Anticipation, learning and welfare: the case of distortionary taxation

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Anticipation, Learning and Welfare: the Case of Distortionary Taxation☆

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Abstract

We study the impact of anticipated fiscal policy changes in the Ramsey economy when agents form expectations using adaptive learning. We extend the existing framework by distortionary taxes as well as elastic labour supply, which makes agents' decisions non-predetermined but more realistic. We detect that the dynamic responses to anticipated tax changes under learning have oscillatory behaviour. Moreover, we demonstrate that this behaviour can have important implications for the welfare consequences of fiscal reforms.

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1. Motivation

Nowadays, fiscal policy is usually accompanied by legislation and implementation lags. These lags create a non-negligible span of time between the announcement and effective date of a fiscal policy change. This gives individuals in the economy the opportunity to anticipate the tax changes. The economic literature denotes this aspect of fiscal policy either anticipated fiscal policy or fiscal foresight. From our reading, those two terms are equivalents and will be used as such.[1]

When agents anticipate, their resulting actions may to some extent depend on the way they form expectations about the future. The standard assumption of expectations in economics is perfect-foresight / rational expectations (RE). This assumption might be questioned. One prominent deviation of RE that imposes weaker requirements on the agent's information set when making his decisions, is the learning literature (see Evans and Honkapohja (2001) for the foundations of this approach). The main idea is that agents form expectations about future values of variables they cannot observe by engaging in a kind of statistical inference when making their economic choices.

Although the learning approach has gained significant popularity in some areas of macroeconomics, anticipated fiscal policy has, until recently, been neglected. A pioneering contribution to the study of the consequences of anticipated fiscal policy when agents learn factor prices, has been made by Evans et al. (2009). They demonstrate the adaptive constant gain learning approach in several deterministic economic environments, taking changes in lump-sum taxation

[1]Recently Leeper (2009, p.11ff.) has listed empirical evidence for fiscal foresight and reemphasized the relevance of expectations for sound fiscal policy. Furthermore, Leeper et al. (2009) is another good example of empirical evidence of fiscal foresight. Therein they also demonstrate the challenges for econometricians that aim to quantify the impact of fiscal policy actions and at the same time account adequately for fiscal foresight.

as an example. The choice of a constant gain therein is motivated by the fact that fiscal policy moves may state structural change. First Evans et al. (2009, p.932ff.) consider permanent, temporary and repeated tax changes in an endowment economy with a balanced-budget policy. The core message of their results is that under learning, anticipated fiscal policy changes have instant effects on key variables as in the perfect foresight case, but the transition paths are remarkably different from the latter. This result, at least with regard to the volatility of key variables' time paths may not come as a surprise. It is well known that constant gain learning causes excess volatility compared to the case of RE (see Evans and Honkapohja (2001, p.49) for an illustration). Thereafter, Evans et al. (2009, p.941ff.) turn attention to the scenario of debt financing of anticipated fiscal policy changes and find that, given agents understand the structure of government financing but have to forecast factor-prices on decentralized markets, the so-called "near Ricardian equivalence" holds under learning. Finally, Evans et al. (2009, p.943ff.) introduce the adaptive learning approach to the basic Ramsey model. For an anticipated balanced-budget permanent tax change they once more confirm that under learning the time paths of key variables are strikingly different from their perfect foresight counterparts.

In subsequent work, Evans et al. (2010) focus on Ricardian equivalence in the basic Ramsey model with anticipated fiscal policy under learning. Most important, Evans et al. (2010, p.8ff.) formally proof that the assumption of RE is not necessary for the classic Ricardian equivalence result. Furthermore, Evans et al. (2010, p.10ff.) provide new departures from the Ricardian equivalence proposition. First, if government expenditures are endogenous, i.e. depend on a fiscal rule, then Ricardian equivalence holds only under RE but fails under learning. Second, Ricardian equivalence breaks down, if the expected interest rates depend on changes in the level of public debt.

Building on the contribution of Evans et al. (2009), we aim to generalize their analysis of anticipated fiscal policy under learning into an economy featuring distortionary taxes and elastic labour supply. Thus, our theoretical key contribution is to derive the dynamic paths of key variables for permanent changes in distortionary taxes in a deterministic version of the prominent Ramsey model. In addition, our second key contribution is to numerically examine fiscal policy reforms, in the presence of several tax instruments.

Note that there are fundamental differences between lump-sum taxation and distortionary taxation: a labour income tax under inelastic labour supply does not affect household margins and therefore causes no distortion, but under elastic labour supply the labour income tax affects the intra-temporal choice between consumption and leisure of the household and may cause an intra-temporal distortion. Next, a capital income tax has the potential to cause up to two types of distortion. First, the capital income tax in any case affects the inter-temporal household Euler equation. In case of elastic labour supply, the capital income tax also affects the intra-temporal choice between consumption and leisure of the household due to its distortion of the consumption choice. Finally, a consumption tax may also cause an inter-temporal distortion by affecting the household Euler equation, but there is an important difference compared to capital income taxation. The consumption tax affects the price of consumption in both periods considered in the household Euler equation whereas the capital income tax always affects only the price of next period's consumption in the household Euler equation. Loosely speaking, a consumption tax can distort consumption and investment decision via the household's Euler equation, only when it is changed, i.e. time-varying, whereas a capital income tax always causes distortions in the

Ramsey economy. Thus, we may expect that the dynamics of the economy for a capital income tax reform may be fundamentally different from the economic dynamics for a consumption tax reform.[2]

Furthermore, the assumption of elastic labour supply implies that endogenous variables such as factor prices as well as employment and consumption are not predetermined as in Evans et al. (2009, p.943ff.) or in Evans et al. (2010), but determined simultaneously in each period.

Next to the analytical derivations, we also calibrate our model and calculate welfare consequences for several policy experiments under perfect foresight as well as under learning. For this purpose, we make use of the welfare measure proposed by Lucas (1990) and also applied by Cooley and Hansen (1992) (for discrete time), which takes into account the whole transition path between the initial and new steady-states associated with initial and changed tax rates. Thus, putting it differently, we ask, to what extent the excess volatility caused by constant gain learning affects the well-being of households compared to the perfect foresight case. Using such a measure of welfare consequences, may even allow comparison of results for learning dynamics to previous studies such as Lucas (1990), Cooley and Hansen (1992) or Garcia-Milà et al. (2010). All these studies evaluate and rank various distortionary tax reforms according to their welfare consequences under perfect foresight, but do not consider the case of learning.

Our main results are as follows. When we assume that agents use adaptive learning rules to forecast factor prices, our model predicts oscillatory dynamic responses to anticipated permanent tax changes. The source of the oscillations are expectational errors. In addition, policy experiments indicate that these

[2] Note that a consumption tax may also be a desirable subject of study, as it has special stability properties. See Giannitsarou (2007) for the details.

volatile responses may have a major impact on the welfare consequences of tax reforms. In particular we consider experiments that improve welfare but do so to a much lower extent under learning compared to perfect foresight.

Note that our approach links the learning literature to that part of the public finance literature that is concerned with the welfare consequences of different types of taxation. See Chamley (1981) for an example of a comparative statics analysis or Judd (1987) for differences in unanticipated and anticipated changes in factor taxes. In addition, there have been studies in stochastic set-ups, like Cooley and Hansen (1992). With regard to the implementation of anticipated optimal fiscal policy an example is Domeij and Klein (2005) or its extension for public goods and capital by Trabandt (2007). Moreover, Garcia-Milà et al. (2010) have recently conducted research on welfare consequences of fiscal policy experiments in the spirit of Cooley and Hansen (1992) in a heterogeneous agents model.

The remainder of the paper is organized as follows. In Section 2 we outline the economic model, derive optimality conditions and detail our approach of learning. Section 3 compares the dynamics with and without elastic labour supply for the case of lump-sum tax changes. This section also provides sensitivity analysis for some structural parameters. In Section 4 we consider changes in distortionary taxation and present a numerical welfare analysis of selected policy experiments. Section 5 concludes and points out directions for further research.

2. The Model

Our economy is a version of the Ramsey economy outlined in detail in Ljungqvist and Sargent (2000, p.305ff.). The capital stock k_t evolves according to the economy-wide resource constraint

$$k_{t+1} = F(k_t, n_t) - c_t - g_t + (1-\delta)k_t, \tag{1}$$

where $F(k_t, n_t)$ is the economy's production function (equalling output) showing that the firm sector uses capital k_t and labour n_t as inputs to produce the single good of the economy (see Section 2.2 for the details). Output can either be consumed by households (c_t) or the government (g_t) or added to the capital stock. Capital is assumed to depreciate at a constant rate δ.

2.1. Households

With regard to the household sector, we assume a continuum of households, where we normalize the size of the economy to unity and each household faces the problem

$$\max_{c_t, n_t} E_t^* \left\{ \sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + \eta \log(\bar{L} - n_t) \right] \right\} \tag{2}$$

s.t.

$$\begin{aligned} k_{t+1} + \frac{b_{t+1}}{R_t} + (1+\tau_t^c)c_t &= (1-\tau_t^l)w_t n_t + (1-\tau_t^k)r_t k_t + (1-\delta)k_t \\ &\quad + b_t - \tau_t + \pi_t, \end{aligned} \tag{3}$$

where all variables are in per capita terms. Thus, the variable k_{t+1} denotes the stock of capital in period $t+1$ and b_{t+1} is the level of government debt holdings chosen in period t. Furthermore, r_t is the rental rate of capital and R_t is the gross real interest rate in period t. The level of consumption chosen in period t is indicated by c_t. Next, $\tau_t^\bullet$ denotes a distortionary tax either on consumption, labour income or capital income[3]. The real wage in period t is given by w_t and $l_t = \bar{L} - n_t$ denotes leisure. In consequence, n_t is labour supply of the household. τ_t is a per capita lump-sum tax and $\pi_t = 0$ is the profit under perfect competition among firms. Furthermore, the parameter $\eta \geq 0$ measures the elasticity of labour supply and the parameter β is the common discount rate.

