Life-cycle consumption: can single agent models get it right?

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Abstract

In the quantitative macroeconomics literature, single agent models are widely used to explain "per-adult equivalent" data, which are obtained at the household level. In this paper we suggest a simple framework to understand the sources of bias when these models are used to make predictions for aggregate consumption. In both a theoretical and a quantitative exercise, we find that economies of scale in consumption inside the household are positively related to the bias introduced by the single agent approach in predicted consumption profiles over the life-cycle. We also do an external validation exercise, which suggests that economies of scale inside the household are rather large, pointing out the need to approach life-cycle consumption with models that consider households rather than single agents.

Keywords: Consumption, Life-Cycle Models, Households

JEL classification: D12, D91, E21, J10

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1 Introduction

To understand life-cycle consumption, single agent models are frequently used given their tractability. In the quantitative macroeconomic literature, a standard approach entails extracting per-adult equivalent consumption facts from household survey data and use them as targets to be replicated by single agent models, which are also calibrated using per-adult equivalent household income. Some papers in this vein include Krueger and Perri (2006), Blundell, Pistaferri, and Preston (2008), Kaplan and Violante (2009) and Guvenen and Smith (2010).[1]

However, this approach faces the inherent challenge that consumption decisions might depend on household size and composition through non trivial channels. Cubeddu and Ríos-Rull (2003) for example, demonstrate that changes in marital status over the life-cycle affect aggregate savings in the same order of magnitude as idiosyncratic income uncertainty. Although this approach has the benefit of considering explicitly multi-person households, its drawback is that the model structure becomes very complicated and computationally intensive to solve. The same criticism can be made of models where demographic transitions occur endogenously, as in Aiyagari, Greenwood, and Guner (2000) and Mazzocco, Ruiz, and Yamaguchi (2007). In this paper, we abstract from these difficulties and propose a simple framework in order to understand the sources and magnitudes of bias when single agent models are used to make predictions for aggregate household consumption or consumption related measures (i.e. welfare). Specifically, we are interested in predictions from the standard incomplete markets model, which has become a workhorse in modern macroeconomic analysis.

We follow Attanasio, Banks, Meghir, and Weber (1999) and Gourinchas and Parker (2002) and perform our analysis by extending the standard incomplete markets model to allow for deterministic changes in household size and composition during the life-cycle and let these changes affect optimal decisions on consumption and savings in a unitary model approach. But unlike these two papers, which use a general 'demographic' taste shifter in the utility function, we propose a formulation where economies of scales inside the household are considered explicitly through equivalence

[1] Other papers present mixed empirical strategies. For example, Storesletten, Telmer, and Yaron (2004) use household income *per person* from the PSID, while trying to match the cross-sectional variance in *total* household consumption (without controlling for household size/composition). Another approach is in Fernández-Villaverde and Krueger (2010), who study durable consumption using an equilibrium life-cycle model. They use worker information to parameterize income profiles and contrast the results from their model with per-adult equivalent household consumption from the CEX.

scales.[2] This setup accommodates both the case of single households (the *Single* model) and the case where household size varies during the life-cycle (what we label the *Demographics* model). Although the *Single* model provides predictions for a single/bachelor consumption only and the *Demographics* model predicts total household consumption, a common practice in the literature is to transform household into individual consumption and vice versa through equivalence scales, making predictions directly comparable.

Using a simple two period model of household consumption, we show theoretically that single agent models introduce bias in predicted household consumption profiles: agents in these models ignore the fact that the relative price of consumption across periods in which family size is changing is affected by economies of scale inside the household.

We also perform a quantitative exercise and find that differences between household consumption data from the *Demographics* and *Single* models can be substantial for mean consumption profiles but not for inequality. The differences are increasing in the amount of economies of scale present in the household. Intuitively, the bigger the economies of scale, the bigger the price effects in life-cycle consumption induced by changing household size which are ignored by the *Single* approach. If there are low economies of scale in consumption, a "household" would be just a collection of individuals sharing a physical address but nothing more: hence, modeling the economy as if everyone lived in a single household would not entail significant loss of accuracy in predictions.

