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Trend inflation and Monetary policy rules: Determinacy analyses in New Keynesian model with capital accumulation

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Abstract

The effects of positive trend inflation is analyzed in the framework of the standard New-Keynesian model with Calvo price setting and capital accumulation. It is build on the work of Duport (2001) and Ascari and Ropele (2007) who separately considered effects of capital accumulation and trend inflation in the similar context. It is shown that the simultaneous presence of positive inflation and capital accumulation greatly affects determinacy property of equilibrium in this setup. Namely, in order to maintain stability in addition to actively react to inflation monetary authorities should react to output fluctuations but not to a great extend. Overreaction to output may lead to indeterminacy. We also show that for a large set of plausible parameters standard Taylor rule leads to indeterminacy. Alternative monetary policy rules are also analyzed.

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1 Introduction

Most of the existing New Keynesian general equilibrium literature within Calvo-pricing framework ignores a positive trend in the inflation process. Some papers, e.g. Clarida et al (2000) assume log-linearization around zero inflation steady state. The main reason for this assumption is the analytical convenience. But it is obvious that equilibrium inflation even in the developed countries is positive. For example, Schmitt-Grohe and Uribe (2007) used the post-war data for the US GDP deflator and estimated the steady state inflation level at about 4.2

A popular technique to eliminate steady state inflation in the equilibrium solution was proposed by Yun (1996). Yun assumed that firms which do not have an opportunity to re-adjust prices simply index them by the steady state inflation. Christiano et al (2001) proposed indexation by the previous period's inflation. But these assumptions seem to be empirically non-realistic: it was shown that not all firms adjust prices each period. Bils and Klenow (2004) produced an empirical study and showed that many prices remain constant during a long period of time. More importantly, from the theoretical point of view the concept of price indexation is not in line with the idea of "sticky" prices: the menu costs of price

adjustment are still significant. Also, there is no such a phenomenon in the state-dependant sticky pricing literature.

The most recent literature deals with the problem in two ways. Some papers propose partial indexation based on the idea that firms face not only menu costs but also information costs. As a result, some fraction of them decides to avoid high information costs and to re-adjust prices by the known inflation of the previous period. But Cogley and Sborne (2008) have shown that assumption of stationary trend inflation eliminates the significance of backward-looking indexation component in estimated Phillips curve.

Another stand of emerging literature deals with positive trend inflation without any indexation. Ascari (2004) derived the New-Keynesian Phillips curve (NKPC) in the presence of low positive trend in the inflation process. His major result is that the current inflation depends not only on the expectations of the next period inflation but also on inflation expectations over the longer horizon. He also showed that non-zero trend influences the dynamics of the solution of the general equilibrium model with staggered Calvo and Taylor - type pricing. Cogley and Sborne (2008) estimated modified NKPC and showed that it does not require backward-looking component - the positive trend in inflation process itself generates enough persistence in the model. Finally, Ascari and Ropele (2007) generalized the optimal monetary policy literature in the case of low trend inflation. One of their main results is that the presence of low trend inflation has an impact on the determinacy of rational expectations equilibrium.

Hornstein and Wolman (2005) and Ascari and Ropele (2009) analyze Taylor principle implication under low positive trend inflation. Ascari and Ropele (2009) and Coibion and Gorodnichenko (2008) emphasized potentially destabilizing role of central bank's policy based on responding to output gap. And from the point of uniqueness of equilibrium better policy for monetary authorities is adding interest rate smoothing component to the rule or responding to output growth rather than output gap in the policy rule.

This paper follows works by Ascari and Ropele (2007) and Coibion and Gorodnichenko (2008) and aims to find out what are the restrictions for monetary authorities in presence of capital accumulation process on policy rules to guarantee the determinacy of equilibrium. It studies the impact of the trend inflation on equilibrium when the monetary policy is conducted via simple interest rules (Taylor rule). We analyze the general solution in the model with capital accumulation process. Alternative monetary policy rules are also analyzed.

The idea of "active" monetary policy rule to be used by Central Bank is very popular for more than a decade since the paper by Kerr and King (1996). Active monetary rule is the one which responds to inflation with more than one-to-one changes in interest rate. This type of policy guarantees the uniqueness of the equilibrium solution of the system for standard New Keynesian model with capital accumulation. But this class is not by any mean complete. For example, Duport (2001), Carlstrom and Fuerst (2005) argued that "activeness" of monetary policy rule is not enough to avoid indeterminacy in the presence of pre-determined variables in the model. A combination of endogenous capital accumulation and sticky prices can also lead to multiple equilibria. As a result monetary policy rule which guarantees the uniqueness

of the solution is more sophisticated than simple "active" one. Dupor (2001) in continuous-time model demonstrates that a passive monetary rule is necessary and sufficient for local equilibrium determinacy. Carlstrom and Fuerst (2005) in discrete-time model have shown that monetary policy must respond very aggressively to inflation to generate determinacy. It is worth mentioning that all the research cited above was made within the framework of zero trend inflation.

To our knowledge Hornstein and Wolman (2005) is the only paper which simultaneously analyzes the presence of capital and positive trend inflation in the model. The difference of their analyses from ours is that they utilized firm-specific capital in the model. The resulting model became too complicated to be solved under the assumption of Calvo-pricing scheme. That is why all their analysis is made in the framework of Taylor (1980) type staggered prices. Their results perfectly correspond the existing literature: small positive inflation changes the region of parameters under which rational expectations equilibrium is unique.

In this paper we analyze the effect of the simultaneous presence of the trend inflation and capital accumulation on the determinacy of the general equilibrium solution. For this purposes we derive and calibrate a standard New-Keynesian model in continuous time. As in the most of the existing literature, monetary authorities stabilize inflation and output gap around long-run targets using nominal interest rate.