$E_t^*\{\bullet\}$ denotes subjective period t expectations for future values of variables. Households apply this operator, if they do not have perfect foresight.[4] This assumption is commonly used in the learning literature. Furthermore, note that we abstract from aggregate uncertainty, i.e. we conduct our analysis in a deterministic economy. Thus, if households do not have perfect foresight, their expectations are so-called point expectations, i.e. agents base their economic choices on the mean of their expectations, see Evans and Honkapohja (2001, p.61). In Section 2.4 below we outline our concept of learning. An important aspect of this concept is that forecasts of single variables are independent of each other. In consequence, we can assume that for any two variables X and Y it is true that $E_t^*\{XY\} = E_t^*\{X\}E_t^*\{Y\}$ holds.

Now, we detail the household's decisions. Each household solves the Lagrangian

$$\begin{aligned}\mathcal{L} \;=\; & E_t^* \sum_{t=0}^{\infty} \beta^t \{\log(c_t) + \eta \log(\bar{L} - n_t) \\ & -\lambda_t[k_{t+1} + \frac{b_{t+1}}{R_t} + (1+\tau_t^c)c_t - (1-\tau_t^l)w_t n_t - (1-\tau_t^k)r_t k_t - (1-\delta)k_t \\ & -b_t + \tau_t]\}\end{aligned}$$

[3] We use the symbol $\bullet$ as a placeholder throughout our analysis.

[4] Recall that under perfect foresight agents fix their current and future choices once and for all. This will no longer be the case under the assumption of learning, as we outline in what follows.

with first-order conditions

$$\frac{\partial \mathcal{L}}{\partial c_t} \quad : \quad \beta^t \left\{c_t^{-1} - \lambda_t(1+\tau_t^c)\right\} \stackrel{!}{=} 0 \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} \quad : \quad \beta^t \left\{-\lambda_t\right\} + \beta^{t+1} E_t^* \left\{\lambda_{t+1}\left[(1-\delta) + (1-\tau_{t+1}^k) r_{t+1}\right]\right\} \stackrel{!}{=} 0 \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial b_{t+1}} \quad : \quad \beta^t \left\{-\lambda_t R_t^{-1}\right\} + \beta^{t+1} E_t^* \left\{\lambda_{t+1}\right\} \stackrel{!}{=} 0 \tag{6}$$

$$\frac{\partial \mathcal{L}}{\partial n_t} \quad : \quad \beta^t \left\{-\eta(\bar{L} - n_t)^{-1} - \lambda_t[-(1-\tau_t^l) w_t]\right\} \stackrel{!}{=} 0. \tag{7}$$

From (4) and (6) we get the household Euler condition

$$c_t^{-1} \quad = \quad \beta R_t E_t^* \left\{ c_{t+1}^{-1} \frac{(1+\tau_t^c)}{(1+\tau_{t+1}^c)} \right\}, \tag{8}$$

(5) and (6) yield the no-arbitrage condition for capital and bonds

$$R_t \quad = \quad \left[(1-\delta) + (1 - E_t^*\left\{\tau_{t+1}^k\right\}) E_t^* \left\{r_{t+1}\right\}\right], \tag{9}$$

and from (4) and (7) we get the consumption leisure trade-off

$$n_t \quad = \quad \bar{L} - \frac{\eta(1+\tau_t^c) c_t}{(1-\tau_t^l) w_t}. \tag{10}$$

2.2. Firms

In our economy, there is a unit continuum of firms who compete perfectly. Each firm in each period t rents capital at given price r_t and labour at given price w_t and produces the numeraire good with constant returns to scale production function

$$y_t = F(k_t, n_t) = Ak_t^{\alpha} n_t^{(1-\alpha)} \tag{11}$$

where $\alpha \in (0, 1)$. The optimal firm behaviour requires that

$$r_t \overset{!}{=} \frac{\partial y_t}{\partial k_t} = A\alpha k_t^{\alpha-1} n_t^{1-\alpha}, \tag{12}$$

as well as

$$w_t \overset{!}{=} \frac{\partial y_t}{\partial n_t} = A(1-\alpha) k_t^{\alpha} n_t^{-\alpha}, \tag{13}$$

i.e. each production factor earns its marginal product. Finally, we have the per capita national income identity

$$\begin{aligned} y_t &= r_t k_t + w_t n_t, \\ \pi_t &= y_t - r_t k_t - w_t n_t = 0, \end{aligned} \tag{14}$$

which means zero profits, as one can expect from perfect competition.

2.3. Government

The government finances its expenses on goods and debt repayment by tax revenues and the issuance of new bonds in each period t

$$g_t + b_t = \tau_t^c c_t + \tau_t^l w_t n_t + \tau_t^k r_t k_t + \tau_t + \frac{b_{t+1}}{R_t}.$$

For the remainder, we will assume that the government operates a balanced-budget rule in each period t, thus tax revenues will fully cover expenses such that bonds are in zero net supply as a direct consequence. Thus the government sets $g_t, \tau_t^c, \tau_t^l, \tau_t^k$ and τ_t constrained by

$$g_t = \tau_t^c c_t + \tau_t^l w_t n_t + \tau_t^k r_t k_t + \tau_t \tag{15}$$

in each period t.

2.4. Learning

Now, we aim to detail our concept of learning that was elaborated in Evans et al. (2009, p.943ff.). For completeness we restate the crucial assumptions on learning. Under learning, households are supposed to know the entire history of endogenous variables. They observe the current period value of exogenous variables and they know the state variables. Furthermore, they know the structure of the economy with regard to the fiscal policy sector. Agents understand the implications of the announced policy change for the government budget constraint. They are also convinced that the intertemporal government budget constraint will always hold (see Evans et al. (2009, p.944)). It is decentralized markets for labour and capital, where agents are not in possession of perfect foresight. Actual factor prices are not observable. Thus, agents forecast factor prices such as interest rates and wages $r_{t+j}^e(t), w_{t+j}^e(t), j \geq 1$, by making use of constant-gain steady-state adaptive learning rules[5]

$$r_{t+j}^e(t) = r^e(t) \quad \text{and} \quad w_{t+j}^e(t) = w^e(t), \tag{16}$$

[5]Here we apply the same short-hand notation as Evans et al. (2009). Thus for any variable say z, its period t expected future value in period $t+j$ derived by a learning rule may either be denoted $E_t^*\{z_{t+j}\}$ or equivalently $z_{t+j}^e(t)$. An additional notation we introduce is $z_{t+j}^p(t)$ which denotes the agent's planned choice of the variable z in period $t+j$ based on expected values formed via the learning rule in period t.

where

$$
\begin{aligned}
r^e(t) &= r^e(t-1) + \gamma(r_{t-1} - r^e(t-1)) \\
w^e(t) &= w^e(t-1) + \gamma(w_{t-1} - w^e(t-1)),
\end{aligned} \tag{17}
$$

where $0 < \gamma \leq 1$ is the gain parameter.[6] Our choice of this specific learning rule is motivated by two well known arguments in the learning literature. First, as Evans and Honkapohja (2001, p.332) outline, choosing a constant gain learning rule is the appropriate choice for agents, when they are aware of structural change, as in such a learning rule agents discount past data exponentially. Note that rules (17) are equivalent to $r^e(t) = \gamma \sum_{i=0}^{\infty}(1-\gamma)^i r_{t-i-1}$ and $w^e(t) = \gamma \sum_{i=0}^{\infty}(1-\gamma)^i w_{t-i-1}$.[7] Second, the timing of the learning rule, i.e. that agents' update in period t uses data up to period $t-1$, is chosen in order to avoid simultaneity between $r^e(t)$ and r_t as well as $w^e(t)$ and w_t (see for example Evans and Honkapohja (2001, p.51)). Think of simultaneity in this context as a situation in which agents' expectations affect current values of aggregate endogenous variables and vice versa, which may potentially introduce some strategic behaviour.

Such a learning rule yields a sequence of so-called *temporary equilibria*, which consist of sequences of (planned) time paths for all endogenous variables. These sequences satisfy the learning rule above, the expectation history, household and firm optimality conditions, the government budget constraint and the economy-

[6] The gain parameter measures the responsiveness of the forecast to new observations, see Evans and Honkapohja (2001, p.18). Be aware that in our model the gain parameter is exogenous. See Branch and Evans (2007) for a recent example where agents can choose the gain parameter.

[7] Time series analysts may recognize that the learning rules are similar to a exponential smoothing method.

wide resource constraint given the exogenous variables as well as the current stock of capital in each period. These plans are revisited and potentially altered in each period after expectations have been updated.

3. Base Case: Lump-Sum Tax

Before pursuing our core issue, i.e. the case of distortionary taxation, we would like to illustrate the applied methodology for the case of lump-sum taxation for two reasons: first, we want to illustrate the consequences of the introduction of elastic labour supply compared to the case of inelastic labour supply as assumed in Evans et al. (2009, p.943ff.) and its effect on the dynamic paths of the key variables such as consumption and capital, given their calibration (see Table 1 below); second, below in Subsection 3.2, we aim to present a sensitivity analysis for the very basic version of the model under examination.

