Paying for prominence

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Paying for Prominence*

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Abstract

We investigate three ways in which firms can become "prominent" and thereby influence the order in which consumers consider options. First, firms can affect an intermediary's sales efforts by means of commission payments. When firms pay commission to a salesman, the salesman promotes the product with the highest commission, and steers ignorant consumers towards the more expensive product. Second, sellers can advertise prices on a price comparison website, so that consumers investigate the suitability of products in order of increasing price. In such a market, equilibrium prices are lower when search costs are higher since a firm's benefit from being investigated first increases with search costs. Finally, consumers might first consider their existing supplier when they purchase a new product, which suggests a relatively benign rationale for the prevalence of cross-selling in markets such as retail banking.

Keywords: Consumer search, e-commerce, price comparison websites, cross-selling, mis-selling, commission sales.

1 Introduction

In many markets, consumers are initially imperfectly informed about the deals available, and must invest effort to find out where to obtain a reasonable product at a reasonable price. In a few situations it makes sense to suppose that consumers search randomly

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through available options. In many circumstances, however, consumers consider options in a non-random manner, and might choose first to investigate those sellers or products which have high brand recognition, which are recommended by an intermediary, which are prominently displayed inside a retail environment, which are known to have a low price, or from which the consumer has purchased previously.

In Armstrong, Vickers, and Zhou (2009) and Zhou (2011), we examined how a firm's profits and its incentive to choose its price depend on whether it is "prominent" in a consumer's search process or not. Armstrong *et al.* (2009) considered a situation in which one firm is sampled first by all consumers, and then the remaining firms are sampled randomly, while Zhou (2011) considered the case where firms were sampled in a known, deterministic order. We used a search model with differentiated products which was first developed by Wolinsky (1986). In this framework, a prominent firm's profit is greater, although its price is lower, than that of its harder-to-find rivals.

Other work, including Arbatskaya (2007), Armstrong *et al.* (2009, section 4) and Xu, Chen, and Whinston (2011), examined the impact of prominence when firms supply a homogenous product, but where consumers differ in their cost of search. In such a setting, expected prices must be lower in less prominent positions—that is to say, a prominent firm sets a higher price—otherwise a consumer would never invest in costly search to find another firm.[1] In this situation, rational consumers must somehow be *compelled* to search through the options in the designated order, or they must find that sampling the prominent option is sufficiently less costly than sampling others, for otherwise they would be better off visiting the cheaper, less prominent sellers first.

A fundamental issue concerns the source of prominence, and in this paper we examine three ways in which firms can become prominent in search markets. To this end we study a variety of stylized models of such markets, including markets with homogenous products and with product differentiation. In section 2 we consider a setting where firms buy prominence by offering financial inducements to intermediaries. We consider two variants of this situation. In the first, firms pay sales commissions to a salesman (not conditioned on whether a product is made prominent), as is often the case in one-to-one sales environments such as for financial services. The salesman chooses to promote the product with the highest commission, and in equilibrium he steers the less informed consumers towards the more expensive product. This could be construed as a form of "mis-selling". In the second

[1]This prediction is verified in McDevitt (2011), who finds that plumbing firms whose chosen name begins with "A" charge higher prices on average than their rivals.

situation sellers compete to offer a lump-sum payment to an intermediary, and whichever firm offers the most is placed in the prominent position. (This might apply to publishers competing to be chosen as the "book of the month" by a bookshop.) Market performance can be improved by the ability of firms to buy prominence, as the firm which is willing to pay the most to be promoted is often the firm which consumers would most like to encounter first.

Next, in section 3 we suppose that firms can advertise their prices, for instance on a price-comparison website. Although consumers need also to investigate a product's suitability, consumers will first investigate those suppliers who advertise low prices. Thus, instead of firms achieving prominence by means of high commissions, firms here become prominent when they choose low retail prices. In such a market, in contrast to most search markets, higher search costs induce lower equilibrium prices since a firm's benefit from being investigated first increases with search costs. Finally, in section 4 we discuss how in some markets it is plausible that consumers exhibit "default bias", in the sense that people who are already customers of one firm may first consider this firm when they decide about subsequent products. In the retail banking market, for example, when consumers need products such as a mortgage or insurance policy, they often consider their existing bank first. In such cases, an incumbent firm is prominent in its customers' future buying decisions. Because prominent firms enjoy greater profits than less prominent rivals, a firm will buy prominence by competing aggressively for a customer's initial purchase.

Of course, there are other routes to prominence. For instance, a taxi firm can call itself "A1 Taxis" to be listed first in an alphabetic directory.[2] Advertising is another common method of achieving prominence. With an advertising campaign firms can make their product or brand prominent in a consumer's mind, so that a consumer is more likely to consider that product first when she decides what to buy. Bagwell and Ramey (1994) propose a model with homogeneous products in which some consumers choose first to investigate the firm which advertises the most intensively, which is thereby prominent in these consumers' search decisions. (In their model, advertising contains no information about price or product attributes, but is done purely to influence search behavior.) Owing to assumed scale economies, when a firm has greater demand it offers a lower price, and therefore it is indeed rational for these consumers to coordinate on the firm which advertises

[2] McDevitt (2011) documents how a high proportion of firms in certain "home emergency" markets choose their name so as to appear early in the phone book. In the most extreme case, 46% of locksmiths in Chicago have names which begin with "A" or a number.

the most. Because a firm enjoys a discrete jump in demand when it advertises even slightly more than its rivals, in equilibrium firms choose their advertising intensities according to a mixed strategy. Haan and Moraga Gonzalez (2011) present a related model with product differentiation in which a firm which advertises more intensively than a rival is more likely (but not certain) to be considered first by consumers. Since a firm's profit increases when a greater proportion of consumers sample it first, firms have an incentive to buy prominence in this way. In their basic symmetric model, all firms advertise with the same intensity, with the result that consumer search ends up being random and advertising expenditures are pure waste.

A way to pay for prominence which has recently been analyzed extensively, for instance by Chen and He (2006), Edelman, Ostrovsky, and Schwarz (2007), Varian (2007) and Athey and Ellison (2011), concerns sponsored links on search engines. In broad terms the seller which pays the most for a specific search term on a search engine will be prominently displayed on the results returned when someone types in that search term. Since internet users often click on prominent links first (either because this is their rule of thumb, or because they have learnt that sellers who are prepared to pay the most to be prominent are often the most relevant), it is worthwhile for sellers to pay for prominence in this way. We discuss related issues in more detail in the next section.

2 Paid Promotion

In some markets, intermediaries highlight one product from among the available options which a consumer should consider first. Examples include search engines which list some results more prominently, financial advisors and other one-to-one sales advisors who choose the order in which they present options to consumers, doctors who recommend a course of medical treatment, stores which put some products on prominent display at eye level or with greater shelf space,[3] shopping malls which put a particular store in a prime position, a motoring magazine which favourably reviews a particular car, or a bookseller promoting its "book of the month". While the hope is that better or cheaper products will be chosen

[3]Dreze, Hoch, and Purk (1994) report results from experiments in stores—their own and previous research—about how a product's position on the shelves, which products are adjacent, and the area taken up by the product on the shelf all matter for a consumer's propensity to purchase. For instance, they report a study showing that a 100% increase in shelf space for a product induces a 20% increase in sales. Table 4 in their paper shows that vertical location is more important than horizontal location on the shelf, and being placed at eye level could double the chance of buying a brand of breakfast cereal.

for special treatment in these ways, a natural worry is that the intermediary will promote an unsuitable or expensive product when paid to do so.[4]

There are at least two natural formats for the financial incentives which sellers provide to the intermediary: (i) the intermediary is paid per sale on a commission basis, regardless of whether the product is prominent or not, and (ii) the incentive takes the form of a lump-sum payment to the intermediary conditioned on the product being made prominent.[5] In many cases, especially in a one-to-one sales environment as is common with financial services, marketing efforts can be hard to monitor and format (i) is more likely to apply. Given the menu of commission rates he is offered, the salesman then decides which product to promote. Note that format (i) can be implemented in a retailing context when a supplier offers its product to a store with a specified wholesale price and a specified retail price. The margin between the retail and the wholesale price is then the "commission" paid to give the store an incentive to favour its product.[6] Format (ii) applies best to situations in which a supplier can monitor the marketing efforts of the intermediary (e.g., a publisher knows that its book was indeed the "book of the month").

We discuss these two cases in turn.

Commission sales. To investigate this first situation, we study a variant of Varian (1980) whereby Varian's framework is modified to allow the intermediary (or "salesman" for brevity in the following) to steer the uninformed portion of consumers towards a particular product. In more detail, two sellers supply a homogenous product which all consumers value at v. A fraction λ of consumers costlessly observe the two retail prices offered by the sellers (and buy from the lowest-price seller) and a fraction $1-\lambda$ of consumers only

[4] Anecdotal evidence suggests that some bookstores "recommend" books before they have even been read. One UK bookstore was alleged in 2006 to charge publishers £50,000 a week to guarantee a book "a prominent position in the store's 542 high street shops and inclusion in catalogues and other advertising". A trade body suggested that 70 per cent of publisher promotional budgets were spent on so-called "below-the-line" schemes operated by bookshops rather than more traditional advertising. For more details, see the article in the (UK) *Sunday Times* by Robert Winnett and Holly Watt titled "£50,000 to get a book on recommended list", 28 May 2006.