The main findings of the paper are the following. First, there is a significant set of plausible parameters in which the equilibrium is indeterminate under active policy. Second, the monetary policy should be not "very" active in order to guarantee the determinacy of the solution. Third, if the policy is conducted with incorporation of interest rate smoothing - passive monetary policy rule is preferable for determinacy reasons. But responding to output growth rather than to output gap in the policy rule requires more than one to one change in interest rate in response to inflation.

The rest of the paper is organized as follows. Section 2 describes the theoretical model with capital accumulation process. Section 3 provides analysis of the model. Section 4 analyses the determinacy under alternative policy rules: interest rate smoothing and output growth targeting. Then, section 5 concludes.

2 The Model

To analyze the consequences of positive trend inflation, we adopt the framework of Blanchard and Kiyotaki (1987), developed in Woodford (2003) and Gali (2008) and add capital accumulation process. The model is the standard version of sticky prices general equilibrium model. The model economy is composed of continuum of infinitely-living consumers-producers of one final and continuum of intermediate goods. We use standard functional forms for preferences and technology and assume perfectly competitive labor market.

2.1 Households

The representative infinitely-lived household seeks to maximize the objective function:

$$\int_0^{\infty} \left(\ln C_t - \frac{L_t^{1+s}}{1+s} \right) \exp(-\theta_t) dt \quad (2.1)$$

where C_t is the consumption of the single final good, and L_t denotes hours of work or employment, θ is a discount factor and s is the inverse of the labor supply elasticity. The instantaneous utility function is additively separable in consumption and labor.

The budget constraint takes the form

$$\left(\frac{\dot{B}}{P_t} + \dot{K} \right) \leq W_t L_t - C_t + (r + \delta) K_t + \frac{i_t B_t}{P_t} + T_t \quad (2.2)$$

where i is an interest rate, B_t is a quantity of one-period, nominally riskless discount bonds, K_t is aggregate capital in the economy, W_t is real wage, $r + \delta$ is real user cost of capital and T_t represents lump-sum additions or subtractions to the income, expressed in real terms. The maximization of (2.1) subject to (2.2) leads to the following solution:

$$W_t = L_t^s C_t \quad (2.3)$$

$$\frac{\dot{C}}{C_t} = i_t - \pi_t - \theta \quad (2.4)$$

$$r_t = i_t - \pi_t \quad (2.5)$$

where $\pi_t = \frac{\dot{P}}{P_t}$ is the inflation rate.

2.2 Firms

The economy produces a single final good and a continuum of intermediate goods indexed by j where j is distributed over the unit interval ($j \in [0, 1]$). The production of final good is perfectly competitive and is subject to the following production function

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{\eta-1}{\eta}} dj \right]^{\frac{\eta}{\eta-1}} \quad (2.6)$$

where Y_t is the quantity of the final good produced, $Y_{t,j}$ is the quantity of intermediate good produced by the firm j and η stands for elasticity of substitution between different inputs. As it was mentioned above, the final good is consumed by the household.

Perfect competition in a final good's sector ensures that the demand which faces the firm j is given by

$$\frac{Y_{t,j}}{Y_t} = \left(\frac{P_{t,j}}{P_t} \right)^{-\eta} \quad (2.7)$$

$P_{t,j}$ is a price of intermediate good j and P_t is an aggregate price which takes the following form

$$P_t = \left[\int_0^1 P_{j,t}^{\frac{1}{\eta-1}} dj \right]^{\eta-1} \quad (2.8)$$

There is a monopolistic competition in the markets for intermediate goods: each intermediate good is produced by a single firm subject to the constant return to scale production function

$$Y_{j,t} = K_{j,t}^\alpha L_{j,t}^{1-\alpha} \quad (2.9)$$

where $L_{j,t}$ is the labor-input and $K_{j,t}$ is the capital input for the production of the firm $Y_{j,t}$ and in the equilibrium

$$L_t = \int_0^1 L_{j,t} dj \quad (2.10)$$

$$K_t = \int_0^1 K_{j,t} dj \quad (2.11)$$

The aggregate capital accumulation process is given by

$$\dot{K} = Y_t - C_t - \delta K_t \quad (2.12)$$

where δ is depreciation rate.

As in Calvo (1983), firms are not allowed to change their prices unless they receive a random "price change signal". It is expressed by exogenous Poisson process with arrival rate ψ and expected time between price changes $\frac{1}{\psi}$. At the moment of realization t_0 the probability that the firm will not have an opportunity to readjust prices during the time period is given by

$$\psi \int_{t_0}^{t_0+\Delta t} \exp(-\psi(\tau - t_0)) d\tau = 1 - \exp(-\psi\Delta t)$$

and is independent of t_0 .

A firm chooses its reset price $X_{j,t}$ to maximize the present value of all future profits subject to Calvo pricing scheme

$$\int_{t_0}^{\infty} \left[\frac{Y_{j,t} X_{j,t} - C(Y_{j,t})}{P_t} \right] \frac{U'(C_t)}{U'(C_{t_0})} \exp(-\psi(t - t_0)) \exp(-\theta(t - t_0)) dt \quad (2.13)$$

and

$$C(Y_{j,t}) = MC_t Y_{j,t} \quad (2.14)$$

$$\frac{\partial Y_t}{\partial K_t} = \frac{(r_t + \delta)}{MC_t} \quad (2.15)$$

$$\frac{\partial Y_t}{\partial L_t} = \frac{W_t}{P_t} \frac{1}{MC_t} \quad (2.16)$$

where $C(Y_{j,t})$ is a cost function and MC_t is real marginal cost which in case of constant return to scale production function equal to average cost of production.

