Firm-Heterogeneity, Persistent and Transient Technical Inefficiency

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5. January 2011

Online at https://mpra.ub.uni-muenchen.de/30737/
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This version February 2011

(This is a rough draft. Contact the authors for more recent versions.)

Abstract

This paper provides a new model that disentangles firm effects from persistent (time-invariant/long-term) and transient (time-varying/short-term) technical inefficiency.

Keywords: Bayesian analysis; Markov Chain Monte Carlo; Technical efficiency.

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Acknowledgments: Tsionas is indebted to Athens University of Economics and Business for partial financial support through research grants under PEBE I, II, and III.
1. Introduction

In recent years there have been many contributions in estimating efficiency from panel data models that utilize the data in increasingly more efficient ways. Since the standard panel data models in Kumbhakar and Lovell (2000) and some other excellent surveys¹, there has been increasing interest in the use of panel data for estimating technical efficiency more accurately, under less demanding assumptions and using more flexible models. Heterogeneous technologies have been the focus of fruitful research including random coefficient stochastic frontier models, latent class or mixture models and Markov switching models. Another important line of research has been the formulation and estimation of true fixed/random effect models proposed by Kumbhakar and associates and recently examined more thoroughly by Greene.

In a standard panel data model, the focus is mostly on controlling firm effects (heterogeneity due to unobserved time-invariant factors). This notion is adapted from the earlier panel data models (Pitt and Lee 1981; Schmidt and Sickles 1984; Kumbhakar 1987) in which inefficiency is treated as time-invariant. The only innovation in the efficiency models was to make these firm effects one-sided so as to give them an inefficiency interpretation. Models were developed to treat these firm effects as fixed as well as random. Several models have been developed based on the assumption that all the time-invariant (fixed or random) effect is (persistent) inefficiency (e.g. Schmidt and Sickles 1984; Pitt and Lee 1981). This is in contrast to the ‘true’ random or fixed effect models by Greene (2005a, 2005b) in which firm-specific effects are not parts of inefficiency. The models proposed by Kumbhakar (1991), Kumbhakar and Heshmati (1995), Kumbhakar and Hjalmarsson (1993, 1995) are in between. These models treat firm effects as persistent inefficiency and include another component to capture time-varying technical inefficiency. Since none of these assumptions outlined above may be wholly satisfactory, we introduce a new model that may overcome some of the limitations of earlier approaches. In this model we decompose the time-invariant firm effect as a firm effect and a persistent technical inefficiency effect.

Among many panel data models the inefficiency specification used by Battese and Coelli (1995) is most frequently used in empirical studies. Their model allows inefficiency to depend on some exogenous variables so that one can investigate how exogenous factors influence inefficiency. Although this model is designed for cross-sectional data, it can readily be used for panel models. The panel data model due to Battese and Coelli (1992) is somewhat restrictive because it only allows inefficiency to change over time exponentially.² Furthermore, these models mix firm effects with inefficiency. Two other models, viz., the ‘true-fixed’ and ‘true-random’ effects frontier models for panel data (Greene 2005a, 2005b) have become popular in recent years. These models separate firm effects (fixed or random) from inefficiency, where inefficiency can either be iid or can be a function of exogenous variables.

Some of the models that are widely used in the literature can be summarized in the following table (Kumbhakar et al. (2011)).

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¹ See Greene (2004a, b, c) and Greene (1980a, b, 1999) for useful and excellent introductions to current problems and achievements in the literature. Greene (1993) provides an overview and excellent introduction.

² Wang and Ho (2011) generalized the Battese-Coelli formulation in which the temporal pattern of inefficiency is made firm-specific by specifying it as a function of covariates that can change both temporally and cross-sectionally.
Table 1: Main characteristics of some of the panel data models

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
<th>Model 6</th>
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<tr>
<td>General firm effect</td>
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<td>No</td>
<td>Fixed</td>
<td>Random</td>
<td>No</td>
<td>Random</td>
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<tr>
<td>Technical inefficiency</td>
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<td></td>
<td></td>
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<tr>
<td>Persistent</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Transient</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Overall technical inefficiency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Symmetric error term</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: ¹ Time-inv. mean inefficiency models include determinants of inefficiency in the mean function. ² Zero truncation models assume inefficiency distribution to be half-normal. ³ Hetero. (Homo.) refers to models in which variances are functions of covariates that are both firm-specific and time-varying (constant).