[5] In terms of online advertising, which is one of the leading ways to pay for prominence, the first form of payment is akin to per-click charging, where the advertiser pays the search engine each time someone visits the advertiser's website. The second form is more akin to so-called per-impression charging, where an advertiser pays for the right for (say) a banner advert on a particular website, and pays each time a consumer visits that website.

[6] See Cassady (1939) for an early discussion of this point.

consider a single product and buy if the retail price of that product is below v. The salesman has the ability to steer the $1-\lambda$ less informed consumers to buy either of the products. Suppose that a firm chooses its retail price, p, and commission rate, b, simultaneously (and simultaneously with the rival seller). A firm pays commission b to the salesman every time a sale of its product is made. We assume that the salesman has no ability to alter the retail price, for instance by reducing the price below the stipulated price from the seller by contributing a portion of his commission to the consumer. (Later, we discuss the impact of relaxing this assumption.)

First, note that the salesman will choose to promote the high-commission product, regardless of how the two retail prices compare (as long as prices do not exceed v). This is because the salesman's marketing effort cannot influence the choice of the informed consumers at all, but fully determines the choice made by the uninformed consumers. Hence, the salesman will direct the uninformed consumers towards the product which pays a higher commission rate.

Second, there are no equilibria in which firms set either a deterministic price or a deterministic commission rate (or both). For instance, if its rival is known to offer the deterministic retail price p, then a firm enjoys a discrete jump in its demand if it slightly undercuts this price since it then sells to all the informed consumers. Likewise, if its rival offers a deterministic commission rate, a firm has an incentive to pay a slightly higher commission, since then the salesman directs all the uninformed consumers to its product. We therefore look for a (symmetric) mixed-strategy equilibrium. The following preliminary result (proved in the Appendix) shows that in any such equilibrium high commission payments are associated with high retail prices:

Lemma 1 *Let $\Omega \subset [0,v]^2$ be the support of the joint distribution of (p,b) in a mixed strategy equilibrium. Then Ω cannot include two pairs (p_1,b_1) and (p_2,b_2) with $p_1 < p_2$ and $b_1 > b_2$.*

In essence, this result indicates that there is a deterministic and increasing relationship between a firm's choice of b and p. Since high commissions are associated with high retail prices, the salesman promotes the highly priced product due to the high commission he then receives. This could be interpreted as "mis-selling", since uninformed consumers are directed to buy the more expensive product.[7]

The next result characterizes the equilibrium in more detail:

[7] This model could also apply to sponsored search auctions, where the highest bidder for a particular keyword was listed first, and each advertiser pays its own bid each time someone clicks its webpage. Such

Proposition 1 *There is a symmetric mixed-strategy equilibrium in which each firm chooses its retail price p according to c.d.f. $G(p)$ with support $[p_{\min}, v]$, where $G(\cdot)$ satisfies*

$$(1-\lambda)[1-G(p)] = 1 - \frac{1}{1+\left(\frac{1-\lambda}{\lambda}\right)^2\left(\frac{v}{p}-1\right)} \tag{1}$$

and

$$p_{\min} = (1-\lambda)v \ ,$$

and chooses its commission as a deterministic function of its price so that $b = B(p)$, where

$$B(p) = \frac{1-\lambda}{\lambda}(p - p_{\min}) \ . \tag{2}$$

It is useful to discuss the main ideas involved in the construction of this equilibrium. Suppose that firm 2 follows the proposed equilibrium strategy. Then, since the salesman promotes the higher-commission product, firm 1's expected profit if it chooses the pair (p, b) is

$$\begin{aligned}\pi(p,b) &= (p-b)[(1-\lambda)\Pr(\tilde{b} < b) + \lambda \Pr(\tilde{p} > p)] \\ &= (p-b)\left[(1-\lambda)G\left(B^{-1}(b)\right) + \lambda(1-G(p))\right] \ . \end{aligned} \tag{3}$$

In equilibrium a firm will be indifferent between all choices $(p, B(p))$ for $p \in [p_{\min}, v]$ and for a given p in the support, a firm's expected profit is maximized over all possible commission rates b by choosing $b = B(p)$. Therefore, a firm's profit must be locally "flat" in all directions at the point $(p, B(p))$. In particular, a firm's profit in (3) must be unchanged (to first order) if it increases both p and b equally by some small amount ε starting at $b = B(p)$. Since this change has no impact on the firm's margin $p - b$, from (3) we require that

$$\frac{d}{d\varepsilon}\left[(1-\lambda)G\left(B^{-1}\left(B(p)+\varepsilon\right)\right) + \lambda(1-G(p+\varepsilon))\right]_{\varepsilon=0} = 0 \ ,$$

i.e., that

$$(1-\lambda)\frac{g(p)}{B'(p)} - \lambda g(p) = 0$$

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a system is similar to the early system for selling online advertising, as described by Edelman *et al.* (2007, pp. 245-246). Since that time, sponsored search positions have usually been allocated with a generalized second-price auction, so that the i_{th} highest bidder for a particular keyword is listed in the i_{th} position but pays the $(i+1)_{th}$ highest bid per click. In our simple duopoly model, this corresponds to a situation in which the highest bidder is made prominent, but pays the losing bidder's commission rate. (However, such a mechanism would not be practical in many one-to-one sales environments.)

so that

$$B'(p) \equiv \frac{1-\lambda}{\lambda} \text{ for all } p \in [p_{\min}, v] \ .$$

Thus, the commission rate is a linear (affine) function of a firm's price.[8] When a firm offers the lowest possible commission rate b which might be offered in equilibrium, it knows for sure it will not be made prominent by the salesman, in which case it should offer no commission at all. We deduce that in equilibrium we must have $B(p_{\min}) = 0$, and so the commission schedule is as in (2).

For now, assume that the firm chooses commission $b = B(p)$ in (2) when it chooses its price p, and consider its incentive to choose p. From (3), its profit with p is

$$\frac{1}{\lambda}\left[(2\lambda-1)p + (1-\lambda)p_{\min}\right]\left[(1-\lambda)G(p) + \lambda(1-G(p))\right] \ .$$

Since in equilibrium the price $p = v$ is sometimes chosen, this profit must be equal to $\frac{1-\lambda}{\lambda}\left[(2\lambda-1)v + (1-\lambda)p_{\min}\right]$. To maintain the firm's indifference over price, the c.d.f. G must satisfy (1), and $p_{\min}$ is then determined from the condition $G(p_{\min}) = 0$. Note that $G(p)$ in (1) is increasing in p for $p_{\min} \leq p \leq v$. One can also check that $p - b > 0$ for all $p_{\min} \leq p \leq v$, so that a firm always obtains a positive margin when it makes a sale.[9] It remains to verify (as we do in the Appendix) that a firm has no unilateral incentive to deviate to (p, b) with $b \neq B(p)$.

In this equilibrium, the uninformed consumers are directed to buy the more expensive product. A firm either serves the uninformed consumers (when it happens to set the higher retail price) or the informed consumers (when it is the cheaper supplier). When λ is close to 1, so that most consumers are well informed, then neither the firms nor the salesman can extract significant profit. When almost all consumers are uninformed ($\lambda \approx 0$), retail prices are approximately equal to the monopoly price v, but almost all of this price is extracted by the salesman, who can steer almost all consumers with his marketing efforts.

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[8] This linearity is the chief modelling advantage of assuming there are just two sellers. If there were $n > 2$ sellers, the corresponding formula becomes

$$B'(p) = \frac{1-\lambda}{\lambda}\left(\frac{G(p)}{1-G(p)}\right)^{n-2}$$

so that there is a nonlinear relationship between b and p which depends on the (endogenous) form of $G(\cdot)$. This complicates the analysis considerably compared to this duopoly setting where we can deduce the shape of $B(p)$ without first calculating $G(p)$. Note that with more than two firms, the commission schedule $B(p)$ is strictly convex in p.

[9] Note that $p > B(p)$ if and only if $(1-2\lambda)p < (1-\lambda)^2 v$. If $\lambda \geq 1/2$, this inequality surely holds. If $\lambda < 1/2$, then $(1-\lambda)^2/(1-2\lambda)$ is increasing in λ and it equals 1 at $\lambda = 0$, so the inequality also holds.

This trade-off can also be seen by noting that a firm's equilibrium profit is $\Pi = \lambda(1-\lambda)v$, which reaches its maximum at $\lambda = \frac{1}{2}$.

