Coordination structures

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Abstract

We study a coordination problem where agents act sequentially. Agents are embedded in an observation network that allows them to observe the actions of their neighbors. We find that coordination failures do not occur if there exists a sufficiently large clique. Its existence is necessary and sufficient when agents are homogenous and sufficient when agents differ and their types are private. Other structures guarantee coordination when agents decide in some particular sequences or for particular payoffs. The coordination problem embodied in our game is applied to the problems of revolts and bank runs.

Keywords: social networks, coordination failures, multiple equilibria, revolts, bank runs

JEL Classification: C72, D82, D85, G21, Z13

1 Introduction

Coordination failures occur when agents fail to coordinate on an equilibrium in which they are better-off than in an alternative equilibrium in which they end up. There are many socioeconomic situations in which it may occur (as in many Keynesian models (Cooper and John, 1988), bank runs (Diamond and Dybvig,

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1983) or revolts (Chwe, 2000), among others). The idea was presented by Jean-Jacques Rousseau as an illustration of a problem which has become known as the stag-hunt game. In this game, hunting the stag is risky. This is the case because the payoff it yields depends on whether the hunters manage to coordinate. If they do so, then their payoff is higher than the one related to the safe choice of going for the hare. However, if they fail, the payoff is lower.

We study coordination problems that are characterized by a risky action, which is the optimal choice only if it is taken by the sufficient amount of agents. They decide in sequence, according to an order of decision exogenously given. We show how the coordination problems disappear if certain information structures exist that allow the observability of actions among the agents. We study the case with heterogenous agents: there may exist agents who differ with respect to their different preferred risky action and/or threshold. The main objective in our paper is to characterize the structures that guarantee that coordination emerges. In this sense, our aim is very closed to Chwe (1999, 2000), as explained later.

The agents that we study are embedded in a social network that allows to observe actions.[1] If two agents are linked, then the one who acts later in the order of play observes the other agent's decision who is aware that her action will be observed. The effect of a link, in this case, is to change the interaction among agents, from simultaneous to sequential. The link allows to observe the action, but neither the type, nor the threshold nor the position in the order of decision are observed. We say that a network allowing the observation of actions is an *observation network*. We look for observation networks that imply that the coordination game played has a unique equilibrium in which all agents take their preferred risky action, independently of the order of decision. If this occurs for any possible payoffs that allow for the type of coordination failures that we study, we say that these observation networks are *coordination structures*.

We find that, when agents are homogenous, the existence of a clique of given size is necessary and sufficient in guaranteeing coordination: coordination failures can be sustained in equilibrium if such a clique does not exist, while in any observation network where the clique exists the unique equilibrium is the efficient one. This is the case for any payoffs of the type that we study and for any sequence of decision. We extend our model to different agents with private types, and we find a sufficient condition on the size of a clique that guarantees that, for any payoffs and sequence of decision, only the efficient outcome can be sustained in equilibrium. For this case, however, it is not a necessary condition, and we find other structures that also guarantee the coordination. However, these structures are formed by a higher number of nodes forming several cliques.

[1] Typically, social networks are used to model the interactions among agents. As far as we know, Choi et al (2011) and Kiss et al (2009) are the first papers that use networks to modelize the actions that agents are able to observe.

When agents are heterogenous and type is private information, we discuss also the case of observation networks that can be sufficient for guaranteeing coordination for certain sequences of decision or certain payoffs, and we name these networks quasi-coordination structures. In this context, we show with an example that more connections may generate coordination failures, a fact that is absent of coordination structures[2].

As an illustration of the type of situations we study, imagine a group of people (e.g. professors in a department) who decide which operating system are going to use[3]. They prefer one system or another depending on how many partners choose it (for instance, because of the possibility of sharing software or knowledge about the operating system). For simplicity, consider that if sufficient agents are choosing Linux, then everyone prefers to use it; if there are not sufficient agents who use Linux, people would prefer Windows. Such a situation is characterized by a threshold, which makes profitable the risky action of using Linux. As a concrete case, suppose that there are four individuals and that they prefer to use Linux over Windows if at least 3 of them use it. Suppose that the network structure is common knowledge and consider the social structures represented in Figure 1.

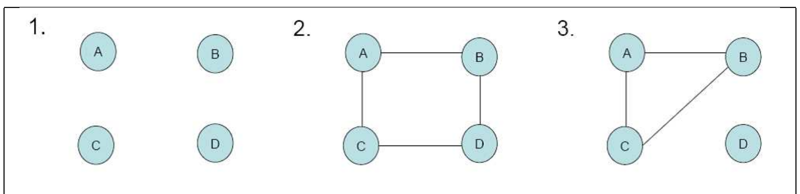


Figure 1

In network 1 individuals are not connected. It means that they are not able to observe each other. If they are sufficiently optimistic and believe that the other individuals will adopt Linux, then all of them adopt it. But in case of pessimism the opposite happens. Therefore, picture 1 represents an observation network that does not lead with certainty to the socially efficient outcome, so this network is not a coordination structure. Individuals are more connected in network 2, but still this structure does not enable coordination always. If any of the individuals observes two others who use Linux, then she adopts it as well. But if only one other Linux's user is observed, then it is not clear what is the optimal choice: B may find optimal not to adopt Linux even if she observes that A has adopted it. It is the case, because even if she adopts Linux and this is observed by D, B does not know with certainty if C will adopt and upon observing only one

[2] Any observation network that contains a coordination structure is also a coordination structure.

[3] The literature on adoption studies how the market ends coordinating in one or another product. See Farrell and Klemperer, 2007.

other agent adopting, D may choose not to adopt. In other words, there is no profitable deviation from the strategy that has "do not adopt Linux when observing zero or one other agent adopting it, and adopt it otherwise". Thus, nobody starts to adopt and everyone may end up using Windows. The network 3 - even though it has less links than the network 2 - enables coordination, because A,B and C will adopt Linux in any equilibrium. Beliefs for them do not play any role, independently of the order in which they act. This is the case because upon observing that two other individuals have adopted Linux, the optimal response for the third is to adopt it as well. By backward induction, the best response when observing that somebody has already adopted Linux is to adopt as well. As a consequence, if any of the three individuals does not observe any action (so she knows to be the first to act), then her best choice is to adopt and induces the other two individuals to follow suit. Since the network structure is common knowledge, individual D knows that the other three individuals will coordinate, so she adopts as well, even although she does not observe anyone and her action is not observed by the others. In Proposition 1, we show in general that in case of one type and a homogeneous threshold t a group of completely connected agents (a clique) of size t is necessary and sufficient to achieve uniquely the Pareto-superior equilibrium.

We also study a more general setup in which there are different types (different preferred operating systems in our example) as well as situations in which the threshold changes from individual to individual (personalized thresholds)[4]. For instance, some people may want to use Linux if they are sure that a few other people use it, while others adopt Linux only if many others use it. Theorem 1 indicates that for every agent there exists a minimal size of the clique that ensures that she takes her risky action. As a consequence, every individual takes her risky action in any order of play, in any sequential equilibrium, if there exists in the society a clique of size as the largest of these minimally required cliques.

Our setup extends naturally to various examples. We study three of them. Revolts succeed if enough individuals join them and they can be modelled as coordination problems. Several papers (e.g. Granovetter, 1978 and Chwe, 1999 and 2000, among others) study what are the necessary condition for a revolt to succeed. Bank runs also may be viewed as coordination failures (Diamond and Dybvig, 1983). When a liquidity shock occurs, some depositors have urgent liquidity need (impatient depositors) and they want to withdraw their money immediately. Other depositors without immediate liquidity needs (patient depositors) prefer to wait if other patient depositors wait as well. If many patient depositors try to withdraw, the bank suffers a bank run and it becomes optimal for the patient depositors to withdraw. In this situation, patient depositors play among them a multiagent stag-hunt game. This example has two types, both of them with a homogeneous threshold. We show that even though types are unobservable a sufficiently large clique allowing extensive

[4] We assume in this case that this information is private.

mutual observation of actions entails that no bank run should occur in equilibrium. The difficulty of the formation of a large clique may explain why bank runs happen.

Our paper is related mainly with Chwe (1999, 2000) and Granovetter (1978), who studied the emergence of revolts, and Choi et al (2011), which study how different observation networks facilitate coordination among agents. Chwe (1999, 2000) analyzes how must be the structure of a communication network in order to enable the revolts. In the model, he analyzed the minimal requirement on the structure in order to ensure that the revolution may occur, for the case in which agents know only the willingness to participate of their neighbors. The condition for coordination requires that agents know through the network that sufficient other agents are also willing to revolt. Chwe shows the key relevance of cliques in the formation of the local common knowledge that enables the revolts. Our aim is similar in the sense that we characterize also a family of networks under which coordination emerges. Moreover, the type of coordination problem that we study is similar to the one studied by Chwe. However, our models differ in several basic aspects. Chwe studies a case in which agents have information only about her neighbors, and this information is precisely what the network transmits. The information of our agents is completely different, since it allows them to know the actions of their predecessors who are also neighbors, although they may ignore the type of her neighbors. However, we assume certain knowledge over these types, since we analyze the case where agents know that there exist enough people for coordination. Therefore in the model by Chwe there exists local knowledge of types but complete uncertainty about the global distribution, while in our model we assume knowledge about the global distribution of types but uncertainty about its local configuration. Chwe characterizes how must be the structure for generating a sufficient information to the agents (there are sufficient people for the coordination) that in our model we assume that exists. So the structures in Chwe guarantees the existence of enough information such that the equilibrium of coordination exists. We show that the coordination in the risky action is in fact the unique equilibrium under certain structures when the network allows the observation of previous actions of the neighbors.

With respect to Granovetter (1978), the environment that we study is very similar, although our objective differs significantly from his. He assumes that each previous action can be observed and analyzes how the distribution of thresholds must be so that a revolution occurs. Agents may have personalized thresholds and they revolt after observing that sufficient other agents do so. The agents studied by Granovetter are rational in the sense that they participate only if they consider it is beneficial for them, and respond to the actions they observe. He characterizes how must be the individual distribution of thresholds in order to generate the collective action, and how it determines the amount of people that take part on it. In our model, we also allow for agents with different utilities, such that each agent requires a different amount of other agents participating for willing to take part on it. However, our agents are called to participate just once, and they decide strategically: they take into account what they observe and also the strategies of the other players. Moreover, in the model of Granovetter the agents observe all past actions, while our aim is precisely to characterize the minimal requirements on observation of actions in order to ensure the coordination in the risky action.

The potential multiple interactions generate that multiplicity of equilibria emerge in many contexts related with networks. For instance, it was the case in one of the first approaches to the formation of networks (Aumann and Myerson, 1988). This leaded to the use of several equilibrium refinements to study the process of formation (Jackson and Wolinsky, 1996; Bala and Goyal, 2000). Problems of coordination emerge also in the study of games that are played by agents embedded in given networks (Ellison, 1993; Bramoullé and Kranton, 2005; Galeotti et al, 2010), or when agents decide at the same time about the formation of the network (Jackson and Watts, 2002; Goyal and Vega-Redondo, 2005). Our approach differs since we study the problem of coordination among all the agents with a fixed network that allows them to mutually observe their actions.

Choi et al. (2011) show the impact of different observation networks in the coordination of agents in an experiment. Agents had to decide simultaneously during three rounds wether to contribute or not to a public good. They were embedded in a network that allowed them to observe the actions of their neighbors only if they were linked. They find that different structures have different impact in the possibilities of coordination. In their environment, they show that the agents, depending on their network position, decide to delay or commit. By contrast, we study a different game where agents are called once to decide and therefore these effects are absent. In Kiss et al. (2009), the effect of having links in an observation network is analyzed in a bank run experiment. It is shown that links may promote coordination but also failures, depending on the particular action observed.

The observation of other agents' actions has an effect on coordination that has been studied in several strands of the economic literature. In herding models (e.g. Banerjee ,1992, or Bikhchandani, Hirshleifer and Welch, 1992) agents face a binary decision and receive a private signal about the quality of the alternatives before deciding. They also observe the actions of their predecessors. They use the information conveyed by these sources in order to choose the alternative that is expected to give the higher utility. In our setup, there is no uncertainty about the utility the alternatives yield, but the issue is to find the information structure that ensures that sufficient agents take their risky action and enjoy the highest possible utility. However, a similarity with herding arises when an agent takes into account that many other agents with low threshold may coordinate in their risky action. In this case, since the number of other agents choosing the risky action is sufficient, she will also choose the risky action, so a herd of risky action forms. However, notice that the reasons and mechanisms in herding papers are very different from ours. Costain (2007) shows in a model that nests global games and herding models that if most agents may observe a few previous actions instead of playing a simultaneous-move game, then in the face of fundamental uncertainty multiplicity of outcomes is prevailing. Our paper - without considering fundamental uncertainty - asks how the social structure enabling observability of previous actions should be so that agents choose optimally.

The rest of the paper is organized as follows. In the following section we define formally our model. Section 3 states our main results and in section 4 we discuss some particular cases. Section 5 applies the model to riots, revolts and bank runs and section 6 concludes. Most of the proofs are relegated to the Appendix.

2 The model

Let be $N = \{1, 2, ..., n\}$ the set of agents. These agents are embedded in an *observation network* Γ that connects them. An observation network Γ is a collection of pairs ij such that if ij belongs to Γ, then agents i and j are linked and are able to observe each other's actions (one of them plays after observing the action of the other), with $i, j \in N$. For convenience, we assume that an agent is always neighbor of itself, i.e. $ii \in \Gamma, \forall i \in N$. We assume that the network is undirected, $ij \in \Gamma \iff ji \in \Gamma$ and common knowledge[5]. Agents that are linked are called neighbors. We define the set of neighbors of i as $N_i \subset N : \forall j \in N_i \rightarrow ij \in \Gamma$. A key network concept throughout the paper is the concept of clique. Given a network Γ a clique is a subset of agents $q \subset N$ such that they are completely connected, i.e. $ij \in \Gamma, \forall i, j \in q$.

2.1 The game

Payoffs of agents depend on their actions and actions of others and are given by a utility function u. We study generalizations of the stag-hunt game, where there is a coordination equilibrium that dominates the rest of payoffs. Each of our players is characterized by a threshold t, such that if at least t agents choose the action preferred by the player, she obtains a payoff higher than with any other option. We label this action as the risky action, since it generates the highest payoff but only if the agents are able to coordinate among themselves.

[5] The requirement of common knowledge about the network structure is strong. Results similar to those in the paper can be obtained with a less demanding condition, the existence of knowledge about the possible relations among the neighbors. We use a common knowledge structure because of its simplicity (a similar approach is used in Chwe, 2000).

2.1.1 Homogeneous case

We name homogeneous case to the one where all the agents share the same risky action and threshold t. Importantly in this case, every agent knows that the rest of players share her threshold. The threshold t represents the amount of agents that are required taking the risky action in order to make it the optimal decision. In this version of the problem, agents face a binary decision. Let $A = \{\alpha, \beta\}$ be the pure action set. We name α the risky and β the safe action, meaning that agents highest payoff is obtained when choosing α if sufficient other agents choose it also. The preferences that we study are represented as follows:

$$u_i \left(a_i = \alpha, \sum_{j \in N} I_{a_j = \alpha} \geq t \right) > u_i \left(a_i = \beta \right) > u_i \left(a_i = \alpha, \sum_{j \in N} I_{a_j = \alpha} < t \right), \tag{1}$$

where $I_{a_j = \alpha}$ is the indicator function and a_i is the action taken by agent i. Action β is the safe action, yielding a fixed utility independently of the other agents' choices. Note that the utility does not depend on the position in the network or the order of decision, it is only a function of the actions taken by the agents.