Finally, we compare our setup with the preference structure estimated in Attanasio, Banks, Meghir, and Weber (1999) and infer the amount of economies of scale present in the household. This external validation exercise suggests sizeable economies of scale, as prescribed by Nelson (1993) and similar to those found in Hong and Ríos-Rull (2009). Following our argument, we conclude that the study of life-cycle consumption should depart from the usage of *Single* models.

The structure of the paper is as follows: in Section 2 we discuss our proposed preferences for the household and present theoretical predictions in a stylized two period framework. In Section

[2] Equivalence scales are functions of household size and composition and typically used to deflate total household information (like consumption and income) by a number less than the actual household size. This approach has gained importance in the macro literature: in the 2010 special issue of the Review of Economic Dynamics, equivalence scales are used to obtain consistent per-adult equivalent cross sectional facts for a wide range of countries (See Krueger, Perri, Pistaferri, and Violante (2010) for a general description and Heathcote, Storesletten, and Violante (2010) for the US economy). Another example is Fernández-Villaverde and Krueger (2007), who discuss the properties of different types of equivalence scales and their effect on life-cycle consumption profiles.

3 we discuss the model we use to quantify these theoretical predictions. In Section 4 we show the quantitative features of the model and the calibration strategy while Section 7 shows our main quantitative results. Section 2 presents our identification exercise, while in the last section, we conclude.

2 Demographics in a Life-Cycle Model

In this section we investigate consumption in a two period model with a deterministic change in household size. In particular, we assume that the household size is one in the first period ($N_1 = 1$), e.g. a young person living alone, and larger than one in the second period ($N_2 > 1$), e.g. because a child is born. The household receives an income stream y_1 and y_2, and can borrow up to the natural borrowing constraint and save at an interest r which without loss of generality is set to zero. Similarly, the discount factor is set to one, i.e. from the perspective of period one the utility in period two is not discounted.

A standard approach in macroeconomics (obviously with more than two periods) is to assume that households consist only of a single member, the bachelor household. Accordingly, such a model cannot predict household consumption which is however the format for empirically observed consumption data. Usually, this data is therefore divided by an equivalence scale which transforms total household consumption into a per-adult equivalent consumption, against which the predictions of the bachelor household are compared.

The three mechanisms through which household size affects the intra-temporal rate of transformation between expenditures and consumption services, and that are captured partially through equivalence scales, are family/public goods, economies of scale, and complementarities. See for example, Lazear and Michael (1980).

To ensure consistency between the model and the data, the income fed into the model is cleaned for household size effects in a similar fashion by dividing household income with an equivalence scale.[3] Fernández-Villaverde and Krueger (2007) list a summary of representative equivalence scales

[3]There are obviously other methods to create per-adult equivalent information from household data. An alternative method is to estimate household size/composition effects directly from micro data using least squares regressions. Although studying heterogeneity in household size/composition (which is a pre-requisite to understand the regression methodology) is beyond the scope of our paper, this approach generates adjustments that can be trivially converted to an ad-hoc equivalence scales.

which are all normalized to one for single person households and are increasing in household size by less than one. In our concrete setup this implies that the equivalence scale ϕ_t equals one in the first period and is larger than one in the second period, i.e. $\phi_1 = 1$ and $\phi_2 > 1$. We label this approach as the *Single* model and the corresponding optimization problem is thus

$$\max_{c_{1,S},c_{2,S}} U = u\left(c_{1,S}\right) + u(c_{2,S}) \tag{1}$$

subject to

$$c_{1,S} + c_{2,S} = \frac{y_1}{\phi_1} + \frac{y_2}{\phi_2} \equiv Y_S. \tag{2}$$