This analysis presumes that the salesman cannot affect a seller's retail price, and thus there is a form of (minimum) retail price maintenance applied by sellers. Suppose instead that the salesman can reduce the price a consumer pays for a product by contributing a portion of his commission b. In the equilibrium in Proposition 1, does the salesman have an incentive to offer such a discount? Suppose the high-price seller chooses (p_H, b_H) while the low-price seller chooses (p_L, b_L), where b_i and p_i are related by (2). If the salesman does not alter the retail prices, his expected commission payment is

$$(1-\lambda)b_H + \lambda b_L \tag{4}$$

as the informed consumers buy the low-price product and the uninformed consumers buy the expensive product. The alternative strategy involves the salesman reducing the retail price of the expensive product (with a high commission) so that *all* consumers choose to buy it. To do this entails discounting p_H by $(p_H - p_L)$ to make it competitive with the cheaper product. When he does this, his net commission payment from each consumer is

$$b_H - (p_H - p_L) \ .$$

However, this net commission payment is lower than the payment in (4) he receives when he does not distort retail prices if $\lambda(b_H - b_L) < p_H - p_L$. One can check from (2) that this must be true. We deduce that in the equilibrium in Proposition 1, the salesman has no incentive to discount either firm's retail price, even when he has the ability to do so.

There are at least two natural benchmarks with which to compare the equilibrium outcome when commissions are paid. The first benchmark is when there is no salesman, and the uninformed consumers buy randomly from the two firms. In this case the framework reduces to Varian (1980)'s model. Again, there is no pure strategy equilibrium in prices, and if its rival chooses its price according to c.d.f. $\hat{G}(\cdot)$, a firm's profit when it chooses price p is modified from (3) to be

$$p\left[\frac{1-\lambda}{2} + \lambda(1-\hat{G}(p))\right] \ .$$

To maintain the firm's indifference over price, this profit must always equal $\frac{1}{2}(1-\lambda)v$, the profit when the firm sets the highest possible price. Therefore, the c.d.f. for prices with random search is given by

$$1-\hat{G}(p) = \frac{1-\lambda}{2\lambda}\left(\frac{v}{p} - 1\right) \tag{5}$$

with the lowest possible price being $\hat{p}_{\min} = \frac{1-\lambda}{1+\lambda} v$. One can check that $\hat{G}(p) > G(p)$ for $\hat{p}_{\min} < p < v$, where G is described in (1), and so retail prices are higher, in the sense of first-order stochastic dominance, when firms pay commissions to a salesman to promote their product relative to the situation with random search. This is due in part to the sales commissions which artificially inflate the marginal cost of selling a product. However, whether firms enjoy greater profits when they pay commission is ambiguous. Without commission payments, each firm makes expected profit $\hat{\Pi} = \frac{1}{2}(1-\lambda)v$, while in the regime with commissions a firm makes expected profit $\Pi = \lambda(1-\lambda)v$. Thus, more profit is obtained with commission payments when $\lambda > \frac{1}{2}$, so that the uninformed consumers are in the minority. But when the uninformed consumers are in the majority, the two firms end up playing a prisoner's dilemma due to the fierce competition to become prominent.

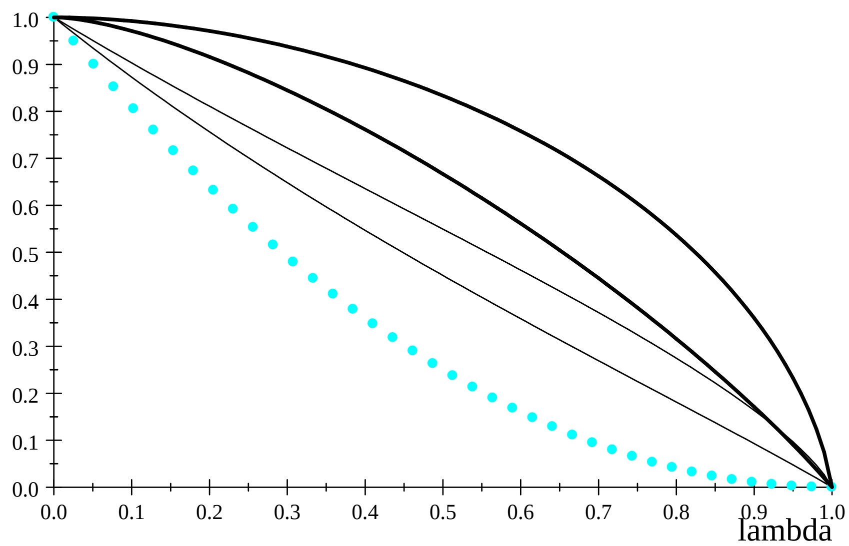

Figure 1: Expected prices and commissions

Figure 1 plots the expected prices paid in these two regimes as a function of λ, the proportion of informed consumers. (Here, $v = 1$.) The two bold lines depict expected prices when commissions are paid, where the upper of these lines is the expected price paid by the uninformed and the lower line is the expected price paid by the informed consumers.[10] The dotted line represents the expected commission paid to the salesman.[11] The two feint

[10] When commission is paid, the expected price paid by an uninformed consumer is the expected value of the maximum of two i.i.d. draws from the c.d.f. in (1), while the expected price paid by an informed consumer is the expected value of the minimum of two such draws.

[11] This is simply the commission payment in (2) evaluated at the expected price paid by all consumers, which is λ times the price paid by the informed consumers plus $1-\lambda$ times the price paid by the uninformed consumers.

lines depict the corresponding prices in the Varian model where no commissions are paid and search is random.[12] The two regimes have the same outcome for consumers when $\lambda = 0$ (when the monopoly price $p = v$ is chosen for sure) and when $\lambda = 1$ (when the competitive price $p = 0$ is chosen). However, for intermediate values of λ, the prices paid in the commission regime are substantially higher than when no commissions are paid. Indeed, in most cases an uninformed consumer in the no-commission regime pays a lower price than even the informed consumers do in the commission regime.

The second benchmark with which to compare the outcome with commission payments is to suppose that the salesman is necessary for consumers to buy the product (unlike Varian's model with random search), but now the salesman is paid by consumers rather than by sellers.[13] Suppose that when the salesman is paid by consumers, say in the form of a lump-sum consultation fee, he directs the uninformed consumers to the cheaper product. (This might be because, all else equal, he has a small intrinsic preference for selling the correct product to consumers.) In this case, all consumers buy the cheaper product and in Bertrand fashion the two sellers are forced to set retail prices equal to cost (zero in this case). The outcome for consumers then depends on how much they have to pay the salesman for his advice. One assumption is that the consultation fee is set equal to the revenue the salesman received under the commission regime, so that the salesman is indifferent between the two regimes, perhaps because the advice industry needs to be supportive of a policy shift from a commission-based model to a consumer-fee model. In this case, the expected total price (the price for the product plus the fee to the salesman) paid by any consumer is simply the dotted line on Figure 1. From the figure it follows that all consumers are better off when they pay the salesman compared to when sellers pay the salesman. In fact, they are also better off when they pay the salesman than when they search randomly (where prices are the feint lines on the figure).

This section has described a model where firms attempt to influence a salesman's marketing efforts by means of commission payments. The salesman gives prominence to the product which pays the highest commission, and in equilibrium this entails steering unin-

[12] The expected price paid by the informed consumers is again the expected value of the minimum of two prices, but this time taken from the c.d.f. in (5), while the expected price paid by the uninformed consumers is just the expected value of one price draw from the same distribution.

[13] The UK regulator, the *Financial Services Authority*, published rules in March 2010 concerning how financial advice can be remunerated. The rules state that an advisor will not be able to accept commission for recommending products, and the consumer fee for advice must be agreed between the consumer and the advisor, rather than between the seller and the advisor. See FSA (2010) for further details.

formed consumers towards the more expensive product. The outcome for consumers, both informed and uninformed, is poor: worse than the situation without commission payments where the uninformed shop randomly, and far worse than a situation in which consumers pay directly for advice.[14]

Lump-sum payment for prominence. Consider next the case where the financial inducement for prominence is a lump-sum payment, conditional on whether the product is promoted or not. In more detail, suppose that there are two symmetric firms, and an intermediary auctions off the right to the prominent position. The highest bidder obtains the prominent position and pays its bid, while the loser pays nothing. Once the prominent position is awarded, the two firms then choose their retail prices. (In contrast to the commission sales case, here it is often natural to assume that firms observe who has won the prominent position before setting their prices.) Suppose that when firms choose their equilibrium prices, the prominent firm makes profit π_H (excluding the lump-sum payment to the intermediary), while the less-prominent firm makes profit $\pi_L < \pi_H$. Given that the prize of prominence is awarded to a single firm, each firm is willing to pay up to $\pi_H - \pi_L$ for the right to be prominent. The result of the auction is that both firms bid up to this amount, and the prize is awarded at random to one firm, who is then made prominent. Since the prominent firm has paid a lump-sum fee equal to $\pi_H - \pi_L$, the net profit of each firm is equal to π_L. Suppose that π_0 is each firm's equilibrium profit with random search and $\pi_L < \pi_0$ so that the profit of the less-prominent firm is lower than the firm would make in a regime in which consumers search randomly. Then when the intermediary auctions off the right to be prominent the two firms are forced to play a prisoner's dilemma and in equilibrium both firms are worse off relative to the case where no firm was prominent.[15]

Whether the intermediary mis-sells the prominent product in this context is ambiguous, and depends in part on whether products are homogenous or differentiated. As discussed

[14] Inderst and Ottaviani (2010) present an alternative model of potential mis-selling, where the salesman advises consumers about the suitability of a product rather than its price. There, no consumers are informed, and must rely on the salesman to advise them about which product to buy. The salesman has only a noisy signal about the suitability of a product, and he has an intrinsic preference to recommend the suitable product to a consumer. However, this preference can be overturned if sellers set high enough commissions.