One can imagine this setup as the generalized version of the example in the Introduction. In a large Department, everybody may prefer Linux (the risky action), but only if enough other agents adopt it as well. Otherwise, they prefer the safe option and use Windows.

2.1.2 General set up

In general, agents differ in their preferred action and in their threshold. Returning to our example about the operating systems, some individuals may prefer Linux, while others prefer Mac OS. There may be individuals that would adopt Linux whatever the others do while other people would need to observe that some have adopted an operating system before choosing it as well. And others would adopt upon observing a high number of earlier adopters. The type of an agent consists of the preferred operating system and the number of individuals adopting it that prompts her to adopt as well.

In this general environment, the set of actions if formed by a set of possible risky actions $\bar{A} = \{\alpha', \alpha'', ...\}$ and the safe action β. We name $A = \left\{\bar{A}, \beta\right\} = \{\alpha, \alpha', \alpha'', ..., \beta\}$ the set of pure actions. For simplicity, we are assuming that the safe action is the same for everybody[6]. Let define $a_i \in A$ as the action chosen by agent i and $a = \{a_i : \forall i \in N\}$ as an action profile, which contains an action for every agent.

At the beginning of the game, nature reveals privately to each agent her preferred action and threshold, which we name the type of the agent, $\tau_i = \{\alpha_i, t_i\} \in A \times \mathbb{N}$. We assume that the amount of agents of each

[6] If agents would have different safe actions, they could be perfectly identified if they are observed choosing it. In this way, it is more difficult to infer the type of each agent from her action.

type is fixed, commonly known and given by k

$$\begin{aligned} k &: (A \times \mathbb{N}) \to \mathbb{N} \\ k(\alpha, t) &= \#\{i : \tau_i = (\alpha, t)\}. \end{aligned}$$

and therefore type is randomly assigned to each agent as in a sampling without replacement.

This is a strong assumption, since it means that the agents know exactly the amount of agents of each type that exist, although they ignore the exact type of each agent (except the own type). A more standard assumption would be one where types are randomly drawn from a given distribution. However, we use our assumption because it allows us to focus in pure coordination problems, as explained later. We assume that nature assigns types unconditionally, so that ex-ante all agents can be of a given type with the same probability, given by k.

Consistency requires

$$\sum_{(\alpha,t)\in A\times\mathbb{N}} k(\alpha, t) = n. \tag{2}$$

Let denote by

$$K(\alpha, t) = \sum_{t' \leq t} k(\alpha, t')$$

the number of agents whose risky action is $\alpha \in A$ and who have a threshold that is t at most.

We say that $\alpha \in A$ is the risky action of agent i if that agent's highest payoff is obtained when she chooses α and sufficient other agents do the same. If not enough agents choose it, then the safe action is preferred:

$$u_i(\bar{a}') > u_i(\bar{a}'') > u_i(\bar{a}''') \tag{3}$$

$$\text{with } \left\{\bar{a}' : \left(\bar{a}'_i = \alpha, \sum_{j\in N} I_{\bar{a}'_j=\alpha} \geq t_i\right)\right\}, \{\bar{a}'' : \bar{a}''_i = \beta\}, \bar{a}''' \notin \{\bar{a}', \bar{a}''\} \tag{4}$$

Note that in this definition of the problem, the threshold t may vary across agents. Action β is the safe action, yielding a fixed utility independently of the other agents' choices. If there is only one preferred action and all the agents share the same threshold, this utility is the one of the homogenous case.

We restrict our attention to situations in which coordination is possible, i.e.

$$\max K(\alpha, t) \geq \max\{t' : k(\alpha, t') > 0\}, \forall \alpha \in A$$

This condition implies that coordination is a rational outcome: the highest threshold of each risky action is smaller than the amount of agents who prefer that action. This implies specifically that if all agents who prefer the action are choosing it, they are in fact behaving optimally. Our assumption of a fixed amount of agents of each type allow us to study pure coordination problems. Since the amount of agents of each type is known, we are restricting our attention to environments where coordination is always an equilibrium outcome: every agent would best respond choosing her own risky action if everyone else were choosing also it, and it is known that it is possible. Therefore any equilibria different from the efficient one is always the result of a pure coordination failure. Our aim is to characterize the conditions over the observation network such that the efficient equilibrium is the only one possible.

2.1.3 The coordination game

Agents act in a consecutive manner according to the order of decision $\theta(N)$, which assigns a position to each agent. Let $\theta(N) = \{\theta_i : \forall i \in N\}$ such that $\theta_i \in \{1, 2, ..n\}$ and $\theta_i \neq \theta_j, \forall i, j \in N$. Then, $\theta_i = 5$ indicates that agent i is the fifth to take the decision. Let $\Theta(N)$ denote the set of all possible orders of decision, which is equivalent to the set of possible permutations on $\{1, 2, ..n\}$. This order is randomly assigned by the nature according to some probability $P(\theta(N))$, which is assumed to be common knowledge: agents ignore the precise order of decision[7], but they know actions of neighbors who have already decided[8]. The order of decision and the observation network define the information set of each individual. For instance, an empty network corresponds to a simultaneous game, since nobody observes no other action: in this case, information sets only contain the private type of each individual. Links allow observation, transforming the game into a sequential one. In the other extreme case, the complete network, all the previous actions belong to the information set of the agent and she is able to infer perfectly her exact position in the sequence of decisions Notice that the connections in the network and the position in the order of decision make agents heterogeneous.

A coordination game starts with the nature selecting the order of decision with the probability $P(\theta(N))$. We assume that this distribution is common knowledge (but not the precise order). The network and the order of decision determine the information set of each agent, ψ_i. It includes the type of the agent and the actions played by those neighbors who have decided before the agent:

$$\psi_i = \{\tau_i, \{a_j : ij \in \Gamma; \theta_j < \theta_i; \theta_j, \theta_i \in \theta(N)\}\}. \tag{5}$$

Thus, agent i observes the actions of her preceding neighbors in the order of decision selected by the Nature. The knowledge of the network enters the information set by allowing the observation of neighbor's actions. The set of all possible information sets in which agent i could be is denoted by Ψ_i.

[7] Except if $P(\theta(N))$ is degenerated.

[8] Note that given this information, agents have partial information about their position.

A strategy is a mapping from all possible information sets into actions. We allow for mixed strategies, so

$$s_i : \Psi_i \rightarrow \triangle \{A\}.$$

The history of the game up to agent i contains the order of decision, the actions up to agent θ_i and the type of each agent, and is defined as follows:

$$H_i = \left\{\theta(N); \{a_j, \forall j : \theta_j < \theta_i; \theta_j, \theta_i \in \theta(N)\}_{j \in N}; \{\tau_{j'}, \forall j' \in N\}\right\} \tag{6}$$

The belief of agent i is the probability that she assigns to each history that may occurred before she takes her decision, given her information set. Hence, the belief is defined as

$$\pi_i = \Pr(H_i \mid \psi_i) \geq 0 : \sum_{H_i} \Pr(H_i \mid \psi_i) = 1. \tag{7}$$

Now we are ready to define the coordination game.

Definition 1 *A coordination game is defined by* $N, \Gamma, A, k, P(\theta(N))$ *and* $u(.)$ *that satisfies condition (3).*

A coordination game is defined by the observation network Γ, that connects the agents, the probability distribution over the different orders of play as well as the utility function that depends on the actions, which satisfies condition (3).

The following example clarifies each of the elements in a coordination game.

Example 1 *There are three agents,* $N = \{B, G, Y\}$*. All of them prefer Linux if at least two of them adopt it. Otherwise they would rather use Windows than being the only person using Linux. They are set in an observation network that links agents* B *and* G*, so* $\Gamma = \{BG\}$*. The action set for all of them is* $A = \triangle\{L, W\}$*,* L *denoting the choice of Linux (*W *denoting Windows). The utilities are given by:*

$$u_i(L, \sum_{j \in N} I_{a_j = L} \geq 2) = 4,$$

$$u_i(W) = 3,$$

$$u_i(L, \sum_{j \in N} I_{a_j = L} < 2) = 1.$$

The extensive form of the coordination game is as follows:

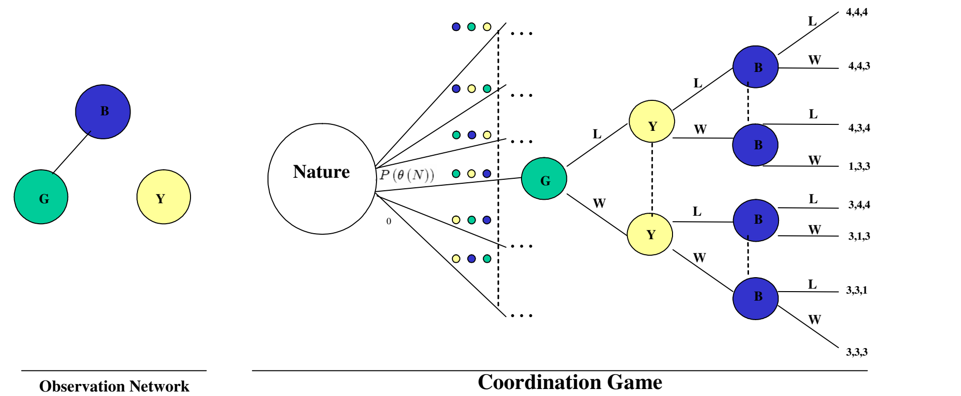


Figure 2

The coordination game associated with the network starts with the Nature selecting one order of decision with probability $P(\theta(N))$, we illustrate the order GYB. This is the case in which $\theta(N) = \{3, 1, 2\}$ (remember that we have defined $N = \{B, G, Y\}$). Figure 2 shows that B is the third one who decides, and observes the action taken by G but not the action taken by Y, who is the second to decide. This is a homogenous case, with

$$k = \begin{cases} 3 \text{ for } (a,t) = (L,2) \\ 0 \text{ otherwise} \end{cases}$$

We use sequential equilibrium as the solution concept. A sequential equilibrium is defined by a profile of behavioral strategies in each information set and beliefs such that the strategies are best responses to the strategies of the other agents conditional on beliefs, and the beliefs are consistent with Bayes rule for some sequence of completely mixed strategies that converges to the equilibrium ones.

Definition 2 *Let (Σ', Π') be an assessment, a profile of behavioral strategies and beliefs for each player in each of her information sets. The assessment (Σ^*, Π^*) is a sequential equilibrium (Kreps and Wilson, 1983) if*

1. *(Σ^*, Π^*) is consistent:*

$$\begin{aligned} (\Sigma^*, \Pi^*) &= \lim_{n\to\infty} (\Sigma_n, \Pi_n), \{(\Sigma_n, \Pi_n)\} \subseteq \Phi^0, \\ \Phi^0 &= \left\{ \begin{array}{c} (\Sigma, \Pi) : \Sigma \text{ is completely mixed and} \\ \Pi(x) = \frac{P^{\Pi}(x)}{P^{\Pi}(\psi(x))} \end{array} \right\} \end{aligned}$$

where x is a decision node included in the information set $\psi(x)$, and $P^{\Pi}(x)$and $P^{\Pi}(\psi(x))$ are the probability assigned to x and to $\psi(x)$ respectively,by the system of beliefs Π.

2. (Σ^*, Π^*) *is sequentially rational:*

$$E\left(u_i|\Sigma^*_{-i}, \sigma^*_i\left(\bar{\psi}_i\right), \Pi^*, \bar{\psi}_i\right) \geq E\left(u_i|\Sigma^*_{-i}, \sigma_i\left(\bar{\psi}_i\right), \Pi^*, \bar{\psi}_i\right), \forall i \in N, \forall \bar{\psi}_i \in \psi_i$$

Our aim is to find the observation networks that lead to the Pareto-superior equilibrium in the coordination game defined by N, Γ, A and k, for any $\Theta(N), P(\theta(N))$ and $u(.)$.

Definition 3 *An observation network Γ is a coordination structure for N, A and k, if for all sequence of decision and utilities, $\forall P(\theta(N))$ and $\forall u(.)$, all agents take the risky action over the equilibrium path in any sequential equilibrium of the associated coordination game.*

For a set of agents in which it is known how many of them prefer each option, if Γ is a coordination structure, the unique equilibrium implies the efficient coordination. The observation network allows that some individuals in the game play sequentially instead of simultaneously, which generates that some Nash equilibria of the simultaneous version can not be sustained as sequential equilibria. We can understand the concept of coordination structure as the social network such that, if it is known to exist, an agent not belonging to it would choose the risky action, since she can infer that the unique option is that the structure promote the coordination in the efficient outcome.

3 Coordination structures

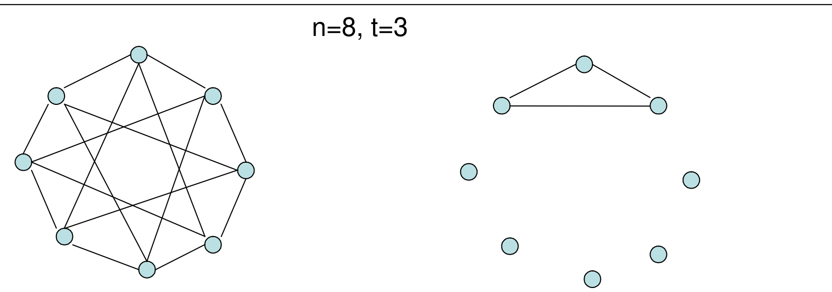
3.1 Homogenous case

First we present the result for the homogenous case:

Proposition 1 *In the homogenous case with threshold t, an observation network Γ is a coordination structure if and only if there exists a subset of agents $\{q \in N : \#q = t\}$ that forms a clique.*
Proof. *See Appendix.* ■

A clique is sufficient by an argument of backward induction. If there exists a clique of size t, in any order the last agent in the clique chooses the risky action if in her information set observes $t-1$ risky actions. Agent in position $t-1$ in the clique best responds to a strategy of this type by choosing the risky action if she observes $t-2$ risky actions, and so on. Therefore, any agent in the clique in position r among those that also belong to the clique, chooses the risky action when observing $r-1$ risky actions. So the first agent in the clique chooses it, and any agent out of the clique best responds to these strategies by choosing the risky action, since there are t agents who choose it. Everyone chooses the risky action over the equilibrium path. We prove that the clique is necessary by constructing an equilibrium assessment in which, over the equilibrium path, everyone chooses the safe action when the clique does not exist. If everyone believes that everyone else is going to choose the safe action, everyone choosing the safe action can be sustained in equilibrium, if the clique does not exist. This is the case because an initial agent cannot know if her action will be observed by sufficient people who are also mutually observing their actions in order to be sure that the first best is obtained. Note also that any structure that contains the clique is also a coordination structure. In this sense, a clique of size t can be said to be minimal and sufficient.

Figure 3 illustrates the importance of the clique. In this example, there are two different network structures of $n = 8$, and it is required that at least 3 agents take the risky action so that it becomes profitable.



This network has 16 links but no clique of size 3. It is not a *coordination structure*. We can find a sequential equilibrium with a *coordination failure*

This network has 3 links forming a clique of size 3. It is a *coordination structure*. All agents choose the risky action in any sequential equilibrium. *Coordination failures* may not occur.

