Post-Keynesian modelling: where are we, and where are we going to?

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Post-Keynesian modelling: where are we, and where are we going to?

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This paper presents the current "state of the art" of Post-Keynesian modelling, as well as the most important issues raised by it. We first present a new formal statement of the Keynes' model, highlighting the importance of the "static model of a dynamic process", and insisting on the influence of "true uncertainty" and of the "views concerning the future." The paper then analyses the three most important classes of Post-Keynesian contemporary models: the Kaleckian models of growth; the Minskian models showing the destabilizing impacts of financial variables on the economy; and the path-dependant models insisting on the nature of time in economics, and on the absence of any "natural" anchor. We argue that, whereas the current Post-Keynesian models have a lot in common with Keynes' model, none of them encompasses all its rich and realistic properties, and a synthetic dynamic Post-Keynesian model is desirable, and has still to be framed. The main barriers to this synthesis are underlined.

Keywords: Post-Keynesian modelling; Keynes' "static model of a dynamic process"; Kaleckian models; Minskian models; Models of path dependency.

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1. Introduction

Post-Keynesian models are mainly concerned with issues relating to economic cycle, income distribution and growth theory. Because of the leading role of demand, the long-run dynamics of these models sharply differs from the mainstream models. In particular, investment in capital goods has a positive impact on aggregate demand, while it increases simultaneously the productive capacity in a virtuous circle *a la* Kaldor. Other important aspects of Post-Keynesian models are Kalecki's emphasis of the impact of income distribution on aggregate demand and growth; Minsky's concern with financial instability; and Kaldor and Robinson's insistence on the fact that equilibria are necessarily path-dependent.

The common feature of all these models lies in Keynes' finding of the leading role of effective demand. Indeed, Post-Keynesian dynamic models are concerned with the effects on effective demand of changes in such variables as technology, income distribution or financial issues which are usually taken as constant in static and short run dynamic models. Static models based on given production conditions therefore are essential to long-run models, for they provide the equilibrium value of the key variables as a function of the variables which change in the long period[2]. Growth models hence derive the motion of the system as a result of the changes in the production conditions and other variables. This is the way (Keynes, 1936, p 247) goes about things: the "skill and quantity of available labour, the existing quality and quantity of available equipment, the existing technique, ..." are taken as given, while the "object is to discover what determines at any time the national income of a given economic system ...".

The methodological usefulness of representing a dynamic process by means of a static model has been pointed out accurately in the Post-Keynesian literature. The method mainly consists in taking long-term expectations as given, so that the functional relations involved by the determination of effective demand can be considered in a tractable way. Once those relations have been identified, it becomes possible to consider the dynamics involved by changing expectations and supply conditions. There have been a few attempts to formalize

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[2] Though dynamic models deal with changes in variables, they start necessarily from a point in time, with initial values: X_0,Y_0, Z_0… the consistency of which requires a theory of equilibrium values at date (0). Dynamics then considers the effects on equilibrium of the changing technology, income distribution, expectations, institutions... This is the why the 'static' model is of paramount importance.

static/short run models, of which Chick (1983) and Palley (1996) are to be mentioned[3], though the former is reluctant to set the model in equations, while the latter offers a rich thirteen-equation system. The reason to such discrepancy rests on the controversial issue of whether it is possible or not to represent a static model of a dynamic process by means of a system of simultaneous equations. The question is dealt with in section 2 of the paper, where a formal framework embracing numerous methodological issues is proposed. Section 3 then analyses the three most important types of contemporary Post-Keynesian dynamic frameworks, and discusses some difficulties related to these models, and how these models are related to Keynes' static model. Section 4 briefly concludes.

2. Keynes's static model of a dynamic process: a statement

In spite of strong empirical evidence of structural instability, the mainstream[4] nowadays continues postulating that individuals make optimal intertemporal decisions, as if it was sensible to base long-term decisions on the current knowledge of a system which is likely to change at any time[5]. By contrast, Keynes built his *General Theory* by reconsidering the functioning of competitive markets in the absence of any "natural" anchor for expectations, that is, in presence of "true uncertainty." Keynesian uncertainty does not mean that agents do not try to predict the value of such and such decisive variable; it means that the usefulness of forward-looking expectations is much weaker than it is usually supposed in the mainstream approach. Keynes admits that people make use of all available information, but whatever the kind of probabilistic tools they may use, expectations can be systematically wrong, as the past events never give enough information about what the future will be (non ergodicity[6]). As individual decisions have no optimal solution, decisions have to be made on the basis of the "views concerning the future". While the mainstream only reckons the Pareto-optimal solution (which may be a 1st best or 2nd best … because of imperfect competition), Keynes'

3 See also Fontana and Setterfield (2009).

4 By "mainstream" we refer to the "new macroeconomic consensus".

5 Asensio (2008) collects recent empirical and theoretical material about strong uncertainty which questions the New Consensus' foundations from its own point of view.

6 Ergodicity is defined as the dynamic stability of a stochastic process; see Davidson (2002, pp. 39-69).

theory produces a different equilibrium for every state of the "views concerning the future"[7]. Thus *The General Theory* is basically more general than the mainstream's theory, and more realistic, since uncertainty is not reduced to a weakened concept. The introduction of "true uncertainty" in economic reasoning is the essential innovation, which led Keynes to such a revolutionary finding as a competitive equilibrium with non clearing labour market.