In this paper we consider a more general model of the following form (Model 6 in Kumbhakar et al. (2011)) of the production function:

\[ y_{it} = x_{it}^\prime \beta + \alpha_i + v_{it} - u^*_it - \eta^*_i \]

where the dependent variable is output (in log) and the input variables are represented by the vector \( x \). Note that the model in (1) has four error components. If we denote the composed error \( \epsilon_{it} = \alpha_i + v_{it} - u^*_it - \eta^*_i \) where the superscript (+) indicates nonnegative value of the corresponding error component, we can give a meaningful interpretation of each of the error component. First, random noise component is \( v_{it} \) which is similar to the noise component in a standard regression model. Second, the persistent (long-run) technical inefficiency component is \( \eta^*_i \). Third, short-run or transient technical inefficiency is allowed by \( u^*_it \). Fourth, the firm-specific random effect component (to capture heterogeneity) is given by the \( \alpha_i \) component.

If one uses a cost function formulation the formulation in (1) will be of the form

\[ y_{it} = x_{it}^\prime \beta + \alpha_i + v_{it} - u^*_it - \eta^*_i \]

where the dependent variable is cost (in log) and the regressors are input prices and outputs (in log).

In terms of technical inefficiency, we decompose the overall inefficiency into a long-run or persistent component (\( \eta^*_i \)) and the short-run or transient component (\( u^*_it \)). This decomposition is desirable in panel data as indicated before (see also the arguments in Tsionas (2006)). The decomposition proposed here is more flexible and does not rest on parametric assumptions on the dynamics of \( \eta^*_i + u^*_it = U^*_it \). Moreover, we allow for firm-effects (\( \alpha_i \)) whose existence can be tested once we allow for a decomposition of technical inefficiency into permanent and transient or short-run components.
It is clear that many of the models listed in Table 1 above can be derived as special cases from (1). Our objective in this paper is to estimate the most general formulation, i.e., Model 6 which is based on equation (1) above. We consider a Bayesian MCMC approach in estimating the model. Both simulation results and results from real data are provided.

2. Econometric model

We consider the model in (1b), i.e.,

$$y_{it} = \alpha_i + x'_{it} \beta + \eta_i + v_{it} + u_{it},$$

where $x_{it}$ is a vector of covariates (including an intercept), $\eta_i \sim N\left(0, \sigma^2_\eta\right)$, $v_{it} \sim N\left(0, \sigma^2_v\right)$, $u_{it} \sim N\left(0, \sigma^2_u\right)$, all are iid as well as independent of $x_{it}$ and the usual random effect $\alpha_i \sim N\left(0, \sigma^2_\alpha\right), t=1,...,T,$ and $i=1,...,n$. Usually the firm-specific effects $\alpha_i$ are nuisances but cannot be ignored when the purpose is to separate permanent inefficiency, $\eta_i$, from transient or short-run inefficiency, $u_{it}$.

We rewrite the model as

$$y_{it} = \left(\alpha_i + \eta_i^*\right) + x'_{it} \beta + \left(v_{it} + u_{it}^*\right) \equiv \delta_i + x'_{it} \beta + \xi_{it}^*.$$  \hspace{1cm} (2)

In this notation we seemingly have a standard panel data model with firm-specific effects $\delta_i$, and an “overall” error term $\xi_{it}^*$. It is well known that we can separate the “overall” error term $\xi_{it}^*$ into noise and “technical inefficiency”. One relevant question in many empirical applications concerns the behavior of the “technical inefficiency” component $u_{it}^*$. Without firm-specific effects $\delta_i = \delta^*$, for all $i=1,...,n$) it is clear that the component $u_{it}^*$ provides a measure of technical inefficiency. Since the model does not allow for firm-specific heterogeneity it is clear, nonetheless, that this “technical inefficiency” component includes aspects of both firm characteristics as well as “true” technical inefficiency. One possible variant of the model is to allow for firm-specific heterogeneity by assuming:

$$y_{it} = \delta_i + x'_{it} \beta + \left(v_{it} + u_{it}^*\right).$$ \hspace{1cm} (3)