Figure 3

The network structure on the left has 16 connections but no clique of size 3. Given our result, in any sequential equilibrium in the structure on the right, the Pareto-dominant equilibrium emerges. In the structure on the left, coordination failures are a likely outcome.

3.1.1 Relevance of cliques: consistency

This result supports the importance of cliques in the emergence of coordination from a new point of view. Cliques, usually understood as the representation of groups, enable coordination because the agents are able to signal their decision in those structures. Importantly for our result, it holds for any utility function: it does not depend on u: it is possible to find a coordination failure equilibrium if the clique does not exist, and it is not possible if the clique exist, for all possible functions that satisfy 1. This has relevant implications. Imagine the case of a society deciding wether to revolt against a dictatorship. Our homogenous case would describe a society where everyone wants to revolt, it is known, but it is required a certain amount of people revolting in order to support these revolts. In this situation, in a simultaneous set up (i.e., an empty network in our model), both situations are equilibria, everyone or nobody revolting. In principle, which equilibrium emerges would depend on the relative payoffs[9]. For instance, if the penalty if the revolt do not success is very strong, we would expect that people do not revolt.

However, once the clique exists, relative payoffs do not matter, and everyone would choose the risky action even if the penalty for failure is very strong. This intuition make clear that a dictatorship has an interest in destroying such type of cliques, since penalties for failure would not matter. This also suggests that we should observe this kind of efficient coordination when there exist these large cliques that allow the observation of actions. This point of view gives an additional explanation for the cities as the usual starting points of revolts, since in the cities there are large amounts of individuals able to observe each other actions, i.e. embedded in large observational cliques.

How important are the assumptions that we use for this result? Is it consistent? We argue that it is the case and that the existence of a clique really plays a crucial role because of several reasons. With respect to the previous literature, in a closed but different environment, Chwe (2000) obtained a result that is connected with ours (we detail this connection in a later section). Importantly he showed that the existence of cliques has a key role in generating the common knowledge that allows coordination (among other results). We assume the existence of that common knowledge and show that cliques play also a crucial role for obtaining coordination as the *unique* expected outcome.

One of the main points is the independence of the result with respect to utilities: if the clique exists and the benefit of coordination is arbitrarily small, the unique equilibrium implies that coordination emerges; if the clique does not exists, even if many agents are able to observe among themselves and to others, and even if the benefit of coordination is arbitrarily large with respect to the failure, it is possible to construct the coordination failure equilibrium.

The clique is sufficient for coordination for any possible sequence of decision, but it is also necessary. In this homogenous case, no matter how the structure or the payoffs are, no matter who starts to play, the

[9] This kind of simultaneous situations have been succesfully analyzed with *global games* (Carlsson and Van Damme, 1993). These models show the importance of the relative riskness of each equilibrium in determining which one is selected.

existence of the clique guarantees the efficient coordination, and the non existence of the clique guarantees the existence of a coordination failure. In fact, the existence of the clique is necessary and sufficient for any sequence of decision: there exists no structure in which the coordination failure may not be sustained, even if the first agent who decides is observed by many other people (but not forming a clique of size t).

Another aspect that reinforces the consistency of the result is the type of equilibrium that we use. Along the paper, for consistency, we use the concept of sequential equilibrium, which is a relatively strong concept that requires the existence of very rational beliefs out of equilibrium. Even using this concept, we find that the coordination failure can be sustained when the clique does not exist. When proving uniqueness of the equilibrium when the clique exist, we in fact prove that it is the case for arbitrary beliefs out of equilibrium. That is, we prove that coordination is the unique equilibrium outcome for any weak perfect Bayesian equilibrium. Under our point of view, the fact of obtaining uniqueness of equilibrium with a very few demanding equilibrium concept as WPBE, and of obtaining multiplicity even under sequential equilibrium, reinforces the consistency of the result, supporting the real relevance of cliques in the emergence of coordination.

3.2 General case

The general version of the model allows to study the case when the agents ignore if they are observing people who share their preferences or not. In this general case, we assume that preferences (i.e., the utility function of each individual) are private information but that the amount of agents of each type is fixed and common knowledge[10]. We start our analysis with some illustrating examples of the simplest case. Imagine a society formed by n agents, where $n-1$ are willing to take the action α and the other agent has a degenerated decision, choosing always β. We name her the "unwilling" agent. If types are private information, how must

[10]Note that common knowledge about the type distribution implies knowledge about private type in the homogenous case, which is therefore nested in the general model.

be the coordination structures if agents that prefer α have threshold $t = 2$? Figure 4 shows some examples.

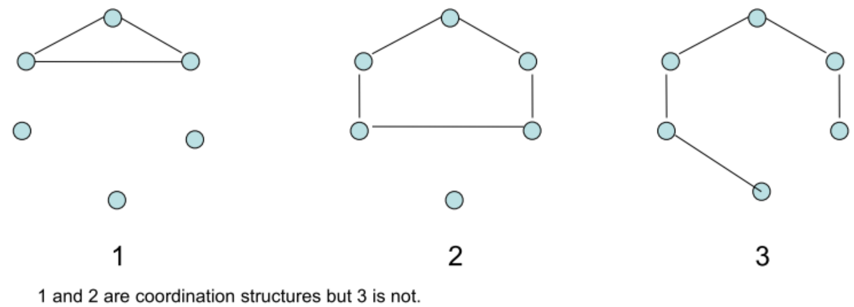


Figure 4

If there is in the society an observation network as 1 or 2, in any sequential equilibrium, for any payoffs, and for any order selected by the nature, agents willing to take α choose it on the equilibrium path. Let explain why. Since the agents have threshold $t = 2$,any of them would best respond by choosing α if observes that a predecessor has chosen also α. In the network 1, the first agent in the clique who has to choose, knows that her action is observed by at least one agent willing to take the action (she is observed by 2 agents, and only 1 in the society does not want to choose α). Therefore, she best responds by choosing α, since then the first best is obtained. The second agent in the clique, when choosing, if she observes that the first one has not chosen α must believe with probability 1 that she was the unwilling agent, and assigns probability 1 to be observed by a willing agent. Therefore, in any equilibrium path, the willing agents in the clique choose α, and agents who does not belong to the clique best respond to these strategies by also choosing α. A similar argument can be applied in the network 2. However, in 3 a different equilibrium can be sustained. Imagine that everyone is playing β after observing β and that penalization of choosing α if nobody else chooses it is extremely high. If the first agent who decides is not in the extremes, she will choose α since she knows that is observed by, at least, one willing agent. But if the first agent is in one of the extremes, if she plays α, with positive probability she is observed by the unwilling agent, who would choose also β. Given the prescribed strategy, if the order of play is according to the sequence, nobody else would choose α, and the initial agent prefers β. Therefore it is possible to sustain an equilibrium failure. The existence of the structure 3 does not guarantee the non existence of coordination failures.

Figure 5 shows other coordination structures when threshold is $t = 3$ and there exists only one unwilling agent.

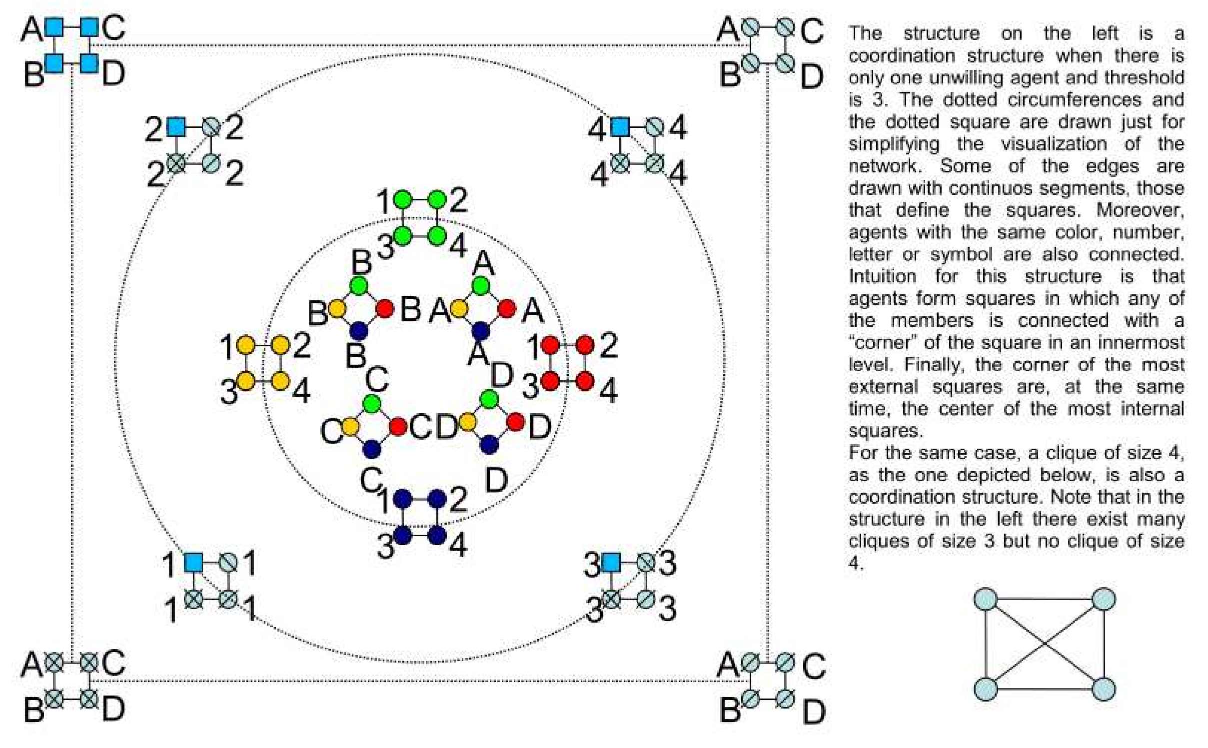


Figure 5

In this situation of threshold $t = 3$, a clique of size 4 as the one in the right corner, formed by 4 nodes and 6 links, is a coordination structure. The argument is related with the one used for the clique of size 3 when threshold was $t = 2$. The figure in the left, with 64 agents and 320 connections is "minimal" since, if just one link is deleted, a coordination failure may occur in equilibrium (for certain payoffs and order of play). It is a coordination structure with cliques of size 3. The figure is formed by squares where all agents are connected to a "center" of the square, who is at the corner of another square. In a square connected to a center, it can be proved that, if the first agent who decides is the center and she is of the willing type, then every willing agent chooses the action α. In the structure in the left, every agent is the center of a square, and her neighbors in the square are at the same time the center of a different square. Therefore, the first agent who decides chooses the action α if she is of the willing type. If she is not of the willing type, the following agent who decides is in the center of some square and is of the willing type, and so on. In any order of decision, for any payoffs, every willing agent chooses α in that structure. Therefore, if the structure exists and it is known, any willing agent who does not belong to it would also take the risky action over the equilibrium path.

3.2.1 General results

The previous examples show some coordination structures for a very particular case: one where all the society shares the same risky action and threshold except for one individual. We have found some simple structures, as cliques, and some other complex structures. Here we explore if we can obtain more general results. We find that the existence of one clique sufficiently large guarantees the coordination.

Let focus in the case of agents who share the same risky action α but with different thresholds t. Given our environment, an agent knows that there are sufficient people if the clique is of a size at least $n - K(\alpha, t) + t$. This is the case since $K(\alpha, t)$ are the amount of people in the society who wants α with a threshold smaller or equal than t, $n - K(\alpha, t)$ are all individuals in the society who would not take α optimally although there were t agents choosing it. The following lemma comprises this insight:

Lemma 1 *An agent of type $\{\alpha, t'\}, t' \leq t$ takes the risky action on any equilibrium path if there exists a clique of size $n - K(\alpha, t) + t$*

Proof. *See Appendix* ■

If there exists a clique of size $n - K(\alpha, t) + t$, it is sure the existence of t agents willing to choose α. Note that the clique of size 3 in Figure 4 and the clique of size 4 in Figure 5 can be shown to be coordination structures according to this Lemma. There are significant differences with the homogenous case. In particular, in the equilibrium of coordination, it is likely that an agent observes actions different from her risky action. What Lemma 1 adds to our knowledge is that, in any sequential equilibrium and given the existence of the clique, any equilibrium beliefs of the agent must incorporate probability 0 to the event of those non-risky action being taken by agents like the individual (other agents who share her own risky action). In the Appendix we prove that this is the case.

When having different thresholds, however, a second effect must be incorporated, related with a "cascade effect". Because of Lemma 1, an agent of type (α, t) takes the risky action if there exists a clique of size at least $n - K(\alpha, t) + t$. But note that an agent of type (α, t) takes the risky action also if there are t agents with a threshold smaller than t (say $\bar{t} < t$). In such a case, the clique required by agents with threshold $\bar{t}$ is also sufficient for the agents with threshold t. All these insights are summarized in the following theorem:

Theorem 1 *An agent of type $\{\alpha, t\}$ takes the risky action on any equilibrium path if there exists a clique*

$q \subset N$ of size $\#q = q(t)$ where

$$\begin{aligned} q(t) &= q_{\alpha}(r_{\alpha}(t)), \qquad (8) \\ \textit{where } r_{\alpha}(x) &= \min\{N - K(\alpha, t) + t : \forall t \geq x\} \\ q_{\alpha}(x) &= \begin{cases} r_{\alpha}(x) & \textit{if } r_{\alpha}(y) < x \\ r_{\alpha}(y) & \textit{if } r_{\alpha}(y) \geq x \end{cases}, y = \max\{t : y < x\} \end{aligned}$$

where $q_{\alpha}(x)$ is constructed iteratively from the lowest threshold x for the action α.

Proof. *See Appendix.* ■

Theorem 1 shows that if a clique of certain size exists, then the agent takes the risky action in any sequential equilibrium. This is the case for an agent of given type, independently of how many different preferences exist (in risky actions or threshold). So this result can be applied for environments where people differ in their objectives of coordination. The result of Lemma 1 is incorporated in $r_{\alpha}(x)$, which includes that an agent takes the risky action when the condition of the Lemma 1 is met by the agents with some threshold equal or larger than her threshold. The possibility of existing more than t agents choosing the risky action with a threshold smaller than t is incorporated in $q_{\alpha}(x)$, which is constructed recursively and says that, if there are more than t agents with a threshold smaller than $t' < t$, the size of the clique required by agents with threshold t is the same than the one required by the agents with threshold t'. The following example of a society of 25 agents who share the same risky action α but differ in their thresholds, and 1 agent who prefers β', illustrates the result.

