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Abstract

An important question in insurance is the amount of coverage to purchase. A standard microeconomic model for insurance shows that full insurance is optimal. I present a different model where the decision variable is the number of put options and show that full insurance is still optimal, but the number of put options required to achieve this is larger than the endowment of risky assets. The model I present is based on a binomial model for a financial market, where the put option represents insurance.

Keywords Insurance, put option, binomial model, risk averse, risk neutral

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1 Introduction

An important question in consideration of insurance is the amount of coverage to purchase. In microeconomics terms, a risk averse agent in a two-state risk neutral world will purchase enough insurance that their payoff in each state is the same. I present a model with one risk averse agent, two states, and risk neutral pricing; the agent holds an endowment of stock and can purchase an amount of one put options. I show the optimal portfolio has the same wealth in each state, but the number of put options is greater than one. This is important because some markets, such as agricultural insurance, restrict how an agent can trade insurance contracts. The new model is a portfolio allocation problem where the put option is an insurance contract, which opens the world of option trades and mathematical finance to understand how economic agents use insurance.

2 Standard Result

There are two ways to motivate the standard model for demand for insurance (Rees and Wambach, 2008): an agent holds an asset that has a certain value, but may suffer a loss; an agent trades state contingent wealth in a complete market with risk neutral pricing. I review the state contingent wealth approach because it is succinct. To determine how much insurance should the agent buy, the model starts by listing state contingent wealth. Since there are two states, there are W_1 and W_2. The probability of each state is known, 1-π and π respectively. The objective function is expected utility. The budget constraint is set by the endowment of risky asset:

$$W_1 = W_0 \qquad W_2 = W_0 - L$$

A crucial assumption is that premium (P) depends on coverage (C) by a pricing factor (p) of the form: $P = C\,p$. Also, I use W^* to denote the optimal amount of wealth in each state. With this notation, we can understand the crucial result of this modelling approach (Rees and Wambach, 2008, p.21):

$$p = \pi <==> W_1^* = W_2^*$$

This result means that the agent should trade insurance such that there is no uncertainty in their payoff. This occurs because insurance is priced by risk neutral calculation, but the agent has risk neutral preferences; the agent attributes higher value to insurance than the marketplace. This standard assumption can be explained by theories of supply for insurance, such as the Arrow Lind Theorem (Rees and Wambach, 2008, p.53).

3 New Results

The motivation for this model is an agent who holds an asset (stock) that has uncertain future value; what amount of puts will maximize expected utility? I suppose the agent can trade put options on the stock, with one strike price and terminal time. I use a binomial model where the two possible states of the world are stock price increase, and stock price decrease: $S_T = uS$ with probability p; $S_T = dS$ with probability q. I assume the put option has initial price: $P\_0 = E((K – S_T)^+) = q (K - dS).$

The budget constraint for this model is set by the initial wealth $W_0 = S_0 + X_0$, where S is the endowment of one unit of stock and X_0 is the endowment of cash to be used for trading put options. The agent is allowed to trade such that $X_0 = a B_0 + b P_0$, where a,b are the choice parameters. This budget constraint allows me to rewrite the problem with one variable, b. The terminal wealth is $W_T = S_T + aB_T + bP_T$. Now we can list state contingent wealth: if stock goes down, then terminal wealth is $W_T = dS + a B_T + b (K - dS)$; if stock goes up, then $W_T = uS + a B_T$. The formula for expected utility is:

$$E(U(W\_T)) = p\ U(uS + a\ B\_T) + qU(dS + a\ B\_T + b\ (K - dS)\ ).$$

To simplify what follows, I assume B_0 = 1 and r = 0. Thus, B_T = 1 and a = X_0 - b P_0. Now I state the portfolio problem with one variable:

$$MAX_b\ [\ p\ U(uS + X_0 – b\ P_0\ ) + q\ U(dS + X_0 – b\ P_0 + b\ (K - dS)\ )\ ]$$

This is very similar to the standard model, except: $W_0 = uS$, $W_0 - L = dS$, $-P = a$, and $C = b\ (K - dS)$. The following result, Table 1, shows that the optimal wealth in each state is equal – the same as the standard model (Rees and Wambach, 2008, p.21). The *b* is the number of put options bought by the agent, *U(uS)* is the utility when stock price goes up, *U(dS)* is utility when stock goes down, and *E(U)* is the expected utility. Notice that the underlined entry *b=3.5* is the optimal value of *b*.

b	*U(uS)*	*U(dS)*	*E(U)*
1.00	2.45	1.89	2.34
1.50	2.43	2.00	2.35
2.00	2.42	2.10	2.35
2.50	2.40	2.20	2.36
3.00	2.38	2.28	2.36
<u>3.50</u>	<u>2.36</u>	<u>2.36</u>	<u>2.36</u>
4.00	2.34	2.43	2.36

Table 1: calculation of state contingent wealth and expected utility

The parameter values I use to calculate this result are: $r = 0; B_0 = 1; S_0 = 10; uS_0 = 12; dS_0 = 5; K = 7$ (put option strike); $Prob(S_T = uS_0) = 0.8; Prob(S_T = dS_0) = 0.2.$ When the agent initially has zero dollars cash to finance the trade in put options, $X_0=0$, the position in bonds is: $a = -1.4$. If the agent is endowed with two units of stock, then the optimal allocations are: $a = -2.8, b = 7$.

