z(t,\cdot) = \begin{cases} h(t,\cdot) \text{ if } \ t \in S \\ f(t,\cdot) + \frac{\varepsilon\delta}{2\mu(B)} \text{ if } \ t \in A \end{cases}$$

As mentioned above, by monotonicity of preferences $U_t(z(t,\cdot)) > U_t(f(t,\cdot))$ for every $t \in B$. Additionally, properties (P3) and (P4) allow us to ensure that $z(t,\cdot)$ is $\Upsilon_t(S)$ - measurable for every $t \in B$. More precisely, condition (P2), which is implied by (P3), guarantees that $z(t,\cdot)$ is $\Upsilon_t(S)$ - measurable for every $t \in S_\epsilon$, whereas (P4) leads us to confirm that $z(t,\cdot)$ is $\Upsilon_t(S)$ - measurable for every $t \in B \setminus S_\epsilon$. Finally, $\displaystyle\int_B (z(t,\cdot) - e(t,\cdot))\, d\mu(t) \leq 0$.

Therefore, we have constructed an arbitrarily large coalition B. By using Theorem 3.1 once again, we conclude the proof.

Q.E.D.

4 Information and coalitions' size

In this Section we aim at specifying a general rule where the information sharing among traders is regulated on the size of the coalition they belong to.

For this purpose, we consider a partition $(M_j, j \in J)$ of the interval $M = [0, \mu(I)]$, where I is the set of agents and a collection $\mathcal{F}$ of information sharing rules indexed by the same set J, that is, $\mathcal{F} = (\Upsilon^j, j \in J)$.

We define a new information sharing rule Υ, such that the information that each agent can dispose of when she becomes a member of a coalition S, that is $\Upsilon_t(S)$, depends on the size of the coalition itself through the sets M_j. Precisely, we define Υ as follows.

Given a coalition $S \subseteq I$, let j be the index in J such that $\mu(S) \in M_j$; then:

$$\Upsilon_t(S) = \Upsilon_t^j(S) \text{ , for each } t \in S. \tag{1}$$

The intuition behind this setup is the following: in the market there are some relevant sizes for the coalitions identified by the sets $(M_j)_{j \in J}$. When trader t takes part in coalition S, she has access to specific information given by a sharing rule according to the size of the coalition S itself.

We state the following assumption on the collection $\mathcal{F}$:

(F) There exists an index $o \in J$ such that $\Upsilon^o \succeq \Upsilon^j$ for every $j \in J$ and Υ^o satisfies properties (P3) and (P4).

The following equivalence result holds:

Theorem 4.1 *Consider the asymmetric information economy $\mathcal{E}$ and suppose that the information sharing rule Υ defined by (1) satisfies assumption (F). Then, the Υ-core coincides with the Υ^o-core, that is, $\mathcal{C}^{\Upsilon}(\mathcal{E}) = \mathcal{C}^{\Upsilon^o}(\mathcal{E})$.*

Proof. It is easy to see that $\mathcal{C}^{\Upsilon^o}(\mathcal{E}) \subseteq \mathcal{C}^{\Upsilon}(\mathcal{E})$. To prove this, let f be Υ-blocked by a coalition $S \subseteq I$ via allocation g. Since Υ^o is finer than any Υ^j in the collection $\mathcal{F}$, it holds that $g(t,\cdot)$ is $\Upsilon^o_t(S)$-measurable, for every $t \in S$. Then f is Υ^o-blocked by the coalition S via the same allocation g.

To prove that $\mathcal{C}^{\Upsilon}(\mathcal{E}) \subseteq \mathcal{C}^{\Upsilon^o}(\mathcal{E})$ assume that f does not belong to $\mathcal{C}^{\Upsilon^o}(\mathcal{E})$. Hence, there exists a coalition S and an allocation $g : S \times \Omega \longrightarrow \mathbb{R}^\ell_+$ such that:

(i) for every $t \in S$, $g(t,\cdot)$ is $\Upsilon^o_t(S)$-measurable,

(ii) $\displaystyle\int_S g(t,\omega)d\mu(t) \leq \int_S e(t,\omega)d\mu(t)$ for every $\omega \in \Omega$ and

(iii) $U_t(g(t,\cdot)) > U_t(f(t,\cdot))$ for every $t \in S$.