Example 2 *Let suppose a set of agents $N = \{1, 2, ..., 26\}$ which play a game of coordination. The type of agents is given by*

$$k(.,.) = \begin{cases} 5 \textit{ for } (\alpha, 2) \\ 5 \textit{ for } (\alpha, 6) \\ 8 \textit{ for } (\alpha, 10) \\ 2 \textit{ for } (\alpha, 14) \\ 5 \textit{ for } (\alpha, 21) \\ 1 \textit{ for } (\alpha', 1) \\ 0 \textit{ otherwise} \end{cases}$$

We can compute here the size of the clique required by each agent in order to be sure that she chooses the *risky action in any equilibrium:*

t	$k(\alpha, t)$	$N - K(\alpha, t)$	$N - K(\alpha, t) + t$	$r_\alpha(t)$	$q_\alpha(r_\alpha(t))$
2	5	21	23	18	18
6	5	16	22	18	18
10	8	8	18	18	18
14	2	6	20	20	18
21	5	1	22	22	22

Table 1

If there is a clique of size 23*, agents with threshold* $t = 2$ *know that there are at least 2 people with threshold 2 in the clique, and therefore, they are going to take the risky action. Agents with threshold at most ten (*$t \leq 10$*) know that if there is a clique of size* 18*, it includes with certainty at least 10 agents with a threshold equal or smaller than 10. Therefore, those agents with threshold at most 10 in the clique coordinate on their efficient action in any equilibrium, and - since the network is common knowledge - also any other agent with the same threshold in the society will do so. This is the direct effect of a clique of size* 18*.*

Finally, the "amplifying" effect generates that agents with threshold 14 take the risky action also when there is a clique of size 18*. This occurs because this clique generates that all agents with a threshold smaller or equal than 10 take the risky action in any equilibrium path; and since they are 18 agents, the effect of the clique affects also to those agents with threshold at most 18 (in this case, the 2 agents with threshold 14). However, the agents with threshold 21 will not necessarily follow suit. They require the existence of a clique sufficiently large to know that 21 agents will coordinate: in this case, a clique of size* 22*.*

Note that, in the example, if there exists a clique of size 22, any agent chooses the risky action. This means that an observation network Γ including a clique of size 22 is, in fact, a coordination structure. Nevertheless, if the highest threshold was 18 instead of 21, then a clique of size 18 would be a coordination structure. Therefore we can obtain a sufficient condition for the existence of a coordination structure, that follows directly from the Theorem 1:

Corollary 1 *A network* Γ *is a coordination structure in the incomplete information case if there exists a subset of agents* $\left\{s \subset N : \#s = \max\left\{q_\alpha\left(r_\alpha(t)\right)\right\}_{\forall \alpha \in A, \forall t}\right\}$ *that forms a clique.*

Proof. *If there exists a clique of size* $\left\{s \subset N : \#s = \max\left\{q_\alpha\left(r_\alpha(t)\right)\right\}_{\forall \alpha \in A, \forall t}\right\}$*, given Theorem 8, for any agent in* N *there exists a clique sufficiently high such that she takes the risky action over any equilibrium path. Therefore, everyone takes the risky action and* Γ *is a Coordination structure.* ■

Agents of each type require a clique of a given size for choosing the risky action for sure. Corollary 1 says that if the largest of the required cliques exists, everyone chooses her risky action. And therefore Γ is a coordination structure. This can be applied when in the society there are groups of agents who differ in their risky action α. In these conditions, Corollary 1 shows that if a sufficiently large clique exists, the unique equilibrium outcome is the Pareto-efficient one.[11]

This result provides us with a sufficient condition to obtain coordination in the case of heterogenous agents with private types. It is required the existence of a clique larger than in the homogenous case. The reason is that now, initial agents who would potentially coordinate require that their action is observed by much more people than before in order to be sure that they are coordinating with sufficient people of the same type. At the same time, people who plays lately in the sequence of decision requires also to have enough observations over the past in order to identify correctly those agents who are not choosing the risky action as agents of a type different from their own type. When an agent has information of the types of the agents she is observing to act (the homogenous case), we have shown that still a clique of size t is necessary. So that, this represents a lower bound on the requirements for coordination. We can imagine that in the case of private information, the existence of a clique of size t is necessary. Figures 5 and 6 show some of these structures, formed by small cliques of the size that would be required in the case of known types. In the Discussion in the following section, we argue how can be constructed this family of coordination structures and show them for the simplest cases.

It is interesting to note that the first individuals to decide are not always those whose threshold is the lowest. However, this does not impede coordination, because agents' choices are strategic and consequently even an agent with a high threshold takes the risky action if the observation network enables coordination. The ability to observe other actions works as mechanisms of coordination. When people know that there is a sufficient amount of other agents, and that they are mutually observing their respective actions, they are able to signal their type to the rest of members of the group by choosing their preferred action. This happens only if the group is sufficiently large to ensure that enough people with the same type will observe the action: coordination failures are less likely when there are larger groups of people able to observe each

[11] The result relies on the fact that coordination in the risky action generates the highest possible payoff. It is worthy to note that this does not include related situations, as the one that follows. Imagine the revolt case with two different groups that prefer different regimes. Each group prefers to revolt in favour of their candidate. This means that if clique are sufficiently high, both groups could revolt, and then it may be the beginnig of a fight in which both groups would be worse off than in the non revolting case. This situation is not nested in our model, since then the coordination in the risky action do not generate the hisghest possible payoff, since depending on the rest of agents it can generate a lower payoff than the safe action. In fact, under these payoffs, may appear different equilibria if the clique is sufficiently high, depending on the order in which different agents take their decissions.

other (even if they do not communicate).

Figure 6 illustrates the case of a coordination structure when there are different risky actions (although for simplicity, we assume that all thresholds are the same). Let suppose a society of 24 individuals who have to dress for a party. We assume that each one has a preferred color to dress, preferring 7 of them to dress in green, 6 in red, 6 in pink and 5 in blue. However, they prefer it only if at least 2 people more are dressing their preferred color. If not, they prefer to dress in black. Formally, this is a case where $N = \{1, 2, ..., 24\}$, $A = \{green, red, pink, blue, black\}$ and

$$k(\alpha, t) = \begin{cases} 7 \text{ if } (\alpha, t) = (green, 7) \\ 6 \text{ if } (\alpha, t) = (red, 6) \\ 6 \text{ if } (\alpha, t) = (pink, 6) \\ 5 \text{ if } (\alpha, t) = (blue, 5) \\ 0 \text{ otherwise} \end{cases}$$

In this situation, what observation networks are coordination structures?

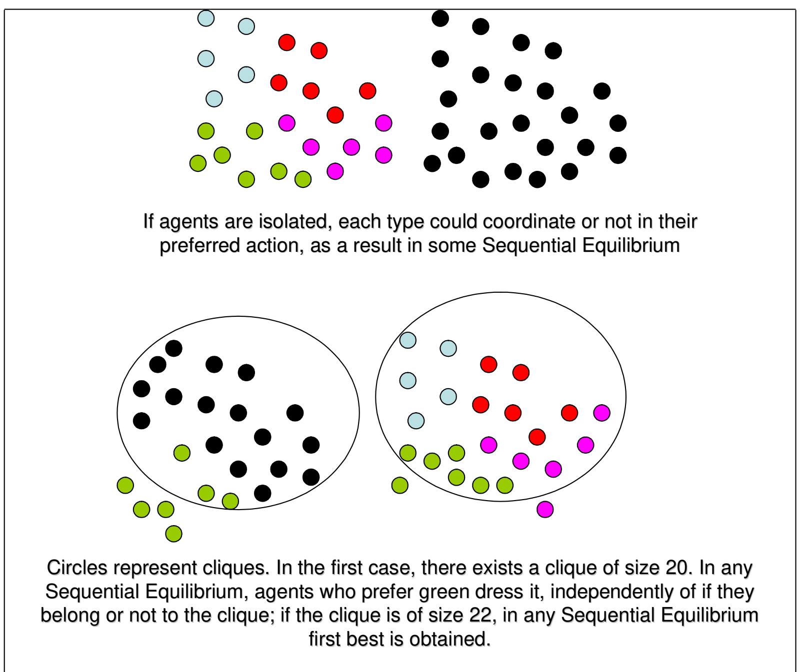


Figure 6

Theorem 1 shows that, if there exists a clique of size 20, every agent whose preferred action is green chooses it in any sequential equilibrium. It is not necessary true for the other agents. A clique of size 22 is a coordination structure, since the agents who prefer blue are those who require the highest clique, and it is of size 22. If such an observation structure exists, in any order of play, for any payoffs, every agent chooses her risky action over the equilibrium path and the first best is obtained: all agents will dress in their favorite colors. It is important to remember that we are analyzing a situation where agents do not communicate. Our conditions are sufficient only if the individuals in the clique are able to observe how each other finally dresses.

The clique required by Theorem 1 and Corollary 1 may be very large, which is typically not likely. However, the kind of social interaction that we are studying is mild: a link between two subjects only implies that they are able to observe their actions. This can be simply the case of living in the same neighborhood. Imagine once more the case of revolts against dictatorships. People may prefer go to the streets if enough other people are going to revolt. We require that these actions are observable. One can imagine that a city would be in fact a clique that connects all its citizens, since in a city, each agent would observe if other agents go to the streets. Our result implies that if there exist a clique sufficiently large, coordination emerges. In the revolt environment, we would say that if there exists a city sufficiently large, where people are able to mutually observe their actions, we expect that revolt occurs. We argue that this mechanisms can give some insights on the cities being places where revolts generate. In fact, we are saying that if people know that there exist a city sufficiently large, where there are for sure sufficient people willing to revolt, everyone would decide to revolt, even people who is not in the city, and even people who require high thresholds for participating. The difference with respect to the homogenous case is that now when the agents take the decision they do not know who of their neighbors are willing to take it, but we show that also in this situation the unique outcome is the efficient one when a sufficiently large clique exists. The key point is to know that there exist sufficient people willing to revolt.

4 Discussion

The existence of a sufficiently high clique guarantees the coordination of agents for any payoffs and for any sequence of decision. Now we discuss some other possibilities, the case of coordination structures formed by more complex structures with many smaller cliques, as the one depicted in Figure 5, or structures that guarantee the coordination for particular payoffs or sequence of decisions, a case that we name Quasi-coordination structures.

4.1 Coordination structures with smaller cliques

An agent that has to coordinate with a part of the society in order to obtain her first best takes her risky action if she knows that there exists a sufficiently large clique, as determined by Theorem 1. However, in contrast with the homogenous case, it is not a necessary condition: it is possible to find structures with cliques of smaller sizes that also guarantee the coordination in the risky action for any payoffs and sequence of decision. Figures 4 and 5 illustrate the case when there is 1 agent in the society who will play always the safe action and the required threshold is $t = 2$ or $t = 3$ respectively. Here we discuss how can be constructed these structures. We focus in the simplest case, when there is a group of agents with the same risky action α, the same threshold t and the rest of the society chooses always the safe action β. We name n_α the cardinal of agents who prefer α if at least t agents choose it and n_β the cardinal of agents who always choose β. Abusing the notation we say that an agent is of type α or of type β respectively. In this kind of situations, following Corollary 1, a clique of size $n_\beta + t$ is a coordination structure. In fact, any larger structure including a clique of size $n_\beta + t$ is a coordination structure. But what about structures formed by smaller cliques?

What can be known about any sequential equilibrium, for any order and payoffs? Any equilibrium strategy profile must include that an agent of type α chooses the action α in all those information sets where she observes $t - 1$ actions α. For the simplest case, when $t = 2$, an agent of type α chooses it if she knows that is followed by another agent of her type. If in the society there is only one agent of type β, $n_\beta = 1$, an agent of type α who has two neighbors who have not played yet will choose action α in equilibrium. The simple structure where we can know that there will be a initial player with two neighbors is a triangle, the clique of size 3. For this case, one clique of size 3 is sufficient, in fact, because of Lemma 1.

Can we found other coordination structures for this case? Yes, as depicted in Figure 7, an square is also a coordination structure: an agent of type α has two neighbors and will choose α; consistent beliefs imply that, if she does not play α, must be the type β agent with probability 1. The square is an example of coordination structure formed by cliques of size 2 for the case $t = 2$, $n_\beta = 1$.

For the case of $t = 2$, $n_\beta = 2$ each agent requires to have 3 successors in order to play the risky action if is of type α. In a clique of size $\#q = t + n_\beta = 4$, every agent has 3 followers, and therefore if the first one plays β consistent beliefs imply that she is a type β agent. In order to obtain a coordination structure, we require agents who have 3 successors. In this way, an agent of type α will choose α in any sequential equilibrium, because having 3 successors she knows that at least 1 is of type α. It can be got with a clique of size 4 or with other structures with more nodes and connections but cliques of smaller size. In Figure 7 we have drawn different coordination structures when $t = 2$ and the amount of agents of type β is between 1 and 4. Basically, for the case of $t = 2$, we obtain coordination structures if there are sufficient agents with

$n_\beta + 1$ neighbors.

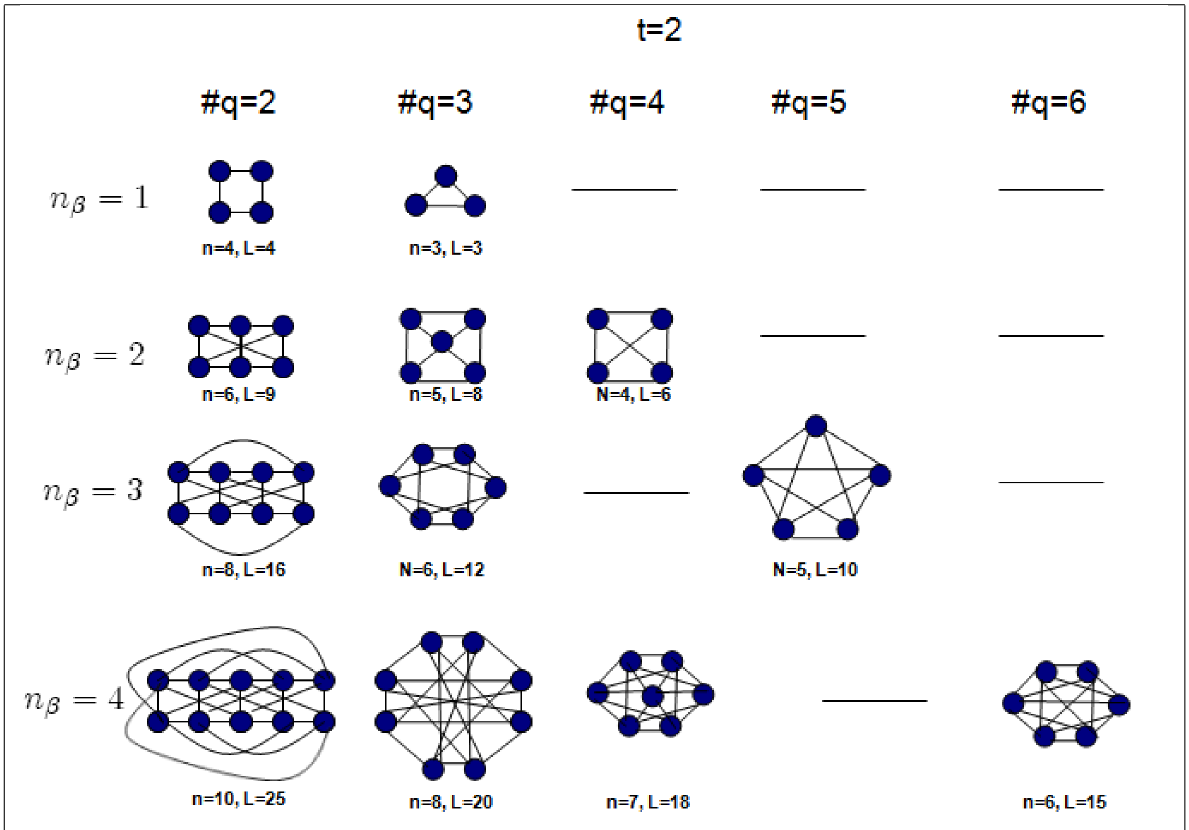


Figure 7

When coordination is more difficult, that is, when it is required more agents for obtaining coordination, coordination structures become more complex. In Figure 8 we have drawn the required coordination structures formed by cliques of size 3 for the case when $t = 3$ and $n_\beta = 1$ and $n_\beta = 2$. In this cases, cliques of size 4 and 5 would be sufficient, following Corollary 1. But structures become much more complex when using smaller cliques. In the simplest case, when $t = 3$ and $n_\beta = 1$, a square with all corners linked with a "center"[12] generates that, if the center is the first who plays, she will choose α if is of such type. For obtaining a coordination structure, we must ensure that each of the agents is the center of a square. Figure 8 shows how looks a network where it occurs. When $n_\beta = 2$, the center of an hexagon whose opposite corners are connected would also choose α if she is of such type and is the first who play. Therefore, we can

[12]Note that a in asquare with all corners connected to a center the highest clique is of size 3.

construct a coordination structure if we guarantee that each node is the center of an hexagon of that type. Note that in such an hexagon, the highest clique is of size 2, and therefore, if agents connect to some center, the highest clique remains of size 3.