Let us now derive the dynamic paths under learning for an anticipated lump-sum tax change. Consequently we assume all other types of taxation away, i.e. $\tau_t^c = \tau_t^l = \tau_t^k = 0$. The Euler equation (8) is standard

$$c_t^{-1} \quad = \quad \beta(c_{t+1}^p(t))^{-1}\left[(1-\delta) + r_{t+1}^e(t)\right]$$

and forward substitution of this yields

$$c_{t+j}^p(t) \quad = \quad \beta^j D_{t,t+j}^e(t) c_t, \tag{18}$$

where we define $D_{t,t+j}^e(t) \equiv \Pi_{i=1}^j[(1-\delta) + r_{t+i}^e(t)]$. One can think of this term as "expectations of the interest rate factor $D_{t,t+j}$ at time t" (see Evans et al. (2009, p.933)). Next, we notice that the consumption leisure trade-off in this case is

$$n_t \;=\; \bar{L} - \frac{\eta c_t}{w_t}. \tag{19}$$

Given the adequate transversality condition for capital[8]

$$\lim_{T \mapsto \infty} \left(D^e_{t,t+T}(t)\right)^{-1} k^p_{t+T+1}(t) \;=\; 0, \tag{20}$$

the inter-temporal budget constraint of the consumer is

$$\begin{aligned} c_t + \sum_{j=1}^{\infty} \frac{1}{D^e_{t,t+j}(t)} c^p_{t+j}(t) \;=\;& [(1-\delta)+r_t]k_t + w_t n_t - \tau_t \\ &+ \sum_{j=1}^{\infty} \frac{1}{D^e_{t,t+j}(t)} \left[w^e_{t+j}(t) n^p_{t+j}(t) - \tau^e_{t+j}(t)\right], \end{aligned}$$

which by the virtue of (18) and (19) yields

$$\begin{aligned} c_t \frac{(1+\eta)}{(1-\beta)} \;=\;& [(1-\delta)+r_t]k_t + w_t \bar{L} - \tau_t \\ &+ \underbrace{\sum_{j=1}^{\infty} \frac{1}{D^e_{t,t+j}(t)} w^e_{t+j}(t) \bar{L}}_{\equiv SW_1} - \underbrace{\sum_{j=1}^{\infty} \frac{1}{D^e_{t,t+j}(t)} \tau^e_{t+j}(t)}_{\equiv ST_1}. \end{aligned} \tag{21}$$

Equations (12) and (13) hold for firms. Finally, government faces the constraint

$$g_t \;=\; \tau_t \tag{22}$$

in each period t and the economy-wide resource constraint is given by (1).

We now need to think about the policy experiment we will study. We are looking at a scenario of a credible permanent change in taxes announced at the

[8] Note that agents plan to satisfy this condition.

outset of period $t = 1$ and effective from period $t = T_p$ onwards. In particular a tax change from τ_0 to τ_1 at some point in time T_p. The dynamics under perfect foresight are standard.[9] Under learning we can directly follow Evans et al. (2009, p.943ff.). The crucial step is to calculate the infinite sums on the right-hand side of (21), i.e. SW_1 and ST_1. Directly following the appendix in Evans et al. (2009, p.951ff.) we calculate

$$SW_1 = \frac{w^e(t)\bar{L}}{r^e(t) - \delta}. \tag{23}$$

With regard to ST_1, we have[10]

$$ST_1 = \frac{\tau_0}{r^e(t) - \delta} + (\tau_1 - \tau_0)\frac{[(1-\delta) + r^e(t)]^{t-T_p}}{1 - [(1-\delta) + r^e(t)]^{-1}} \tag{24}$$

for $1 \leq t < T_p$ and

$$ST_1 = \frac{\tau_1}{r^e(t) - \delta}. \tag{25}$$

for $t \geq T_p$. From (21) follows that we have

$$\begin{aligned} c_t &= \frac{(1-\beta)}{(1+\eta)}\{[(1-\delta) + r_t]k_t + w_t\bar{L} - \tau_0 + \frac{w^e(t)\bar{L}}{r^e(t) - \delta} \\ &\quad - \frac{\tau_0}{r^e(t) - \delta} - (\tau_1 - \tau_0)\frac{[(1-\delta) + r^e(t)]^{t-T_p}}{1 - [(1-\delta) + r^e(t)]^{-1}}\} \end{aligned} \tag{26}$$

[9]Ljungqvist and Sargent (2000, p.305ff.) illustrate the analytical derivations and numerical simulation alternatives for the perfect foresight case. We will simply make use of the DYNARE toolbox throughout all calculations to compute dynamics under perfect foresight. Note that this toolbox employs linearization methods.

[10]See Appendix A.2 for details on derivations and Appendix A.1 for an illustration of the timing, which is relevant for all derivations.

for $1 \leq t < T_p$ and

$$c_t = \frac{(1-\beta)}{(1+\eta)}\left[[(1-\delta)+r_t]k_t + w_t\bar{L} - \tau_1 + \frac{w^e(t)\bar{L}}{r^e(t)-\delta} - \frac{\tau_1}{r^e(t)-\delta}\right] \quad (27)$$

for $t \geq T_p$. Given a calibration, we can then compute the dynamics of consumption and other endogenous variables.

3.1. Inelastic Labour Supply vs. Elastic Labour Supply

We believe that it is of importance to use a model that features elastic labour supply in order to calculate welfare implications of fiscal policy reforms adequately. Completely inelastic labour supply is a quite unrealistic assumption itself and at least some moderately elastic labour supply should be considered. Moreover, inelastic labour supply implies that agents' choices of current period endogenous variables are in fact predetermined as is pointed out in Evans et al. (2009, p.944). In order to illustrate differences in the dynamics of endogenous variables based on the assumption of inelastic and elastic labour supply, we return to the simulation exercise of Evans et al. (2009, p.943ff.). Note that $\tau_t^c = \tau_t^l = \tau_t^k = \delta = 0$ and $\eta = 0$ imply that $n_t = \bar{L}$ (i.e. inelastic labour supply) for all t (see equation (19)). Therefore, we are exactly in the same scenario as in Evans et al. (2009, p.943ff.). Although we do not fully agree with the calibration of Evans et al. (2009), we will stick to their calibration in this subsection to keep our results comparable. We will indicate, when we deviate from their calibration later on. The basic reason for this disagreement is the combination of parameters $\beta = 0.95$ and $T_p = 20$. These parameter choices imply that a government, which in reality is usually in charge of a legislation period of four to six years, may announce a tax policy change that will be effective in 20 years' time. From our perception of political execution and our confidence in fiscal policy makers' ability to commit, this appears to be unrealistic in most cases.

For the moment, we calibrate the model according to Table 1 below.

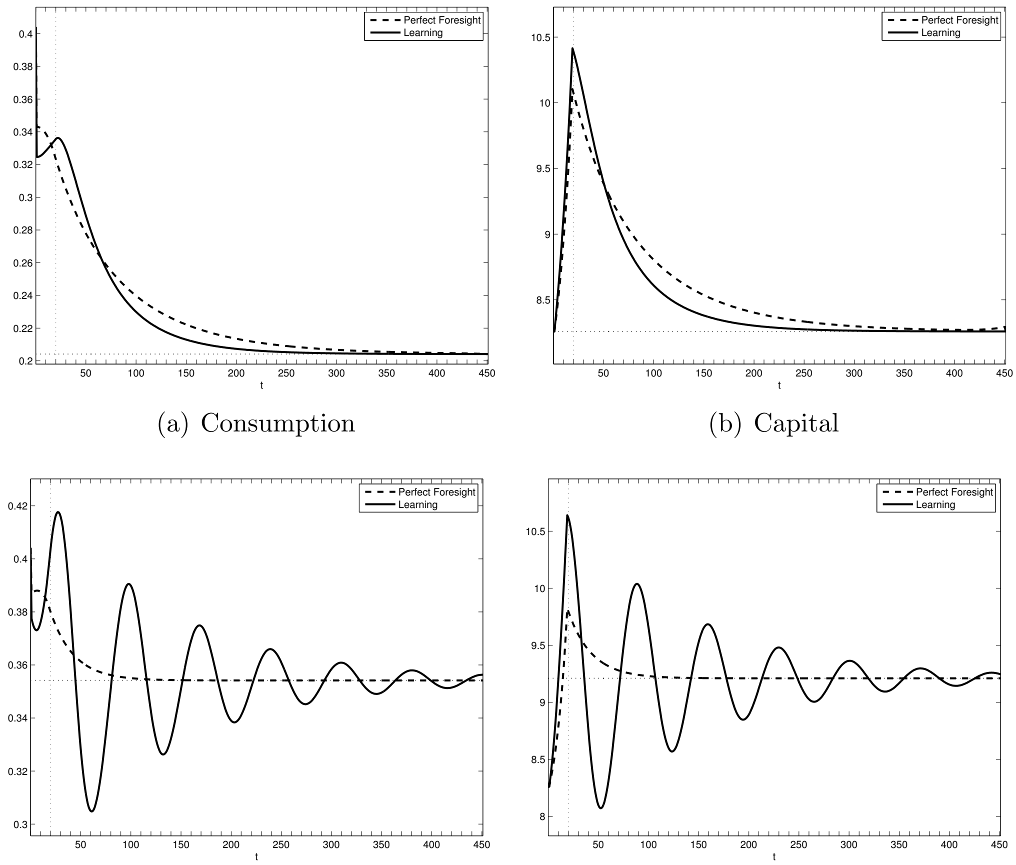
Insert Table 1 here.

The policy experiment considered in Evans et al. (2009, p.943ff.) is a permanent increase in government purchases from $g_0 = \tau_0 = 0.9$ to $g_1 = \tau_1 = 1.1$ that is announced credibly in period $t = 1$ and will be effective from period $T_p = 20$ onwards. It is assumed that the economy is in steady-state in period $t = 0$. Simulations in Evans et al. (2009, p.943ff.) for consumption and capital are recalculated (with $\eta = 0, \bar{L} = 0.5182$) and displayed in Figures 1(a) and 1(b) below. Furthermore, Figures 1(c) and 1(d) exhibit the dynamics for elastic labour supply with $\eta = 2.00$ and $\bar{L} = 1.00$, values that match $n_0 = 0.5182$ and $g_0 = 0.9$ in this set-up.[11]

Two distinct features emerge from Figure 1. First, when we compare the dynamic paths of consumption (as well as capital) under perfect foresight and learning, they are different from each other no matter with or without elastic labour supply. Therefore, it may be quite important to consider learning when evaluating fiscal policies as learning is a more realistic assumption of human behaviour from our point of view.[12] Second, obviously the learning paths in Figures 1(a) and 1(b) for inelastic labour supply are strikingly different to the ones under elastic labour supply in Figures 1(c) and 1(d). In particular, elastic labour supply yields much more volatility in the time paths of consumption and

[11]Note that $n_0 = 0.5182$ corresponds to 12.44 hours per day. This appears to be quite unrealistic, but we choose those numbers in order to achieve comparable magnitudes in Figure 1 below.