The modelling we propose here reconciles both the aim of representing Keynes' static model in equations (as in Palley, 1996), and the willingness to preserve its generality (as in Chick, 1983). The model is a set of conditions involving supply and demand sides in four macroeconomic markets. These conditions express individual decisions and constraints, whose compatibility is ensured *at any time* by the adjustment of prices and quantities to equilibrium values (table 1). Insofar as no particular specification has been given to the resulting functional relations, the degree of generality of the theory is preserved. Our model is close to Chick's approach in spirit, and to Palley's model as regards the main equations, with a few noticeable differences. First, the model is more synthetic than Palley's one, which formalizes short-run dynamics that are not of primary importance to Keynes' theory. Additional equations represent a financial sector with loans, bonds and stocks in Palley's model, while our presentation omits stocks and allows for an implicit account of the market for bonds in relation to the aggregate budget constraints. More importantly, while Palley and Chick have the long-term rate of interest determined by the banking sector and the money demand interaction[8], no discussion is made as to whether this rate meets the "conventional expectation of the future". Yet, Keynes' theory suggests that if the current market rate is higher (lower) than the convention and the rate of interest therefore is expected to decrease (increase), the demand for money alters, so that the current rate eventually meets the convention. Therefore, the long-term rate of interest is "a highly conventional … phenomenon. For its actual value is largely governed by the prevailing view as to what its value is expected to be. *Any* level of interest which is accepted with sufficient conviction as *likely* to be durable *will* be durable; subject, of course, in a changing society to fluctuations for

[7] "Or, perhaps, we might make our line of division between the theory of stationary equilibrium and the theory of shifting equilibrium -meaning by the latter the theory of a system in which changing views about the future are capable of influencing the present situation" (Keynes 1936, p. 293).

[8] While, in Fontana and Setterfield (2009), pure horizontalism (total accommodation) is assumed for the sake of simplicity, in Palley (1996, p. 111), the supply curve may be positively sloped in the 'accommodationist' as well as in the 'structuralist' case, but the rationale of course differ.

all kinds of reasons round the expected normal" (Keynes, 1936, p. 203). This questions the ability of monetary policy to really control the long-term interest rate. Hence, we suppose that the monetary base increases as a result of lower refinancing short rates policy, and that, as a matter of consequence, lower long-term bank rates starts boosting the demand for credit. If the liquidity preference then increases as a result of the long-term rate decreasing below the "conventional expectation", banks may be able to sell more credit without reducing their interest rates substantially, for bonds and other non-bank loan rates tend to rise in this case, in order to compensate for the increasing liquidity preference. Even if "the monetary authority were prepared to deal both ways on specified terms in debts of all maturities, and even more so if it were prepared to deal in debts of varying degree of risk", there would be "limitations on the ability of the monetary authority to establish any given complex of rates of interest for debts of different terms and risk…" (Keynes, 1936, pp. 205-207). Some of these limitations would only arise in extreme circumstances (virtually absolute liquidity preference when rates are considered too low; breakdown of stability in the rate of interest – owing to a flight from the currency or other financial crisis); but others apply in normal circumstances (the intermediate cost of bringing the borrower and the lender together, the allowance for risk required by the lender, including liquidity risk). Hence, whilst the Post-Keynesian endogenous money approach is right when it states that banks do deliver the amount of credit money that is demanded at the current interest rate, and whilst it is also correct to say that the long-term rate is exogenous in the sense that it does not result from a market clearing process, it is not certain that the central bank has enough control as to set the rate at the level it decides[9]. Accordingly, the equilibrium rate of interest in our model is given, ultimately, by the market convention.

Lastly, Chick and Palley share Keynes' statement that nominal wages rigidity is stabilizing in the sense that there would be no equilibrium in the absence of such a rigidity (Keynes 1936, pp. 303-304), so that they take the nominal wage as given[10]. But they do not

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[9] According to Lavoie (1999, p.2), "monetary authorities have the ultimate say on the convention." But he also points out that the spreads between the long-term rates and the overnight rate vary according to the liquidity preference of the commercial banks and the participants in the financial markets. See Asensio (2011 a) for a detailed discussion.

[10] Palley (1996, chapter 4) discusses several wages adjustment processes, which illustrate the ambiguous effects that Chick pointed out. In those special cases there is an endogenous equilibrium wage, but in chapter 13, where the full macro model is stated with the higher degree of generality, the nominal wage is taken as given.

deal with rigid wages in terms of the equilibrium outcome of some stabilizing forces. Following Asensio (2011b), we adopt a concept of equilibrium which involves institutional stabilizers besides the potentially destabilizing market forces.

Table 1 summarizes the way equilibrium values emerge within both theories.

Table 1
Keynes's static model - the four macro-market equilibrium conditions (equilibrium prices and quantities in bold)

	Mainstream	***The General Theory***
Goods market	*Supply*: $Y_s=f(N,K)$ *(diminishing marginal returns)* *Demand*: C(i,…) and I(i;…) adjust to the supply of goods (given G) by means of i, along with the bonds market (Say's law): $\rightarrow \mathbf{Y^*}=Y_s, \forall Y_s$ The market clears Price level: $\mathbf{p^*}=p^T$	*Demand*: $Y_d=C+I+G=Y_d(i,E,\ldots)$ Effective demand (Y_d^e) is (approximately) equal to Y_d.[11] *Supply*: equal to eff dem: $Y_s=Y_d^e, \rightarrow$ (2.1) $\mathbf{Y^*}=Y_d(i^*,E,\ldots)$ The market may clear on different positions, according to the 'views about the future' (which influence I, E, and therefore Y_d^e and Y*) (2.2) Price level: $w/p=f'_N \rightarrow \mathbf{p^*}=w^*/f'_N$
Labour market	*Demand*: $f'_N=\omega \rightarrow N_d(\omega)$ *Supply*: $N_s(\omega)$ Competitive real-wage clears the market: $\mathbf{N^*},\boldsymbol{\omega}^*$ (nb: $\mathbf{w^*}=p^*\omega^*$)	*Demand*: $Y=f(N,K) \rightarrow N_d=g(Y^*,K)$ *(diminishing marginal returns)* *Supply*: rationed, $N_s=N_d \rightarrow$ (2.3) $\mathbf{N^*}=g(Y^*,K)$ In the face of unemployment, flexible money-wage would be destabilizing through the negative effects on E, Y_d... $\rightarrow$Institutional and social forces stabilize the money-wage: $\mathbf{w^*} \equiv w_{I\&M}$
Money market	*Demand*: L(Y,p,i) *Supply*: adjusts so that p is at the targeted level, given i* and $Y^* \rightarrow \mathbf{M^*}$ The market clears. Remark: in the 'short short-run' p may deviate from the target temporarily for stabilization	*Demand*: L(Y,p,i) *Supply*: $M_s(i)$, adjusts to the demand endogenously[12]: (2.4) $M_s^*(i^*)=L(Y^*,p^*,i^*) \rightarrow \mathbf{M^*},\mathbf{i^*}$ Remark: L (and M*) would shift if i* differed from the convention so that in general, i* (approximately) meets the convention: $\mathbf{i^*} \equiv i_{conv}$

[11] Keynes' model abstracts from possible differences between effective (expected) demand and realized aggregate demand, for "... there is a large overlap between the effect on employment of the realized sale-proceeds of recent output and those of the sale-proceeds expected from current input" (Keynes, 1936, p. 51).