This variant $y_{it} = \delta_i + x'_{it} \beta + \left(v_{it} + u_{it}^*\right)$ would be Greene’s “true effects” model where firm-specific effects $\delta_i$ are separated from technical inefficiency $u_{it}^*$ or $u_{it}^* \equiv \eta_i^*$. By allowing for firm-specific technological heterogeneity, one hopes for better estimates of technical inefficiency per se. Greene showed that when the $\delta_i$’s are fixed effects, the computational side of the incidental parameters problem can be addressed. A relatively simple Gauss–Newton iteration is required with respect to the fixed effects (with analytical first and second derivatives) in the log–likelihood function implied by the convolution of $v_{it} + u_{it}^*$ or $v_{it} + u_{it}^*$. 
Apart from computational considerations, however, the impact of the incidental parameters problem on small-sample distributions of parameters and technical efficiency estimates is something that needs to be investigated. Although under a log–likelihood implied by the convolution of $v_{it} + u_{it}^*$ a relatively simple Gauss – Newton iteration with respect to the fixed effects is feasible, does not imply that small-sample properties of technical inefficiency are satisfactory. In fact the asymptotics with respect to $T$ will be required to establish consistency and asymptotic normality, so in samples of length $T=10$ or 20 it is not clear what is the performance of these estimators.

In fact, alternative estimators are possible. For example, if we rewrite the model as $y_{it} = \left( \eta_{it}^* + u_{it}^* \right) + x_{it}' \beta + \left( \alpha_i + v_{it} \right)$, one may consider the convolution of $\alpha_i + v_{it}$ (a normal distribution under an assumption of random effects for $\alpha_i$ and $v_{it}$) and maximize the likelihood using $\zeta_{it}^* = \eta_{it}^* + u_{it}^*$ as a parameter subject to the constraint $\zeta_{it}^* \geq 0$. Although a simple Gauss – Newton iteration is no longer possible, solution of these nonlinear programming problems is possible.

In Greene’s true effect model, however, it would not be possible to identify short-run inefficiency, that is any adjustment after $\eta_{it}^* = u_{it}^*$ has been accounted for. In the alternative nonlinear programming estimator above, one would encounter the same problem. In Greene’s true effect model where the likelihood is formed using a convolution of $v_{it} + u_{it}^*$ one could identify short-run inefficiency $u_{it}^*$ but long-run inefficiency would be merged with the overall fixed effect and would, thus, become unidentifiable.

Given these shortcomings, the question is whether we can usefully employ a more general model of the form:

$$y_{it} = \left( \alpha_i + \eta_{it}^* \right) + x_{it}' \beta + \left( v_{it} + u_{it}^* \right) \equiv \delta_{it} + x_{it}' \beta + v_{it} + u_{it}^*, \quad (4)$$

where long-run inefficiency ($\eta_{it}^*$) is separated from its short-run counterpart ($u_{it}^*$) when we allow for noise ($v_{it}$) and a firm-specific effect ($\alpha_i$) which is not related to time – invariant or persistent technical inefficiency.

Kumbhakar (1991) proposed a model of the form:

$$y_{it} = \alpha_i + \lambda_i + x_{it}' \beta + v_{it} + u_{it}^*. \quad (5)$$

This model (where all firm and time effects are random) is, essentially, Greene’s true effect model (when $\lambda_i=0$). Kumbhakar’s (1991) model can be thought of as a random effect panel data specification with a convolution of $v_{it} + u_{it}^*$. It is clear that a Gauss – Newton iteration would facilitate estimation in the context of a convoluted likelihood by treating both $\alpha_i$ and $\lambda_i$ as fixed effects in the light of Greene’s analysis. But again, this model delivers only estimates of “overall” technical inefficiency $u_{it}^*$. Its decomposition into permanent and transitory components is not possible unless one is willing to resort to relative concepts.

The decomposition of technical inefficiency into persistent and short-run components requires a specific assumption about the overall firm effects $\delta_{it} = \alpha_i + \eta_{it}^*$.
and such assumptions naturally require random effect specifications for both components. The same is true for $\xi_{it} = v_{it} + u_{it}$.