The maximization problem leads to the following solution

$$X_{j,t} = \frac{\eta}{\eta - 1} \frac{\int_t^\infty e^{-(\theta+\psi)(\tau-t)} P_\tau^\eta MC_\tau (1/C_\tau) d\tau}{\int_t^\infty e^{-(\theta+\psi)(\tau-t)} P_\tau^{\eta-1} (1/C_\tau) d\tau} \quad (2.17)$$

where $X_{j,t}$ denotes a reset price. Using definition of $X_{j,t}$ one can rewrite (*asa function of reset – price* :

$$P_t^{1-\eta} = \psi \int_{-\infty}^t e^{\psi(\tau-t)} X_{j,t}^{1-\eta} d\tau \quad (2.18)$$

2.3 Policy

Fiscal policy is conducted with lump-sum taxes and subsidies so that $B_t = 0$. Monetary policy is conducted using simple Taylor-type interest rule. Namely, monetary authorities target inflation and output to their steady state levels.

$$i_t - i^* = (1 + a_1) (\pi_t - \pi^*) + a_2 (\ln Y_t - \ln Y^*) \quad (2.19)$$

where i^* , π^* , Y^* are non-stochastic steady states levels of interest rate, inflation and output.

2.4 Non-stochastic steady state

Equilibrium is given by 3-13 and 15-20. Non-stochastic steady-state is derived from the assumption that $\dot{C} = 0$ and $\dot{X} = \dot{P} = \pi^*$. The steady states levels of variables i^* , MC^* , $\frac{Y}{K}^*$ and $\frac{X}{P}^*$ are given by

$$i^* = \theta + \pi^* \quad (2.20)$$

$$\frac{X^*}{P} = \left(\frac{\psi}{\psi - \pi(\eta - 1)} \right)^{\frac{1}{\eta-1}} \quad (2.21)$$

$$MC^* = \left(\frac{\eta - 1}{\eta} \right) \left(\frac{\theta + \psi - \pi\eta}{\theta + \psi - \pi(\eta - 1)} \right) \left(\frac{X}{P} \right)^* \quad (2.22)$$

$$\left(\frac{Y}{K} \right)^* = \frac{\theta + \delta}{\alpha} \left(\frac{1}{MC} \right)^* \quad (2.23)$$

$$\left(\frac{C}{Y}\right)^* = 1 - \frac{\delta}{\left(\frac{Y}{K}\right)^*} \quad (2.24)$$

As shown in Ascari and Ropele (2007) the presence of steady-state inflation lowers Y^* .

2.5 Log-linearization

Standard log-linearization procedure of equations 3, 4, 6 and 15-21 around non-stochastic steady-state, described in the previous section leads to the following system

$$\dot{x}_t = (\psi + \theta - \eta\pi^*)(xp_t - mc_t) + \pi^*(1 - \eta)z_t \quad (2.25)$$

$$\dot{p}_t = (\psi + (1 - \eta)\pi^*)(xp_t) \quad (2.26)$$

$$\dot{c}_t = i_t - \pi_t \quad (2.27)$$

$$i_t = (1 + a_1)\pi_t + a_2y_t \quad (2.28)$$

$$\dot{k}_t = \left(\frac{Y}{K}\right)^* y_t - \left(\frac{C}{K}\right)^* c_t - \delta k_t \quad (2.29)$$

$$r_t = i_t - \pi_t \quad (2.30)$$

$$y_t - l_t = w_t - mc_t \quad (2.31)$$

$$y_t - k_t = \frac{r_t}{\theta + \delta} - mc_t \quad (2.32)$$

$$mc_t = \alpha r_t + (1 - \alpha)w_t \quad (2.33)$$

$$w_t = c_t + sl_t \quad (2.34)$$

$$\dot{z}_t = -\pi_t + (\theta + \psi - \pi^*(\eta - 1)z_t) + \frac{\dot{c}_t}{\eta - 1} \quad (2.35)$$

where $z_t = (\theta + \psi - \pi^*(\eta - 1)) \int_t^\infty \exp - (\theta + \psi - \pi^*(\eta - 1)) \left(p_\tau - p_t - \frac{c_\tau - c_t}{\eta - 1} \right) d\tau$

and the lower case letters denote logs of initial values in deviations from their steady-state levels except interest rates and inflation which are deviations from steady-state in levels. For nominal variables (x and

p) small case stands for log deviations from balanced-growth path levels deflated by steady state inflation level. For example, $x_t = \log[X_t \exp(-\pi^* t)]$

New-Keynesian Phillips curve is derived from 26-27 and takes the following form, analogously to Ascary (2004) in discrete time:

$$\dot{\pi}_t = (\theta - \pi^*)\pi_t - \varphi m c_t - \varphi' \pi^* (\eta - 1) z_t \quad (2.36)$$

$$\varphi' = \psi - \pi^* (\eta - 1), \varphi = \varphi' (\varphi' + \theta - \pi^*) \quad (2.37)$$

As in Ascary (2004) today's inflation depends not only on expectations of its change today which is the case of convenient Philips curve but on expectations of all path of future inflation

The equilibrium solution of (27-37) results in the following system:

$$AX = BY$$

$$\text{where } Y_t = \begin{bmatrix} k \\ c \\ \pi \\ z \end{bmatrix}_t, X_t = \begin{bmatrix} l \\ w \\ r \\ i \\ mc \\ y \end{bmatrix}_t$$

$$A = \begin{bmatrix} -s & 1 & 0 & 0 & 0 & 0 \\ -(1-\alpha) & 0 & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & \frac{-1}{\theta+\delta} & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -a_2 \end{bmatrix} B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1+a_1 & 0 \end{bmatrix}$$

Given equations

$$\dot{Y} = CY + DX = JY \quad (2.38)$$

$$\text{where } C = \begin{bmatrix} -\delta & -(\frac{C}{K})^* & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & \theta - \pi^* & -\pi^* (\eta - 1) \varphi' \\ 1 & 0 & \frac{a_1}{\eta - 1} - 1 & \varphi' + \theta \end{bmatrix} D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & (\frac{Y}{K})^* \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\varphi & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{a_2}{\eta - 1} \end{bmatrix}$$

and $J = C + DA^{-1}B$.