[12] In the pure horizontalist case, perfect accommodation makes Ms independent of i.

	purposes, which supposes that monetary authorities deviate i from i*	→The market may clear on different positions according to the liquidity preference, and the capacity of monetary authorities to influence the convention.
Bonds market	Implicit (aggregate balanced budgets)*	Implicit (aggregate balanced budgets)*

* See Asensio (forthcoming a) for a formal statement.

Y	output, 'volume'[13]
Y_d^e	effective demand,'volume'[15]
C	consumption, 'volume'[15]
I	investment, 'volume'[15]
G	government expenditures (exogenous), 'volume'[15]
N	employment
M	quantity of money
K	capital stock[15] (given)
p	output price[15]
w	money wage
$w_{I\&M}$	money wage that market and institutional forces lead to (given)
i	long-term interest rate
i_{conv}	conventional long-term interest rate (given, but authorities may have some influence)
E	marginal efficiency of capital (MEC) schedule[14] (given)
ω	real wage

'e' superscript denotes the expected value of a variable.

In the mainstream's model, the real wage and the employment level are determined in the labour market. Provided there is no hindrance to competitive forces, and provided the price setting is common knowledge, the nominal wages setting ensures that the price setting does not affect the equilibrium real wage: $w^*=p^*\omega^*$. Hence, the money-wage is proportional to the price of goods and varies along with it: $dw^*/w^*=dp^*/p^*+d\omega^*/\omega^*$. The basic mainstream model assumes that investment in capital goods is completely financed through borrowing in the bonds market, at a real expected long-term rate of interest (the natural equilibrium rate: $r^*=r_{nat}$). Again, the nominal-rate setting is aimed at increasing interests in due proportion, so as to compensate for expected inflation: $i^*=r^*+(dp/p)^e$. Competitive forces

[13] This heterogeneous aggregate is very hard to measure (as well as the capital stock and the output price). Measuring it in wage units (as Keynes did) does not really solve the problem (see Hayes, 2007 and Ambrosi, 2009). As it has become usual, we refer to the 'volume' of output, the capital stock and the price index in both models in order to make them easy to compare. Notice that the volume of output equals real income insofar as producers' expectations of aggregate demand are correct (see Chick and Tily, 2007, p. 8, footnote 9).

[14] Notice that the money-wage expected change is likely to influence E. This is an important feature of the dynamics of Keynes' equilibrium, along with expectations, technology.... It is omitted here for the sake of simplicity (as at equilibrium, institutions are deemed to have stabilized the money-wage at a given level).

in this market ensure both investment and saving equalization, that is, the bonds market clearing and, as a matter of consequence, $Y_d=Y_s \ \forall Y_s$ (Say's law), that is, the goods market clearing as well. Given the level of employment and the existing (exogenous) capital stock, firms set the output level in accordance with the existing technology: $Y_s=f(N,K)$, which determines the equilibrium level of output: $Y^*=f(N^*,K)$, insofar as Say's law ensures $Y_d=Y^*$ as well.

In the modern endogenous-money version of the mainstream model, the central bank aims at stabilizing the price of goods at a desired level: p_T (exogenous), which requires the money supply to be adapted by the central bank to the amount agents demand given that price level[15]. Therefore, the equilibrium quantity of money is endogenous $M^*=M_d\ (p_T,Y^*)$. More (less) money would lead to higher (lower) prices than the desired level. Standard models allow for short run non-neutrality, through inflation deviations from the target that produce unexpected inflation. But in the long run neutrality holds, for either authorities implement the target, or agents eventually learn about the central bank's strategy if it tries to keep employment beyond the "natural level" continuously. In both cases it would only raise expected inflation to effective inflation, without any real effect in the economy.

In Keynes' theory, the equilibrium interest rate is not aimed at adjusting aggregate saving and investment (which nevertheless equalize, but not through the bonds market); it is determined by the "conventional expectation" of the future rate[16] which may be influenced by the central bank, but is not under its complete control. As the rate of interest does not adjust, firms must decide the levels of employment and production in accordance with expected aggregate demand. Therefore, in the goods market, $Y^*=C^e+I^e+G^e=Y_d{}^e$, which, according to Keynes' argument on short run expectations, is approximately the same as $Y^*=C^*+I^*+G=Y_d(i^*,E,\ldots)$ (2.1): production adjusts to effective demand. On the "labour market", the demand by firms results from that output level, given the technology: $N^*=N_d(Y^*,K)$ (2.3). Firms manage so that the marginal product of labour matches the real wage, but in Keynes's theory, this does not result from firms hiring decisions; it results from their pricing in the goods market: $p^*=w^*/f_{N^*}$', where the equilibrium value of money wage is an outcome of the interaction of institutions and market forces (2.2).

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15 See Romer (2000).

16 This expected rate includes expected inflation, so that a conventional real rate r* can be deduced from $i^*=r^*+(dp/p)^e$.

Unemployment happens when, given the global propensity to consume, the conventional interest rate compared with the MEC schedule does not allow for a sufficient amount of investment. There is no systematic force that could spontaneously eliminate unemployment, since it does not depend on wages. The only way wages could influence output is through the demand side effect of an expected change of wage which would influence the MEC schedule (rather negatively according to Keynes, 1936, chapter 19, though the possibility that a positive impact leads to full employment is not discarded as a special case). But in any case, as institutions do stabilize $w_{I\&M}$ in case of unemployment, so that the whole system does not fall into cumulative depression, any change of wages in such a context is the result of some institutional force (Asensio forthcoming b). This is to consider the active role of institutions at any given time, besides the influence of the institutional framework on the long run dynamics.