[12]This is the core message of Evans et al. (2009).



(a) Consumption

(b) Capital

(c) Consumption with elastic n_t

(d) Capital with elastic n_t

Figure 1: Consumption (a) and capital (b) dynamics under learning (solid curve) and perfect foresight (dashed curve) with inelastic labour supply as in Evans et al. (2009, p.943ff.) as well as consumption (c) and capital (d) dynamics under learning (solid curve) and perfect foresight (dashed curve) with elastic labour supply. The dotted horizontal line indicates the (new) steady state, the dotted vertical line indicates period T_p.

capital (as well as other variables in the model) compared to the inelastic labour supply case. In fact, the variables oscillate around their steady-state until they converge to it.

In our opinion, the reason for the significant differences in the dynamics under learning between elastic and inelastic labour supply can be explained by considering the initial responses of variables to the anticipated policy change and its consequences for the expectations formation process of agents in turn.

In Figure 1 the decline of current wealth due to an anticipated higher lump-sum tax rate from date T_p onwards causes households to save more instantly which results in an initial drop in consumption, which can be seen from (26). Given elastic labour supply, this drop in consumption leads to a rise of labour supply via (10). As the stock of capital is predetermined and cannot respond instantly, the increase of labour supply clearly implies a rise in the rental rate of capital and a fall in the real wage via (12)-(13). The capital stock in the subsequent period will be larger due to (1).

Note that under learning the expectations about factor prices as specified in (16)-(17) do not initially respond. This causes a difference between the factor price and its expectation, i.e. an expectational error. Exactly this error triggers the oscillating learning process that starts in the subsequent period.

Next, consider the algebra of the learning rule (16)-(17). Although, expectations of factor prices may hit the new steady-state value, the agent simply does not realize that it is the new steady-state value. Furthermore, the learning rule (16)-(17) forces the agent to update expectations in each period and the second term in the learning rule (16)-(17) will not be zero. Put differently, there is some persistence of expectational errors. Therefore, if expectations of a factor price used to be above (below) the new steady-state, then after hitting the new steady-state, the expectations will be below (above) the new steady-state until the learning rule (16)-(17) again forces the expectations to hit the new steady-state. Note that when the expectations of a factor price hit the new steady-state from above (below) they do so because the actual value of this factor price must be below (above) the new steady-state at this moment. Thus, unlike in the perfect foresight case, the expectations of a factor price cannot stop evolving when they reach the steady state. In sum, the persistence of expectational errors is the fundamental reason for the oscillations. If the expectational errors diminish over time, the economy will converge to the new steady-state as it is the case under perfect foresight. Otherwise, we may observe explosive paths of the economy.

Now, one can perfectly explain the evolution of actual variables in the subsequent periods. Recall that households own capital. Given the rise in the rental rate of capital in the initial period, the expectational error in the related learning rule (16)-(17) is positive. Thus, the expected value of the rental rate of capital increases after the update in the subsequent period. This causes households to save even more and to consume even less compared to the previous period. Thus, next periods capital stock will be larger and the actual rental-rate of capital will be lower than in the current period.

In analogy, households supply labour and given the fall of the real wage in the initial period, the expectational error in the related learning rule (16)-(17) is negative. Thus, the expected value of the real wage decreases after the update in the subsequent period. The lower expected real wage causes households to supply even more labour compared to the previous period.

Notice that these movements in actual variables continue until the signs of the expectational errors change. Then the movements of actual variables proceed in the opposite directions. Furthermore, the evolution of these variables in turn explains the evolution of all remaining variables such as output.

In addition, inspection of (10) and (26) makes clear that the oscillations are not caused by the assumption of elastic labour supply. When we assume inelastic labour supply the initial drop in consumption is simply larger as in the case of elastic labour supply. The result of the sensitivity analysis below underlines this fact.

In order to sum up, under learning an anticipated tax change will cause initial responses in non-predetermined variables. These responses inevitably cause expectational errors. The magnitude of the expectational errors is related to the model assumptions such as (in-)elastic labour supply, the type of the tax that is changed, the magnitude of the tax change or the implementation date. Small expectational errors mean fast convergence and small oscillations, while large expectational errors result in large oscillations or even divergence. With regard to the example in Figure 1, we conclude that it is not the different assumption about labour supply per se that causes the differences in the dynamics, but the fact that expectational errors in the two cases are different.

3.2. Sensitivity Analysis

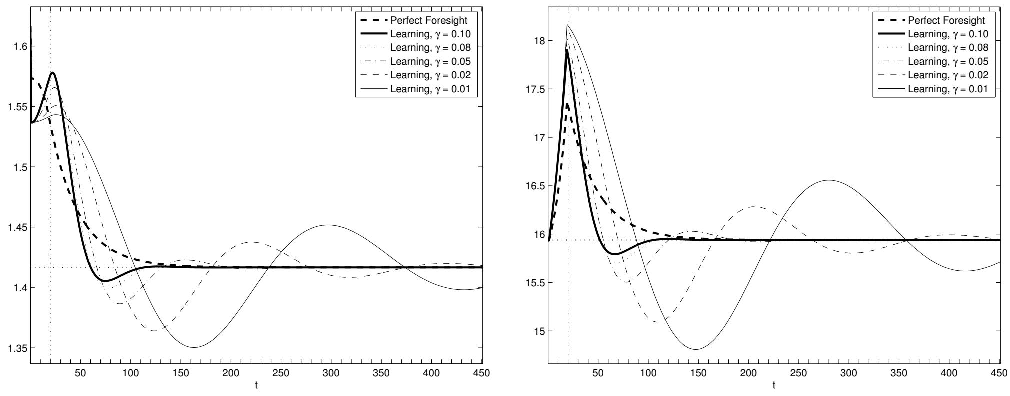
Compared to the previous literature on welfare evaluation of tax reforms, our learning approach introduces two additional structural parameters. One is γ, the gain parameter and a second one is T_p, the period, in which the pre-announced tax change becomes effective. Therefore, we are interested in how these two parameters affect the dynamic properties of the model.

3.2.1. Sensitivity Analysis for the Gain Parameter

No matter what calibration, one usually has to choose a gain parameter γ in the adaptive learning literature. In this subsection we would therefore like to illustrate the consequences of different choices of the gain parameter. The sole empirical estimate we are aware of is provided by Milani (2007, p.2074) for quarterly frequency and is $\gamma = 0.0183$. This number indicates that agents use approximately $1/\gamma \approx 55$ quarters of data. But a reason to be cautious to use the estimate of Milani (2007, p.2074) is that it is based on a data set containing output, inflation and the nominal interest rate, whereas in our setting agents forecast the rental rate of capital and the real wage. Next, Milani (2007, p.2074) mentions that for constant gain learning a range of $\gamma \in [0.01, 0.03]$ is commonly used. Evans and Honkapohja (2009, p.154) note a range of $\gamma \in [0.01, 0.06]$ as known estimates.

Below we will present sensitivity of the dynamics under learning for $\gamma \in \{0.01, 0.02, 0.05, 0.08, 0.10\}$. We do so for the original numerical analysis of Evans et al. (2009, p.943ff.) ($\bar{L} = 1.00$, $\eta = 0.00$), as in this case, there is inelastic labour supply and we can focus solely on the possible fluctuations introduced by varying the gain parameter γ. Note that the two thick lines in Figures 2(a) and 2(b) exactly replicate the Figures 8 and 9 in Evans et al. (2009, p.943ff.).

In Figure 2(a) we observe that the smaller the gain γ, the smaller the increase in consumption until the period of the tax change T_p (after the initial drop). Furthermore, as we recognize from Figure 2(b), the smaller the gain γ, the larger the increase in capital accumulation until the period of the tax change T_p. However, in both Figure 2(a) and 2(b), we observe that with decreasing γ the dynamics fluctuate around the steady-state with increasing amplitude and it takes an increasing number of periods to converge to the steady-state. These observations are partly at odds with what Evans and Honkapohja (2001, p.332) report: "a larger gain is better at tracking changes but at the cost of a larger variance". In our case it holds, that, the smaller the gain, the larger the volatility.



(a) Consumption (b) Capital

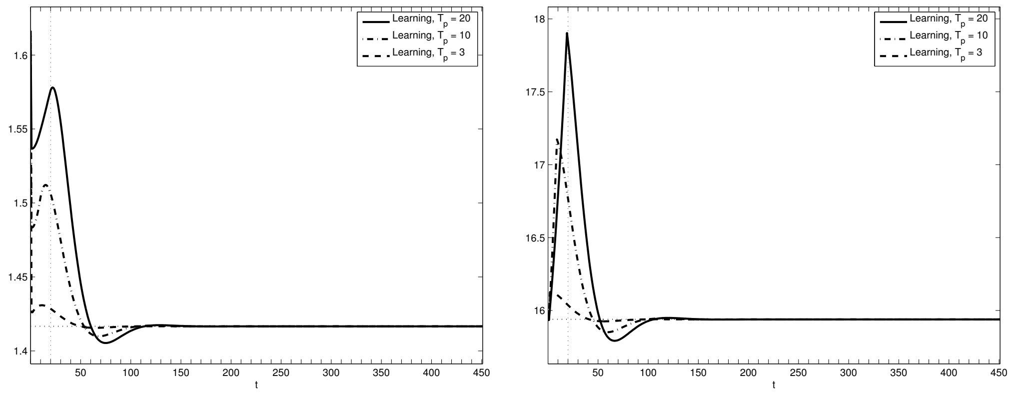
Figure 2: Consumption (a) and capital (b) dynamics under learning and perfect foresight with inelastic labour supply as in Evans et al. (2009, p.943ff.) for alternating values of γ. The dotted horizontal line indicates the (new) steady state, the dotted vertical line indicates period T_p.