3. Numerical inference procedures

Given the model in (1) our random effect assumptions are as follows:

$$v_{it} \sim iidN\left(0, \sigma_v^2\right), \quad u_{it}^* \sim iidN\left(0, \sigma_u^2\right), \quad \alpha_i \sim iidN\left(0, \sigma_\alpha^2\right), \quad \eta_i^+ \sim iidN\left(0, \sigma_\eta^2\right).$$

All random components are mutually independent as well as independent of $x_{it}$. For Bayes analysis what remains is to specify a prior $p\left(\beta, \sigma_v, \sigma_u, \sigma_\alpha, \sigma_\eta\right)$.

Implementing Gibbs sampling is straightforward but it will not have good mixing properties and will collapse under the most favorable conditions.

So we need to consider other strategies, for example a reparametrization. We begin with the $\delta$-parametrization.

We consider $\delta_i = \alpha_i + \eta_i^+$ whose distribution is well known:

$$p_{\delta_i}(\delta) = \frac{2}{\sigma_\delta} \varphi\left(\frac{\delta}{\sigma_\delta}\right) \Phi\left(\frac{\lambda \delta}{\sigma_\delta}\right), \quad \text{where } \sigma_\delta^2 = \sigma_\alpha^2 + \sigma_\eta^2, \quad \text{and } \lambda = \sigma_\eta / \sigma_\alpha. \quad (6)$$

Suppose $R_{it} = y_{it} - x_{it}' \beta - u_{it}$. Then for each $i$, the conditional posterior distribution of $\delta_i$ will be:

$$p(\delta_i | \Theta_{-\delta}, Y, X, u) \propto \exp\left[-\frac{(R_i - \delta_i)^T (R_i - \delta_i) - \delta_i^2}{2 \sigma_\delta^2}\right] \varphi\left(\frac{\lambda \delta_i}{\sigma_\delta}\right), \quad \text{where } \Theta_{-\delta} \text{ denotes all parameters except the } \delta_i \text{s.}$$

The above may seem impossible to simulate but in fact it is log-concave so special rejection techniques can be used, requiring only the first and second derivative of this function, which are easy to find. The following strategy has been found extremely effective: Given the mode $\delta^*_i$ and an estimate, say $s^2$ from the inverse negative second derivative at the mode, we generate a random draw $\delta_i \sim N\left(\delta^*_i, s^2\right)$. The draw is accepted with probability

$$\frac{p(\delta_i | \Theta_{-\delta}, Y, X, u) / f_N\left(\delta_i, \delta^*_i, s^2\right)}{p(\delta_i' | \Theta_{-\delta}, Y, X, u) / f_N\left(\delta_i', \delta^*_i, s^2\right)} = \frac{p(\delta_i | \Theta_{-\delta}, Y, X, u)}{p(\delta_i' | \Theta_{-\delta}, Y, X, u)} \exp\left[-\frac{(\delta_i - \delta^*_i)^2}{2s^2}\right], \quad \text{where}$$

$f_N\left(x | m, s^2\right)$ denotes the density function of the normal distribution with mean $m$ and variance $s^2$, evaluated at $x$. 

Obtaining a draw for $\sigma^2_\delta$ is easy: $\frac{\sigma^2_\delta}{\delta} \sim \chi^2(n)$ given the $\delta$'s from previous step.

Obtaining draws for $\beta$ and $u$ is also quite straightforward along with $\sigma^2_u$ and $\sigma^2_\varepsilon$, as usual, conditional on the $\delta$'s. Here, we don't want to do more integrations with respect to $u$. That would be easy, but it would destroy the simplicity of the Gibbs sampler.

As we describe below, in an artificial example with $n=100$, $T=5$ and reasonable values for the parameters to obtain efficiency measures close to the true ones (75% correlation and parameters hitting precisely the true ones).

Apparently we can get good estimates of “transient inefficiency” $u$ in this way unconditional on any knowledge of parameters or the random effects ($\alpha$ and $\eta^*$). We can obtain $\delta$ and its characteristics (as well as good estimates of $\delta$) we can readily obtain from the MCMC scheme. If one is interested only in short-run inefficiency then this completes the analysis. Under the assumption that firm-specific effects $\alpha_i$ are “small” when permanent inefficiency $\eta_i$ is introduced, one can perhaps proceed under the assumption that $\eta_i \approx \delta_i$. But this cannot always be the case so we next describe a computational scheme to complete the analysis.