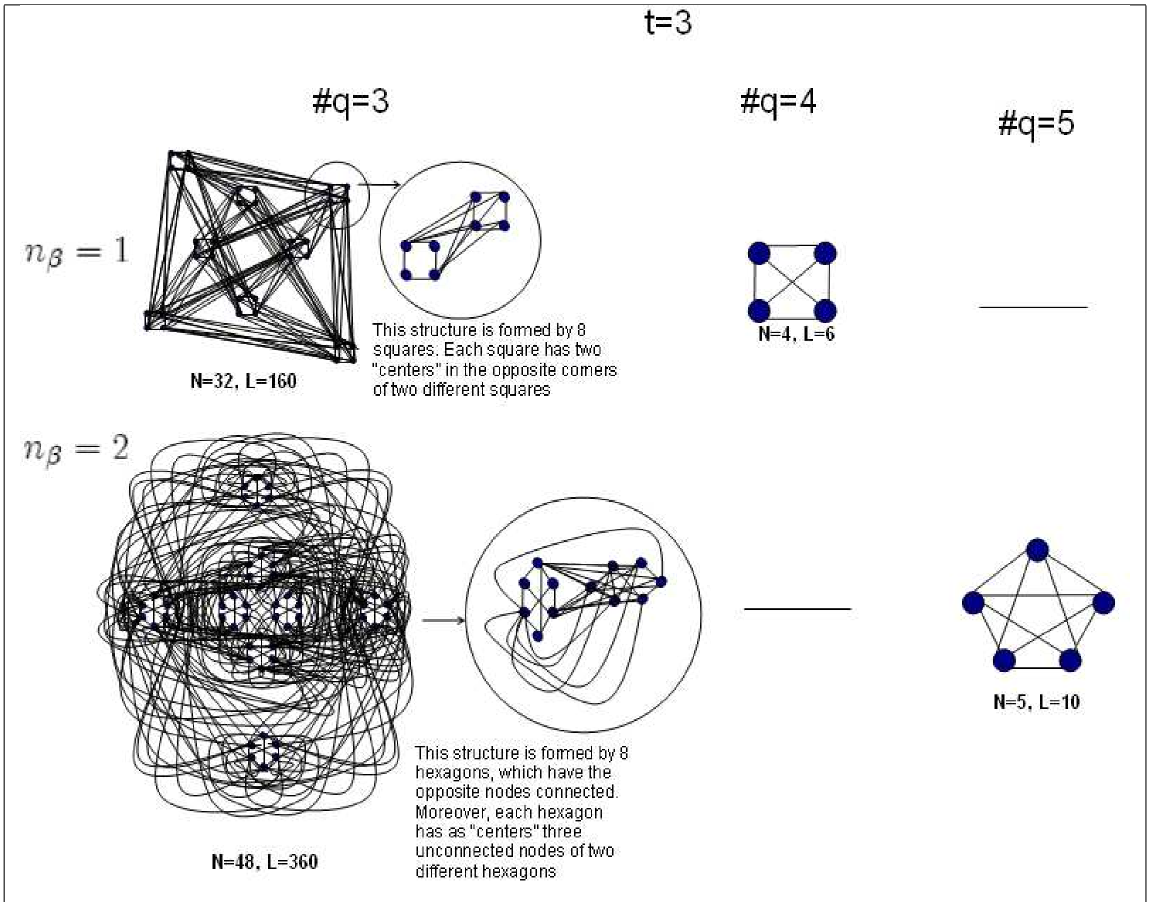


Figure 8

Figure 8 illustrates the complexity that can require an observation network in order to be a coordination structure when size of the cliques is limited. Corollary 1 guarantees the existence of a coordination structure with a relatively low requirements in terms of amount of nodes and connections. Moreover, it guarantees the existence of some coordination structure when there are sufficient agents for the coordination: if $t \leq F_a(t)$, that is, if there are enough agents for coordination, a complete network always contains a clique of size $n - F_a(t) + t$, and we can guarantee that, at least the complete network, is a coordination structure.

4.2 Quasi-coordination structures

Up to now, we have analyzed how are the structures in which coordination problems do not occur for any payoffs and for any sequence of decision. However, in particular situations, it is possible also that the unique equilibrium is the one where everybody plays her risky action with other structures, for particular payoffs and/or sequences of decisions. In this subsection, we study this situation. We focus, as previously, in the simplest coordination problem in a heterogenous society: a society formed by n_α agents that are playing the coordination game among them. These agents prefer α over the safe action β if at least t agents are choosing α. There are also n_β agents who always choose action β.

The agents who face the coordination problem are homogenous with respect to their threshold, so that the utility of agents of type α can be reduced to the payoffs $u_{\alpha,\geq t} > u_\beta > u_{\alpha,<t}$, that represent the utility they get if choose α or β, depending on wether finally there are less than t people choosing α or not. Since we deal now with specific payoffs, we assume that agents are expected utility maximizers.

4.2.1 Quasi-coordination structures for certain payoffs

We say that a network Γ is a Quasi-coordination structure for certain payoffs if, for any possible sequence of decision, any agent chooses always her risky action over the equilibrium path, under certain restrictions over the payoffs. The following proposition shows that an isolated clique of size $\#q = t$ can be sufficient. An isolated clique $q \subset N$ is a set of individuals such that $\forall i \in q, ij \in \Gamma \iff j \in q$. It occurs when the benefit of the risky action is sufficiently high.

Proposition 2 *In any sequential equilibrium, any agent of type α chooses α over the equilibrium path, for any sequence of decision, if there exists an isolated clique $q \subset N$ of size $\#q = t$ when*

$$\frac{u_{\alpha,\geq t} - u_{\alpha,<t}}{u_\beta - u_{\alpha,<t}} > \frac{(n_\alpha + n_\beta - 1)!\,(n_\alpha - t - 1)!}{(n_\alpha + n_\beta - t - 1)!\,(n_\alpha - 1)!} = \frac{1}{\tilde{p}} \text{ and } n_\alpha > t$$

Proof. *See Appendix.* ■

This proposition reveals that, even when types are uncertain, a clique of size t may be sufficient for coordination. This is true if $u_{\alpha,\geq t}$ and $u_{\alpha,<t}$ are sufficiently large with respect to u_β. In this circumstances, an agent who is not connected to the clique assigns a probability sufficiently high to the event of existing t agents of type α in the clique such that they are going to coordinate among them (in the Appendix we prove that it is the case). In fact, $\tilde{p}$ is the probability of having t agents of size α in the clique conditional on having one agent of type α out of the clique.Therefore, any agent who is not connected to the clique best responds by choosing α, and everyone choose the risky action over the equilibrium path. In general, when

there are higher cliques, the probability of having sufficient agents of type α' in the clique increases, and there are a higher set of utilities under which coordination would emerge for any sequence of decision. This reasoning connects with Lemma 1: when there exists a clique of size $n - K(\alpha, t) + t$, it includes for sure a set of t agents with threshold t. Since it occurs with probability 1, every agent chooses the risky action for general coordination payoffs $u_{\alpha,\geq t} > u_\beta > u_{\alpha,<t}$.

Non-isolated: more connections may destroy coordination Interestingly, in this case we have that more connections could potentially generate the existence of coordination failures. Our general results of coordination structures show some kind of networks such that, if the structure exists, coordination emerges always, independently on wether there are more connections or not. It is not the case with quasi-coordination structures for certain payoffs. Note that Proposition 2 requires the existence of an *isolated* clique. When it exists, nobody knows what is occurring in the clique and agents believe that coordination occurs with sufficient probability, and they take the risky action. But if agents are observing what occurs in the clique, with positive probability we have in it agents who are not of type α, and agents observing actions in the clique would play simultaneously with the rest of agents of type α. In this case, under some sequences of decision, agents of type α would choose the safe action over some equilibrium path (specifically, when they play after agents in the clique, and agents in the clique are not of type α). In the Figure 8 we present two different networks. For the case in which $n_\alpha = 4$, $n_\beta = 3$, we have that a clique of size 3 includes 3 agents of type α with probability $p = \frac{4}{35}$. An agent of type α who does not belong to the clique believes that it is formed by agents of type α with probability[13] $\tilde{p} = \frac{1}{20}$. If utilities are given by $u_{\alpha,\geq t} = 21, u_\beta = 1, u_{\alpha,<t} = 0$, the agents of type α who are not connected to the clique in the network in the left choose the risky action in any equilibrium, and agents in the clique best respond to it by also choosing α, even if they observe that some agent in the clique does not choose α.

[13] It is the probability of the 3 agents in the clique being of type α conditional on one agent out of the clique being of type α. Note that in such circumstances, there are 3 agents of type α and 3 agents of type β that can be in the clique, and the probability of everyone being of type α is given by $\tilde{p} = \frac{3}{6} \cdot \frac{2}{5} \cdot \frac{1}{4} = \frac{1}{20}$

Figure 9

However, in the figure in the right all the agents are connected to the clique. If the sequence is such that the agents in the clique are the first to decide and they are of type β, all agents out of the clique know that they are playing simultaneously with the rest of agents of type α and the failure of coordination may occur in equilibrium, just by assuming that in such information set anyone of them chooses β. In this sense, more connections, more information, may be detrimental for coordination[14].

4.2.2 Quasi-coordination structures for certain sequences of decision

We have shown that smaller networks may be quasi-coordination structures for certain payoffs. Another interesting issue is if we can find quasi-coordination structures that guarantee the efficient equilibrium for certain sequences of decision but general payoffs. The answer is positive and we show now some examples, using the same framework that in the rest of this subsection. Suppose that there are n_α and $n_\beta = 1$ individuals, and that threshold is $t = 2$. The central agent in a segment of three nodes chooses the risky action in this context. Therefore, if in the order of decision the central individuals play the first and there are two segments of three individuals, the risky action is played by everyone. Figure 9 illustrates some examples for the cases in which $t = 2$ or $t = 3$ and $n_\beta = 1$ or $n_\beta = 2$. We have written a part of the sequence of decision with the ordinal position of the different nodes. This networks are quasi-coordination structures for any possible payoffs if the nodes with a assigned position play in that position, and for any order of decision of the nodes for those we have not written a position. If the sequence of decision is known, since at least t agents will choose α, any agent of type α in the society who is not included in those structures chooses it as well.

[14] It is not the case for coordination structures, but it may occur with the quasi-coordination structures for certain payoffs.

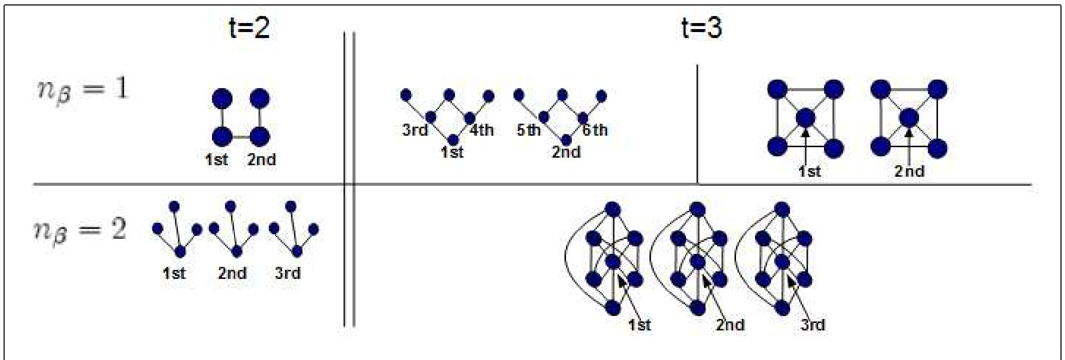


Figure 10

Quasi -coordination structures for certain sequences of decision can be specially relevant if we think in an extended version of the coordination game where we would endogeneize the sequence of decisions. Specifically, the conditions in which the extended model would have a unique efficient equilibrium can be explored. If it is the case, we could obtain the efficient coordination with less demanding requirements over the social structure.

5 Applications

In this section we relate our model to the minimal sufficient networks in Chwe (1999, 2000), the threshold models by Granovetter (1978) and apply it in a bank run model à la Diamond and Dybvig (1983).

5.1 Networks that allow the revolts are sufficient for them

The aim of this paper is very close to the one by Chwe (1999, 2000), as explained in the introduction. Our work characterizes the type of structures where coordination emerges among agents when the network enables observability of actions and type distribution is known, although type may be private information. Chwe characterized how must be the structure of a communication network in order to allow for the coordination. Both approaches stress the importance of cliques in the generation of this coordination.

In the model of Chwe there is a set of agents who have to decide whether to revolt (r, the risky action) or not (s, the safe action). Agents are of type willing (w) or unwilling (x). An agent of the willing type prefers the risky action to the safe one if all the other agents are choosing the risky action; an agent of the unwilling type, always prefers the safe action. Utilities of agents of willing type are assumed to be supermodular in the action of the others, i.e., the difference in utility for an agent of type w between r and s is increasing in the amount of other agents who choose r: for an agent of type w, the action of revolting is increasingly interesting when there are more people participating in the revolt.

The agents are embedded in a communication network. When two agents are connected through a directed link $i \to j$ (that we represent as ij), it means that i "talks" to j. This communication process allows in practice that the private type of i is revealed to j. The network structure is common knowledge, so that if two mutually connected agents talk to another one, say i and j are mutually connected and talk to k, this means that k knows the types of i and j, as well as that they talk between themselves and know their respective types. In this sense, the network generates "local common knowledge".

In this environment, Chwe studies the structure of the minimal sufficient networks for coordination. The main concern in his work is how is a communication network that enables coordination with independence of the believes of the agents. In order to study this, Chwe analyzes in which situation all the agents would take part in the revolt for the case in which every agent is of the willing type. He analyzes in which case each agent has incentives to revolt if she only has information about her neighbors. Chwe shows that the minimal networks that enable coordination can be described by a set of cliques in "sequence", such that there exists some leading cliques who decide to revolt by themselves and who are followed by other cliques. This type of structure allows that people recognize that there exist sufficient people willing to revolt, who at the same time know that there are sufficient people willing to revolt.

The concept of sufficient network of Chwe requires that revolting is a possibility *for any* distribution of the unknown types. In particular, in a minimal sufficient network, an agent knows the types of those neighbors that talk to him and this allows him to know that there exist enough people for coordination (given her preferences): a minimal sufficient network in the sense of Chwe guarantees the existence of one equilibrium where agents decide to revolt, i.e., take their risky action. Given their information, agents know that every agent has information on the existence of enough people willing to participate, and they best respond by choosing the risky action if anyone else chooses the risky action.