Inspection of the learning rule (16)-(17) explains this fact. In the alternative representation the term $(1-\gamma)^i$ one can observe that the smaller the gain, the stronger the discounting of past data. Thus, the more unimportant is past data for agents expectation formation. One can also think of this as agents having more confidence in their initial expectations. But, if these expectations are wrong, and they have to be once a tax change occurs, then they have bigger errors over time and need longer to learn the new steady-state.

Summing up, we find that for the parameter range considered in this sensitivity analysis, the choice of the gain parameter γ is not crucial for the shape of the dynamic response.

3.2.2. Sensitivity Analysis for the Implementation Date

Another issue that may be of interest is the implementation date T_p. As mentioned above a tax policy change that is going to be effective in 20 years time appears to be unrealistic from our point of view. Therefore, we examine sensitivity of dynamics under learning for various implementation dates, in particular $T_p \in \{3, 10, 20\}$. Figures 3(a) and 3(b) below display the results.



(a) Consumption

(b) Capital

Figure 3: Consumption (a) and capital (b) dynamics under learning with inelastic labour supply as in Evans et al. (2009, p.943ff.) for alternating values of T_p. The dotted horizontal line indicates the (new) steady state, the dotted vertical line indicates period $T_p = 20$.

In Figure 3(a) we observe that the shorter the distance between the announcement date and implementation date of the tax change, the higher the initial drop in consumption and the lower the increase in consumption until the implementation date thereafter. Focusing on capital, in Figure 3(b) we observe that with decreasing distance between the announcement date and implementation date of the tax change, the level that capital reaches until the implementation date, is also lower. Finally, for implementation in three years time, i.e. $T_p = 3$, learning dynamics are not significantly different from $T_p \in \{10, 20\}$, but lower in scale. Overall, we observe that the shorter the distance between announcement date and implementation date of the tax change, the earlier the learning dynamics approach the steady-state, but, at least for the parameter range considered herein, the nature of dynamics is not seriously affected.

Thus, we learn that in the subsequent numerical analysis, next to the elasticity of labour supply η (and the commonly known candidate parameters β and δ), the choice of the gain parameter γ as well as the implementation date T_p may also be crucial in achieving convergence on the one hand and determining the magnitude of volatility of the dynamics on the other hand. But these choices may not affect the general nature of the dynamics. Furthermore, our experience with β and δ suggests that they strongly affect the scale of results, next to their impact on convergence.

In order to summarize, there are three important insights from the analysis above. First, there are at least qualitative differences between the case of inelastic labour supply ($\eta = 0$) and elastic labour supply ($\eta > 0$). Therefore, if one regards the latter assumption as more realistic, a model that allows for elastic labour supply is a more appropriate framework to study anticipated fiscal policy under learning. Second, our sensitivity analysis suggests that the choice of the gain parameter γ and the implementation date T_p does not affect the nature of transition paths so we consider ourselves free to choose any of the values considered in the sensitivity analysis.[13] Finally and most notably, we observed at least a qualitative difference in the dynamics under learning compared to the dynamics under perfect foresight. The former appear to be much more volatile than the latter. This stylized fact, from our point of view, justifies the quantification and comparison of welfare cost of anticipated fiscal policy reforms under learning and under perfect foresight. In order to be able to mimic, at least to some extent, a realistic fiscal policy reform, we will introduce distortionary taxes. Before we look at complex fiscal policy reforms, we qualitatively inspect isolated changes in

[13] In particular, in the subsequent analysis, we will choose $\gamma = 0.08$ and $T_p = 8$, which will correspond to 8 quarters.

distortionary taxes and the resulting dynamics for each type of tax. Thereafter, we analyze more sophisticated fiscal policy reforms with regard to their welfare costs in a realistic calibration.

4. The Case of Distortionary Taxation

After the base case of lump-sum taxation, we now study the case of distortionary taxes. In the remainder, we will assume elastic labour supply. We first derive the dynamic paths of the economy in presence of multiple types of taxes.[14] Thereafter, we evaluate some specific tax reforms with regard to welfare, given our suggested calibration.

Let us now assume that $\tau_t^c, \tau_t^l, \tau_t^k \in [0,1]$ and $\tau_t \neq 0$ for all t. The Euler equation (8) now changes to

$$c_t^{-1} = \beta(c_{t+1}^p(t))^{-1}\left[\frac{(1+\tau_t^c)}{(1+\tau_{t+1}^{c,e}(t))}\right][(1-\delta)+(1-\tau_{t+1}^{k,e}(t))r_{t+1}^e(t)]$$

and forward substitution of this expression yields

$$c_{t+j}^p(t) = \beta^j D_{t,t+j}^{k,e}(t)\left[\frac{(1+\tau_t^c)}{(1+\tau_{t+j}^{c,e}(t))}\right]c_t, \tag{28}$$

where we define $D_{t,t+j}^{k,e}(t) \equiv \Pi_{i=1}^{j}[(1-\delta)+(1-\tau_{t+i}^{k,e}(t))r_{t+i}^e(t)]$. Furthermore, notice that the consumption leisure trade-off is now given by (10). Given the adequate

[14]Note that in an earlier version of this paper we also presented the dynamics for the case where only one type of distortionary taxation is present. Each, labour income tax, capital income tax or consumption tax was raised by 10%. For the labour income tax and capital income tax we found that in both cases there are again oscillations. Compared to the lump-sum case the magnitude is much larger and the it takes more time for convergence to the new steady-state. The dynamics of the consumption tax reform coincide for perfect foresight and learning. This result depends on our utility specification with regard to consumption, that is log-utility.

transversality condition for capital

$$\lim_{T \mapsto \infty} \left(D^{k,e}_{t,t+T}(t) \right)^{-1} k^p_{t+T+1}(t) \;\; = \;\; 0, \tag{29}$$

the inter-temporal budget constraint of the consumer is

$$\begin{aligned}(1+\tau^c_t)c_t + \sum_{j=1}^{\infty} \frac{1}{D^{k,e}_{t,t+j}(t)}(1+\tau^{c,e}_{t+j}(t))c^p_{t+j}(t) = [(1-\delta)+(1-\tau^k_t)r_t]k_t \\ +(1-\tau^l_t)w_t n_t - \tau_t \\ +\sum_{j=1}^{\infty} \frac{1}{D^{k,e}_{t,t+j}(t)}[(1-\tau^{l,e}_{t+j}(t))w^e_{t+j}(t)n^p_{t+j}(t) - \tau^e_{t+j}(t)],\end{aligned}$$

which by the virtue of (28) as well as (10) yields

$$\begin{aligned}\frac{(1+\eta)}{(1-\beta)}(1+\tau^c_t)c_t &= [(1-\delta)+(1-\tau^k_t)r_t]k_t + (1-\tau^l_t)w_t\bar{L} - \tau_t \\ &\quad +\sum_{j=1}^{\infty} \frac{1}{D^{k,e}_{t,t+j}(t)}[(1-\tau^{l,e}_{t+j}(t))w^e_{t+j}(t)\bar{L} - \tau^e_{t+j}(t)] \\ &= [(1-\delta)+(1-\tau^k_t)r_t]k_t + (1-\tau^l_t)w_t\bar{L} - \tau_t \\ &\quad +\sum_{j=1}^{\infty} \frac{1}{D^{k,e}_{t,t+j}(t)}[w^e_{t+j}(t)\bar{L} - \tau^{l,e}_{t+j}(t)w^e_{t+j}(t)\bar{L} - \tau^e_{t+j}(t)] \\ &= [(1-\delta)+(1-\tau^k_t)r_t]k_t + (1-\tau^l_t)w_t\bar{L} - \tau_t \\ &\quad +SW_2 - ST_2 - ST_3. \end{aligned} \tag{30}$$

For firms nothing changes compared to the base case in Section 3. Finally government now faces the constraint (15) in each period t. The economy-wide resource constraint is again given by (1).

We now consider the scenario of a permanent (simultaneous) change in (some of the) taxes at some point in time T_p. The dynamics under perfect foresight are again standard. Under learning we again follow the approach Evans et al. (2009, p.943ff.). The infinite sum on the right-hand side of (30) is

$$SW_2 = \sum_{j=1}^{\infty} \frac{1}{D^{k,e}_{t,t+j}(t)} w^e_{t+j}(t)\bar{L}. \tag{31}$$

Given (16) and (17), for $1 \leq t < T_p$ we calculate[15]

$$SW_2 = \frac{w^e(t)\bar{L}}{[(1-\tau^k_0)r^e(t)-\delta]} + w^e(t)\bar{L} \times \left[\frac{[(1-\delta)+(1-\tau^k_1)r^e(t)]^{t-T_p}}{1-[(1-\delta)+(1-\tau^k_1)r^e(t)]^{-1}} - \frac{[(1-\delta)+(1-\tau^k_0)r^e(t)]^{t-T_p}}{1-[(1-\delta)+(1-\tau^k_0)r^e(t)]^{-1}}\right] \tag{32}$$

and for $t \geq T_p$ we calculate

$$SW_2 = \frac{w^e(t)\bar{L}}{[(1-\tau^k_1)r^e(t)-\delta]}. \tag{33}$$

ST_2 on the right-hand side of (30) is

$$ST_2 = \sum_{j=1}^{\infty} \frac{1}{D^{k,e}_{t,t+j}(t)} \tau^{l,e}_{t+j}(t) w^e_{t+j}(t)\bar{L}. \tag{34}$$

Given (16) and (17), for $1 \leq t < T_p$ we calculate[16]

$$ST_2 = \frac{\tau^l_0\ w^e(t)\bar{L}}{[(1-\tau^k_0)r^e(t)-\delta]} + w^e(t)\bar{L} \times \left[\frac{\tau^l_1\ [(1-\delta)+(1-\tau^k_1)r^e(t)]^{t-T_p}}{1-[(1-\delta)+(1-\tau^k_1)r^e(t)]^{-1}} - \frac{\tau^l_0\ [(1-\delta)+(1-\tau^k_0)r^e(t)]^{t-T_p}}{1-[(1-\delta)+(1-\tau^k_0)r^e(t)]^{-1}}\right] \tag{35}$$

[15] See Appendix A.3 for details on derivations of SW_2.