3 Analysis of determinacy

It is considered that an adequate monetary policy rule should imply a unique equilibrium. The logic is the following. If the policy conducted does not lead to unique equilibrium random shocks which are

not connected with fundamentals of the economy may affect its dynamics. In other words, unnecessary fluctuations may be introduced to the economy and may increase its volatility. So, it may be desired that optimal monetary policy should not give rise to indeterminacy of equilibrium.

3.1 Methodology

According to Blanchard and Kahn (1980) the system of equations described above has a unique solution if and only if the number of eigenvalues of matrix J with negative real part is equals to the number of pre-determined variables. With the presence of capital accumulation process capital is the only predetermined variable in the model. So, the equilibrium of the model is unique if and only if one eigenvalue of matrix J has a negative real part. If the number of such eigenvalues (k) is greater than one, the equilibrium is indeterminate and the dimension of equilibrium space equals to $k-1$.

The intuition behind the fact that the presence of the steady inflation can affect the determinacy of equilibrium is easily seen by looking at derived Philips curve. When inflation is greater then the discount factor the slope of Phillips curve (37) changes and become negative and thus Blanchard-Kahn condition may be violated: there is a large chance that inflation process may become stationary autoregression under virtually any expectation formation mechanism and this fact may bring about multiplicity of equilibria.

The solution of the system (38) even if can be derived analytically is too complicated to provide intuition. That is why the solution of the model was obtained using numerical methods with the standard calibration of parameters.

3.2 Calibration of basic parameters

Models parameters are calibrated according to values accepted as plausible in the business cycles literature: α , the Cobb-Douglas parameter of production function, is fixed at $1/3$; δ , depreciation of capital rate, is set to 0.08 per annum and θ , consumer's discount factor, is assumed to be 0.02 per annum. For the analytical simplicity, consumer's instantaneous utility function is linear with respect to labor: in this case s equals 0 which means that labor supply is infinitely elastic as for example in Hansen's indivisible labour model (1985).

According to the recent empirical findings by Bils and Klenow (2004) and Nakamura and Steinsson (2008) we set the Poisson parameter of Calvo scheme (ψ) to correspond to opportunity for firms to change their prices every 6 and 12 month. The same logic was used in Coibon and Gorodnichenko (2008). The shorter time period was proposed in the first paper (4-5 months) and longer time horizon (10-11 months) corresponds with recent paper by Nakamura and Steinsson. We also adopt two different values of firm's steady-state mark-ups: 10% and 20%. This means that corresponding values of η equals to 11 and 6. To illustrate how the increase in steady state inflation influences the determinacy of equilibrium we analyze the steady state inflation level of 2%, 4% and finally 7%.

Finally, the solution of the system (38) was obtained according to the calibration of basic parameters presented in Table1:

Table 1: Calibration of basic parameters

Parameter	Definition	Value
π^*	Steady-state inflation rate	2%; 4%; 7%
ψ	Inverse of average time of price fixity	1; 2
η	Elasticity of substitution among intermediate goods	11;6
α	Capital share in output	1/3
δ	Depreciation rate	0.08
s	Inverse of labor supply elasticity	0
θ	Discount factor	0.02

3.3 Analysis of indeterminacy regions and policy implications

The results of solution are presented on Figures 1-4. The axes are a_1 and a_2 values—the monetary policy rule coefficient

To compare and illustrate how our results corresponds to the main finding by Duport (2001) and Carlstrom and Fuerts (2005) first of all we analyze the case of zero-trend inflation. The first graph on Figures 1-4 represents eigenvalues regions for this case and we are interested in colored zone - determinacy. One can see that monetary policy which responds (actively) only to inflation variation leads to indeterminate equilibrium. To guarantee the uniqueness of equilibrium monetary authorities should also respond to variations in output. And the case of passive to the inflation policy rule and no reaction to the output gap provides the economy with a determinate equilibrium. This result perfectly corresponds with one presented in papers stated above for the continuous time case.

From Figures 1-4 one can see that monetary policy parameter's regions which guarantee the determinacy of equilibrium become smaller with an increase of steady state inflation. Higher firm's mark-ups lead to broader range of possibilities for monetary authorities to respond to variation in output and inflation which leads to a unique equilibrium.

The result, that active monetary policy rule in response to inflation does not provide a determinant equilibrium, obtained for zero trend inflation holds. In absence of the policy reaction to output fluctuations and active in response to inflation the economy is guaranteed to be in sunspot equilibrium. According to results presented on Figures 1-4, if trend inflation is greater than discount rate monetary authorities should react not too active to changes in output. The idea that monetary authorities still should react to output gap contradicts findings of Ascary and Ropele (2009) but they did not include capital accumulation process in the theoretical model. Also, our result about the reaction to output extends the analysis of Duport (2001) who argued that the equilibrium is determinant once the reaction to inflation by interest

rate is lower than one to one (as graphs for zero inflation case clearly show). We argue that the reaction parameter to the output gap matters: if it is too large then either equilibrium becomes indeterminate again. The policy should not overreact to fluctuations of output.