The aim in our paper is to characterize the set of structures in which, when we assume that the existence of that equilibrium is known, only the efficient outcome can be sustained in a (sequential) equilibrium. In our set up the revolting equilibrium (the Pareto efficient one) always exists: our assumption is that agents know that there are enough people for coordination, because every of them is homogenous or because they know the fixed amount of agents of each type, which allows the coordination. In this sense, the cases that we study are specifically the situations that allow coordination in the sense of Chwe. We add conditions over the observation network that generate that coordination becomes the unique equilibrium outcome.

How related are then the sufficient networks by Chwe and the coordination structures of the present work? Applying our concepts of sequential decision, we show know that if the communication network which is sufficient in the sense of Chwe exists and everyone is of the willing type, given that information of types is restricted to those transmitted by the network, all agents choose the risky action over any sequential equilibrium, with independence of the order in which they are called to decide. This result holds when observation of actions occurs in the same direction that communication transmission:

Proposition 3 *Suppose a set of agents N embedded in a sufficient network Γ such that everyone is willing to revolt in the sense of Chwe (2000). Suppose that the nature selects an order of play according to $P(\theta(N))$ and that if $ij \in \Gamma$, a_i is in the information set of j if $i < j$. Then every agent chooses to revolt in the path of any weak perfect Bayesian equilibrium.*

Proof. *See Appendix.* ■

In a sufficient network, there exists "leading cliques" where in some equilibrium any agent in the clique best responds to the action of revolting by also revolting. There are other cliques whose members "are talked" by the participants in the leading clique. Those "followers" best respond to the action of members in their clique and in the leading clique also revolting.

The result by Chwe states that an equilibrium exists where everyone revolts (if everybody is willing to). The communication structure reveals private types to agents in the leading clique. Therefore, any of the members of the clique knows that there are sufficient people for coordination in such a clique. For any order, we know that the last one who decides in the clique would best respond to anyone else choosing to revolt by also revolting. And we can do the backward induction argument used for the sufficiency part of the result of coordination structures. For any order, in equilibrium, agents in cliques which follow the leading clique have to best respond to strategies of agents in the leading clique which imply that they revolt. And therefore, it can be shown that every agent will revolts in a sufficient network.

Our results reinforce those by Chwe, showing that coordination must emerge when cliques exist, with independence of the relative payoffs. Revolts are not only a possibility, they become the unique possibility.

5.2 Riots are not necessarily initiated by the most "radical" people

Granovetter (1978) modeled the conditions required for the emergence of collective behavior starting from an individual distribution of preferences. In his model, agents decide to join a riot depending on how many agents are taking part on it. Given a distribution of those thresholds, Granovetter studied how many agents actually take part in the revolt. Interestingly, two very similar societies could generate completely different outcomes.

Agents decide in a dynamic context. Initially, the agents who have threshold 0 go to the streets. The threshold represents the amount of people that an agent requires in order to join the riot. People with threshold 0 (or very low) are the initial "instigators". They want to participate in the revolts independently of others, and when they are in the streets, other people whose willingness is not so high, also decide to participate. This generates a cascade effect that determines how many people is finally in the streets. Let $h(t)$ be the amount of people with threshold t and $H(t) = \sum_{k=0}^{t} h(k)$. In the model by Granovetter, the amount of people that joins the riots is given by $n^* = \max\{H(t) : k \leq H(k), \forall k \leq t\}$. The reason is that people with a certain threshold enters to the riots when they observe that at least as many people as their threshold is taking part on it.

This process of collective behavior also shows that two very similar societies in their microstructure (preference distribution, as stated by $h(t)$) can generate very different aggregated behavior. Granovetter illustrates this fact showing two similar societies formed by 100 citizens[15]. In one of them each agent i has threshold i for $i \in \{1, 2, ..., 100\}$. The other society is exactly the same but agent i has threshold 2. In the first society, everyone takes part in the riots: the agent 1 starts, then the agent 2 (who observes 1 people revolting), then the agent 3 (who observes 2 people already revolting), and so on. In the second society, nobody revolts, since there exists no "initial demonstrator". Granovetter emphasizes two points: two societies which are basically identical can generate outcomes hugely different; and the key role that initial demonstrators play in generating social movements.

Based on Granovetter's assumptions, we study the effect of strategic behavior when the agents have information about the aggregate distribution of types. Connecting the case studied by Granovetter with our model, we assume that agents are called once to decide by the nature in some exogenously order and that the distribution of type $h(t)$ is common knowledge. Since in the model of Granovetter agents respond to the actions of their predecessors, we assume that all actions are known. In our language, we would say that a complete observation network Γ is connecting the agents. Under this circumstances, we ask how many agents take part in the revolts, and if there is some difference depending on wether initial agents called to decide are those with low or high thresholds. The following Proposition answer these questions:

Proposition 4 *For any sequence of decision, each agent i with a threshold $t_i \leq \max\{t : H(t) \geq t\}$ chooses the risky action over the path of any sequential equilibrium.*

[15] The original threshold model by Granovetter stablish decisions with respect to the proportion of the citizens that take part. We use a version were thresholds depend on the amount of agent that participate. Both models are equivalent.

Proof. *This Proposition follows from Lemma 1, for a unique risky action when there exists a complete observation network.* ■

In the line with our results over coordination structures, the revolt emerges for any order in which agents are called to decide. When threshold distribution is known, all the agents take the risky action over the equilibrium path, independently of the order in which they are called to decide: in this sense, collective action is not necessarily initiated by those agents with the lowest threshold, while it is the case when distribution is unknown. Therefore, we can infer from the model that unexpected collective actions, that is, those that occur when people did not know about the existence of sufficient willing people, must be initiated by a set of initial "instigators" (as named by Granovetter), who start the process and that generate that people realize that there exist sufficient people willing to take part. But when it is known the existence of sufficient people willing to take part, i.e. it is known the existence of a certain distribution although people do not know which threshold each particular individual has, we show that social movements are initiated by any of the willing people, independently of her threshold.

Another important difference between ignoring or knowing the distribution is the amount of people that take part on the revolts. When distribution is unknown, people go to the streets only if there exists a certain amount of previous instigators, and the amount of total people who participate is given by the first crossing point between the type distribution and the 45^o line, when we plot the distribution $H(t)$ against thresholds, as shown by Granovetter. This is the case because before that crossing point, people observes that an amount of people higher than their threshold is participating in the revolt. But the people with higher thresholds do not observe that and stay at home. When distribution is known, all the people who knows that in the society there are sufficient other people willing to take part in the revolts, would in fact take part (and even if they are the first to decide and do not observe anything). In the graph, the amount of people would be given by the last point where distribution $H(t)$ where over the 45^o line. Figure 10 illustrates this with an example:

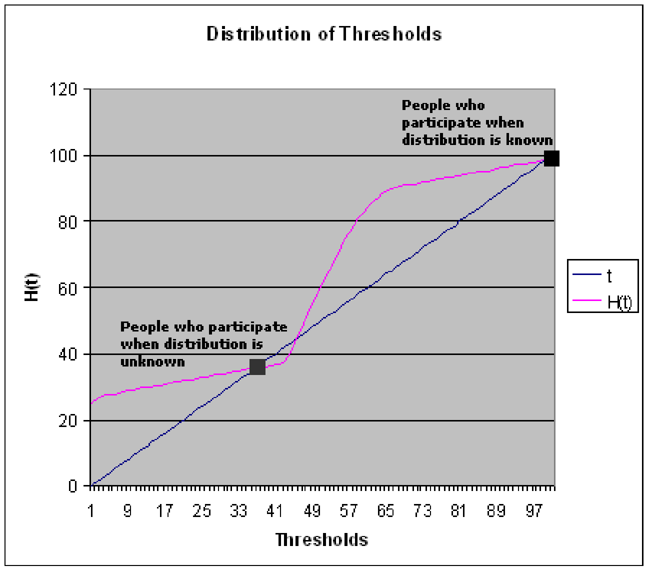


Figure 11. Example of distribution of thresholds and amount of people who participate in the revolts when distribution is known or unknown.

5.3 Local banks may be immune to runs

In this section we adapt our general setup to the canonical model of depositor decision by Diamond and Dybvig (1983). We find how must be connected a given society in order to exclude bank runs as the result of coordination failures. The required clique is decreasing in the profitability of the long run investment. This possibility of observing other depositors actions is not likely in large banks but it is possible in local banks focused in a given community. In such situation, it is likely that decisions on wether withdraw the money or not from the bank are observed in the local community. Therefore, since the community may act as an observation network that guarantees coordination, the result suggests that small local banks may be immune to bank runs as coordination failures[16].

5.3.1 The environment

Let $N = \{1, 2, ..., n\}$ denote the set of depositors. There are three time periods denoted by $t = 0, 1, 2$ and depositor i's consumption in period t is denoted by $c_{t,i} \in \mathbb{R}_+$. Depositors may be patient and impatient.

[16] Obviously, bank runs may occur as the result of problems with the fundamentals of the bank.

Impatient depositors only care about consumption at $t = 1$ whereas patient depositors value consumptions at $t = 1, 2$. Depositor i's utility function is given by

$$u_i(c_{1,i}, c_{2,i}, \lambda_i) = u_i(c_{1,i} + \lambda_i c_{2,i}). \tag{9}$$

If $\lambda_i = 1$ (0), depositor i is (im)patient. The utility is strictly increasing, strictly concave, twice continuously differentiable and to satisfy the Inada conditions. The relative risk-aversion coefficient $-c_i u_i''(c_i)/u_i'(c_i) > 1$, for every $c_i \in \mathbb{R}_+$, and all $i \in N$.

The number of patient depositors is assumed to be constant and given by $p \in [1, n]$. The remaining depositors are impatient. The number of patient and impatient depositors is common knowledge.

At $t = 0$, each depositor $i \in N$ has one unit of a homogeneous good which she deposits in the bank. The bank has access to a constant-return-to-scale productive technology which pays a gross return of one unit for each endowment liquidated at $t = 1$, and a fixed return of $R > 1$ for each endowment liquidated at $t = 2$.

The bank acts in the interest of the depositors and tries to maximize their expected utility. If the bank could observe each depositor's consumption preferences, then she would be able maximize the sum of depositors' utilities with respect to $c_{1,i}$ and $c_{2,i}$ subject to a resource constraint and p. The optimization problem is the following:

$$\begin{aligned} &\max_{c_{1,i},c_{2,i}} \ (n-p)u_i(c_{1,i}) + pu(c_{2,i}) \ \text{ s. t.} \\ &(n-p)c_{1,i} + [pc_{2,i}/R] = n. \end{aligned} \tag{10}$$

The solution to this problem is

$$u_i'(c_{1,i}^*) = Ru_i'(c_{2,i}^*), \tag{11}$$

which - as in Diamond and Dybvig (1983) - implies that $R > c_{2,i}^* > c_{1,i}^* > 1$. This is the unconstrained efficient allocation. The bank insures against the privately observed liquidity risk, which is only realized at the beginning of $t = 1$, by offering a simple demand-deposit contract that implements the unconstrained efficient allocation. The simple demand-deposit contract offers to pay $c_{1,i}^*$ to any depositor i who withdraws at $t = 1$ as long as the bank has funds. Any patient depositor i who waits until $t = 2$ receives a pro rata share of the funds available then. Let $\eta \in [0, p]$ be the number of depositors who wait at $t = 1$. Given η, depositor i's consumption at $t = 2$, if she waits is

$$c_{2,i}(\eta) = \begin{cases} \max\{0, \frac{R(N-(N-\eta)c_{1,i}^*)}{\eta}\} \text{ if } \eta > 0 \\ 0 \text{ if } \eta = 0 \end{cases} \tag{12}$$

If $\eta = p$, that is, only impatient depositors withdraw at $t = 1$, then $c_{2,i}(\eta) = c^*_{2,i}$ and patient depositors enjoy a higher consumption than impatient ones. However, if η is too low, then to withdraw at $t = 1$ is better also for patient depositors since to wait until $t = 2$ yields them strictly less than $c^*_{1,i}$: if the number of patient depositors who keep the money in the bank is below $\bar{\eta}$, a threshold value for η, then their period-2 consumption is strictly below $c^*_{1,i}$.

Lemma 2 *There exists* $1 \leq \bar{\eta} \leq p$ *such that for all* $i \in N$,

$$c_{2,i}(\bar{\eta} - 1) < c^*_{1,i}, \text{ for any } \eta \leq \bar{\eta} - 1, \text{ and}$$
$$c^*_{1,i} \leq c_2(\bar{\eta}), \text{ for any } \eta \geq \bar{\eta}. \tag{13}$$

Based on the equality of $c^*_1 = c_2(\bar{\eta})$ we obtain that

$$\bar{\eta} = \frac{Rn(c^*_1 - 1)}{c^*_1(R - 1)}.$$

The value of $\bar{\eta}$ is the threshold for patient depositors. Notice that this threshold is the same for all of them. Hence, in this application there are two types: $\tau(0, 1)$ and $\tau(1, \bar{\eta})$. That is, impatient depositors always choose to withdraw, independently of what other depositors do, whereas patient depositors prefer to wait if at least $\bar{\eta}$ other depositors wait.

5.3.2 Information and decisions

At the beginning of period 1 liquidity types (patient or impatient) are realized privately. Let $\Lambda^N = \{0, 1\}^N$ and $\lambda^N = (\lambda_1, ..., \lambda_N)$ denote the type vector of the depositors that satisfies $\sum_{i=1}^{N} \lambda_i = p$. After the realization of types, depositors contact the bank sequentially at $t = 1$ according to the order of decision given by $\theta(N)$.[17] The depositors are embedded in an *observation network* Γ. Depositors choose if they want to withdraw (action 0) or to keep their money deposited (action 1) and they observe the choices of their neighbors who precede them.[18] Depositor i's information set is defined as

$$\psi_i = \{\tau_i, \theta(N), \{a_j : ij \in \Gamma; \theta_j < \theta_i; \theta_j, \theta_i \in \theta(N)\}\}. \tag{14}$$

[17] We define here $\theta(N)$ only to conform to the original setup. Knowing the exact order of decision is not necessary to obtain the result.

[18] We use "to keep the money deposited" and "to wait" as synonyms.

Note that depositor i observes previous actions enabled by the network, but she does not observe types. A strategy is a mapping from all possible information set into actions. We allow for mixed strategies, so

$$s_i : \Psi_i \to \triangle \{0, 1\} .$$

The bank pays immediately to those who choose to withdraw. Consumption in period 2 is determined by equation 12.

The bank does not know the network and consequently cannot condition the payment to depositors on it. The bank has to respect the sequential service constraint, so the bank cannot make depositors wait and condition payment on information which is not available at the time the depositor wants to withdraw.

We find that bank runs as coordination failure do not occur in equilibrium if the depositors are sufficiently connected.

Proposition 5 *In the finite-depositor version of the Diamond-Dybvig model, if there is a clique of size* $(n-p)+\frac{Rn(c_1^*-1)}{c_1^*(R-1)}$*, then there is no bank run in any sequential equilibrium.*

Proof. *The result follows from Theorem 1, given that* $\frac{Rn(c_1^*-1)}{c_1^*(R-1)}$ *is the threshold such that a patient agent prefers the risky action of not to withdraw over the safe one.* ■

The efficient payoffs that generate the multiplicity of equilibria allowing for a bank run, generate the coordination in the no run equilibrium if the depositors are sufficiently connected. This result shows that small, local banks, whose depositors are able to observe mutually, should not present bank runs as the result of coordination failures. Deposit insurance has been proposed as one of the most effective mechanisms to avoid this undesirable bank runs. Our result shows that small, local bank, may be immune to them just because of the social configuration of their depositors.