[15] Armstrong *et al.* (2009, page 221) show that $\pi_L < \pi_0$ in their particular model with differentiated products. Related effects are found in Choi and Kim (2010), who study the outcome when an internet service provider offers to give transmission priority in its lines to one of two content providers. "Priority" in markets with congestion plays a somewhat similar role to "prominence" in markets with search frictions.

in the introduction, with homogeneous products we expect that the prominent firm will set a higher price than its rivals, and those consumers with a higher search cost will be directed to buy the more expensive product (as in the previous case with commission sales). By contrast, with differentiated products as in Armstrong *et al.* (2009), we expect that the promoted product is offered at a lower price than the rival product. However, although consumers are guided to the cheaper product first, this does not mean that consumer surplus or overall efficiency is increased compared to a situation with random search. Armstrong *et al.* (2009) show in a uniform example that consumers are worse off and welfare declines when one firm is made prominent. This is because the non-prominent firms are induced to charge higher prices and the resulted non-uniform prices across firms lead to less efficient match between consumers and products.

Nevertheless, there are a number of situations in which awarding the prominent position to the highest bidder is likely to lead to more efficient outcomes, especially when sellers differ in the quality or cost of the product they supply. Armstrong *et al.* (2009, section 3) present a model where firms differ in their quality, and show that the firm with the highest quality product is willing to pay the most (as a lump sum) to become prominent, and consumers also benefit when this firm is placed at the start of their search process. In this model, "prominence" acts as a signal of otherwise unobserved product quality, and this improves market performance. This effect was discussed in a specific model in which each firm had linear demand and "quality" was indexed by the vertical intercept of this demand. However, the result applies much more widely. Indeed, it applies whenever the firm from a pool of heterogeneous firms which is prepared to pay the most to become a monopolist is also the monopolist from the pool of firms which consumers would most like to face.

To see this, suppose there are a large number of heterogeneous firms in the market, indexed by $i = 1, 2....$ Specifically, suppose the match utility from firm i is distributed with c.d.f. $F_i(u)$ and that this firm has constant marginal cost of supply c_i. Suppose in the equilibrium with random search that consumers obtain expected consumer surplus V from participating in the market. Since there are many firms, each firm then obtains approximately zero profits. A firm which is placed in the prominent position and which sets price p will sell whenever $u - p \geq V$, and so makes profit $\pi_i = \max_p (p - c_i)(1 - F_i(p + V))$, where p_i maximizes this profit. (In fact, the firm will choose this price p_i even if it is in a less prominent position.) Since non-prominent firms make negligible profits, the firm which is willing to pay the most to be in the prominent position is the firm i with the highest π_i.

Expected consumer surplus when firm i is placed in the prominent position is

$$v_i \equiv \int_{p_i+V}^{\infty} (u - p_i) dF_i(u) + F_i(p_i + V))V = V + \int_{p_i+V}^{\infty} (1 - F_i(u))du \ .$$

In many situations the firm with the highest π_i is also the firm with the highest v_i, so that π and v are positively correlated in the population of firms. To take one simple case, suppose that the only way in which firms differ is in their cost c_i, and each firm generates the same distribution of match utilities. Then the firm with the lowest cost is willing to pay most to be in the prominent position, and this firm also offers the lowest price to consumers. Whenever the highest π_i and the highest v_i coincide, auctioning off the prominent position to the firm which is willing to pay the most will increase consumer surplus and total welfare, relative to the situation with random search.[16,17]

3 Price-Directed Search

Price comparison websites are now a major part of the retailing landscape.[18] As long as a consumer has access to the internet, she can almost costlessly gain access to a list of

[16] Another instance of this situation is seen in the extension of Wolinsky (1986)'s model developed in Eliaz and Spiegler (2011). Here, a product is suitable with some probability q, which differs across firms. If a product is suitable, its value is taken from a common distribution, while if the product is unsuitable its value is zero. A firm's "quality" q does not affect its pricing decision, and the firm with the highest q is willing to pay the most to be prominent, and that is also the firm which consumers would most like to investigate first. Similar effects are also seen in models of sponsored-link auctions, as studied by Athey and Ellison (2011) and Chen and He (2006). There, firms differ in the likelihood of providing a relevant match for a consumer, and an advertiser who is more likely to provide a relevant match is willing to pay more to be listed prominently on the search results page. (There is no effective price competition between advertisers in these papers.) It is therefore rational for consumers to investigate the links in the order presented on the page, as the more relevant links are listed at the top. Again, paying for prominence improves the effectiveness of consumer search.

[17] Of course, if π and v were instead negatively correlated in the population of firms, then when the prominent position is sold to the highest bidder, consumers are made worse off relative to the situation with random search. Jerath, Ma, Park, and Srinivasan (2010) consider a model in which a higher quality firm may prefer to be displayed in a less prominent position than other firms, once the reduced advertising payments are taken into account. Relatedly, McDevitt (2011) shows that plumbing firms whose names start with "A" tend to receive a greater number of complaints from consumers, suggesting they are often lower-quality suppliers.

[18] See Baye, Gatti, Kattuman, and Morgan (2009) and Ellison and Fisher Ellison (2009), and the papers cited there, for background to this topic. In the literature on search in labour markets, recent papers have analyzed "directed search", where employers advertise their wages and workers choose their search order

prices from various suppliers for a wide range of consumer items. Sometimes products may be approximately homogeneous, so that price is mostly what matters for a consumer. In such cases, price comparison websites may go a long way towards achieving competitive outcomes.[19] An important early model of a price comparison website with homogenous products was presented in Baye and Morgan (2001). There, a number of symmetric firms supply a product, and firms can advertise their prices on a website to which (in equilibrium) all consumers have access. There are two groups of consumers: some consumers care only about finding the cheapest supplier, while other "loyal" consumers will only buy from a single firm to which they have a strong brand preference. The website charges a lump-sum listing fee to sellers, and each seller decides whether or not to put its price on the website. In equilibrium, the decision to advertise on the website is random, and this in turn generates prices which are also random. The firm which chooses to advertise and which happens to offer the lowest price on the website will sell to all the price sensitive consumers, and other firms will sell only to their pool of loyal consumers.

For some products, however, price is not the only consideration and a consumer cares also about product suitability. For instance, a traveller may look for a flight from London to New York on a travel website, and find various flights listed in order of increasing price, but only a subset of the flights meet the traveller's needs in terms of airport location, departure time, and so on. Nevertheless, price is still an important factor for the consumer, and it is plausible that the consumer will investigate the options in order of increasing price. In these situations, sellers become "prominent" in a consumer's search process by posting the lowest price on the website.[20]

In theory, one could try to use the search model with product differentiation in Wolinsky (1986) to study this question. However, it turns out that this framework, where a consumer's match utility is independently distributed across firms, apparently does not

accordingly. An important additional feature in the labour market relative to a typical consumer market is that a job vacancy can be filled, so that workers do not necessarily visit the highest-wage employer first if they anticipate that many other workers will apply for the same post. See Rogerson *et al.* (2005, section 5) for discussion of these models.

[19] For the purposes of this discussion, we ignore important caveats to this claim, including the ability that sellers have to "obfuscate" their true price. For instance, a seller might post a low "price" on the website, but then compensate for this by excessive "postage and packing" charges. See Ellison and Fisher Ellison (2009) for further discussion of this point.

[20] By contrast, in Armstrong, Vickers, and Zhou (2009) where price information is imperfect, the direction of causality is reversed, and a firm which happens to be prominent will then choose to offer a lower price than its rivals.

lead to an easily tractable solution for how firms choose prices on a price-comparison website. Instead, in this section we present a variant of Wolinsky's model which is tractable, and which provides insight about the impact of price advertising on market performance.

Suppose that two firms compete to offer a differentiated product in a Hotelling framework. Specifically, the two firms are located at the ends of the unit interval $[0, 1]$, and consumers are uniformly located on this interval, with brand preference parameter denoted $\ell \in [0, 1]$. The valuation of a consumer at ℓ for the product supplied by the firm at 0 is $v - \ell t$, where v is the valuation of the consumer for the ideal product and t is the unit "transport" cost (not to be confused with the search cost) and captures the extent of product differentiation. Similarly, her valuation for the product supplied by the firm at 1 is $v - (1 - \ell)t$. (As usual, we suppose v is sufficiently high that the market is fully covered in equilibrium.) If production cost is normalised to zero, then in the absence of search frictions, so that consumers have full information about their preferred product and the two firms' prices, the equilibrium price in this market is $p = t$.