”FIGURES 1 2 3 4”

Taylor (1993) proposed a monetary policy rule of the form $a_1 = 0.5$ and $a_2 = 0.5$ for annual data. For most parameter calibrations stated above this ”classical” Taylor rule does not lead to a unique equilibrium for trend inflation greater than 2%. This means that the policy based on Taylor principle does not necessarily lead to determinant equilibrium. And as it was stated above good policy should not lead to indeterminacy of equilibrium. This finding extends to all the results from previous literature: Taylor principle does not eliminate indeterminacy in presence of non-zero trend inflation.

Indeterminacy leads to different equilibrium paths for the same fundamentals. So in the indeterminate case there may exist equilibrium which is closer to empirical results that in determinacy case.

4 Alternative monetary policy rules

As it was shown above the Central bank’s policy, based on the Taylor rule does not always provide a determinacy. For a large set of parameters of the rule this policy leads to unpleasant result: the economy may be subject to additional fluctuations. And to use this kind of policy rule to provide the economy with a determinate equilibrium policy makers should have very precise knowledge about the calibration parameters of model which are suitable for their special case. The region of the coefficients of the rule which guarantees the unique and determinate equilibrium is highly dependant on the calibration of the basic model and especially on the level of trend inflation.

The next step is to analyze how the presence of the trend inflation in the model with capital accumulation process affects the determinacy of the solution when the policy is conducted by other types of monetary rules. First, we analyze whether adding interest rate smoothing to the standard Taylor rule affects the determinacy regions. Then the policy rule which responds to inflation and output growth (instead of output gap) deviations from their steady state levels will be examined.

4.1 Interest rate smoothing

The idea of adding the smoothing component into the basic Taylor rule is quite common. So, we are going to find out how this component affects the determinacy of the model solution. This means that the log-linearized policy rule for the continuous time case, expressed in equation 29, is substituted by

$$\dot{i}_t = \lambda((1 + a_1)\pi_t + a_2y_t - i_t) \tag{4.1}$$

where λ stands for level of smoothing and $\lambda = -\ln(\rho)$ where ρ is a standard degree of smoothing degree is a standard degree of smoothing which is assumed in discrete time models of the type $i_t =$

$\rho i_{t-1} + (1 - \rho)(basicrule)$. The case of no smoothing means that λ is going to infinity.

The determinacy analysis is the same as discussed above. The new policy rule is added to the system and λ is assumed to be equal 1, which means that ρ is about 0.8 (for the quarterly data) which is widely assumed in the literature. Other model parameters are calibrated according to the Table 1.

”FIGURES”

The solution of the new system is presented on the Figure 6-7, the case with firms mark-ups equal to 20% is not presented because it does not differ from the case of 10%. The shaded area on the Figures 6-7 represents the determinate equilibria. The white one stands for set of monetary rules parameters which either does not lead to the model solution or the solution is indeterminate.

The results do not crucially differ from the ones without interest rate smoothing. As in Duport (2001) for zero inflation case the passive monetary policy rule in respect to inflation guarantees the existence of unique and determinate equilibrium.

For low levels of trend inflation only passive monetary rule guarantees the determinacy of equilibrium. But with an increase in the level of trend inflation the active monetary rule also becomes suitable for the determinacy reasons although it also requires some level of response to output gap changes as can be seen from Figures 6-7. The regions with determinant equilibrium are much wider than ones without interest rate smoothing. It can be concluded that if the central bank uses interest rate smoothing in its policy rule the passive monetary rule ($a_1 < 0$) is more preferable for determinacy reasons. Monetary authorities can also afford any level of response to output gap fluctuations ($a_2 > 0$) conducting monetary policy. So, interest rate smoothing helps to improve the determinacy of the model solution for a larger set of parameters of monetary policy rule.

4.2 Output growth targeting

An alternative to the classic Taylor rule is proposed by Walsh (2003) and Orphanides and Williams (2006): responding to output growth rather than to output gap. Coibon and Gorodnichenko (2008) have shown that this type of policy rule can help to ensure determinacy in the model without capital accumulation process: determinacy is guaranteed if monetary authorities respond to inflation and output growth by interest rate greater than one to one.

In order to find out how this policy rule affects the determinacy of solution we rewrite the policy rule (29) in the following form

$$i_t = (1 + a_1)\pi_t + a_2\dot{y}_t \tag{4.2}$$

where $a_2 \neq 0$.

The system of equations 28, 29” and 30-37 is resolved with the basic calibration stated in Table 1. We are interested only in the region of monetary policy rules parameters which guarantees the determinacy. In these terms the result is the same for different calibrations of the parameters of the economy. Figure 8 represents the solution for the firm’s mark-ups set to 20% and assumption that firms change their prices

every 12 month. In case of policy based on output growth targeting it is clear from the Figure 8 that the response by the interest rate greater than one to one to the inflation ($a_1 > 0$) and any positive response to output growth ($a_2 > 0$) provide the economy with determinacy. This differs from the results for the model without capital accumulation process: the response to output growth should be more than one to one for the same reasons. On the other hand, any passive to inflation policy rule leads to indeterminacy or to absence of the solution. Surprisingly, the level of trend inflation in case of this particular policy rule does not influence the determinacy regions.

5 Conclusions

This paper investigates the influence of positive trend inflation on the equilibrium determinacy in the typical New-Keynesian model with capital accumulation process. In this model the monetary policy is conducted using simple interest rates rule. The recent findings in literature that "active" monetary policy rule does not guarantee the uniqueness of the equilibrium correspond to our results. We show that presence of capital accumulation affects the indeterminacy region. In short, the result of Duport (2001) that determinacy is restored under policy which is passive in response to inflation. We show that reaction to output gap should be very accurate: the response by interest rate more than one to one in change in inflation requires positive and significant reaction to output gap in order to guarantee the determinacy. We show that if the reaction is larger than some value of this parameter the equilibrium is also indeterminate. In other words, the response to output should be active; but not too active.