A salient feature of Keynes's static model is that i* and w* are not modelled. These variables are endogenous to the economic system, but their equilibrium level depends on expectations and human institutions which could hardly be modelled with a high degree of generality in a context of "true uncertainty", although it may be possible and useful to model their action in accordance with the context in specific cases. The openness of the determination of i* is the very reason why Keynes rejected that effective demand sets necessarily at full employment, while the openness of the determination of w* is the very reason why Keynes rejected that unemployment spontaneously vanishes under competitive conditions. The static model accordingly delivers a genuine "shifting equilibrium", insofar as there is no objective anchor for E, i* and w*, with the result that, at any given time, there is a continuum of possible equilibria. Expectations, conventions and institutions determine their position.

3. Post-Keynesian dynamic models

The renewed interest for Post-Keynesian modelling, that has started in the 1970's, has been stronger in the past decade, when it became clearer than ever that the mainstream models are unable to explain economic events in the real world. Schematically, these Post-Keynesian models can be ranked in three classes. First, Kaleckian models of growth take into account the effects of distribution on utilization and profit rates and accumulation. Second, Minskian

models integrate financial variables and show their destabilizing impacts on the economy. Thirdly, path-dependant models try to develop Kaldor's (1934) and Robinson's (1962) work about the nature of time in economics, and the absence of any "natural" anchor[17].

3.1. Kaleckian models of growth and income distribution

The so-called Kaleckian model has undeniably become the most popular one in the Post-Keynesian literature. Due to the extreme flexibility of their structures, these models have generated numerous contributions from the beginning of the nineties (see Blecker, 2002). We present a basic Kaleckian framework, based on three simple equations.

The price level is given by a mark-up, $m > 0$, over unit labour costs:

$$p = (1+m)wl \tag{3.1}$$

w is the nominal wage rate and $l=L/Y$ the labour-output ratio.

National income is divided between the wage bill and aggregate profits: $pY = wL + \Pi$. That gives the profit share in national income:

$$\pi = \Pi / pY = 1 - (w/p)l \tag{3.2}$$

Introducing (3.1) into (3.2), we find the value of π using the mark-up: $\pi = m/(1+m)$. The rate of profit, r, is decomposed between the profit share, π, and capacity utilization, u:

$$r = \frac{\Pi}{pK} = \frac{\Pi}{pY}\frac{Y}{K} = \pi u \tag{3.3}$$

The investment function is:

$$g^d = g_0 + g_u u + g_r r \tag{3.4}$$

[17] Some Post-Keynesians would argue that there is in fact only one Post-Keynesian model: the Kaleckian model of growth and income distribution. For them, path-dependency *a la* Kaldor and financial instability *a la* Minsky would only be desirable features, but not models as such. Nevertheless, even if we wish that a Post-Keynesian synthesis including all those features finally emerges, this is not the case yet – so it is still true at the present stage that these various stands of literature can be characterized as "models."

Where g_0 is meant to represent animal spirits, while g_u and g_r are positive parameters showing the influence of utilization and macroeconomic profit rates on investment planned by firms.

The third equation is the Cambridge saving function, s_π being the propensity to save out of profits.

(3.5) $$g^s = s_\pi \pi$$

Equation (3.5) is based on the usual hypothesis that workers do not save. Equating (3.5) and (3.4) and replacing *r* by (3.3), we find the equilibrium value for the rate of capacity utilization:

(3.6) $$u^* = \frac{g_0}{(s_\pi - g_r)\pi - g_u}$$

Substituting (3.6) into (3.3) and (3.4) gives the profit rate and the rate of accumulation:

(3.7) $$r^* = \frac{g_0 \pi}{(s_\pi - g_r)\pi - g_u}$$

(3.8) $$g^* = \frac{g_0 s_\pi \pi}{(s_\pi - g_r)\pi - g_u}$$

Two fundamental results of this model are fully consistent with Keynes' model: the "paradox of thrift" and the "paradox of costs" (also called the *stagnationist effect*). Firstly, an increase in the propensity to save has a negative impact on economic activity by decreasing *u**, *r** and *g**. Secondly, an increase in the profit share (implying a fall in real wages) lowers the rate of capacity utilization, and therefore has a negative influence on growth. This apparent paradox is easily explained: the rise in the profit share is not important enough to counterbalance the depressing effect on consumption demand, emanating from a fall in *w*/*p*.

The latter result has been challenged many times since Bhaduri and Marglin's (1990) seminal paper. By modifying the investment function, they have put forward the existence of a dual case. Since then, it is established that two possibilities exist: a "stagnationist" case, in which an increase in real wages tends to raise capacity; and an "exhilarationist" case, where a

rise in real wages diminishes the rate of capacity utilization. In the long-run, the paradox of costs becomes a single possibility that occurs under some circumstances.

A more fundamental criticism has been addressed to Kaleckian models, mainly by Marxists and Sraffians (see, for example, Duménil and Levy, 1999). According to them, one can be Keynesian (or Kaleckian) in the short-run but, in the long-run, one has to be Classical, getting rid of the stagnationist case and the negative influence of a fall in real wages. Duménil and Levy claim that long-run models with a realized rate of profit, r^*, different from its normal (or standard) value are inconsistent. Any discrepancy between these two rates (and also between normal and realized rates of capacity utilization, u^*) contradicts the concept of long-run equilibrium in which there are no economic forces that lead the system to change.

There are two main solutions to this critique. Chick and Caserta (1997) introduce the idea of "provisional equilibria", explaining that Kaleckian models rather deal with medium-run issues. In this case, normal and realized rates of profit may diverge, but only in the short and medium-run. Dutt (1997) assumes that the normal rate of utilization depends on the value of the realized (or actual) rate of utilization, involving the endogeneity of normal rates even in the long-run. This leads to a multiplicity of equilibria, many of them keeping the paradox of thrift and the stagnationist case. Lavoie (2003) develops a neo-Kaleckian model with conflict inflation and a target rate of profit. He establishes that, in the long-run, the realized rate of profit and the normal rate of profit targeted by firms may be equal, and that, without assuming a fully adjusted position between the realized rate of capacity utilization and its normal value. In that case, the rate of capacity utilization remains endogenous even in the long-run, thus preserving the paradox of costs. Thus, one can remain Keynesian in the long-run!