Let $k \in J$ such that $\mu(S) \in M_k$. If $k = o$, the proof ends. If $k \neq o$, by making use of Theorem 3.2, we can find a coalition $\tilde{S}$, with $\mu(\tilde{S}) \in M_o$, which Υ^o-blocks f.

Q.E.D.

5 A new characterization for the fine core

Based on the general idea stated in the previous section, we now analyze two specific blocking systems defined by using the traditional ways to share information, which are common, pool and private information. In the first one, the process of information sharing can only take place within coalitions with sizes smaller than an exogenous threshold, whereas traders in the remainder of coalitions keep their initial information or use common information. On the other hand, in the second one, traders only pool their initial information in coalitions whose size is larger than an exogenous threshold.

Both mechanisms make sense from an economic point of view. The first model has a natural interpretation in a frame where the process of information transmission within groups of individuals is costly: the larger the coalition is, the more difficult the communication among its components will be. In this perspective, it can be argued that the cost of communication inside a coalition depend on its size and is lower for small coalitions than for larger ones. The same mechanism can be also interpreted in a different perspective: when a coalition is large enough, it is likely that each member may not have the incentive to reveal her private information, adopting a selfish behavior which results in a situation where all members of the coalition only share common information.

Symmetrically, the second mechanism formalizes the intuition that, when agents join a big coalition, they presume that their own information is known (as the probability of finding members with the same or more information is increasing with the size of the group) and, consequently, they spontaneously share out.

In order to formally define the two mechanisms described above, we start by considering the partition of the set $M = [0, \mu(I)]$ formed by the intervals $M_1 = [0, s), M_2 = [s, b], M_3 = (b, \mu(I)]$, where $s, b \in [0, \mu(I)]$, and $s < b$.

We associate with this partition a collection $\mathcal{F} = (\Upsilon^j, j \in \{1, 2, 3\})$ of three information sharing rules, defined for every $S \in \mathcal{A}$ and every $t \in S$ as follows:

$$\begin{aligned}
\Upsilon_t^1(S) &= \textstyle\bigvee_{t \in S} \Pi_t, \\
\Upsilon_t^2(S) &= \Pi_t, \\
\Upsilon_t^3(S) &= \textstyle\bigwedge_{t \in S} \Pi_t,
\end{aligned}$$

That is, Υ^1, Υ^2 and Υ^3 are the fine, private and coarse information sharing rules, respectively.

Note that assumption (F) holds for the collection $\mathcal{F}$.

Starting from the partition $(M_j, j \in \{1, 2, 3\})$ and the collection $\mathcal{F}$, we construct the information sharing rules Υ^1 and Υ^2 defined by:

$$\Upsilon_t^1(S) = \begin{cases} \bigvee_{t \in S} \Pi_t & \text{if } \mu(S) < s \\ \Pi_t & \text{if } \mu(S) \in [s, b] \\ \bigwedge_{t \in S} \Pi_t & \text{if } \mu(S) > b \end{cases} \tag{2}$$

$$\Upsilon_t^2(S) = \begin{cases} \bigwedge_{t \in S} \Pi_t & \text{if } \mu(S) < s \\ \Pi_t & \text{if } \mu(S) \in [s, b] \\ \bigvee_{t \in S} \Pi_t & \text{if } \mu(S) > b \end{cases} \tag{3}$$

We state the following equivalence result which provides a new characterization for the notion of fine and weak fine core (depending on the requirements to be an informationally feasible allocation). We omit the proof since it is an immediate consequence of Theorem 4.1 and of the definitions of fine and weak fine core.

Proposition 5.1 *Consider the asymmetric information economy* $\mathcal{E}$ *and the information sharing rules* Υ^1 *and* Υ^2 *defined by (2) and (3). It holds that:*

$$\mathcal{C}^{\Upsilon^1}(\mathcal{E}) = \mathcal{C}^{\Upsilon^2}(\mathcal{E})$$

Moreover, when $\mathcal{P}_t = \Pi_t$ *(*$\mathcal{P}_t = \bigvee_{t \in I} \Pi_t$*, respectively) for almost every* $t \in I$*, both the notions coincide with the fine core (weak fine core, respectively).*

We remark that the previous characterization means that we only need that traders within a family of coalitions pool their information in order to obtain the fine core. An analogous proposition could be stated to characterize the private core by requiring that the information associated to a given family of coalitions is the initial information for each of their members whereas agents within the remaining coalitions use any coarser information.

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