We show how the model's results change if monetary authorities conduct policy using interest rate smoothing in their rule. The results also coincide findings by Duport (2001) and propose passive monetary policy rule in respect to inflation changes. Reaction to output growth rather than output gap in the policy rule is also considered. The results differ from ones obtained for the model without capital accumulation by Coibon and Gorodnichenko (2008). Any positive response to output growth in addition to active policy in response to inflation guarantees unique solution.

Monetary policy rule's parameters which lead to determinacy depend on the level of trend inflation (inflation in the steady state). Higher levels of trend inflation reduce the space of parameters value suitable for the unique solution of the model. We have also shown that a "classical" Taylor rule leads to indeterminate equilibrium for a wide range of plausible values of model's parameters. We also add some fundamental disturbances to the model and find out that the reaction of output to these shocks looks alike the empirical results of most of the literature. But to justify the empirical relevance the country's case study will be helpful. So, the future research will be devoted to the comparison of models simulation results with the empirical result for the particular country with inflation target of Central Bank greater than 2%.

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Figure 1. "Mark-up 10%, firms change prices every 6 month"

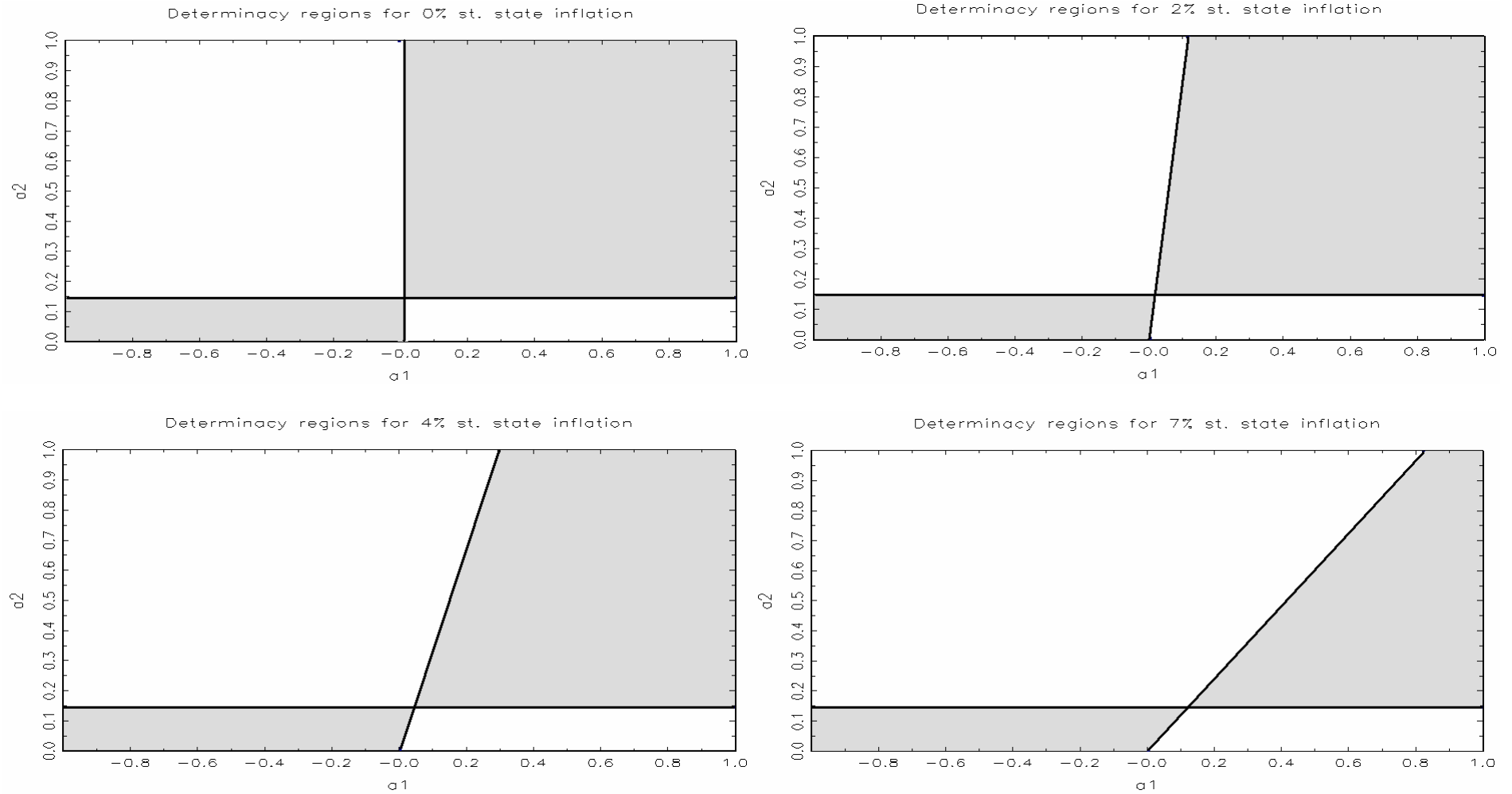


Figure 2. “Mark-up 20%, firms change prices every 6 month”