Not only do the consumption data come in household format but also the household consumption choices are made taking into account household size. As a benchmark we therefore consider a model in which household utility is affected by household size N_t, which we label as *Demographics* model, and the optimization problem is represented by

$$\max_{c_{1,D},c_{2,D}} U = u(c_{1,D}, N_1) + u(c_{2,D}, N_2) \tag{3}$$

subject to

$$c_{1,D} + c_{2,D} = y_1 + y_2 \equiv Y_D. \tag{4}$$

For the utility function we employ the following specification:

$$u(c_t, N_t) = N_t u\left(\frac{c_t}{\phi(N_t)}\right). \tag{5}$$

Household utility $u(c_t, N_t)$ is the product of the utility from per-adult equivalent consumption $u\left(\frac{c_t}{\phi(N_t)}\right)$, and household size N_t, as the per-adult equivalent consumption is enjoyed by each household member. While in the theoretical part of this paper, we will not rely on any specific equivalence scale, our choice of the utility function can be best explained by considering a concrete example, e.g. the widely used OECD equivalence scale which is given by

$$\phi_{OECD} = 1 + 0.7(N_{ad} - 1) + 0.5N_{ch}, \tag{6}$$

with N_{ad} being the number of adults and N_{ch} the number children in the household. According to

Equation (6) it takes \$ 1.7 of consumption expenditures to generate the same level of welfare out of consumption for a two adult household that \$ 1 achieves for a single member household.

We think of this framework as the mildest departure from the *Single* model which considers something akin to household size effects. The only additional twist in the *Demographics* model is that household size and composition affect the (marginal) utility of consumption. As a consequence, changes in household demographics over time impact the intertemporal allocation of consumption. However, and as in the *Single* model, the optimal consumption saving choices are undertaken by a single decision maker who equates marginal utilities over time. Note that the *Single* model directly predicts an per-adult equivalent consumption because the household receives a per-adult equivalent income. In the *Demographics* model, household consumption is predicted which then has to be deflated by the equivalence scale in order to be comparable, i.e. $\frac{c_{1,D}}{\phi_1} = \frac{c_{1,D}}{1}$ and $\frac{c_{2,D}}{\phi_2}$.

We are not the first to let demographics affect household utility. Attanasio, Banks, Meghir, and Weber (1999) and Gourinchas and Parker (2002) introduce such an effect via a taste shifter $[\exp(\xi_1 N_{ad} + \xi_2 N_{ch}) u(c)]$, while Fuchs-Schündeln (2008) uses the same structure as in Equation (5) but multiplies the utility from per-adult equivalent consumption by the equivalence scale ϕ_t instead of household size N_t. Similarly, Cubeddu and Ríos-Rull (2003), and Hong and Ríos-Rull (2007) use a different multiplication factor $[\min\{N_{ad}, 2\}]$ which accounts only for the "head" and the spouse (if present) but not for dependents in the household. The latter three papers do not provide any further justification for their choice of the multiplication factor. As outlined before, the interpretation of an equivalence scale makes the total household size a more natural choice for this multiplicative factor.

The following sections discuss how the *Single* model performs relative to our benchmark, the *Demographics* model.

2.1 Consumption Profiles

Result 1. *The per-adult equivalent consumption profile in the* Demographics *model and* Single *model coincide if* $\phi_2 = N_2$.

This result can be immediately read of from the two Euler equations which for the *Demographics* model is given by

$$u'(c_{1,D}) = \frac{N_2}{\phi_2} u' \left(\frac{c_{2,D}}{\phi_2} \right), \tag{7}$$

and for the *Single* model by

$$u'(c_{1,S}) = u'(c_{2,S}). \tag{8}$$

In both specifications only per-adult equivalent consumption appears. For the *Single* model the consumption levels $c_{1,S}$ and $c_{2,S}$ in fact reflect per-adult equivalent consumption because income as an input to the optimization problem has already been deflated by the equivalence scale. For the *Demographics* model it is obvious in the second period as the household receives the (marginal) utility from per-adult equivalent consumption $\frac{c_{2,D}}{\phi_2}$ which is however also true in the first period because household size is one in period one ($\phi_1 = 1$).