In the following discussion we introduce search frictions into this market. Specifically, we suppose that each consumer must incur a search cost s to visit a firm to discover its price (where necessary) and its match utility (where necessary), as well as to purchase the product. When a consumer visits the first firm, she discovers her brand preference parameter x, which then also reveals her match utility at the other firm. Thus, this is a search model where match utilities are negatively correlated across the two firms, rather than independently distributed as in Wolinsky's model.

The market without the price comparison website. Suppose that consumers choose their initial firm randomly (i.e., they have no idea about their preferred supplier *ex ante* and expect firms to offer the same prices). What is the equilibrium price, p^*, in this market? Consider the incentive of the firm at 0 to choose its price p. Since consumers anticipate that both firms choose price p^*, a consumer who first visits this firm will buy if

$$p + \ell t \leq p^* + (1 - \ell)t + s \ ,$$

where ℓ is her discovered brand preference parameter and s is the search cost for buying from the second seller. (Note that in our Hotelling setting once a consumer has investigated her first seller, she knows the match utility of the second seller and believes this firm's price is p^*. We assume that she nevertheless needs to incur the search cost s in order to buy from the second firm.) Thus, the fraction of those consumers who first visit the firm and who end up buying from that firm is

$$\frac{1}{2}+\frac{s+p^*-p}{2t} \ .$$

If a consumer visits the rival firm first, since consumers anticipate the price p^* at both firms, only those consumers at a distance

$$\ell \le \frac{1}{2}-\frac{s}{2t}$$

from the firm at 0 will choose to investigate it. (Note that if $s \ge t$, then in equilibrium no consumers will search beyond the first sampled firm. To avoid this less interesting case, we assume $s < t$ henceforth.) They will then buy from this firm unless the price they find there is significantly higher than p^* (i.e., the surprise is big enough to drive some of them back to their initial firm, which requires that $p > p^* + s$). Thus, at least for local deviations from the equilibrium price p^* (i.e., for $p \le p^* + s$), the firm's total demand is

$$q=\frac{1}{2}\left(\frac{1}{2}+\frac{s+p^*-p}{2t}\right)+\frac{1}{2}\left(\frac{1}{2}-\frac{s}{2t}\right)=\frac{1}{2}+\frac{p^*-p}{4t}$$

which does not depend on s.[21] This is like the full information Hotelling model, but with differentiation parameter $2t$ instead of t. Therefore, the equilibrium price is

$$p^* = 2t \ .$$

This price does not depend on the search cost s so long as $s > 0$, and so we see that even tiny search frictions cause equilibrium prices to double. (This is reminiscent of Diamond (1971)'s famous result that arbitrarily small search costs can lead to monopoly prices.) The reason is that a firm cannot attract more of those consumers who first visit its rival with a low price, and so a firm's demand elasticity is halved.

The market with the price comparison website. Now suppose that firms advertise their prices to all potential consumers (e.g., on a price comparison website). By symmetry, all consumers will first investigate the firm which posts the lower price, and only go on to buy from the second firm if their discovered brand preference turns out strongly to favour the more expensive product. If the two posted prices are p_L and $p_H > p_L$, where $p_H - p_L < t - s$, then the low-price firm has demand

$$Q_L=\frac{1}{2}+\frac{s+p_H-p_L}{2t}$$

[21] One can check that for a large deviation price $p > p^* + s$, the firm's demand is $q = \frac{1}{2}(\frac{1}{2} + \frac{s+p^*-p}{2}) + \frac{1}{2}(\frac{1}{2} + \frac{p^*-p}{2t})$. That is, the demand function has a kink at $p = p^* + s$. But one can show that taking this into account does not change the equilibrium characterization below even for a small search cost.

and the high-price firm has demand

$$Q_H = 1 - Q_L = \frac{1}{2} - \frac{s + p_H - p_L}{2t} \ . \tag{6}$$

(As before, once a consumer has investigated her first seller, she knows the price and the match utility of the second seller. She nevertheless needs to incur the search cost s to buy from the second firm.) Notice that $Q_L - Q_H = s/t$ when $p_L = p_H$, so that there is a jump in its demand equal to s/t when one firm slightly undercuts its rival.[22] Due to this incentive to undercut and the presence of product differentiation, it is clear that there can be no symmetric pure-strategy equilibria. It is also true that there are no asymmetric pure-strategy equilibria either.[23] In the following result we derive a (symmetric) mixed-strategy equilibrium.

Proposition 2 *Suppose that* $0 < s < t$*. Then the search market with a price comparison website has a symmetric mixed-strategy equilibrium in which each firm chooses its price according to c.d.f.* $H(p)$ *with support* $[p_{\min}, p_{\max}]$*, where* $H(\cdot)$ *satisfies*

$$\frac{1}{2} + \frac{\bar{p} - p}{2t} + \frac{s}{2t}[1 - 2H(p)] = \frac{\Pi}{p} \ . \tag{7}$$

Here Π *is a firm's expected profit in equilibrium and* $\bar{p} = \int_{p_{\min}}^{p_{\max}} p dH(p)$ *is a firm's expected price in equilibrium.*

We now describe how to construct this equilibrium and determine the four unknown parameters $p_{\min}$, $p_{\max}$, $\bar{p}$, and Π. If its rival chooses price with c.d.f. H, a firm's expected demand when it chooses price $p \in [p_{\min}, p_{\max}]$ is[24]

$$Q(p) = \underbrace{\int_{p_{\min}}^{p} \left(\frac{1}{2} - \frac{s + p - \tilde{p}}{2t} \right) dH(\tilde{p})}_{\text{demand when rival charges lower price}} + \underbrace{\int_{p}^{p_{\max}} \left(\frac{1}{2} + \frac{s + \tilde{p} - p}{2t} \right) dH(\tilde{p})}_{\text{demand when rival charges higher price}}$$

[22] Baye *et al.* (2009) find in their dataset that a seller's "click rate" increases by 60 per cent when the seller lowers its price to become the cheapest seller on the price comparison website. Note that Baye *et al.* document that a seller's click rate also depends on its position on the webpage, which reflects a second form of prominence.

[23] To see this, suppose instead that there exists an asymmetric equilibrium with $p_L < p_H$. Since $\frac{\partial Q_L}{\partial p_L} = \frac{\partial Q_H}{\partial p_H}$, the first-order conditions imply $p_L/p_H = Q_L/Q_H$. That is, a lower price is associated with a lower demand, which cannot be the case.

[24] If instead we adopted Wolinsky's model with independent match utilities, expected demand does not have such a simple expression since the demand term inside each integral will be quadratic rather than linear in price.

$$= \frac{1}{2} + \frac{\bar{p} - p}{2t} + \frac{s}{2t}[1 - 2H(p)] \ . \tag{8}$$

Since the firm must be indifferent between choosing all prices in the interval $[p_{\min}, p_{\max}]$, it follows that H satisfies (7). (Notice that this calculation of demand assumes that $p_{\max} - p_{\min} < t - s$, and we will check later that this condition is satisfied in equilibrium.) In particular, the density for a firm's chosen price, $h(p)$, is

$$h(p) = \frac{1}{2s}\left(\frac{2t\Pi}{p^2} - 1\right) \tag{9}$$

which decreases over its support.

To complete the equilibrium characterization, we need to determine the four unknown parameters. Since $H(p_{\min}) = 0$ and $H(p_{\max}) = 1$, expression (7) implies that

$$t + \bar{p} - p_{\min} + s = \frac{2t\Pi}{p_{\min}} \tag{10}$$

and

$$t + \bar{p} - p_{\max} - s = \frac{2t\Pi}{p_{\max}} \ . \tag{11}$$

Given the density h in (9), it follows that $\bar{p}$ must satisfy

$$\bar{p} = \frac{1}{2s}\int_{p_{\min}}^{p_{\max}} \left(\frac{2t\Pi}{p} - p\right) dp = \frac{1}{2s}\left(2t\Pi \log\frac{p_{\max}}{p_{\min}} - \frac{1}{2}(p_{\max}^2 - p_{\min}^2)\right) \ . \tag{12}$$

The final condition is derived from the fact that a firm does not want to set its price higher than $p_{\max}$. If a firm chooses $p \geq p_{\max}$, it will surely be the more expensive firm, and its expected profit will be

$$\pi(p) \equiv p\left(\frac{1}{2} - \frac{s + p - \bar{p}}{2t}\right) \ .$$

To ensure that the firm has no incentive to do this, we require that $\pi(p) \leq \pi(p_{\max})$ for all $p \geq p_{\max}$. Since $\pi(p)$ is a concave function, this requires

$$p_{\max} \geq \frac{1}{2}(t + \bar{p} - s) \ . \tag{13}$$

On the other hand, given the other firm's equilibrium strategy, a firm's expected profit is constant for $p \in [p_{\min}, p_{\max}]$, so its derivative with respect to p evaluated at $p_{\max}$ (from the left-hand side) is zero, which from (8) implies

$$\lim_{p \to p_{\max}^-} Q(p) + pQ'(p) = 0 \Rightarrow \frac{1}{2}(t + \bar{p} - s) - p_{\max} = sp_{\max}h(p_{\max}) \ . \tag{14}$$

From (13) and (14), we conclude that

$$h(p_{\max}) = 0$$

and

$$p_{\max} = \frac{1}{2}(t + \bar{p} - s) = \sqrt{2t\Pi} \ . \tag{15}$$

(The second equality in (15) used (11).) The four unknowns $p_{\min}$, $p_{\max}$, $\bar{p}$ and Π can then be solved from (10), (12) and the two equalities in (15).[25] In the Appendix, we show that this system does have a unique solution, and the requirement $0 < p_{\max} - p_{\min} < t - s$ is satisfied.