6 Conclusion

We introduce in our paper the notion of *observation network* to model the type of social structure that allows agents to observe their mutual actions. We characterize the structure of such networks that generate that agents coordinate in the efficient equilibrium in games related with generalization of the stag-hunt game. We provide necessary and sufficient conditions on the size of the clique such that, if it exists, the efficient coordination emerges as the unique outcome, and the social network is considered a *coordination structure*. We apply our model to revolutions and bank runs, but it also applies to any other situation where coordination failures may emerge, as problems of product adoption. We find that the existence of cliques play a crucial role in guaranteeing the coordination of the agents, and so our analysis naturally complements the one by Chwe (1999, 2000).

We study situations in which it is known that coordination is possible, so the problems that we study are pure problems of coordination. With this aim, we focus on the case in which the amount of agents of each type is known. All agents know that efficient coordination is a possible outcome, and the networks that we characterize avoid the problem of coordination failure. A different approach would occur if we assume that types are randomly drawn from some distribution. In this situation, agents would not know with certainty if there are sufficient agents for getting coordination. The role of an observation structure in such circumstances is out of the scope of this paper, but is an interesting line of future research.

The unique equilibrium prediction when the observation network meets our conditions is an issue that can be tested. We have carried out some related experiments for the bank run environment (Kiss, Rodríguez-Lara and Rosa-García, 2009) that provide mixed evidence on the issue: although some structures generate more coordination, agents are affected also by the particular observations. However, more evidence is required in order to be sure on the effects of observation networks in the behavior.

The model invites us to explore many issues that we have not discussed in this article. Although our agents decide sequentially, they decide just once, so dynamic considerations are not studied and they play relevant roles in many coordination issues, as those that we study (bank runs and revolts). In this context, it is also likely that social learning plays a role in the emergence of the efficient equilibrium. Other interesting issue is to study the incentives to formation. Would agents choose a social structure that may lead to a coordination failure? Under which conditions?

Moreover, some of our assumptions are restricting many real life situations. For instance, some of the actions may be observed if they are chosen. In the case of bank runs, typically the action of withdrawing is observed but the action of waiting is not. In the case of revolts, the action that is observed is typically the one of taking part, but not staying at home. Note that in the first case the observed action is the safe one and in the second is the risky one. Our preliminary approach to these issues shows that our results maybe robust when the safe action is hidden but not when the risky action is. Other issue is what occurs with other coordination problems, as for instance for generalizations of the battle of sexes. Or also the relevance of free-riding: how would change the results if once the sufficient amount of people chooses the risky actions, the rest of agents do not have incentives to take it.

These related issues maybe of special interests. We require in our analysis that agents have access to a large amount of information, and to be able to observe the precise order in which actions are chosen. New internet social networks, such as Facebook and Twitter are a way of communicating and they allow to show own actions to other agents, so that each time more individuals are informed about the actions carried out by other individuals. This can generate several effects, including those that our results postulate, that coordination on efficient situations may be facilitated. We have done a first approach to this kind of topics (Kiss and Rosa-García, 2011) analyzing the how coordination is facilitated when agents get information through internet social media in contrast with traditional social media. We find that social media are a better way of coordination and may facilitate revolutions, as it has been argued in the recent Arab revolts. Up to which point this is really a relevant effect and if its relevance can be extended to the related extensions of the analysis of coordination structures must be still studied.

7 References

References

[1] Aumann, R. and R. Myerson (1988): "Endogenous Formation of Links Between Players and Coalitions: An Application of the Shapley Value," In: Roth, A. (ed.) The Shapley Value, Cambridge University Press, 175–191.

[2] Banerjee, A. (1992): "A Simple Model of Herd Behavior", *Quarterly Journal Economics* 107, 797-818

[3] Bikhchandani, S., D. Hirshleifer, and I. Welch (1992): "A Theory of Fads, Fashion, Custom, and Cultural Change as Informational Cascades", *Journal of Political Economy* 100, 992-1026

[4] Bramoullé, Y. and R. Kranton (2007): "Public goods in networks," *Journal of Economic Theory* 135, 478-494.

[5] Choi, S., Gale, D., Kariv, S., Palfrey, P., 2011: "Network architecture, salience and coordination", *Games and Economic Behavior*, forthcoming. doi:10.1016/j.geb.2011.01.001

[6] Chwe, M. (1999): "Structure and Strategy in Collective Action", *American Journal of Sociology*, 105, 128-156.

[7] Chwe, M. (2000): "Communication and Coordination in Social Networks ", *The Review of Economic Studies*, 67, 1-16

[8] Cooper, R., and A. John, (1988): "Coordinating Coordination Failures in Keynesian Models," *Quarterly Journal of Economics* 103 (Aug. 1988), 441-464.

[9] Costain, J. S. (2007): "A herding perspective on global games and multiplicity", *The B. E. Journal of Theoretical Economics*, Vol. 7 : Iss. 1 (Contributions), Article 22

[10] Diamond, D.W., P.H. Dybvig (1983): "Bank runs, deposit insurance and liquidity", *Journal of Political Economy* 91, 401-419

[11] Ellison, G., (1993): "Learning, local interaction, and coordination", *Econometrica* 61 (5), 1047–1071.

[12] Farrell, J. and P. Klemperer (2007): "Coordination and lock-In: Competition with switching costs and network effects." In *Handbook of Industrial Organization*, Volume 3. Elsevier, 1967–2072.

[13] Galeotti, A., S. Goyal, M. Jackson, F. Vega-Redondo, L. Yariv (2010): "Network games", *Review of Economic Studies*, 77, 218-244

[14] Goyal, S. and F. Vega-Redondo (2005):."Network formation and social coordination", *Games and Economic Behavior*, 50, 178–207.

[15] Granovetter, M. (1978): "Threshold Models of Collective Behavior", *The American Journal of Sociology* 83, 1420-1443

[16] Jackson, M. and A. Watts (2002): "On the formation of interaction networks in social coordination games", *Games and Economic Behavior* 41, 265–291.

[17] Kiss, H. J., J. Rodríguez-Lara and A. Rosa-García (2009): "Do Social Networks Prevent Bank Runs?", IVIE WP-AD 2009-25

[18] Kiss, H. J. and A. Rosa-García (2011): "Why do Facebook and Twitter facilitate revolutions more than TV and radio?", *mimeo*

8 Appendix

Proposition 1: In the homogenous case with threshold t, an observation network Γ is a coordination structure if and only if there exists a subset of agents $\{s \in N : \#s = t\}$ that forms a clique.

Proof. Sufficiency: an observation network Γ is a coordination structure if there exists a subset of agents $\{s \in N : \#s = t\}$ that forms a clique.

We prove this by backward induction. Let $\{\sigma^* (\varphi), \Pi_i (H_i|\varphi)\}$ be an assessment that defines a sequential equilibrium. Let be $\{q \subset N : \#q = t\}$ a set of t agents completely connected under Γ, i.e. a clique of size t.

From now on, let name as agent $i, i = 1, 2, .., t$ the agent who is in the $i-th$ position in the clique according to $\theta(n)$. In any sequential equilibrium assessment $\{\sigma^*(\varphi), \Pi_i(H_i|\varphi)\}$, the agent $i = t$ chooses α in any information set φ_t in which she observes $t-1$ risky actions by her predecessors in the clique. Agent in position $i = t-1$ best responds by choosing α in any information set φ_{t-1} where she observes $t-2$ risky actions by her predecessors in the clique. Applying this recursively, agent i in the clique best responds to the equilibrium strategy by choosing α in those information sets φ_i where she observes $i-1$ risky actions chosen by her predecessor in the clique. Then the agent $i = 1$ chooses the risky action although she does not observe anything and in any sequential equilibrium: the t agents in the clique choose the risky action over the equilibrium path. Any agent who does not belong to the clique best responds by also choosing α. Therefore, if there exists a clique of size t, every agent chooses her risky action over the equilibrium path.[19] q.e.d.

Necessity: an observation network Γ is a coordination structure only if there exists a subset of agents $\{s \in N : \#s = t\}$ that forms a clique.

For proving this part of the result, we propose a strategy and a consistent belief that define a sequential equilibrium where the strategy profile implies that all the agents choose the safe action over the equilibrium path.

First let define $Q_{\Gamma,k} \subset P(N)$ where $P(N)$ is the power set of N, as the set of cliques of size k in Γ:

$$q \in Q_{\Gamma,k} \rightarrow \begin{cases} q \subset N \\ \#q = k \\ \forall i, j \in q \rightarrow ij \in \Gamma \end{cases}$$

For the information set φ_i of the agent i, we define $Q^-_{\Gamma,k,\varphi_i} \subset Q_{\Gamma,k}$ as the set of cliques of size k in Γ such that the agent i does not observe the action of any agent belonging to the clique,

$$q \in Q^-_{\Gamma,k,\varphi_i} \rightarrow \begin{cases} q \in Q_{\Gamma,k} \\ \forall j \in q \rightarrow j \in \{N \backslash N_i \cup \{j' \in N_i : \theta_j \geq \theta_i|\varphi_i\}\} \end{cases}$$

We define $N_{i,<\theta_i,\varphi_i} \subset N_i$ as the set of neighbors of i with an order of decision previous to i given the information set φ_i,

$$j \in N_{i,<\theta_i,\varphi_i} \rightarrow j \in N_i \cap \{j : \theta_j < \theta_i|\varphi_i\}$$

For the information set φ_i and the clique $q \in Q^-_{\Gamma,k,\varphi_i}$, we define as $\#\alpha_{q,\varphi_i}$ the amount of actions of type α

[19] The strategy in equilibrium implies to choose the risky action over the equilibrium path for arbitrary beliefs. This means that, in fact, we have proved also that there is a unique equilibrium path in any Weak Perfect Bayesian Equilibrium, which is a much softer equilibrium concept.

that belong to φ_i observed by all the agents in q plus the cardinal of q:

$$\#\alpha_{q,\varphi_i}(q,\varphi_i) = \#q + \#\left\{a_j = \alpha : \forall k \in q, j \in N_{i,<\theta_i,\varphi_i} \to jk \in \Gamma\right\}$$

For the agent i, we define as $\bar{\varphi}_i^r$ the set of information sets in which the maximal $\#\alpha_{q,\varphi_i}$ is r:

$$\varphi_i \in \bar{\varphi}_i^r \to \max\left\{\#\alpha_{q,\varphi_i}\right\} = r$$

That is, for $\varphi_i \in \bar{\varphi}_i^r$ we have that r is the maximum of the sum of elements that, forming a clique, observe a certain amount of risky actions that are observed by i.

We prove now that $\sigma^*(\varphi), \Pi^*(H|\varphi)$ where

$$\begin{aligned}
\sigma_i^*(\varphi_i) &= \begin{cases} \beta \text{ if } \varphi_i \in \bar{\varphi}_i^r : r < t \\ \bar{\sigma}_i(\varphi_i) \text{ if } \varphi_i \in \bar{\varphi}_i^{k,m} : r \geq t, \text{ for some optimal } \bar{\sigma}_i \end{cases} \\
\Pi_i(h_i|\varphi_i) &= 0 \ \forall h_i : \#\{a_j \in h_i : a_j = \alpha, j \notin N_i\} = 0, \varphi_i \in \bar{\varphi}_i^r : r < t
\end{aligned}$$

defines a sequential equilibrium for an appropriate $\bar{\sigma}_i(\varphi_i), \Pi_i(h_i|\varphi_i)$ in which the safe action is played over the equilibrium path when there does not exist in Γ a clique of size t.

We show, *first*, that this strategy defines a path where everybody takes the safe action. If there does not exist a clique of size t, any agent who does not observes any risky action is in an information set $\bar{\varphi}_i^r < t$. This is the case because in such case we have that $\#\left\{a_j = \alpha : \forall k \in q, j \in N_{i,<\theta_i,\varphi_i} \to jk \in \Gamma\right\} = 0$ and $\#\alpha_{q,\varphi_i} = \#q$, which is smaller than t, for any clique $q \subset \Gamma$. Therefore any agent who does not observe any action α plays β according to $\sigma^*(\varphi), \Pi^*(H|\varphi)$, and therefore β is played by everyone along the equilibrium path.

Second, note that $\varphi_i \in \bar{\varphi}_i^r : r < t$ are information sets where the agent observes $t-2$ or less actions of type α. This is the case because the agent herself is a clique of size 1 who observes all the actions she observes, and therefore these information sets occur when a maximum of $r-1 < t-1$ actions of type α are observed. Choosing β in those information sets is an optimal decision if it is expected that nobody else is going to play α and nobody else has played it (this last statement occurs with probability 1 according to Π^*).

Third, we show that the strategy is a best response to $\sigma_{-i}^*(\varphi)$ and $\Pi^*(H|\varphi)$. If the agent i plays the safe action her expected payoff is

$$E\left(u_i\left(\beta, \sigma_{-i}^*(\varphi), \Pi_i\right) | \varphi_i \in \bar{\varphi}_i^r : r < t\right) = u_i(a_i = \beta)$$

We focus in the information sets $\varphi_i \in \bar{\varphi}_i^r : r < t$. In those information sets, the beliefs assign probability 0 to other actions of type α being chosen by non observed predecessors of i. Take now the agent j who plays after i. We show that she will play β according to $\sigma^*(\varphi), \Pi^*(H|\varphi)$. Suppose that $r \leq t-2$. Since i is in an information set $\varphi_i \in \bar{\varphi}_i^r : r < t$ and she believes that there exist no predecessor who has played α and is not observed by i, any successor j is, at maximum, in an information set $\varphi_j \in \bar{\varphi}_j^r, r = t-1$, and plays β. Suppose that $r = t-1$. Here we can have 2 different cases, depending on wether i belongs to the clique, $i \in q$, such that any agent in the clique observes the same $r - \#q$ actions or i may not belong to it, $i \notin q$. Suppose that $i \notin q$. In such a case, the first agent who decides j in that clique will be in an information set $\varphi_j \in \bar{\varphi}_j^r : r < t$, since not all the agents in the clique observe the action of i (since $i \notin q$). Finally, suppose that $i \in q$. But then, since $\varphi_i \in \bar{\varphi}_i^r : r = t-1$ and $i \in q$, if i chooses α, the rest of agents in q will be still in an information set $\varphi_j \in \bar{\varphi}_j^r : r = t-1$. This is the case because they observe one risky action more than i but the clique has one individual less. Therefore they would play β according to $\sigma^*(\varphi), \Pi^*(H|\varphi)$. This means that given the beliefs and the strategies, if the agent plays α nobody else is expected to play α and the expected payoff is

$$\begin{aligned} E\left(u_i\left(\alpha, \sigma^*_{-i}(\varphi), \Pi_i\right) | \varphi_i \in \bar{\varphi}_i^r : r < t\right) &= u_i\left(a_i = \alpha, \sum_{j \in N} I_{a_j = \alpha} < t\right) < \\ &< u_i\left(a_i = \beta\right) \end{aligned}$$

And therefore playing β is a best response to $\bar{\sigma}_{-i}, \Pi_i(h_i|\varphi_i)$ in the information sets $\varphi_i \in \bar{\varphi}_i^r : r < t$.