3.2 Minskian frameworks and the issues raised by a Kalecki-Minsky synthesis

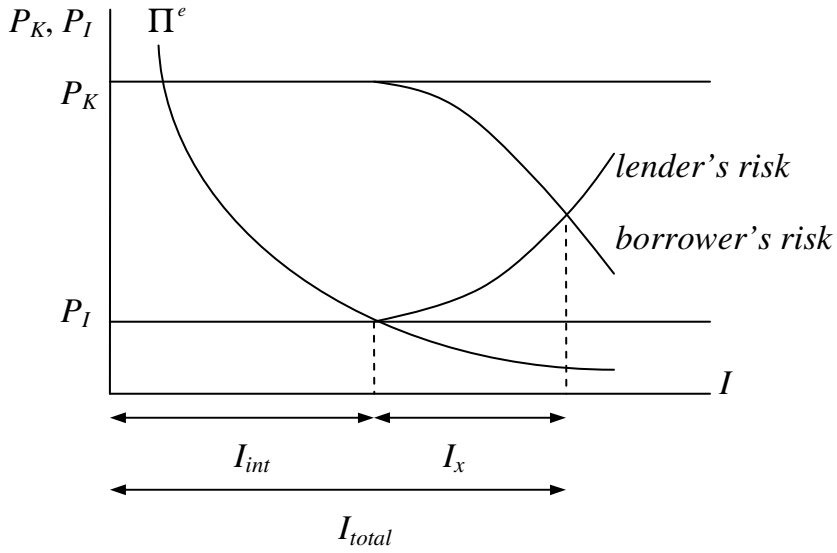
Another rich field in Post-Keynesian research is the work lead on Minskian models, which has become more and more popular since the 2007 financial crisis. Minsky (1975) starts from a microeconomic framework, in which the determination of investment depends on the difference between two prices. The demand price of capital goods, P_K, depends on the firm's expected cash flows. The supply price of these goods, P_I, is given by a mark-up over unit labour costs. The amount of investment a firm can finance internally is constrained by its

expected cash flows, which leads to the equation, $P_I I_{\text{int}} = \Pi^e$. This is represented in figure 2 by a rectangular hyperbola (in which expected profits are assumed to be constant).

As soon as a part of investment is financed by external funds, I_x, two risks appear. On one hand, lender's risk is borne by the banking system and it increases the supply price of capital goods. On the other hand, as indebtedness increases, firms are more fragile since they have to manage uncertain expected profits and certain financial charges. Therefore, borrower's risk, borne by firms, diminishes the demand price of capital goods when the amount of external financing – *i.e.* indebtedness – is raised.

Figure 2

Determination of investment



Minsky (1975, p. 113; 1986, p. 82) claims that during a boom financial fragility appears because borrower's risk and lender's risk become imprudently low, rising the ratio of external financing to internal financing and the burden of debt. At this point, Minsky applies to the macroeconomic level the previous results obtained at the microeconomic level. In consequence, the economic expansion for a country is necessarily associated with higher debt ratios.

The next logical step for future Post-Keynesian researches would consist in integrating financial variables, in a Minskian perspective, to the Kaleckian model. However, this integration raises issues related to Minsky's financial instability hypothesis. Indeed, Lavoie and Seccareccia (2001) show that Minsky did not correctly take into account Kalecki's profit

equation. Following Minsky, we know that financial instability arises from the increase in debt ratio (the fraction of external financing or indebtedness with respect to profits) during the boom. Nevertheless, at the aggregate level, one cannot assume that profits do not change after a rise in investment. Indeed, a rising debt allows for a larger amount of aggregate investment which, in turn, also generates higher profits. This reinforces the capability to pay back higher debt.

Kalecki's determination of the level of profits may be represented through the following set of equations:

$$W + \Pi \equiv C + I \tag{3.9}$$

$$C = C_W + C_C \tag{3.10}$$

$$I = CF + \Delta D \tag{3.11}$$

Equation (3.9) establishes the standard national accounting identities for a closed economy without State. Equation (3.10) indicates that consumption is divided between worker consumption and capitalist consumption. The last equation shows that investment depends on retained cash flows and the flow of new debt. Some simple calculi give the well-known Kaleckian relation indicating that more investment means more profits at the aggregate level:

$$\Pi = C_C + \underbrace{CF + \Delta D}_{I} \tag{3.12}$$

Including indebtedness the sequence of equation (3.12) may be summarized as:

$$\uparrow D \quad \rightarrow \quad \uparrow I \quad \rightarrow \quad \uparrow \Pi$$

Lavoie and Seccareccia (2001, p.83) claim that the appearance of financial fragility depends on the assumption according to which retained earnings have "to grow more slowly than investment". Otherwise, debt ratios will not rise during the economic boom and they even decrease if retained profits grow faster than investment. Then, the different cases are (with $d = D/\Pi$ the ratio of external financing, called I_x in the Minskian diagram):

$$\text{with } \Delta D \quad \rightarrow \quad \Delta I \qquad \text{and} \qquad \begin{cases} \Delta\Pi > \Delta I & \rightarrow & \Delta d < 0 & (i) \\ \Delta\Pi < \Delta I & \rightarrow & \Delta d > 0 & (ii) \\ \Delta\Pi = \Delta I & \rightarrow & \Delta d = 0 & (iii) \end{cases}$$

Minsky's financial instability hypothesis only holds in case (*ii*). Consequently, a synthesis between Kaleckian and Minskian models cannot emerge until a viable solution is proposed.