[16] See appendices A.4 and A.5 for the details on derivations of ST_2 and ST_3.

and for $t \geq T_p$ we calculate

$$ST_2 \;=\; \frac{\tau_1^l \; w^e(t)\bar{L}}{[(1-\tau_1^k)r^e(t)-\delta]}. \tag{36}$$

Finally, ST_3 on the right-hand side of (30) is

$$ST_3 \;=\; \sum_{j=1}^{\infty} \frac{1}{D_{t,t+j}^{k,e}(t)} \tau_{t+j}^e(t). \tag{37}$$

Given (16) and (17), for $1 \leq t < T_p$ we calculate

$$\begin{aligned} ST_3 \;=\; & \frac{\tau_0}{[(1-\tau_0^k)r^e(t)-\delta]} + [\; \frac{[(1-\delta)+(1-\tau_1^k)r^e(t)]^{t-T_p}}{1-[(1-\delta)+(1-\tau_1^k)r^e(t)]^{-1}} \;\tau_1 \\ & - \frac{[(1-\delta)+(1-\tau_0^k)r^e(t)]^{t-T_p}}{1-[(1-\delta)+(1-\tau_0^k)r^e(t)]^{-1}} \;\tau_0 \;] \end{aligned} \tag{38}$$

and for $t \geq T_p$ we calculate

$$ST_3 \;=\; \frac{\tau_1}{[(1-\tau_1^k)r^e(t)-\delta]}. \tag{39}$$

Given (30) we can then compute the dynamics responses for consumption and the other endogenous variables as before. Now, we will conduct several policy experiments numerically and compute welfare measures following the approach of Cooley and Hansen (1992, p.301ff.).[17] Intuitively speaking, we compute the increase in consumption that an individual would require to be as well off as under the equilibrium allocation without taxes. We express that number in percentage of output. First, we will do so for our initial choice of tax levels (see line 1 in Table 3 below). Thereafter, we carry out policy reforms, where we change taxes

[17]We detail the computation in Appendix B.

in a certain way and each time recalculate welfare measure both for learning and perfect foresight. As a result we can then compare the welfare implications for a tax change under perfect foresight against the case under learning. Note that we use the measure of Cooley and Hansen (1992, p.301ff.) for the transition paths. We do so because their measure for static comparison would lead to the same number for perfect foresight and learning, as in both cases the initial and new steady-states are identical.

An additional parameter needs to be chosen. That is the evaluation horizon T. Cooley and Hansen (1992, p.301ff.) choose a horizon $T \geq 2000$ and give no further detail on the motivation of that choice. Garcia-Milà et al. (2010) use $T = 200$ and give no motivation either. We will choose the latter in our welfare evaluations as a time span of 200 quarters or 50 years respectively appears to be more realistic from our point of view. For the series of experiments in Table 3 below, our calibration of the model is according to Table 2 below.

Insert Table 2 here.

We choose the initial tax rates to be $\tau_0 = 0.0000$, $\tau_0^l = 0.2300$, $\tau_0^k = 0.5000$ and $\tau_0^c = 0.0500$. These non-zero tax rates lead to distortions. The first row in Table 3 reveals the welfare loss between the steady-state of the economy without taxes and the steady-state of the economy with our initially chosen tax rates amounts to 73.72%. This number tells us the change in consumption (in percentage of output) which is required so that households in the economy with initial tax levels are as well off as in the case with zero taxes is 73.72%. Be aware that Table 3 also indicates that without taxes our calibration yields a first best steady-state employment of $n_{FB} = 0.4024$, which implies 9.66 hours. With the initial taxes in place, the steady-state employment is $n_0 = 0.4326$, which implies 10.38 hours.

Now we assume a credible pre-announced permanent tax reform that favours capital accumulation, i.e. we lower the capital income tax to a level of $\tau_1^k = 0.2500$. As suggested by Judd (1987), Lucas (1990) and Cooley and Hansen (1992) this is expected to reduce the welfare costs of distortionary taxation. In each experiment reported lines 2 to 4 in Table 3 below, one of the other tax instruments, $\tau_\bullet$, $\tau_\bullet^l$ or $\tau_\bullet^c$ will be raised to a level that ensures that the periodic tax revenue in the new steady-state is the same as in the initial steady-state.[18] The second row of Table 3 indicates that compensating the cut in the capital income tax to τ_1^k by an increase in the labour income tax to τ_1^l leads to a welfare improvement under perfect foresight as well as under learning as both welfare measures decrease. But the numbers also reveal that the magnitude of the improvement differs. Whereas under learning the welfare measure goes down from 73.72% to 72.12%, under perfect foresight it decreases much more to 64.47%.[19] We can also observe that the new steady-state employment n_1 is lower than the initial steady-state employment n_0.

The pattern just described is also true, if we compensate the cut in $\tau_\bullet^k$ by an increase in $\tau_\bullet^c$ or $\tau_\bullet$ as the third and fourth row in Table 3 indicate. It is noteworthy

[18]Note, that as long as the dynamics under learning and perfect foresight differ, one is not able to equalize present values of tax revenues under learning and perfect foresight to the present value of tax revenues in the initial steady-state by manipulating tax rates in the same way. This approach was used in the analysis of Cooley and Hansen (1992) for perfect foresight only, but is not feasible in our case. In addition, we believe that keeping present values constant is not the kind of fiscal policy change that governments conduct in reality. Moreover, we believe that our comparison of welfare costs under learning to welfare costs under perfect foresight is valid even without equalizing present values of the tax revenue.

[19]We would like to emphasize that we set the rate of depreciation to $\delta = 0$ in order to achieve convergence for the dynamics under learning. That might be the reason, why the scale of $\mathcal{W}$ both under learning and perfect foresight is approximately twice the scale as the results in Cooley and Hansen (1992).

that using the the lump-sum tax to compensate for the cut in the capital income tax yields the largest welfare improvement and keeps steady-state employment at the highest level independent of the assumption about expectations.

Thus, experiments 2 to 4 indicate that the resulting welfare improvements of an anticipated tax reform might be much smaller in magnitude under learning compared to its improvements under perfect foresight.

Insert Table 3 here.

---

5. Conclusion

We demonstrate that the responses to anticipated permanent tax changes when agents learn are remarkably different compared to their counterparts under perfect foresight. The dynamics under learning appear to oscillate around the steady-state to which they converge slowly. Thus, there is more volatility under learning.

We argue that the observed oscillations are related to expectational errors. The expectational errors are caused by the anticipated permanent tax change. The persistence of the expectational error in the learning rule of the agents is the fundamental reason for the oscillations.

Moreover, sensitivity analyses show that a smaller gain parameter leads to higher volatility in our framework. This result is at odds with conventional wisdom about the link between the gain parameter and the dynamic responses in the learning literature.

In the subsequent analysis we derive the dynamics in the presence of multiple tax instruments. Policy experiments in this set-up indicate that the magnitude of welfare improvements due to the tax reform considered herein appears to be substantially lower under the assumption of learning compared to the case of perfect foresight. The reason may be the oscillatory behaviour of the dynamics under learning.

Form our point of view these results raise two major issues. First, oscillatory dynamic responses to exogenous shocks are rarely found in actual economic data. This fact questions the suitability of the model herein for policy analysis. Second, given that this model would be suitable for policy analysis, our results indicate that permanent tax changes may lead to lower welfare improvements under learning compared to perfect foresight.

We believe that future research in this area needs to come up with convincing empirical evidence on whether or how agents learn about fiscal policy. In addition, we also need to clarify from actual economic data, how the dynamic responses to anticipated permanent tax changes look like. Are they smooth or oscillatory?

With regard to theoretical considerations, it would also be desirable to derive a version of the model that allows for changing different tax rates at different points in time and therefore allows for public debt accumulation. But this task is beyond the focus of this paper and we aim to pursue that idea in subsequent research.

Furthermore, we think that perfect foresight and the implied once and for all choices of agents on the one hand and learning which implies periodic revision of current and future choices of agents on the other hand are extreme cases. One could also imagine agents that use adaptive learning, but infrequently and with differing interval length update their expectations and revise their current and future choices. Alternatively, agents randomly receive a signal to update their expectations.

In addition, more sophisticated computational methods may allow to calibrate the rate of depreciation different from zero or more realistic values of the elasticity of labour supply and still ensure convergence for the dynamics under learning on the other side. This could facilitate numerical results that are directly comparable to the existing literature in public finance.

A. Model Derivations

A.1. Timing

We believe that the understanding of the timing is crucial to follow the derivations. For time periods indexed by t, discounting periods indexed by j, and an implementation date T_p announced in $t = 1$ and $T \equiv T_p - t$ denoting the number of periods until T_p we got the following picture:

$$\begin{aligned} t &= 1, 2, 3, 4, 5, 6, ... \\ j &= 0, 1, 2, 3, 4, 5, ... \\ T \equiv T_p - t &= 4, 3, 2, 1, 0, -1, ..., \end{aligned}$$

thus for the infinite sum over index j

$$\sum_{j=1}^{T-1} \{\bullet\} + \sum_{j=T}^{\infty} \{\bullet\} \tag{A.1.1}$$

from period $t = 1$ perspective, given exemplary $T_p = 5$ on the line $1 \leq t \leq T_p - 1$, until $j = 3 = T - 1$ we have the old tax rate. Furthermore, on the line $t \geq T_p$ from $j = 4 = T$ onwards we have the new tax rate. Equivalently for the infinite sum

$$\sum_{j=0}^{T-2}\{\bullet\} + \sum_{j=T-1}^{\infty}\{\bullet\} \tag{A.1.2}$$

from period $t = 1$ perspective, given exemplary $T_p = 5$ on the line $1 \leq t \leq T_p - 1$, until $j = 2 = T - 2$ we have the old tax rate. Furthermore, on the line $t \geq T_p$ from $j = 3 = T - 1$ onwards we have the new tax. This allows us later on to replace T with $T_p - t$ for $1 \leq t \leq T_p - 1$ and $T - 1$ with 0 for $t \geq T_p$.