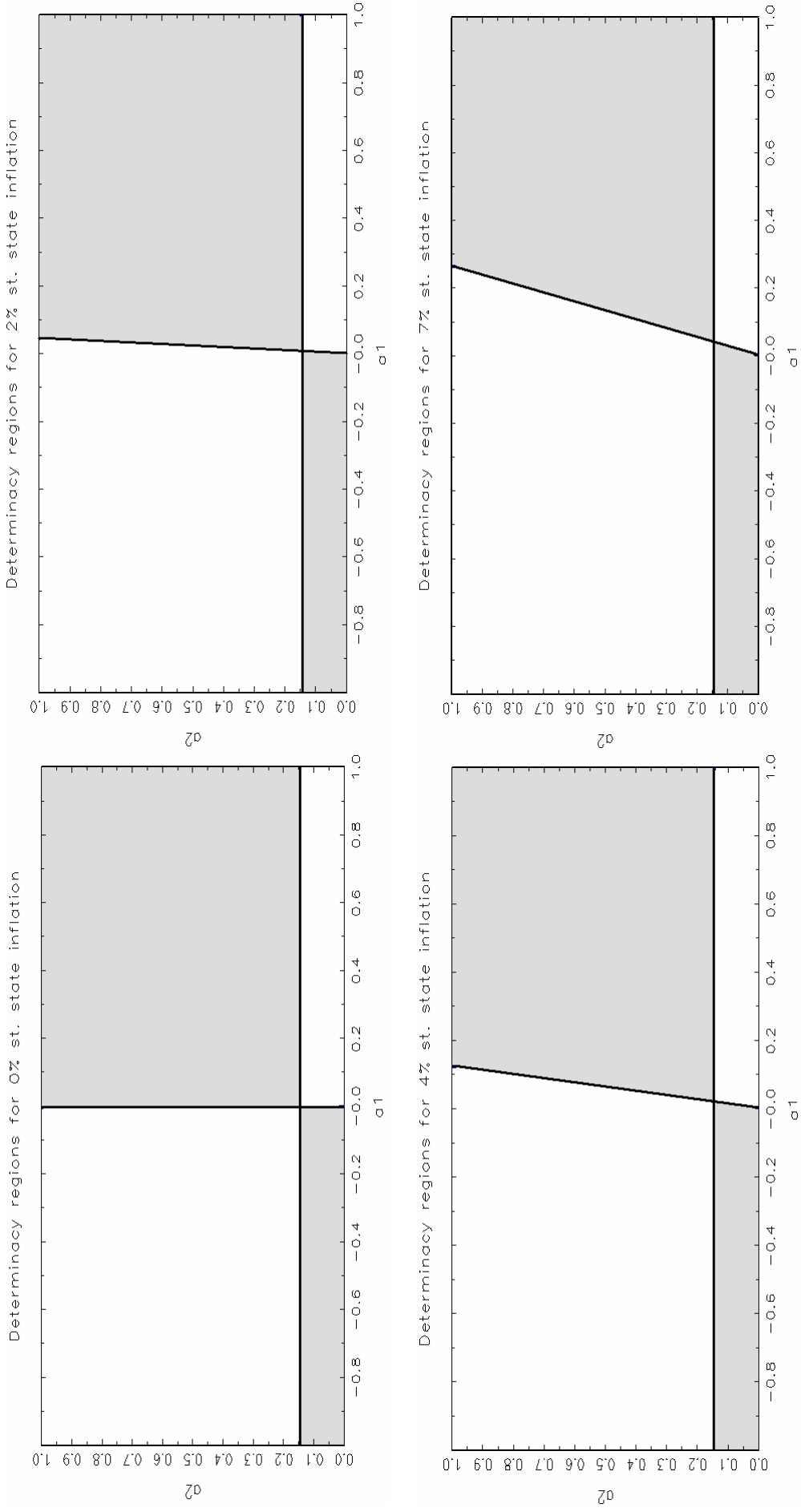


Figure 3. “Mark-up 10%, firms change prices every 12 month”