3.3 Path dependency and hysteresis.

Path dependency frameworks draw on the initial research lead by Kaldor (1934) and Robinson (1962). We take the example of hysteresis because such models have been widely developed throughout the past two decades. The models of hysteresis, at least the models in which the term is used in a proposer sense (see Lang, 2009) are faithful to Keynes' model presented in section 2. They get rid of the idea of a "natural rate" - and hence of the sole determination by supply variables in the middle and the long run. When "proper hysteresis" is introduced, there is no "natural anchor", and the equilibrium reached by the system in the medium/long run will change with the short-run dynamics. Hence, demand and economic policy shocks will have long-lasting effects on the economy. Putting aside the various multiple definitions of hysteresis that do not correspond to this definition[18], two types of hysteresis can in fact be considered as being "proper" ones: hysteresis seen as a process of endogenous change and "genuine" hysteresis.

Hysteresis as a process of endogenous change has been developed mainly by Katzner (1999) and Setterfield (1997). Hysteresis is considered by them as a process characterized by changes in the dynamic path followed by the economic system. The equilibria may never be reached because the variables underlying it change during the dynamic process.

The modelling of the Setterfield-Katzner approach starts from a general characterization of unit root "hysteresis":

(3.13) $$u_t = \nu + \mu \cdot u_{t-1} + \phi Z_t$$

Where u denotes the rate of unemployment, and the coefficients as well as the variable Z are treated as genuinely exogenous[19].

[18] For a characterization of these other multiple definitions, see Lang (2009), chapter 2.

[19] Note that, if one wants to work on the integration of this kind of hysteresis into the Kaleckian models, the variable u could also represent the rate of capacity utilization.

In the Classical model, the economy will end up reaching the steady state, and the previous equation may be solved by setting $u_t = u_{t-1} = u_n$, where the steady-state value corresponds to the "natural rate" of unemployment. Provided that $\mu \neq 1$, the solution is unique:

$$u_n = \frac{\nu + \phi Z^*}{1 - \mu} \tag{3.14}$$

Consider a situation where the dynamics towards u_n is long enough to allow the data underlying the determination of the long-run equilibrium to change. In that case, this long-run equilibrium become useless, and the relevant value of unemployment will become the current rate of unemployment. In this situation, the changes in the data are not autonomous, but arise from the "deep endogeneity" of the independent variable.

Hence, the dynamics of unemployment will be described by the following set of equations:

$$\begin{aligned} u_t &= \nu + \mu u_{t-1} + \phi Z_t \\ u_{t-1} &= \nu + \mu u_{t-2} + \phi Z_{t-2} \\ &\vdots \\ u_1 &= \nu + \mu u_0 + \phi Z_1 \end{aligned} \tag{3.15}$$

It follows that the full dynamics of unemployment can be rewritten:

$$u_t = \mu^t u_0 + \nu \sum_{i=1}^{t} \mu^{i-1} + \phi \sum_{i=1}^{t} \mu^{i-1} Z_i \tag{3.16}$$

At any moment in time, the value of unemployment will depend on its previous adjustment path. This way of seeing the dynamic functioning of the economy is obviously very close to Keynes' view that the factors underlying the "long run" equilibrium will be changing during the process, mainly because the equilibria reached are based on conventions.

"Genuinely hysteretic" systems differ from the Setterfield-Katzner definition. In these systems, the unemployment rate depends on the extremum values of past shocks that have hit the economy, and unemployment reacts in a non-linear way to shocks. In order to illustrate

this, consider the hiring and firing practices of numerous, heterogeneous firms who respond differently to some common aggregate growth shock. Heterogeneity is defined here as the presence of two different threshold values: one for hiring, the other for firing, and these threshold values are different from one firm to another. Let a_i be the threshold in terms of the aggregate growth rate (g) required for the individual firm i to hire, and b_i the threshold for g below which the firm fires. By construction, for every firm i, $b_i < a_i$. Let e_i be an "activity dummy", which gives an indication as to whether the firm employs a person, or not.

The activity dummy function of each firm can be written:

(3.17)
$$e_{i,t} = \begin{cases} 1 \;\; if \;\; e_{i,t-1} = 0 \;\; and \;\; g \geq a_i \\ 1 \;\; if \;\; e_{i,t-1} = 1 \;\; and \;\; g > b_i \\ 0 \;\; if \;\; e_{i,t-1} = 0 \;\; and \;\; g < a_i \\ 0 \;\; if \;\; e_{i,t-1} = 1 \;\; and \;\; g \leq b_i \end{cases}$$

This can be summarized:

(3.18)
$$e_{i,t} = e_{biai}\,[g(t); e_{i,t-1}]$$

This microeconomic framework has two important properties. First, the past history of the system matters: for any value of g bounded by a and b, in order to determine whether the firm employs a person or not, it is necessary to know the "initial" value of growth, and the number of times the system has "switched" over a, or under b. Secondly, there is a remanence effect: if, initially, the firm had not hired, and growth rises over the "hiring" threshold and goes back to its initial value, the firm will hire, and keep its employee even when growth returns to its initial value. Nevertheless, the remanence effect does not depend on the amplitude of variations in the growth rate: what matters more is whether or not the threshold values (a_i and b_i) have been crossed.

At the aggregate level, if there are multiple firms, characterized by their two switching values, and that these are significant variations in these switching values, firms can be represented in Mayergoyz's (1991) diagram. In this diagram (see figure 3), each firm is represented by its two switching values. The set of all firms is represented in the triangle T, defined by the first bisector and the extreme values $b = b_0$ and $a = a_0 = a_{\max}$. By construction, for each firm, $a > b$. $a_{\max}$ is the switching value for the most demanding firm, i.e., the maximum level of employment that the economy can create. The firms which have hired and

kept an employee belong to the domain marked “A” (“active” firms), while the ones that have fired operate inside the region “NA” (“non-active” firms).