A.2. Derivation of ST_1

Here we want to illustrate the methodology we apply in all derivations under learning for the example of ST_1. Starting from

$$ST_1 = \sum_{j=1}^{\infty} \frac{1}{D^e_{t,t+j}(t)} \tau^e_{t+j}(t)$$

we split this infinite sum into

$$ST_1 = \left[\sum_{j=1}^{T-1} \frac{1}{D^e_{t,t+j}(t)} \tau_0 + \sum_{j=T}^{\infty} \frac{1}{D^e_{t,t+j}(t)} \tau_1\right].$$

Next we go back to the definition of $D^e_{t,t+j}(t)$. Given the learning rules (16) and (17) we get

$$D^e_{t,t+j}(t) = \Pi^j_{i=1}\left[(1-\delta) + r^e(t)\right] = \left[(1-\delta) + r^e(t)\right]^j. \tag{A.2.1}$$

Consequently we get

$$ST_1 = \left[\sum_{j=1}^{T-1} \left(\left[(1-\delta) + r^e(t)\right]^{-1}\right)^j \tau_0 + \sum_{j=T}^{\infty} \left(\left[(1-\delta) + r^e(t)\right]^{-1}\right)^j \tau_1\right],$$

or

$$
\begin{aligned}
ST_1 &= [(1-\delta)+r^e(t)]^{-1} \times \\
&\left[\sum_{j=0}^{T-2}\left([(1-\delta)+r^e(t)]^{-1}\right)^j \tau_0 + \sum_{j=T-1}^{\infty}\left([(1-\delta)+r^e(t)]^{-1}\right)^j \tau_1\right].
\end{aligned}
$$

Given the property of a finite geometric series $\sum_{j=m}^{n} f^j = \frac{f^{n+1}-f^m}{f-1}$ for some constant f, we get

$$
\begin{aligned}
ST_1 &= [(1-\delta)+r^e(t)]^{-1} \times \\
&\left[\left(\frac{[(1-\delta)+r^e(t)]^{1-T}-1}{[(1-\delta)+r^e(t)]^{-1}-1}\right)\tau_0 + \left(\frac{-[(1-\delta)+r^e(t)]^{1-T}}{[(1-\delta)+r^e(t)]^{-1}-1}\right)\tau_1\right],
\end{aligned}
$$

which can be rewritten as

$$
ST_1 = \frac{\tau_0}{r^e(t)-\delta} + \frac{(\tau_1-\tau_0)}{[(1-\delta)+r^e(t)]} \frac{[(1-\delta)+r^e(t)]^{1-T}}{1-[(1-\delta)+r^e(t)]^{-1}}. \quad \text{(A.2.2)}
$$

Now, considering the timing outlined in Appendix A.1 above, for $1 \leq t \leq T_p - 1$ we plug in $T_p - t$ for T and get (24)

$$
ST_1 = \frac{\tau_0}{r^e(t)-\delta} + (\tau_1-\tau_0) \frac{[(1-\delta)+r^e(t)]^{t-T_p}}{1-[(1-\delta)+r^e(t)]^{-1}}, \quad \text{(A.2.3)}
$$

and for $t \geq T_p$ we have $T-1=0$, thus we get (25)

$$
ST_1 = \frac{\tau_1}{r^e(t)-\delta}. \quad \text{(A.2.4)}
$$

A.3. Derivation of SW_2

We start from (31)

$$SW_2 = \sum_{j=1}^{\infty} \frac{1}{D_{t,t+j}^{k,e}(t)} w_{t+j}^{e}(t)\bar{L}.$$

Next, we recall the definition of $D_{t,t+j}^{k,e}(t)$. Given the learning rules (16) and (17) we get

$$D_{t,t+j}^{k,e}(t) = \Pi_{i=1}^{j}\left[(1-\delta)+(1-\tau_0^k)r^e(t)\right] = \left[(1-\delta)+(1-\tau_0^k)r^e(t)\right]^j \quad \text{(A.3.1)}$$

for $\tau_{t+j}^{k,e}(t) = \tau_0^k$ and

$$D_{t,t+j}^{k,e}(t) = \Pi_{i=1}^{j}\left[(1-\delta)+(1-\tau_1^k)r^e(t)\right] = \left[(1-\delta)+(1-\tau_1^k)r^e(t)\right]^j \quad \text{(A.3.2)}$$

for $\tau_{t+j}^{k,e}(t) = \tau_1^k$. Thereafter, we split this infinite sum into

$$\begin{aligned} SW_2 &= \bar{L}\left[\sum_{j=1}^{T-1} \frac{1}{D_{t,t+j}^{k,e}(t)} w^e(t) + \sum_{j=T}^{\infty} \frac{1}{D_{t,t+j}^{k,e}(t)} w^e(t)\right] \\ &= \bar{L}[\sum_{j=1}^{T-1}(\left[(1-\delta)+(1-\tau_0^k)r^e(t)\right]^j)^{-1} w^e(t) + \\ &\quad \sum_{j=T}^{\infty}(\left[(1-\delta)+(1-\tau_1^k)r^e(t)\right]^j)^{-1} w^e(t)\ ], \end{aligned}$$

or

$$SW_2 = \frac{w^e(t)\bar{L}}{\left[(1-\delta)+(1-\tau_0^k)r^e(t)\right]}\sum_{j=0}^{T-2}\left(\left[(1-\delta)+(1-\tau_0^k)r^e(t)\right]^{-1}\right)^j$$

$$+\frac{w^e(t)\bar{L}}{\left[(1-\delta)+(1-\tau_1^k)r^e(t)\right]}\sum_{j=T-1}^{\infty}\left(\left[(1-\delta)+(1-\tau_1^k)r^e(t)\right]^{-1}\right)^j.$$

As in Section A.2 above, we exploit the properties of geometric series and derive

$$SW_2 = \frac{w^e(t)\bar{L}}{\left[(1-\delta)+(1-\tau_0^k)r^e(t)\right]}\left(\frac{1-\left[(1-\delta)+(1-\tau_0^k)r^e(t)\right]^{1-T}}{1-\left[(1-\delta)+(1-\tau_0^k)r^e(t)\right]^{-1}}\right)$$

$$+\frac{w^e(t)\bar{L}}{\left[(1-\delta)+(1-\tau_1^k)r^e(t)\right]}\left(\frac{\left[(1-\delta)+(1-\tau_1^k)r^e(t)\right]^{1-T}}{1-\left[(1-\delta)+(1-\tau_1^k)r^e(t)\right]^{-1}}\right).$$

Now we get back to the timing outlined in Appendix A.1 above, for $1 \leq t \leq T_p - 1$ we plug in $T_p - t$ for T and get (32)

$$SW_2 = \frac{w^e(t)\bar{L}}{[(1-\tau_0^k)r^e(t)-\delta]} + w^e(t)\bar{L} \times$$

$$\left[\frac{[(1-\delta)+(1-\tau_1^k)r^e(t)]^{t-T_p}}{1-[(1-\delta)+(1-\tau_1^k)r^e(t)]^{-1}} - \frac{[(1-\delta)+(1-\tau_0^k)r^e(t)]^{t-T_p}}{1-[(1-\delta)+(1-\tau_0^k)r^e(t)]^{-1}}\right] \tag{A.3.3}$$

and for $t \geq T_p$ we have $T - 1 = 0$, thus we get (33)

$$SW_2 = \frac{w^e(t)\bar{L}}{[(1-\tau_1^k)r^e(t)-\delta]}. \tag{A.3.4}$$

A.4. Derivation of ST_2

Starting from (34)

$$ST_2 = \sum_{j=1}^{\infty} \frac{1}{D_{t,t+j}^{k,e}(t)} \tau_{t+j}^{l,e}(t) w_{t+j}^{e}(t) \bar{L}$$

for (A.3.1) and (A.3.2) and $\tau_{t+j}^{l,e}(t)$ is either given by τ_0^l or τ_1^l, we may once more split the infinite sum into

$$\begin{aligned} ST_2 &= w^e(t)\bar{L} \times \\ & [\sum_{j=1}^{T-1} \left(\left[(1-\delta) + (1-\tau_0^k) r^e(t) \right]^j \right)^{-1} \tau_0^l \\ & + \sum_{j=T}^{\infty} \left(\left[(1-\delta) + (1-\tau_1^k) r^e(t) \right]^j \right)^{-1} \tau_1^l \,], \end{aligned}$$

or

$$\begin{aligned} ST_2 = & \frac{\tau_0^l \; w^e(t)\bar{L}}{\left[(1-\delta) + (1-\tau_0^k) r^e(t) \right]} \sum_{j=0}^{T-2} \left(\left[(1-\delta) + (1-\tau_0^k) r^e(t) \right]^{-1} \right)^j \\ & + \frac{\tau_1^l \; w^e(t)\bar{L}}{\left[(1-\delta) + (1-\tau_1^k) r^e(t) \right]} \sum_{j=T-1}^{\infty} \left(\left[(1-\delta) + (1-\tau_1^k) r^e(t) \right]^{-1} \right)^j . \end{aligned}$$

Now, the properties of the geometric series allow us to rewrite this as

$$\begin{aligned} ST_2 = & \frac{\tau_0^l \; w^e(t)\bar{L}}{\left[(1-\delta) + (1-\tau_0^k) r^e(t) \right]} \left(\frac{\left[(1-\delta) + (1-\tau_0^k) r^e(t) \right]^{1-T} - 1}{\left[(1-\delta) + (1-\tau_0^k) r^e(t) \right]^{-1} - 1} \right) \\ & + \frac{\tau_1^l \; w^e(t)\bar{L}}{\left[(1-\delta) + (1-\tau_1^k) r^e(t) \right]} \left(\frac{- \left[(1-\delta) + (1-\tau_1^k) r^e(t) \right]^{1-T}}{\left[(1-\delta) + (1-\tau_1^k) r^e(t) \right]^{-1} - 1} \right) . \end{aligned}$$