Now in another parametrization of the model (the $\xi$-parametrization) we can write

$$y_{it} = \alpha_i + x_{it}' \beta_i + \eta_i + u_{it} = x_{it}' \beta_i + \eta_i + u_i + \xi_{it},$$

where $\xi_i = \alpha_i + v_i$.

If we denote $\xi_i = [\xi_{i1}, \ldots, \xi_{iT}]'$, then $\xi_i \sim N_T \left(0_{(T \times 1)}, \Sigma \right)$ where $\Sigma = \sigma^2_u J_T + \sigma^2_\varepsilon I_T$, $J_T = t_T L_T'$. In this parametrization, the posterior conditional distribution of $\eta_i$ is:

$$\eta_i \sim N_{m_i} \left( m_i, \varphi^2 \right),$$

where $\varphi^2 = \sigma^2_\eta \left(1 + \sigma^2_\varepsilon L_T' \Sigma^{-1} L_T \right)^{-1}$, $m_i = \varphi^2 L_T' \Sigma^{-1} D_i$, $D_i = y_i - X_i' \beta - u_i$.

From these draws we can, finally compute

$$\alpha_i = \delta_i - \eta_i,$$

as well as the following posterior conditional distributions:

$$\frac{\alpha' \alpha}{\sigma^2_\alpha} | \Theta_{-\alpha}, y, X \sim \chi^2(n),$$

and

$$\frac{\eta' \eta}{\sigma^2_\eta} | \Theta_{-\eta}, y, X \sim \chi^2(n).$$
The parameter $\sigma_\delta^2$, needed before when drawing the $\delta$'s can be drawn easily as $\sigma_\delta^2 = \sigma_\sigma^2 + \sigma_\eta^2$. It must be noted that the posterior conditional distribution of $\sigma_\delta^2$ is not chi–squared.

4. Artificial examples and sampling performance of Bayes estimators

4a. Artificial examples

We consider an artificially generated data set with $n=100$, and $T=5$. We have a constant term and a covariate that was generated as independent standard normal, and $\sigma_\sigma = 0.1$, $\sigma_u = 0.2$, $\sigma_v = 0.2$ and $\sigma_\eta = 0.5$. The MCMC scheme was implemented using 15,000 iterations the first 5,000 of which are discarded to mitigate start-up effects while in the computation of all statistics we take every other tenth draw to mitigate autocorrelation.

First, we present the marginal posterior distributions of the scale parameters in Figure 1. The true parameter values are of course in the region of high posterior probability mass, as one would expect.

Second, we are concerned with estimates of the efficiency measures say $\hat{u}_u$ and $\hat{\eta}_\eta$. Such estimates can be provided readily as $\hat{\eta}_\eta = S^{-1} \sum_{s=1}^{S} \eta_{s,\eta}^{(s)}$, where $\eta_{s,\eta}^{(s)}$ denotes the $s$th draw for $\eta_s$ and $S=1,000$ in our case after the burn-in and skipping phases. Similarly we have $\hat{u}_u = S^{-1} \sum_{s=1}^{S} u_{s,u}^{(s)}$. These estimates are provided in Figure 2 where they are plotted against the true values that were generated according to the true values of the parameters.

Specifically, the correlation coefficient between $\hat{\eta}_\eta$ and its true values is 0.856, while the correlation between $\hat{u}_u$ and its true value is 0.754. These are not the simple correlations between $\hat{\eta}_\eta$ and $\eta_s$ but posterior means of the correlation coefficient, say $\rho_\eta$, or $E(\rho_\eta \mid y, X)$. Specifically, for each draw $s$, we compute the correlation coefficient between $\eta_s$ and $\eta_{s,\eta}^{(s)}$, which we denote by $\rho_{s,\eta}^{(s)}$. The correlation coefficient is $E(\rho_\eta \mid y, X) = S^{-1} \sum_{s=1}^{S} \rho_{s,\eta}^{(s)}$ so it reflects fully parameter uncertainty.
Figure 1. Marginal posterior distributions of scale parameters
Figure 2. Marginal posterior estimates of efficiencies versus true values
4b. Simulation experiment