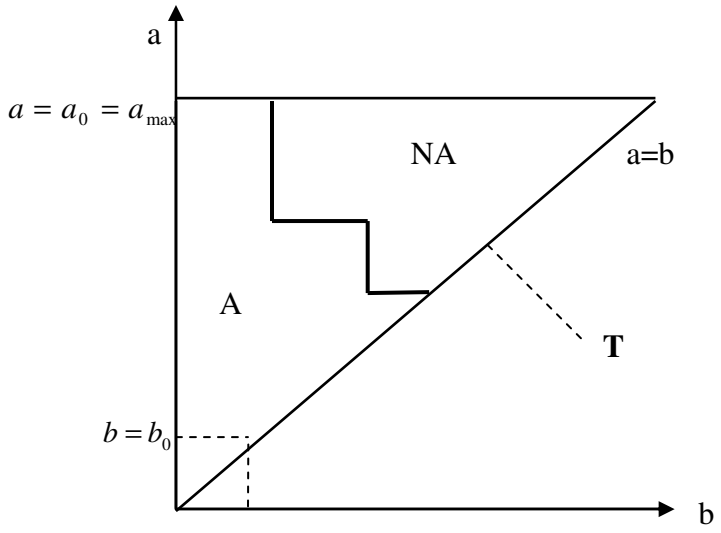


Figure 3: The border between the « A » and « NA » areas in Mayergoyz’s diagram

It can be shown easily (see Cross, 1993) that the frontier between the two areas A and NA will necessarily take the form of a staircase. The coordinates of the stairs correspond to the past local non-dominated minima and maxima of the growth rate.

The economy depicted here will retain a selective memory of the sequence of past growth shocks. The memory is erasable: only non-dominated shocks remain in the memory bank of the system. The remanence effect is also present at the macroeconomic level: two different growth shocks of the same size but of opposite signs will never bring the system back to its initial position. After a positive (negative) growth shock, some firms will become active (inactive), and will remain active (inactive) even after a negative (positive) shock of the same magnitude, whatever the magnitude of the shock. The reactions of the economy to positive and negative shocks are asymmetric, and, as a consequence, the fluctuations of unemployment in reaction to growth shocks will not be the same during booms as during recessions.

At any time, given the distribution of firms between the two domains (active/inactive), and given the past values of the growth rate, the rate of unemployment will be written:

(3.19) $$u_t = \iint_{A(t)} e_{ab}\,[g(t)|I_{t\text{-}1)}\, f(a,\, b)\mathrm{d}a\mathrm{d}b$$

Where $g(t)$ is the growth rate of the economy at time t, and e_{ab} is the activity dummy of firm i. $f(a,b)$ describes the distribution or density function of firms, each firm being characterized by its pair of switching values (a,b) . I_{t-1} is the information set on the state of the system at time *t-1*. *h[g(t)]* is the "hysteretic transformation of growth."

In the presence of "genuine hysteresis", all the main properties of the Classical framework are violated. There are major economic policy consequences: "equilibrium rates are shaped, inter alia, by nominal variables" (Cross, 1995, p. 197). Hence, "aggregate demand policies can shift the equilibrium rate of unemployment upwards and downwards. The broad negative association between equilibrium unemployment and inflation captured in the original Phillips curve is reintroduced, but in the form of a series of trajectories rather than a single curve. The particular trajectory taken and the reduction of equilibrium unemployment than can be "bought" by any given increase in steady inflation, will depend on the inherited memory of shocks" (*Ibid*, p. 198).

At this stage, the relevant question becomes which one of these two definitions of hysteresis we should use. On one hand, the Setterfield-Katzner definition of hysteresis is that it can be included in rich structural models with complex macroeconomic interactions – as the Kaleckian model. This is not the case of "genuine" hysteresis, because of their complexity.

On the other hand, the Katzner-Setterfield definition raises issues that still have to be solved. First of all, the choice of the "starting point" of the dynamics may always be seen as arbitrary, though our static model presented above may give useful clues. More importantly, determining which variables can be considered as being "deeply endogenous", and how to model these variables is not trivial. Last, it may be hard to implement this definition of hysteresis empirically. And yet, the fact that these tasks have not been undertaken yet does not mean that they are impossible to do, but rather that they remain on the agenda of future Post-Keynesian modelling. After all, perfecting tools in order to evaluate empirically "strong hysteresis" has for a long time been seen as a rather tricky task. And yet, theses tools have been developed since the beginning of the 2000's, amongst others, by Cross et al (2000) and Piscitelli and Hallet (2002). At the present stage, this is the major advantage of "strong

hysteresis" models over the Setterfield-Katzner hysteresis type, all the more than the empirical tests of "genuine" hysteresis produce positive results[20].

On the whole, in Kaldorian-Robinsonian models of path dependency, the unemployment rate (and hence rate of capacity utilization) change with the path followed by the system. As a consequence, history matters, and there are multiple and changing equilibria. As in Keynes' "static model of a dynamic process", there is no need for prices being sticky in order to explain the fact that nominal variables can influence real ones, and "true uncertainty" is a major feature of these models. The most exciting conclusion of hysteresis models is that fiscal and monetary policies are efficient, provided that they are huge enough.

4. Conclusion

The recent dynamic models and Keynes' static model of a dynamic process have a lot in common. Supply is determined by demand. Diminishing wages can have a destabilizing effect on the economy, through their influence on aggregate demand. In the Minskian models, the money supply adjusts to the money demand endogenously. The market may clear on different equilibrium positions, according to the liquidity preference and the capacity of monetary authorities to influence the convention, and according to the stabilizing role of institutions.

The major issue, at the present stage, is that, despite all the progresses of modelling that have taken place during the past decades, none of the Post-Keynesian models encompass *all* the rich and realistic properties of Keynes' static model. How would a complete dynamic Post-Keynesian model look like? Arguably, the baseline would be Kaleckian. The model would include endogenous money and financial instability bred by the behaviour of financial actors in periods of stability. It would also be hysteretic: the equilibria would change with the dynamics of the economy.

Some recent papers (Cassetti, 2006; Dutt, 2009) have tried to introduce hysteresis into the Kaleckian models. The main issue with these recent works is that they usually consider too a simplistic definition of hysteresis, where the equilibrium of the economic system changes all the time. This issue can be dealt with by choosing one of the proper definitions of hysteresis proposed in this paper. The main problem with the papers trying to incorporate

[20] See for example De Peretti and Lang, 2009

financial instability *a la* Minsky in the Kaleckian model is the fact that Minsky did not correctly take into account Kalecki's profit equation. This serious difficulty is probably the most important challenge for a Post-Keynesian synthesis, but the challenge is worth it. Post-Keynesians really need to be able to propose a simple, teachable and realistic alternative to the unrealistic but very simple and teachable mainstream models.