Fourth, the belief is dynamically consistent in the information sets $\varphi_i \in \bar{\varphi}_i^r : r < t$. Suppose that any agent j in an information set $\varphi_j \in \bar{\varphi}_j^r : r < t$ plays the completely mixed strategy $(\varepsilon, 1-\varepsilon)$, representing the probability of playing (α, β). Note that this strategy converges to $\sigma_j^*\left(\bar{\varphi}_j^r\right)$ when $\varepsilon \to 0$. Since the first agent who decides is in an information set $\varphi_j \in \bar{\varphi}_j^r : r < t$, and that all agents play β the subsequent agents are also in such an information set, the belief that assigns probability 0 to any non observed agent having chosen β is consistent with those strategies.

Up to now, we have proved that the strategy is optimal over all the information sets in which $\varphi_i \in \bar{\varphi}_i^{k,m}$: $k < t-m$. In the other information sets, the agents would choose any best response; however, optimality of the equilibrium strategy over the equilibrium path is independent of what occurs on those information sets. Note that the existence in those information sets of the equilibrium is guaranteed by standard arguments. Since we allow for mixed strategies, the equilibrium in those information sets could be the same that in a reduced version of the game where the nature chooses directly α instead of the agents in the information sets $\varphi_i \in \bar{\varphi}_i^r : r < t$.

Thus a coordination failure may be sustained, *q.e.d.* ■

One may ask yourself why an strategy of this type would not be sustained if a clique of size t exists. Imagine, for instance, that it is argued that the action β is played in any information set $\varphi_i \in \bar{\varphi}_i^{k,m} : k \leq t-m$, which implies to choose the safe action in a clique of size t, when $m = 0$ actions of type α are observed. But note that this strategy is not optimal for $m = t-1$. In such a case, it would imply that β is played if a clique of size $k = 1$ exists, and it is trivially formed by i. But then i would choose the action β after observing $t-1$ actions of type α. Therefore, it can not be an equilibrium strategy.

Lemma 1: An agent of type $\{\alpha, t'\}, t' \leq t$ takes the risky action on any equilibrium path if there exists a clique of size $n - F(\alpha, t) + t$

Proof. If there exists a clique q of size $\#q = n - F(\alpha, t) + t$ it includes at least t agents of type $(\alpha, t'), t' \leq t$. Let $\{\sigma^*(\varphi), \Pi_i(H_i|\varphi)\}$ be an assessment that defines a sequential equilibrium. We focus on an agent $i \in q : \tau_i = (\alpha, t'), t' \leq t$ and in the information sets over the equilibrium path We describe the equilibrium assessment for this agent. In order to abbreviate the notation, we group in $\varphi_{k,r}$ the information sets of i when the information set is over the equilibrium path (1), she has observed k actions of neighbors who belong to the clique (2), and r of those actions are actions α (the action preferred by i) (3)

$$\varphi_i \in \varphi_{k,r} \subset \Psi_i \rightarrow \left\{ \begin{pmatrix} P(\varphi_i|\sigma^*) > 0 \ (1) \\ \#\{j \in q : \theta_j < \theta_i\} = k|\varphi_i \ (2) \\ \#\{j \in q : \theta_j < \theta_i, a_j = \alpha\} = r|\varphi_i \ (3) \end{pmatrix} \right.$$

First, if the agent is in an information set $\varphi_i \in \varphi_{k,t-1}$, she best responds by choosing α. It is the case since her threshold is $t' \leq t$. Therefore $\sigma_i^*\left(\varphi_i \in \varphi_{k,t-1}\right) = \alpha$.

Note now that $\sigma_i^*\left(\varphi_i \in \varphi_{t-2,t-2}\right) = \alpha$. In such an information set, agent i has observed $t-2$ actions (and everyone is of type α), and therefore there is with probability 1 at least one other subject in the clique who has not still decided of type $(\alpha, t'), t' \leq t$. This is the case because i is the agent $t-1$ and therefore she has still $N - F(\alpha, t) + 1$ succesors in the clique. Given $\sigma_i^*\left(\varphi_i \in \varphi_{k,t-1}\right)$ and any consistent belief, i best responds by choosing α.

But then, since the beliefs must be consistent with the strategy, in an information set where an agent observes that someone does not choose α after the $t-2$ first agents in the clique choosing α, must assign probability 0 to that agent being of type $\tau_i = (\alpha, t'), t' \leq t$ (given that the information set is reached with positive probability, the action observed in such information set is chosen by an agent of a different type with positive probability). For an agent i in an information set $\varphi_i \in \varphi_{t-1,t-2}$ in which the last agent is the one who has not chosen α, consistent beliefs imply that there exist still one agent of type $\tau_i = (\alpha, t'), t' \leq t$ in the clique who has not decided. Therefore, in such a case, the agent best responds by choosing α (except if she is the last one).

By iterating this reasoning, consistent beliefs in an information set of type $\varphi_i \in \varphi_{t-1,t-3}$ where the two last agents has not chosen the action α imply to assign probability 1 to having one other agent in the clique of type $\tau_i = (\alpha, t'), t' \leq t$ who has not decided yet. Therefore, equilibrium strategy implies that an agent of type $\tau_i = (\alpha, t'), t' \leq t$ also chooses the risky action in an information set $\varphi_i \in \varphi_{t-1,t-3}$. We can repeat the argument to show that in any information set $\varphi_i \in \varphi_{t-1,r}$ where the $t-1$ actions α occurred at the beginning, agents of type $\tau_i = (\alpha, t'), t' \leq t$ choose the risky action. This argument can also be applied for the information sets $\varphi_i \in \varphi_{t-3,t-3}$ and so on. Finally, this proves that in any sequential equilibrium, if there exists a clique of size $n - F(\alpha, t) + t$, agents of type $(\alpha, t'), t' \leq t$ choose the risky action over the equilibrium path. q.e.d. ■

Theorem 1: An agent of type $\{\alpha, t\}$ takes the risky action on any equilibrium path if there exists a clique of size $q(t)$ where

$$\begin{aligned} q(t) &= q_\alpha(r_\alpha(t)), \\ \text{where } r_\alpha(t) &= \min\{N - F_\alpha(x) + x : \forall x \geq t\} \\ q_\alpha(x) &= \begin{cases} r_\alpha(x) & \text{if } r_\alpha(y) < F_\alpha(x) \\ r_\alpha(y) & \text{if } r_\alpha(y) \geq F_\alpha(x) \end{cases}, y = \max\{\bar{t} : \bar{t} < x\} \end{aligned}$$

, where $q_\alpha(x)$ is constructed iteratively.

Proof. Take an agent of type t. Lemma 1 implies that any agent with threshold t chooses also the risky action r if there exists a clique of size $r_\alpha(t) = \min\{N - F_\alpha(x) + x : \forall x \geq t\}$ (note that it is the smallest size of clique required for thresholds higher than t). Note that if the agents with the threshold immediately lower to t (that we note by t_{-1}) are $F(t_{-1}) \geq t$, any agent with threshold t takes the risky action if there exists a clique sufficiently as large as required by agents of threshold t_{-1}. This possibility is incorporated by $q_\alpha(x)$. ■

Proposition 2: In any sequential equilibrium, any agent of type α chooses α over the equilibrium path, for any sequence of decision, if there exists an isolated clique of size $q \subset N, \#q = t$ if

$$\frac{u_{\alpha,\geq t} - u_{\alpha,<t}}{u_\beta - u_{\alpha,<t}} > \frac{(n_\alpha + n_\beta - 1)!\,(n_\alpha - t - 1)!}{(n_\alpha + n_\beta - t - 1)!\,(n_\alpha - 1)!}$$

Proof. Suppose that

$$\frac{u_{\alpha,\geq t} - u_{\alpha,<t}}{u_\beta - u_{\alpha,<t}} > \frac{(n_\alpha + n_\beta - 1)!\,(n_\alpha - t - 1)!}{(n_\alpha + n_\beta - t - 1)!\,(n_\alpha - 1)!}$$

Let us define

$$\tilde{p} = \frac{(n_\alpha + n_\beta - t - 1)!\,(n_\alpha - 1)!}{(n_\alpha + n_\beta - 1)!\,(n_\alpha - t - 1)!}$$

Then we have that

$$\begin{aligned} &\rightarrow \quad (u_{\alpha,\geq t} - u_{\alpha,<t}) \cdot \tilde{p} > u_\beta - u_{\alpha,<t} \\ &\rightarrow \quad \tilde{p} \cdot u_{\alpha,\geq t} + (1 - \tilde{p}) \cdot u_{\alpha,<t} > u_\beta \end{aligned}$$

and let $\{\sigma^* (\varphi), \Pi^* (H|\varphi)\}$ be an assessment that defines a sequential equilibrium. $\Pi_i (H_i|\varphi)$ are dynamically consistent and therefore they are obtained from $\sigma^* (\varphi)$ whenever possible. The probability of existing t agents of type α in the clique size $q, \#q = t$ is given by

$$P(\tau_j = \alpha, \forall j \in q) = \frac{(n_\alpha + n_\beta - t - 1)! (n_\alpha - 1)!}{(n_\alpha + n_\beta - 1)! (n_\alpha - t - 1)!} = \tilde{p}$$

For simplicity, we name from now on agents in the clique q with her position in the sequence selected by nature among the agents in the clique. So agent $i = 1$, $1 \in q$ is the first agent who decides in the clique and agent $i \in q$ is the $i - th$ who decides in the clique. The probability of having a sequence of $t - i$ agents of type α in the last $t - i$ positions in the clique, conditional on the first $i - th$ individuals being of type α is given by

$$\begin{aligned} \tilde{p}_i &= \frac{((n_\alpha - i) + n_\beta - (t - i) - 1)! ((n_\alpha - i) - 1)!}{((n_\alpha - i) + n_\beta - 1)! (n_\alpha - i - (t - i) - 1)!} = \\ &= \frac{(n_\alpha + n_\beta - t - 1)! (n_\alpha - 1)!}{(n_\alpha + n_\beta - 1)! (n_\alpha - t - 1)!} = \tilde{p} \end{aligned}$$

The agent t in the clique, if observes $t - 1$ predecessors choosing α chooses also α in any equilibrium if she is of type $\tau_t = \alpha$. Suppose that the agent $t - 1 \in q$ has a belief $\tilde{\pi}_{t-1}$ such that the probability of being followed by an agent of type α is $p_\alpha^{t-1} \geq \tilde{p}$. Her best response $\varphi_{t-1} = \tilde{\varphi}_{t-1}$ to an information set where she observes $t - 2$ actions of type α by her predecessors conditional on π_{t-1} is α, since her expected payoff of choosing α is

$$\begin{aligned} Eu\left(\alpha, \tilde{\varphi}_{t-1}, \tilde{\pi}_{t-1}\right) &\geq p_\alpha^{t-1} \cdot u_{\alpha,\geq t} + \left(1 - p_\alpha^{t-1}\right) \cdot u_{\alpha,<t} \\ &\geq \tilde{p} \cdot u_{\alpha,\geq t} + (1 - \tilde{p}) \cdot u_{\alpha,<t} \geq u_\beta \end{aligned}$$

Take an agent $j \notin q$ of type α who does not observe anything, apart from her type $\varphi_j = \{\tau_j = \alpha\}$. The system of equilibrium beliefs Π^* must be consistent applying Bayes rule to the equilibrium strategies $\sigma^* (\varphi)$. A best response of agent $j \notin q$ who does not observe anything but her type must respond assigning probability $\tilde{p}$ to the event of agent $i \in q$ in the clique being in an information set such that she is of type α, she observes $i - 1$ actions of type α and is followed by $t - i$ agents of type α. This is the case because Bayes rule requires that beliefs are consistent with the strategies, and therefore the ex-ante probability must be consistent with the system of beliefs. Therefore agent $j \notin q$ assigns probability $\tilde{p}$ to the event of all agents in the clique being of type α and choosing it according to $\{\sigma^* (\varphi), \Pi^* (H|\varphi)\}$ Best responds for the agents who does not observe anything is therefore α if they are of that type, since $\tilde{p} \cdot u_{\alpha,\geq t} + (1 - \tilde{p}) \cdot u_{\alpha,<t} \geq u_\beta$. Agents who observe some actions must assign probability 0 in her consistent beliefs to the event of an action which is not α being chosen by an initial agent of type α. This means that posterior agents or assign a higher

probability to the event of being t agents in the clique choosing α (if they do not observe actions of type α) or require less agents in the clique being of type α (if they observe actions of type α). And therefore, in equilibrium, any agent who does not observe actions in the clique best responds by choosing α. Since the clique is isolated, every agent chooses α over the equilibrium path and agents in the clique best respond to it by also choosing α, in any information set. ■

Proposition 3: Suppose a set of agents N embedded in a sufficient network Γ such that everyone is willing to revolt in the sense of Chwe (2000). Suppose that the nature selects an order of play according to $P(\theta(N))$ and that if $ij \in \Gamma$, a_i is in the information set of j if $i < j$. Then every agent chooses to revolt in the path of any weak perfect Bayesian equilibrium.

Proof. We briefly state the model by Chwe (2000). There is a finite set of agents $N = \{1, 2, ..., n\}$. There are two types, w (willing to revolt) and x (unwilling). Each person i chooses an action $a_i \in \{r, s\}$ that states for revolting (r, the risky action) and not (s, the safe action). Utility of each person depends on own type and the full profile of actions. If a person is of type x, action s is a dominant strategy. If a person is of type w, her utility is supermodular, i.e., the difference in utility between r and s is increasing in the amount of people who chooses to revolt. A person of type w prefers the action r if everyone else is choosing r. A network Γ is a collection of pairs such that if $ji \in \Gamma$ it means that person j talks to i, i.e. i knows the utility function of j. Γ is common knowledge. Γ is a sufficient network if when everyone is willing to revolt then there exists an equilibrium where everyone revolt, regardless of the belief over the type of unobserved agents. This means that there exists an equilibrium where everyone revolts even if the agent believes that all those non-observed agents are of type s. Let t_i be such that the agent i gets a higher utility by choosing r than s if at least $t_i - 1$ other agents choose r. Suppose that the Nature calls to decide to the agents according to $P(\theta(N))$ and that agent i observe the action chosen by j if and only if $ji \in \Gamma$ and $\theta_j < \theta_i$.

Chwe shows that, if Γ is sufficient, there is a sequence of cliques that cover N. This cliques are hierarchically connected such that there is a leading clique in which their aents talk among them, and everyone talks to the following cliques, and so on. Agents in the leading clique best responds to the rest of the agents in the leading by choosing r if they choose r. This means that the clique contains sufficient people willing to take r. Since agents in the clique know that because they talk among and know their types, by Proposition 1 they choose r in any sequence of decision. The clique in the following position is formed by agents that are talked by all agents in the leading clique, and therefore know the types in the leading clique and in the following clique. They best responds to agents in the leading clique and to the mates in the following clique by choosing r if they choose r. Since in equilibrium all the agents in the leading clique choose r and there are sufficient agents in the following clique for making r optimal, we can apply one more Proposition 1 to the agents in this clique, conditioning on the fact that agents in the leading clique choose r. We can apply recursively this argument for all agents in N to show that every of them chooses r for any $P(\theta(N))$. q.e.d.

$\blacksquare$