We consider a data generating process, where \( y_{it} = 1 + x_{it} + \varepsilon_{it} \), \( x_{it} \) is generated from a standard normal distribution and \( \varepsilon_{it} = \alpha_i + \eta_i + \nu_i + u_{it} \). We use \( \sigma_\alpha = 0.2, \sigma_\eta = 0.5, \sigma_\nu = 0.1, \sigma_u = 0.2 \) (for \( n=50 \)) and \( \sigma_\alpha = 0.1, \sigma_\eta = 0.5, \sigma_\nu = 0.1, \sigma_u = 0.5 \) for sample size \( n=100 \). In our Gibbs samplers we make use of 5,000 iterations the first 1,000 of which are discarded to mitigate the impact of start up effects. Bayesian inference is conducted for 1,000 data sets.

For \( \alpha, \eta, \) and \( u \) we compare their true values with posterior estimates. Specifically these random effects are compared in terms of mean, median and standard deviation. For the true values these statistics can be computed easily. For the estimated parts, we report means, medians and standard deviations of the sampling distributions of the Bayesian estimators.

Table 1. Sampling behaviour of Bayes estimators

<table>
<thead>
<tr>
<th></th>
<th>( \alpha )</th>
<th>( \eta )</th>
<th>( u )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>median</td>
<td>s.d</td>
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<tr>
<td>( n=50, T=5 )</td>
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<td></td>
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<tr>
<td>true</td>
<td>-0.014</td>
<td>-0.040</td>
<td>0.205</td>
</tr>
<tr>
<td>estimated</td>
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<td>0.125</td>
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<tr>
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<tr>
<td>true</td>
<td>-0.014</td>
<td>-0.040</td>
<td>0.205</td>
</tr>
<tr>
<td>estimated</td>
<td>-0.011</td>
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</tr>
<tr>
<td>( n=100, T=5 )</td>
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<td></td>
</tr>
<tr>
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<tr>
<td>estimated</td>
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</tr>
<tr>
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<td>( n=200, T=10 )</td>
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<tr>
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<td>-0.007</td>
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<tr>
<td>estimated</td>
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<td>0.100</td>
</tr>
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</table>

Empirical application

We apply the model to 123 UK manufacturing firms observed for 13 years. The data set is the same as in Nickell (1996) and Nickell et al (1997). The inputs are labor and capital and a translog production function is estimated along with firm effects, long run and short run inefficiency. Bayesian inference has been implemented using the Gibbs sampler with 150,000 iterations the first 50,000 of which are discarded to mitigate possible start up effects. We retain every other tenth draw in the computation of posterior moments of functions of interest, to mitigate the impact of autocorrelation.
Posterior distributions of firm effects, long run and short run inefficiency are presented in the three panels of Figure 3. These posterior distributions are for all firms and all parameter draws of the MCMC scheme. Although posterior short run inefficiency appears to be small (exceeding 0.90) posterior distributions of firm – specific long run inefficiency in Figure 2 provide a different picture. In Figure 4 thirty different firms are provided. It is clear that firm – specific long run inefficiency varies between 0.20 and 0.95 (on the average) so in fact it can be substantial.

In Figure 5 we provide median absolute autocorrelations of all parameters in the MCMC scheme. After discarding every other tenth draw autocorrelation is trivial so MCMC explores the parameter space in a satisfactory manner.

In Figure 6 we present posterior distributions of firm – specific short run inefficiency for thirty randomly selected firms –the same as in the construction of Figure 4. Although the distributions are not the same across firms, it seems that short run efficiency well exceeds 0.90 with sizeable posterior probability. The temporal behavior of short efficiency for the thirty firms is presented in Figure 7. On the average, efficiency shows a declining trend but individual short run efficiencies are between 95% and 98%. Finally, in Figure 8 we present the joint posterior distribution of firm effects and long run efficiency. The relation between the two is clearly negative, implying that there is a certain interplay between what we can classify as pure firm effect and as one-sided firm specific effect. Apparently the correlation is not so high as to be destructive in the sense that formal identification is not empirically possible.


Figure 3. Posterior distributions
Figure 4.

**Posterior distributions of long run efficiency for certain firms**

Figure 5.

**Posterior distributions of short run efficiency for certain firms**
Figure 6.
Figure 7.

Temporal behavior of short run efficiency, $\exp(-u_{11})$

Figure 8. Joint posterior distributions