References

Ambrosi, G. M. (2009) Keynes Pigou and the general Theory, revised manuscript (January 25, 2009) of the 'Keynes Seminar' lecture, held at Robinson College, Cambridge, 22 October 2008.

Asensio, A. (2008) The growing evidence of Keynes's methodology advantage and its consequences within the four macro-markets framework", presented at the 11th SCEME Seminar in Economic Methodology, Stirling. http://halshs.archives-ouvertes.fr/halshs-00189221/fr/

Asensio, A. (2011a) Equilibrium interest rate and financial transactions in Post Keynesian models: Pointing out some overlooked features, *Intervention, forthcoming.*

Asensio, A. (2011b) On Keynes's seminal innovation and related essential features -Revisiting the notion of equilibrium in *The General Theory,* in *Cate, T. (Ed.), Keynes' General Theory: Seventy-five years Later*, Edward Elgar, 2011.

Bhaduri, M., Marglin, S. (1990) Unemployment and the real wage: the economic basis for contesting political ideologies, *Cambridge Journal of Economics*, 14: 375-393.

Blecker, R.A. (2002) Distribution, Demand, and Growth in Neo-Kaleckian Macro Models, in Setterfield, M. (ed.) *Demand-Led Growth: Challenging the Supply-Side Vision of the Long Run*, Northhampton MA: Edward Elgar

Cassetti, M. (2006) A note on the Long-Run Behaviour of Kaleckian Models, *Review of Political Economy*, 18(4): 497-508.

Chick, V. (1983) *Macroeconomics after Keynes*, MIT Press Cambridge, Massachusetts.

Chick, V., Caserta, M (1997) Provisional Equilibrium and Macroeconomic Theory, in *Essays in honour of Geoff Harcourt*, vol. 2., *Markets, unemployment and economic policy*, London and New York: Routledge, 223-37.

Chick, V., Tily, G. (2007) Transfiguration and death: Keynes's monetary theory, *Association of Heterodox Economics 9th Conference*, University of the West of England, Bristol, 13-15, July.

Cross, R. (1993) On the Foundations of Hysteresis in Economic Systems, *Economics and Philosophy*, 9(1):53-74.

Cross, R. (1995) Is the Natural Rate Hypothesis Consistent With Hysteresis?, in Cross, R (ed)., *The Natural Rate of Unemployment: Reflections on 25 Years of the Hypothesis*, Cambridge: Cambridge University Press, 181-202.

Cross, R., Darby, J., Ireland, J., Piscitelli, L. (2000) Hysteresis and Unemployment: a Preliminary Investigation, *Computing in Economics and Finance*, 721.

Davidson, P. (2002) *Financial markets, money and the real world*, Cheltenham: Edward Elgar.

De Peretti, C., Lang, D. (2009) A Strong Hysteretic Model of Okun's Law: Theory and a Preliminary Investigation, *International Review of Applied Economics*, 23(4):445-62.

Duménil, G., Lévy, D. (1999) Being Keynesian in the Short Term and Classical in the Long Term: The Traverse to Classical Long-Term Equilibrium, *Manchester School*, 67(6), 684-716.

Dutt, A.K. (1997) On an Alleged Inconsistency in Aggregate-Supply/Aggregate-Demand Analysis, *Eastern Economic Journal*, 23(4):469-76.

Dutt, A.K. (2009) Path dependence, equilibrium and Economic Growth, *International Papers in Political Economy*, pp. 119-161.

Fontana, G., Setterfield, M. (2009) A simple (an teachable) macroeconomic model with endogenous money, in Fontana, G. and Setterfield, M. (eds), *Macroeconomic theory and macroeconomic pedagogy*, Palgrave Macmillan

Hallett, H.A., Piscitelli, L. (2002) Testing for hysteresis against nonlinear alternatives, *Journal of Economic Dynamics and Control*, 27(2):303-327.

Hayes, M.G. (2007) Keynes's Z function, heterogeneous output and marginal productivity, *Cambridge Journal of Economics*, 31:741–53.

Kaldor, N. (1934) A Classificatory Note on the Determinateness of Static Equilibrium, Review of Economic Studies, l:122-36.

Katzner, D.W. (1999) Hysteresis and the Modeling of Economic Phenomena, *Review of Political Economy*, 11(2): 171-81.

Keynes, J.M. (1936) *The general theory of employment, interest and money*, London, Macmillan.

Lang, D. 2009, *Hysteresis in Unemployment*, VDM Verlag.

Lavoie, M. (1999) Fair Rates of Interest in Post-Keynesian Political Economy, University of Ottawa, http://aix1.uottawa.ca/~robinson/english/wp/fairratecla.pdf.

Lavoie, M. (2003) Kaleckian Effective Demand and Sraffian Normal Prices: towards a reconciliation, *Review of Political Economy*, 15(1):53-74.

Lavoie, M., Seccareccia, M. (2001) Minsky's Financial Fragility Hypothesis: A Missing Macroeconomic Link?, in Bellofiore, R. and Ferri, P. *Financial Fragility and Investment in the Capitalist Economy, The Economic Legacy of Hyman Minsky*, vol. 2, Cheltenham, U.K.: Edward Elgar, 76-96.

Mayergoyz I. D. (1991) *Mathematical Models Of Hysteresis*, Springer Verlag.

Minsky, H.P. (1975) *John Maynard Keynes*, McGraw-Hill Professional.

Palley T. (1996) *Post-Keynesian Economics: Debt, Distribution, and the Macro Economy*, Palgrave Macmillan.

Robinson, J. (1962) *Essays in the Theory of Economic* Growth, Palgrave Macmillan.

Romer, D. (2000) Keynesian Macroeconomics without the LM Curve, *Journal of Economic Perspectives*, 14 (2), 149–69.

Setterfield, M. (1997) *Rapid growth and relative decline: Modelling macroeconomic dynamics with hysteresis*, New York: St. Martin's Press; London: Macmillan.