An interesting observation (proved in the Appendix) is that a higher search cost induces firms to post *lower* prices and so causes equilibrium profits to fall.

Corollary 1 *In the equilibrium characterized in Proposition 2, (i)* $p_{\min}$*,* $p_{\max}$*,* $\bar{p}$ *and* Π *all decrease with* s*, and (ii)* $p_{\max} < t$ *so that firms earn less than in the case with random search without the price comparison website and in the case with perfect consumer information.*

Intuitively, a higher search cost implies that consumers are more reluctant to search beyond the first firm they encounter, so there is a greater benefit to being the firm sampled first, and hence a greater incentive to be the firm with the lower price. To illustrate this feature, Figure 2 depicts the density $h(\cdot)$ for various search costs. (Here, $t = 1$.) In the figure the densities corresponding to lower search costs lie further to the right. When the search cost is close to zero, the firms post prices which are close to the full information outcome ($p \approx 1$ in this example with $t = 1$). When search costs are higher, the support of the distribution of prices is wider, but there is a concentration on the low end of the support. As shown analytically in Corollary 1, in all cases firms chooses prices below 1. (In particular that the presence of the price comparison website acts to reduce prices relative to the situation with random search, where the equilibrium price in this example is $p^* = 2$ for all positive search costs.) Moreover, prices posted on the website are always lower than in the situation where consumers are fully informed about price and product characteristics (where the equilibrium price in this example is $p = 1$).When search costs are so high that consumers never investigate both firms (i.e., when $s \geq t$), then all demand goes to the firm with the lower price, and so the price is driven down to cost (zero in this

[25] We also need to check that a firm has no incentive to charge a price below $p_{\min}$. If a firm chooses $p \leq p_{\min}$, its expected profits are

$$\hat{\pi}(p) \equiv p\left(\frac{1}{2} + \frac{s + \bar{p} - p}{2t}\right) \ .$$

To ensure $\hat{\pi}(p) \leq \hat{\pi}(p_{\min})$ for all $p \leq p_{\min}$, we require that $p_{\min} \leq \frac{1}{2}(t + \bar{p} + s)$, which, however, is automatically true as long as (15) holds and $p_{\min} < p_{\max}$.

case) in Bertrand fashion, although consumers must buy a random product rather than their preferred product.

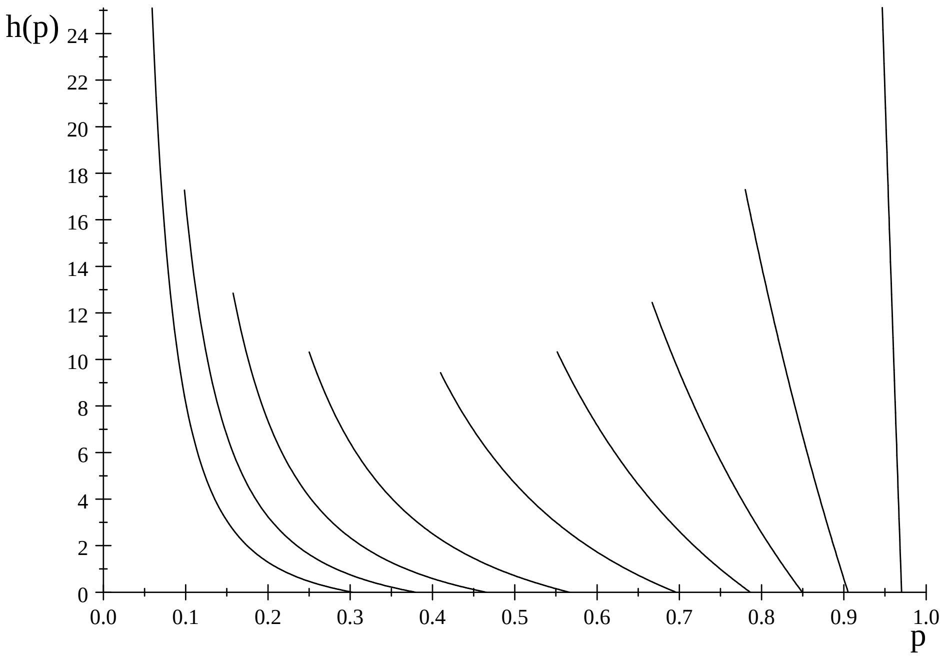

Figure 2: Density $h(p)$ when $s = 0.5, 0.4, 0.3, 0.2, 0.1, 0.05, 0.025, 0.01$, and 0.001

An alternative model of a price comparison website has recently been provided by Zhang (2010). In his model, each product delivers utility v to a consumer if it is "suitable", which occurs with exogenous probability q. If a product is unsuitable, which occurs with probability $1 - q$, it has no value for the consumer. (The suitability of a product is an i.i.d. random variable across firms and consumers.) A fraction of consumers need to incur cost $s > 0$ to determine the suitability of any product, while the remaining consumers costlessly observe all product characteristics. Suppose these firms advertise their prices on a price comparison website to which all consumers have access. Since products are otherwise symmetric, as in our model consumers will optimally search through products in order of increasing price displayed on the website, and they will buy the first suitable product they discover. It is worthwhile for a costly-searcher to keep searching for as long as the price is not so high as to eliminate the benefits of search, and a consumer will investigate a product so long as its price satisfies $q(v - p) > s$. This implies that when the fraction of costly searchers is large enough, the maximum price that any firm will charge is

$$p_{\max} = v - \frac{s}{q} \ . \tag{16}$$

This implies that equilibrium profit can be decreasing in the cost of search, s, just as was the case with the Hotelling model we have presented. The reason, however, is quite different. In our framework, a higher search cost increased the benefit to a firm of being searched first by consumers, and this in turn gives firms a more powerful incentive to win the price-setting contest. In Zhang's model, a high search cost reduces the maximum price which firms can offer in (16), since a consumer must be given an incentive to investigate the risky product.[26]

4 Cross-selling

An important set of markets operate in the following manner: consumers buy an initial "core" (or "gateway") product, and then subsequently they need further products which are also potentially provided by the supplier of their core product. One natural example is retail banking: consumers open a standard bank account when young, and when older they need additional products such as a mortgage, insurance, or a savings account. It is often the case that a bank can "cross-sell" these subsequent products to its pool of existing customers. Table 7.1 in OFT (2010) shows how 88% of UK customers who have a savings account will have it at the same bank as their current account, 53% have a credit card issued by the same bank as their current account, and 27% have a mortgage at the same bank as their current account.

There are a number of reasons why cross-selling can be so successful. The firm may have regular interactions with the customer which facilitates more frequent selling opportunities than rival firms have, or the firm may possess information about its existing customers which helps it target suitable products to its customers. Alternatively, the firm

[26] Wilson (2010) presents an interesting variation on the theme of comparison websites. In his model, firms can choose and advertise their firm-specific "search cost", which is the cost which some consumers need to incur in order to understand the firm's price and how to buy from the firm. (Other consumers are well informed and always buy from the cheapestl firm.) Thus, his model could be interpreted as one of "cost-directed search". Wilson describes an asymmetric equilibrium in the duopoly case in which one firm advertises a search cost of zero and the other advertises a high search cost. (He discusses an example where a car insurance company mounted an advertising campaign to alert potential customers that its products will *not* be found on any price-comparison website.) In this equilibrium, the costly searchers investigate the low-search-cost firm, so that this firm is prominent for these consumers. The reason why one firm has a unilateral incentive to "obfuscate" and make it difficult for consumers to comprehend its offering is that, when it does so, costly shoppers are driven to the rival, which induces the rival to compete less hard for the informed pool of consumers. The result is that both firms increase their prices.

may offer bundle discounts when its customers buy several products. In this section, we suggest another reason for the effectiveness of cross-selling, which is that consumers exhibit "default bias" in the sense that they first consider their existing supplier when searching for subsequent products. That is, a firm is prominent for its existing customers when those customers search for additional products.