In Appendix 1, I provide a payoff diagram for the optimal portfolio reported in Table 1. The portfolio consists of one unit of stock, *a* units of cash, *b* units of put option. The shape of this diagram is determined by the parameters I chose.

4. Discussion

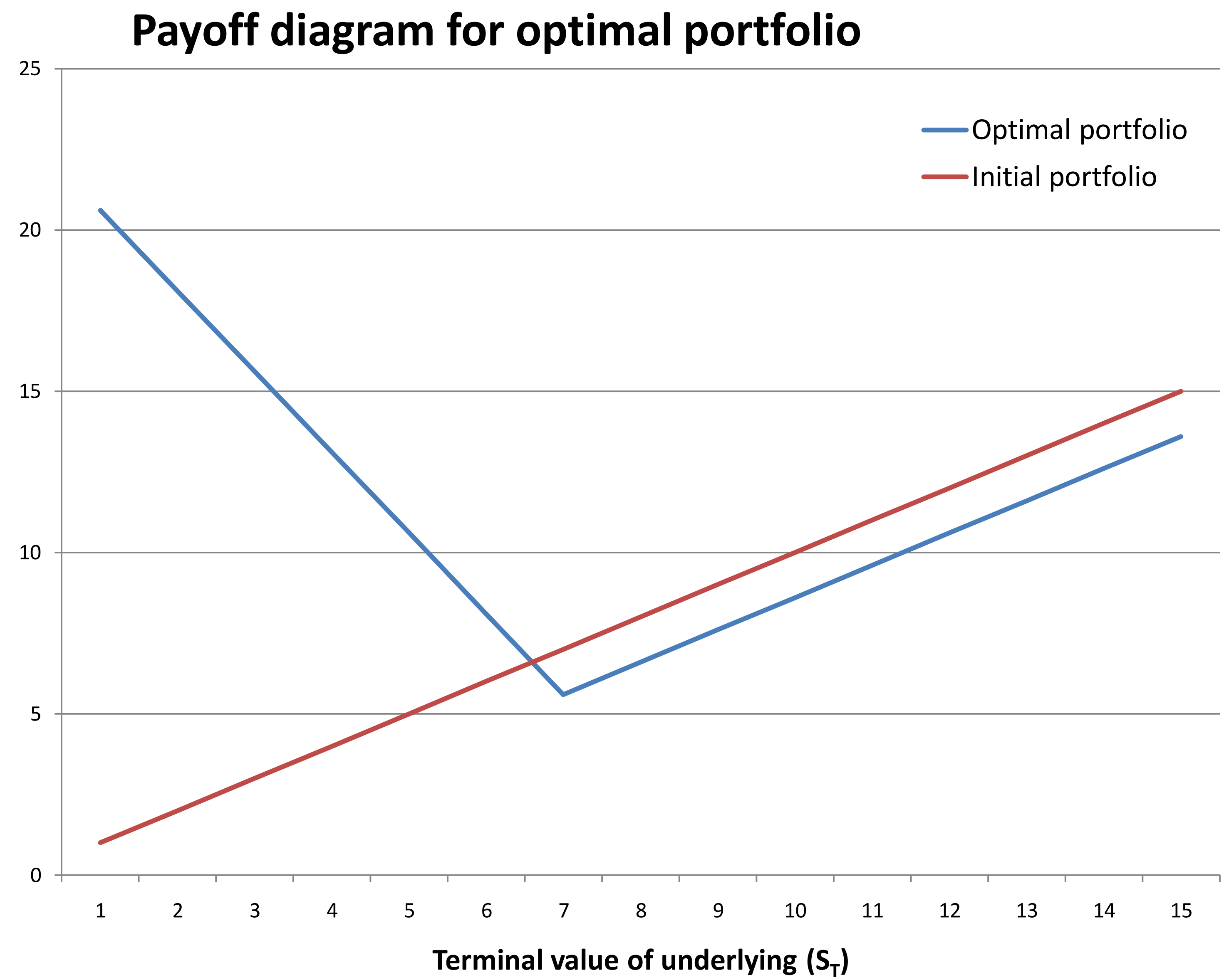
The two models presented here show how agents shift wealth between states. Both models show that the optimal portfolio eliminates all uncertainty in future payoff; this is because the agent is risk averse, but prices are risk neutral. I present a model where an agent holds a risky asset and is allowed to trade put options on it. With a binomial model, I show that the optimal position in put options makes terminal wealth equal in both states. However, to achieve zero uncertainty in terminal wealth the agent must trade more put options than they have endowment of risky assets. This affects the payoff diagram, Appendix 1, in two ways: the payoff shifts downwards everywhere because the agent borrows cash to buy the put option, the payoff kinks upwards below the strike because of the put option.

The result that the optimal amount of put options is larger than the endowment of risky assets is a subtle point that may be important in situations where agents are restricted in the amount of insurance they can trade. I think this is an interesting area and I suspect that that more complicated positions in options, such as straddles, will appear in these portfolio allocation problems when more complicated information structures and Bayesian statistics are used.

References

R. Rees and A. Wambach, The Microeconomics of Insurance, Foundations and Trends in Microeconomics, vol 4, no 1–2, pp 1–163, 2008

Appendix 1



Parameter	Value
X_0	0
B_0	1
r	0
S_0	10
uS_0	12
dS_0	5
P(u)	0.8
K	7
P_0	0.4
a	-1.4
b	3.5