For the timing outlined in Appendix A.1 above, for $1 \leq t \leq T_p - 1$ we plug in $T_p - t$ for T and get (35)

$$
\begin{aligned}
ST_2 &= \frac{\tau_0^l \; w^e(t)\bar{L}}{[(1-\tau_0^k)r^e(t)-\delta]} + w^e(t)\bar{L} \times \\
& \left[\frac{\tau_1^l \; [(1-\delta)+(1-\tau_1^k)r^e(t)]^{t-T_p}}{1-[(1-\delta)+(1-\tau_1^k)r^e(t)]^{-1}} - \frac{\tau_0^l \; [(1-\delta)+(1-\tau_0^k)r^e(t)]^{t-T_p}}{1-[(1-\delta)+(1-\tau_0^k)r^e(t)]^{-1}}\right]
\end{aligned} \tag{A.4.1}
$$

and for $t \geq T_p$ we have $T - 1 = 0$, thus we get (36)

$$
ST_2 = \frac{\tau_1^l \; w^e(t)\bar{L}}{[(1-\tau_1^k)r^e(t)-\delta]}. \tag{A.4.2}
$$

A.5. Derivation of ST_3

Starting from (37)

$$
ST_3 = \sum_{j=1}^{\infty} \frac{1}{D_{t,t+j}^{k,e}(t)} \tau_{t+j}^e(t)
$$

given (A.3.1) and (A.3.2) are true and $\tau_{t+j}^e(t)$ is either τ_0 or τ_1, we again split the infinite sum into

$$
\begin{aligned}
ST_3 = [& \sum_{j=1}^{T-1} \left(\left[(1-\delta)+(1-\tau_0^k)r^e(t)\right]^j \right)^{-1} \tau_0 \\
& + \sum_{j=T}^{\infty} \left(\left[(1-\delta)+(1-\tau_1^k)r^e(t)\right]^j \right)^{-1} \tau_1 \;],
\end{aligned}
$$

or

$$ST_3 = \left[(1-\delta)+(1-\tau_0^k)r^e(t)\right]^{-1}\left[\sum_{j=0}^{T-2}\left(\left[(1-\delta)+(1-\tau_0^k)r^e(t)\right]^{-1}\right)^j \tau_0\right]$$

$$+\left[(1-\delta)+(1-\tau_1^k)r^e(t)\right]^{-1}\left[\sum_{j=T-1}^{\infty}\left(\left[(1-\delta)+(1-\tau_1^k)r^e(t)\right]^{-1}\right)^j \tau_1\right].$$

Given the properties of geometric series we can rewrite the latter as

$$ST_3 = \left[(1-\delta)+(1-\tau_0^k)r^e(t)\right]^{-1}\left(\frac{\left[(1-\delta)+(1-\tau_0^k)r^e(t)\right]^{1-T}-1}{\left[(1-\delta)+(1-\tau_0^k)r^e(t)\right]^{-1}-1}\tau_0\right)$$

$$+\left[(1-\delta)+(1-\tau_1^k)r^e(t)\right]^{-1}\left(\frac{-\left[(1-\delta)+(1-\tau_1^k)r^e(t)\right]^{1-T}}{\left[(1-\delta)+(1-\tau_1^k)r^e(t)\right]^{-1}-1}\tau_1\right).$$

Now given the timing outlined in Appendix A.1 above, for $1 \le t \le T_p - 1$ we plug in $T_p - t$ for T and get (38)

$$\begin{aligned}
ST_3 &= \frac{\tau_0}{[(1-\tau_0^k)r^e(t)-\delta]} \\
&\quad +[\ \frac{[(1-\delta)+(1-\tau_1^k)r^e(t)]^{t-T_p}}{1-[(1-\delta)+(1-\tau_1^k)r^e(t)]^{-1}}\ \tau_1 \\
&\quad -\frac{[(1-\delta)+(1-\tau_0^k)r^e(t)]^{t-T_p}}{1-[(1-\delta)+(1-\tau_0^k)r^e(t)]^{-1}}\ \tau_0\ ]
\end{aligned} \tag{A.5.1}$$

and for $t \ge T_p$ we have $T-1=0$, thus we get (39)

$$ST_3 = \frac{\tau_1}{[(1-\tau_1^k)r^e(t)-\delta]}. \tag{A.5.2}$$

B. Computing Welfare Changes

B.1. Comparative Statics

We follow the approach of Cooley and Hansen (1992, p.301ff.) based on Lucas (1990). Their measure of welfare change for a given policy change is derived by solving

$$U_0 = \log[c_1(1 + x^\bullet)] + \eta \log[1 - n_1] \tag{B.1.1}$$

for x in our case.[20] U_0 is the utility a household obtains in the steady-state without any tax and c_1 and n_1 are the values of consumption and employment at the new steady-state after the tax change either under perfect foresight or learning. It follows that

$$x^\bullet = \frac{\exp(U_0)}{c_1(1 - n_1)^\eta} - 1. \tag{B.1.2}$$

Thus, in general, we need to solve for x for the perfect foresight dynamics and another x^* for the dynamics under learning.[21] Given $x^\bullet$ we can calculate

$$\overline{\mathcal{W}} = \frac{\triangle C}{y_1} = \frac{x^\bullet c_1}{y_1}, \tag{B.1.3}$$

where $\triangle C$ is the restoration value of consumption, which in our case may be interpreted as the total change in consumption required to restore a household to the level of utility obtained under the allocation associated with zero taxes. y_1 is the level of output at the new steady-state.

[20] $x^\bullet$ is either x under perfect foresight or x^* under learning.

[21] Of course we are aware that this must yield the same $x = x^*$ both under perfect-foresight and under learning, but this number may be useful to compare different policy experiments.

B.2. Transition Measure

Again we follow the approach of Cooley and Hansen (1992, p.301ff.) based on Lucas (1990). Their measure of welfare change accounting for transition given a policy change is derived by solving

$$\sum_{t=1}^{T} \beta^t \left\{ \log[c_t(1+x^\bullet)] + \eta \log[1-n_t] - U_0 \right\} \stackrel{!}{=} 0 \tag{B.2.1}$$

for x under perfect foresight and x^* under learning. T is the terminal period, c_t is period t consumption either under perfect foresight or learning and y_t is period t output either under perfect foresight or learning.

$$x^\bullet = \left[\frac{\exp\left(U_0\left[\beta^1 + ... + \beta^T\right]\right)}{\left(c_1^{\beta^1} ... \; c_T^{\beta^T}\right) \times \left[(1-n_1)^{\eta\beta^1} \; ... \; (1-n_T)^{\eta\beta^T}\right]} \right]^{\frac{1}{[\beta^1 + ... + \beta^T]}} - 1.$$

$$x^\bullet = \left[\frac{\exp\left(U_0 \sum_{t=1}^{T} \beta^t\right)}{\Pi_{t=1}^{T} c_t^{\beta^t} \times \Pi_{t=1}^{T} (1-n_t)^{\eta\beta^t}} \right]^{\frac{1}{\sum_{t=1}^{T} \beta^t}} - 1. \tag{B.2.2}$$

Given $x^\bullet$ we can calculate

$$\mathcal{W}^\bullet = \frac{\sum_{t=1}^{T} \beta^t \left\{x c_t\right\}}{\sum_{t=1}^{T} \beta^t \left\{y_t\right\}}, \tag{B.2.3}$$

which will be reported as $\mathcal{W}$ for the perfect foresight dynamics and as $\mathcal{W}^*$ for the dynamics under learning.

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Figure Legends

Figure 1: Consumption and capital dynamics under learning and perfect foresight for a change in lump-sum tax. Consumption (a) and capital (b) dynamics under learning (solid curve) and perfect foresight (dashed curve) with inelastic labour supply as in Evans et al. (2009, p.943ff.) as well as consumption (c) and capital (d) dynamics under learning (solid curve) and perfect foresight (dashed curve) with elastic labour supply. The dotted horizontal line indicates the (new) steady state, the dotted vertical line indicates period T_p.

Figure 2: Sensitivity analysis for consumption and capital dynamics under learning and perfect foresight with regard to the gain parameter. Consumption (a) and capital (b) dynamics under learning and perfect foresight with inelastic labour supply as in Evans et al. (2009, p.943ff.) for alternating values of γ. The dotted horizontal line indicates the (new) steady state, the dotted vertical line indicates period T_p.

Figure 3: Sensitivity analysis for consumption and capital dynamics under learning and perfect foresight with regard to the implementation date. Consumption (a) and capital (b) dynamics under learning with inelastic labour supply as in Evans et al. (2009, p.943ff.) for alternating values of T_p. The dotted horizontal line indicates the (new) steady state, the dotted vertical line indicates period $T_p = 20$.

Tables

Parameter	Value	Parameter	Value
A	1.00	δ	0.00
α	0.33	T_p	20
β	0.95	γ	0.10

Table 1: Parameters similar as in Evans et al. (2009, p.945)

Parameter	**Value**	**Parameter**	**Value**
A	1.00	δ	0.00
α	0.33	T_p	8
β	0.99	γ	0.08
η	0.99	$\bar{L}$	1.00

Table 2: Model calibration for policy experiments $1. - 4.$

no.	$\tau_\bullet$		$\tau^l_\bullet$		$\tau^k_\bullet$		$\tau^c_\bullet$		$\mathcal{W}$		n_{FB}	$n_\bullet$	
	0	1	0	1	0	1	0	1	P	L		0	1
1.	0.0		0.2300		0.5000		0.0500		73.72%		0.4024	0.4326	
2.	0.0		0.2300	0.2931	0.5000	0.2500	0.0500		64.47%	72.12%	0.4024	0.4326	0.3976
3.	0.0		0.2300		0.5000	0.2500	0.0500	0.1027	63.71%	71.28%	0.4024	0.4326	0.4045
4.	0.0	0.0600	0.2300		0.5000	0.2500	0.0500		63.49%	70.23%	0.4024	0.4326	0.4139

0: Value before the tax change   1: Value after the tax change
P: Value under perfect foresight   L: Value under learning

Table 3: Simulation results of various policy experiments.