Consider the following illustrative model. There are many symmetric sellers who compete to sell two products to a population of consumers. Product 1 is the "core" product sold in period 1, and product 2 is an additional item sold in period 2. We suppose that firms cannot commit to their product-2 price in the first period. The discount factor for the second period is δ and the constant marginal cost of producing product $i = 1, 2$ is c_i. As in Armstrong *et al.* (2009), product 2 is assumed to be a search good, and consumers investigate the deals available for that product in a sequential manner. The value of a firm's product 2 is idiosyncratic to consumers, and this value is not observed by consumers at the time they make their product 1 purchase. Specifically, when a consumer investigates a supplier of product 2, she discovers a product with match utility u and price p. Consumers incur the search cost s for investigating each seller of product 2 (including the consumer's supplier of product 1). The match utility u is independently and identically distributed across consumers and across firms and distributed with c.d.f. $F(u)$ and support $[u_{\min}, u_{\max}]$. It is convenient to assume that the hazard rate

$$\frac{f(u)}{1 - F(u)} \text{ increases with } u \ , \tag{17}$$

where $f = F'$ is the density for u. If a consumer buys product 2 from a seller which has match utility u and price p, her surplus is $u - p$. The crucial assumption is that at the start of the second period consumers sample their product 1 supplier first, so that their existing supplier is prominent in a consumer's subsequent product searches. Since, as we will see, a consumer has no strict incentive to search other firms before her own product 1 supplier, this assumption represents a tiny degree of customer inertia or default bias.[27]

In a symmetric equilibrium, each firm will set some price p_1 (to be determined) in the first period. In the second period, firms can potentially set different prices to their existing customers and to customers they poach from rivals. However, as discussed in Armstrong

[27] One way to give consumers a strict incentive to consider their existing supplier first is to assume that they face a smaller search cost when they do so. This might be because the customer already knows that firm's website address, telephone number, or the physical location of the nearest outlet. (See the following footnote for an additional reason why consumers might have a strict incentive to investigate their current supplier first.)

et al. (2009, section 2), when there are many firms, a prominent firm does not want to set different prices to the two groups. In equilibrium, the price for product 2 is

$$p_2 = c_2 + \frac{1 - F(a)}{f(a)} \ , \tag{18}$$

where a is each consumer's threshold for match utility given by

$$\int_a^{u_{\max}} (u - a) dF(u) = s \ , \tag{19}$$

so that the incremental benefit from one more search is equal to the search cost. The size of the markup over cost in expression (18) therefore reflects the magnitude of search frictions in the market for product 2, and given assumption (17) this mark-up increases with s. In the limit as s tends to zero (in which case a tends to $u_{\max}$), the equilibrium price p_2 tends to marginal cost c_2. In equilibrium, $a - p_2$ is a consumer's *ex ante* expected surplus, including her search costs, from participating in the market for product 2.[28]

In the market for product 2, each consumers searches for a product which has match utility u at least equal to the threshold a, starting with her existing supplier for product 1. A customer who purchased a firm's product 1 will therefore buy that firm's product 2 with probability $1 - F(a)$ and so generate expected profit for this firm in the second period equal to $(1 - F(a))(p_2 - c_2) = (1 - F(a))^2 / f(a)$. A customer who purchased a rival firm's core product, however, will generate only negligible profit for the firm in the second period, since there are many alternative sellers that the customer can buy from even if she does not buy from her period-1 supplier.

Consider next the market for product 1. We could also model this as a search market, although this does not add anything of significance to the analysis. Instead, suppose that the market for product 1 is a standard Bertrand market with homogeneous products and full consumer information. Some consumers may be naive or myopic when they buy product 1, and do not recognize that they will likely buy product 2 from the same firm. Such consumers buy their core product purely on the basis of its price. However, even if consumers are forward looking, they anticipate that the product 2 price is as given in (18), regardless of their choice of supplier for product 1. Therefore, these consumers will also base their choice of product 1 purely on the lowest price for product 1 available in the market.

[28] Thus, a consumer finds it worthwhile to engage in search whenever $a - p_2 \geq 0$, which requires that the search cost s not be too large. More precisely, if p_2^M is the monopoly price which maximizes $(p_2 - c_2)(1 - F(p_2))$, then the condition which ensures an active search market is that s is small enough that $a > p_2^M$.

As discussed, each customer generates expected profit in the second period for her core supplier equal to $(1 - F(a))^2/f(a)$. Since this is discounted in period 1 by δ, it follows that Bertrand competition for product 1 leads to the equilibrium price

$$p_1 = c_1 - \delta \frac{[1 - F(a)]^2}{f(a)} \ .$$

Thus, firms in equilibrium offer a below-cost price for the core product in the first period so as to attract more consumers over whom they are prominent in the second period.

The impact of greater search frictions in the market for product 2 is to increase a consumer's cost of searching for a suitable product, to raise the price for product 2, and to decrease the price for product 1. What is the combined effect on consumers? Assuming that product 1 has inelastic aggregate demand, total discounted consumer surplus with search cost s (up to a constant) is $-p_1 + \delta(a - p_2)$. This decreases with s whenever

$$\frac{[1 - F(a)]^2}{f(a)} + \left[a - \frac{1 - F(a)}{f(a)}\right] = a - \frac{F(a)(1 - F(a))}{f(a)}$$

increases with a. However, this is necessarily the case whenever the hazard rate condition (17) is satisfied. Thus, we expect that increased search frictions in the market for product 2 will harm consumers, and policy-makers are justified when they attempt to reduce search frictions.

It is useful to compare the outcome when consumers have this default bias to the situation where there is no such bias and consumers search randomly for product 2. Without a default bias, the equilibrium price for product 2 is unchanged, and given by (18). However, a firm now has no advantage in selling product 2 to its existing product 1 customers, and so there is no incentive to build market share for the core product. Therefore, the equilibrium price for product 1 is simply $p_1 = c_1$, and the impact of this form of default bias is to leave the product 2 product's price unchanged but to *increase* the price for the core product 1. Consumers therefore benefit from this form of inertia.

This outcome has some similarities with markets where consumers incur switching costs when they change their supplier. (See Farrell and Klemperer (2007) for an overview of the literature on switching costs.) In particular, since a firm's existing customers are more likely to buy from it again than are the customers of rival firms, a firm cares about its initial market share and is willing to invest in building up its customer base. Nevertheless, there are significant differences between our approach and markets with switching costs. For instance, in our model customers are not locked-in to their existing supplier in any sense, and a customer incurs no extra cost if she decides to buy subsequent products elsewhere (if the initial search cost in the second period is the same as for subsequent searches). As such, the pattern of prices over time is not so much "bargains then rip-offs" as typically seen in switching cost models, but rather "bargains then the usual price". Moreover, firms have no incentive to set different prices to existing and new customers, and a firm's existing customer base is not exploited.[29] Finally, as discussed above, the presence of inertia benefits consumers in this model. In many two-period switching cost models, consumers are—roughly speaking—left unaffected by the presence of switching costs, since what they gain in the first period is clawed back in the second.

In sum, the model in this section provides a relatively benign rationale for why cross-selling might be prevalent in markets such as retail banking, and why customer inertia might actually boost consumer welfare.

5 Conclusions

In markets with search frictions, a seller can make greater profits when it is "prominent", in the sense that consumers tend to consider its product before they consider rival products. Search costs imply that consumers will buy when a product is merely satisfactory rather than being the best in the market, and this gives an advantage to the first firm in a consumer's search process. This article has discussed three routes by which firms can make their products more prominent in consumer search markets.

First, sellers could pay an intermediary to promote their products. Competition to be prominent induces sellers to pay substantial fees to the intermediary. This can mean that firms are forced to play a prisoner's dilemma and are made worse off compared to a situation with purely random consumer search. In addition, sales commissions raise the marginal cost of supplying a product, and this raises the average retail price paid by consumers relative to random search. Moreover, especially when products are homogeneous, these markets may involve mis-selling, in that consumers who are ignorant or who have high search costs are steered towards the more expensive products. In such cases, the market with sales commission performs poorly for consumers, relative to the case with random search and relative to a situation in which consumers pay the intermediary directly for

[29] If we had a relatively small number of firms, then following the analysis in Armstrong, Vickers, and Zhou (2009), firms would set *lower* period-2 prices to their existing customers than to their rivals' existing customers. This gives a consumer a strict incentive to investigate her existing supplier first. By contrast, see Chen (1997) for a model with switching costs in which firms offer higher prices to their existing customers than they offer to their rival's customers.

advice. Nevertheless, market performance can sometimes be improved when firms pay for prominence, especially when firms are asymmetric in terms of their quality or cost of supply. In such cases, the firm which is willing to pay most to be prominent is often also the firm which consumers would most like to encounter first.

Second, instead of a supplier being made prominent because of costly kickbacks paid to an intermediary, which often results in the prominent firm having the highest retail price, a firm can become prominent if it advertises the lowest retail price. The increasing use of price comparison websites means that many consumers are well informed about prices in the market, and their only search costs involve discovering which products are suitable. In the symmetric model we analyzed, the optimal search strategy for consumers was to investigate products in order of increasing price. Since consumers will only go on to investigate a second product if the cheapest product turns out to be unsuitable, the cheapest firm has a significantly greater market share than its rivals (even if prices are very similar). The resulting intense competition to become prominent drives down retail prices, and consumers benefit when their search process is directed by price advertising. In fact, when search frictions are more pronounced, firms have a greater incentive to be the first firm sampled, and so prices are pushed lower. By comparing the models in sections 2 and 3 we see that different routes to prominence can have very different welfare properties.