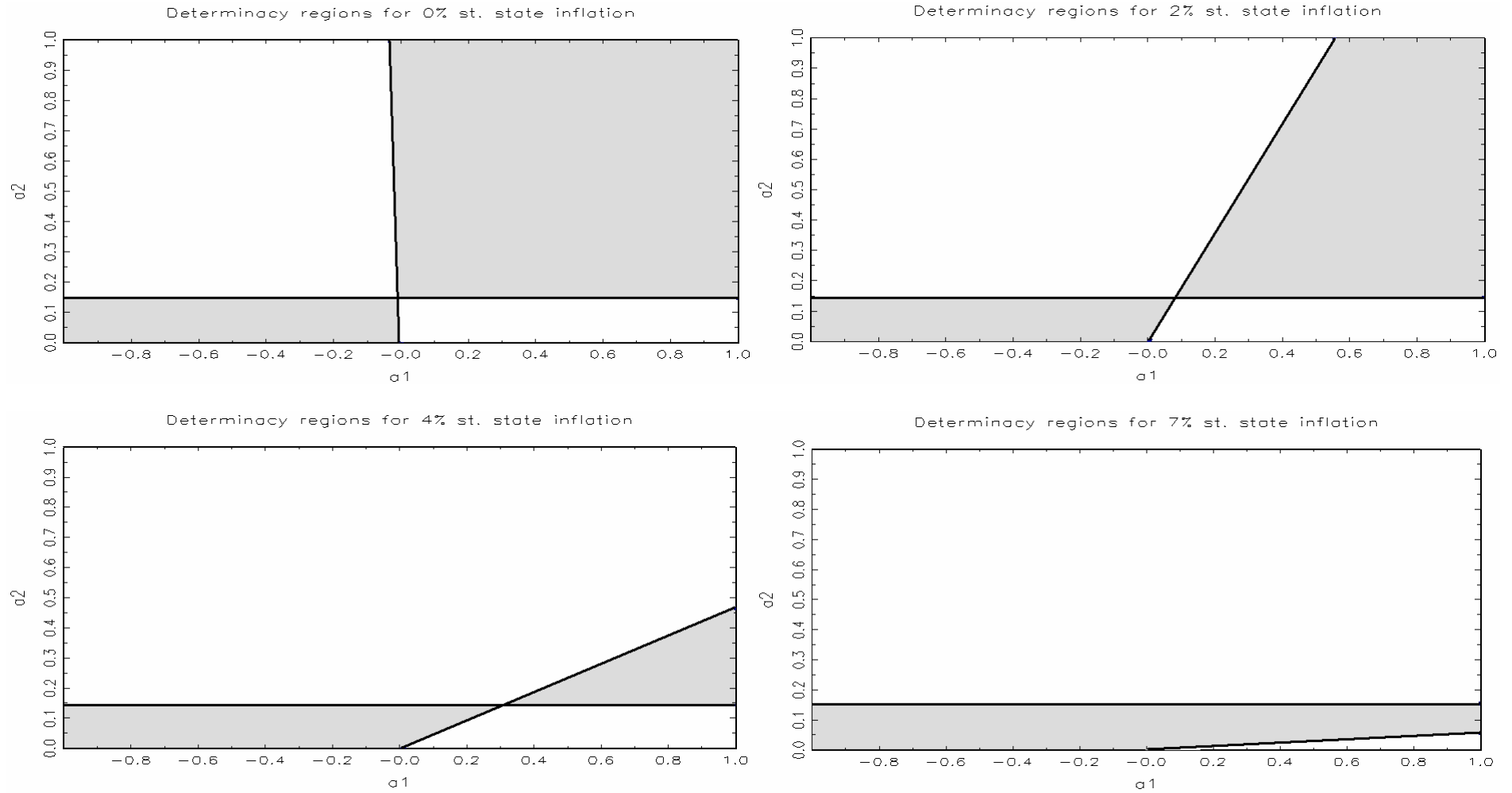
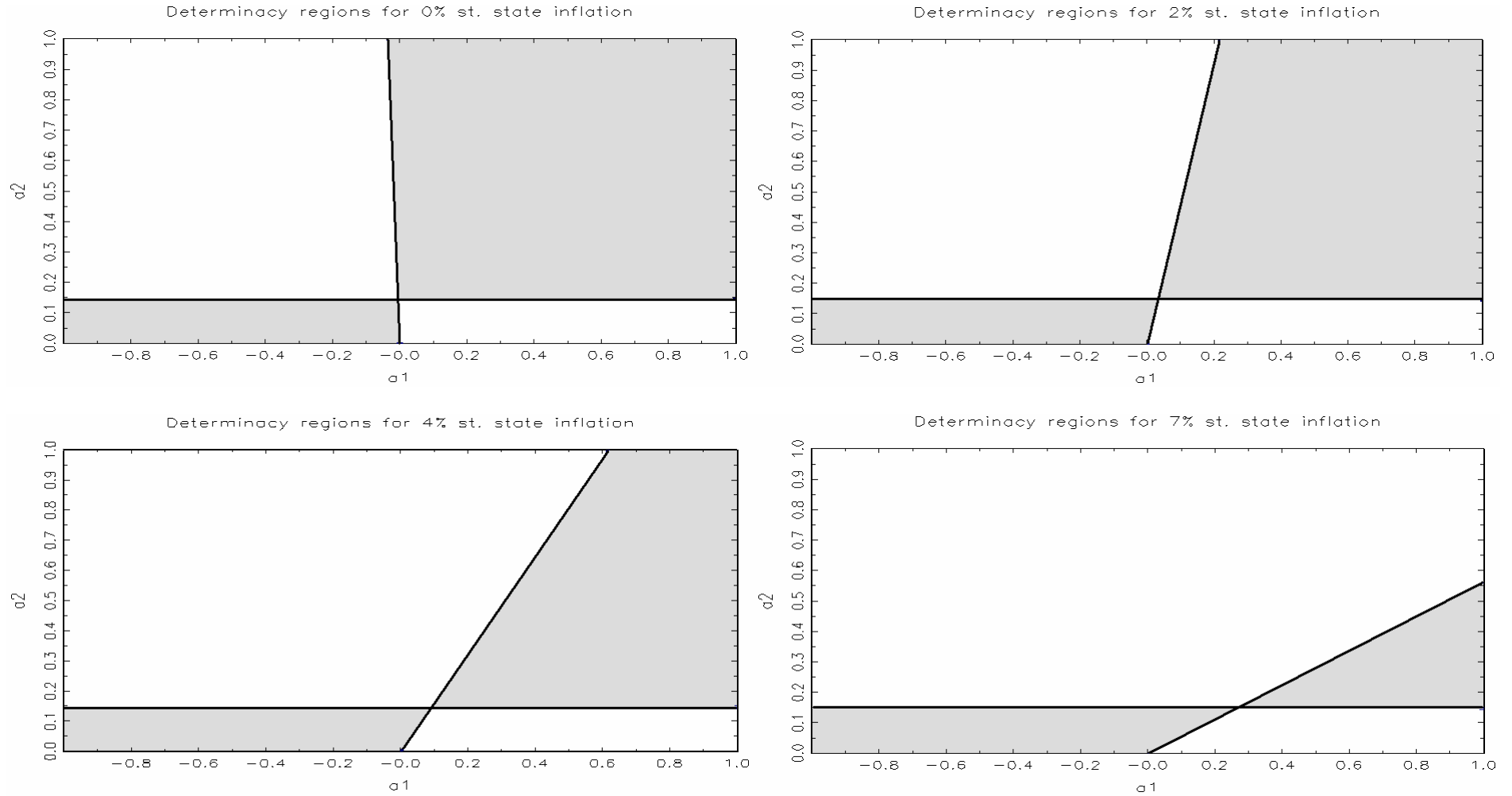


Figure 4 “Mark-up 20%, firms change prices every 12 month”



Alternative monetary rules

Figure: Mark-up 10%, firms change prices every 12m