Finally, we discussed how the notion of prominence can help to explain the prevalence of cross-selling in markets such as retail banking. We supposed that consumers had a small default bias, in the sense that they first investigated their existing supplier (say, the supplier of their main bank account) when considering a second product (such as a mortgage or credit card). Their existing supplier is therefore prominent in a consumer's subsequent search plans. Because a prominent firm obtains greater profits than its harder-to-find rivals, a firm has an incentive to compete hard in the initial core market, and in our model the price for the initial product was driven below cost. Firms pay for later prominence by subsidizing a consumer's initial purchase.

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TECHNICAL APPENDIX

Proof of Lemma 1: Suppose in contrast to the statement of the result that there exist two pairs (p_1, b_1) and (p_2, b_2) in Ω such that $p_1 < p_2$ and $b_1 > b_2$. Then we have

$$(p_1 - b_1)\left[(1-\lambda)\Pr(\tilde{b} < b_1) + \lambda \Pr(\tilde{p} > p_1)\right] \geq (p_1 - b_2)\left[(1-\lambda)\Pr(\tilde{b} < b_2) + \lambda \Pr(\tilde{p} > p_1)\right]$$

and

$$(p_2 - b_2)\left[(1-\lambda)\Pr(\tilde{b} < b_2) + \lambda \Pr(\tilde{p} > p_2)\right] \geq (p_2 - b_1)\left[(1-\lambda)\Pr(\tilde{b} < b_1) + \lambda \Pr(\tilde{p} > p_2)\right] .$$

(Here, these hold with equality if (p_1, b_2) and (p_2, b_1) also lie within Ω.) The first inequality implies

$$\frac{\lambda}{1-\lambda}(b_2 - b_1)\Pr(\tilde{p} > p_1) \geq (p_1 - b_2)\Pr(\tilde{b} < b_2) - (p_1 - b_1)\Pr(\tilde{b} < b_1) ,$$

and the second one implies

$$\frac{\lambda}{1-\lambda}(b_1 - b_2)\Pr(\tilde{p} > p_2) \geq (p_2 - b_1)\Pr(\tilde{b} < b_1) - (p_2 - b_2)\Pr(\tilde{b} < b_2) .$$

Adding the pair of inequalities yields

$$\frac{\lambda}{1-\lambda}(b_1 - b_2)\left[\Pr(\tilde{p} > p_2) - \Pr(\tilde{p} > p_1)\right] \geq (p_2 - p_1)[\Pr(\tilde{b} < b_1) - \Pr(\tilde{b} < b_2)] .$$

This inequality, however, cannot hold since the left-hand side is negative while the right-hand side is positive, which leads to a contradiction. ■

Proof of Proposition 1: Given that firm 2 adopts the equilibrium strategy, firm 1's expected profit, if it sets (p, b) with $0 \le b < p \le v$, is

$$\pi(p,b) = (p-b)\left[(1-\lambda)G\left(B^{-1}(b)\right) + \lambda(1-G(p))\right] ,$$

where

$$B^{-1}(b) = (1-\lambda)v + \frac{\lambda}{1-\lambda}b .$$

A sufficient condition for no profitable deviation is that for any $p \le v$, $\pi(p,b)$ is concave in b and reaches its maximum at $b = B(p)$.

One can verify that $\pi_b = 0$ at $b = B(p)$, and

$$\pi_{bb} = -2(1-\lambda)g\left(B^{-1}(b)\right)\frac{dB^{-1}(b)}{db} + (1-\lambda)(p-b)g'\left(B^{-1}(b)\right)\left(\frac{dB^{-1}(b)}{db}\right)^2 .$$

Then

$$\pi_{bb} < 0 \Leftrightarrow (p-b)\frac{g'\left(B^{-1}(b)\right)}{g\left(B^{-1}(b)\right)}\frac{dB^{-1}(b)}{db} < 2 . \tag{20}$$

From the expression for $G(p)$ in (1), we have

$$g(p) = \frac{(1-\lambda)\lambda^2 v}{\left[\lambda^2 p + (1-\lambda)^2(v-p)\right]^2}; \quad g'(p) = \frac{2(1-\lambda)(1-2\lambda)\lambda^2 v}{\left[\lambda^2 p + (1-\lambda)^2(v-p)\right]^3} ,$$

and so

$$\frac{g'(p)}{g(p)} = \frac{2}{\frac{(1-\lambda)^2}{1-2\lambda}v - p} .$$

By using the expression for $B^{-1}(b)$, (20) holds if

$$\frac{p-b}{\frac{1-\lambda}{1-2\lambda}v - \frac{b}{1-\lambda}} < 1-\lambda .$$

When the denominator in the left-hand side is negative (e.g., when $\lambda > \frac{1}{2}$), this condition must hold. Now suppose the denominator is positive (so λ must be less than $\frac{1}{2}$). Then the condition holds if

$$p < \frac{(1-\lambda)^2}{1-2\lambda}v .$$

This is true since the right-hand side is an increasing function of λ for $\lambda < \frac{1}{2}$ and equals v at $\lambda = 0$. ■

Proof of Proposition 2: The four equilibrium parameters $p_{\min}$, $p_{\max}$, $\bar{p}$, and Π are determined by the four equations in (10), (12) and (15). To solve this system, we introduce a new variable

$$x = \frac{p_{\min}}{p_{\max}} \ .$$

In the following, we will first solve $\bar{p}$ and Π as functions of x from (10) and (15), and then substitute them into (12) to obtain an equation determining x.

Using (15) we can rewrite (10) as

$$2p_{\max} + 2s = p_{\min} + \frac{p_{\max}^2}{p_{\min}} \Rightarrow 2 + \frac{2s}{\sqrt{2t\Pi}} = x + \frac{1}{x} \Rightarrow \Pi = \frac{2s^2}{t(x+1/x-2)^2} \ . \tag{21}$$

From (15) we obtain

$$\bar{p} = 2\sqrt{2t\Pi} + s - t = \frac{4s}{x+1/x-2} + s - t \ . \tag{22}$$

We then rewrite condition (12), by using (15) again, as

$$2s\bar{p} = p_{\max}^2 \cdot \log \frac{p_{\max}}{p_{\min}} - \frac{1}{2}(p_{\max}^2 - p_{\min}^2) \Rightarrow -\frac{2s}{t}\frac{\bar{p}}{\Pi} = 1 - x^2 + 2\log x \ . \tag{23}$$

Substituting (21) and (22) into (23) yields

$$\frac{t-s}{s} = \frac{4 + \frac{1-x^2+2\log x}{x+1/x-2}}{x+1/x-2} \ . \tag{24}$$

The right-hand side of (24) increases with x and tends to zero as $x \to 0$ and tends to $+\infty$ as $x \to 1$. Hence, expression (24) must have a unique solution in the interval $x \in (0,1)$ whenever $t > s > 0$.

To ensure that this equilibrium is well defined, we need to check the condition $p_{\max} - p_{\min} < t - s$ because our equilibrium characterization is predicated on that. This condition holds if and only if $1 - x < (t-s)/p_{\max}$. Since $p_{\max} = \sqrt{2t\Pi} = 2s/(x+1/x-2)$, the condition can be written as

$$1 - x < \frac{t-s}{2s}\left(x + \frac{1}{x} - 2\right) \ .$$

Using (24), this is equivalent to

$$-\frac{1-x^2+2\log x}{2(x+1/x-2)} < 1 + x \ .$$

The left-hand side is an increasing function on $x \in [0,1]$, and it approaches 1 as x tends to 1. So the condition holds for any $x \in (0,1)$. ■

Proof of Corollary 1: (i) Since the right-hand side of (24) is increasing in x, it follows that $x = p_{\min}/p_{\max}$ must decrease with s. Since $4 + \frac{1-x^2+2\log x}{x+1/x-2}$ is decreasing in x and so increasing in s, it follows from (24) that $\frac{1}{s}(x + 1/x - 2)$ increases with s. Thus, (15) implies that both $p_{\max}$ and Π decrease with s. As $x = p_{\min}/p_{\max}$ decreases with s, $p_{\min}$ must fall with s as well. Finally, from (23), we obtain

$$\begin{aligned}
\bar{p} &= -\frac{t\Pi}{2s}(1 - x^2 + 2\log x) \\
&= \left(\frac{s}{x + 1/x - 2}\right)\left(-\frac{1 - x^2 + 2\log x}{x + 1/x - 2}\right) .
\end{aligned} \tag{25}$$

(We used (21) to obtain the second equality.) We know both terms in (25) decrease with s, and therefore $\bar{p}$ does too.

(ii) Intuitively, as $s \to 0$ the situation approaches the perfect information case and so both $p_{\min}$ and $p_{\max}$ tend to t. The result follows as $p_{\max}$ decreases with s. More formally, from (24), we can see that $x \to 1$ as $s \to 0$. From (24), we also have

$$\left(\frac{t}{s} - 1\right)\left(x + \frac{1}{x} - 2\right) = 4 + \frac{1 - x^2 + 2\log x}{x + 1/x - 2} .$$

Thus, as $s \to 0$, $\frac{t}{s}(x + 1/x - 2) \to 2$ which implies $p_{\max} = 2s/(x + 1/x - 2) \to t$. ■