Equation (8) predicts a flat per-adult equivalent consumption profile for the *Single* model. The per-adult equivalent consumption profile in the *Demographics* model is however only flat if $N_2 = \phi_2$ but upward sloping if $N_2 > \phi_2$, i.e. $c_{1,D} = \frac{c_{1,D}}{\phi_1} < \frac{c_{2,D}}{\phi_2}$, while the opposite is true for $N_2 < \phi_2$. The intuition behind this result can be best explained when decomposing the benefit of consuming one additional unit of consumption in the second period in the *Demographics* model which

1. is associated with the marginal utility of per-adult equivalent consumption in period one $\left[u'\left(\frac{c_{2,D}}{\phi_2}\right)\right]$
2. accrues to all household members reflected through the multiplication by household size $[N_2]$
3. has to be divided by the equivalence scale $[\phi_2]$ because each household member does not get the full unit to consume but only the fraction $\frac{1}{\phi_2}$.

The larger household size in period two provides an incentive to allocate more consumption to period two because the household enjoys a larger utility from consuming then because each unit of per-adult equivalent consumption is enjoyed by more individuals (multiplication by N_2). However, in period two every unit of consumption has to be shared with more people which is reflected through the division with the equivalence scale ϕ_2. This in turn reduces the incentive to allocate more consumption to period two. If $\phi_2 = N_2$, the latter two effects cancel out and therefore per-adult equivalent consumption in period one equals per-adult equivalent consumption in period two. This case would however imply that the equivalence scale is larger than the household size and thus no (or for $\phi_2 > N_2$ even decreasing) economies of scale. This contradicts all empirically estimated equivalence scales (see Fernández-Villaverde and Krueger (2007)). Hence, for the only empirically relevant case $\phi_2 < N_2$ per-adult equivalent consumption in period two exceeds per-adult equivalent consumption in period one. Relative to period one, the absolute loss in consumption in period two because of the sharing across household members is outweighed by the fact that each household member enjoys the extra per-adult equivalent consumption. Interestingly, such a configuration provides an additional explanation for the hump observed in per-adult equivalent consumption documented in Fernández-Villaverde and Krueger (2007). The ratio $\frac{N}{\phi_2}$ could be interpreted as a change in the relative price of per-adult equivalent consumption induced by the change in household size (relative to period one) which is ignored in the *Single* model.

The Euler equation (7) gives of course also a clear prediction for the per-adult equivalent consumption profile if the (marginal) utility is multiplied by a number smaller than actual household size as assumed in Cubeddu and Ríos-Rull (2003), Hong and Ríos-Rull (2007), and Fuchs-Schündeln (2008). If the equivalence scale is smaller/equal/larger than the assumed multiplication factor, per-adult equivalent consumption in the *Demographics* model is upward sloping/flat/downward sloping.

2.2 Consumption Levels

Result 2. *Life-time per-adult equivalent consumption in the* Demographics *model coincides with life-time per-adult equivalent consumption in the* Single *model, if in the* Demographics *model period two household consumption $c_{2,D}$ and period two household income y_2 coincide.*

Life-time per-adult equivalent consumption from the *Demographics* model can be written as

$$C_D = c_{1,D} + \frac{c_{2,D}}{\phi_2} = \underbrace{y_1 + y_2 - c_{2,D}}_{c_{1,D}} + \frac{c_{2,D}}{\phi_2} = y_1 + y_2 - \frac{\phi_2 - 1}{\phi_2} c_{2,D} \tag{9}$$

while in the *Single* model life-time per-adult equivalent consumption equals life-time per-adult equivalent income Y_S (see also Equation (2)):

$$C_S = c_{1,s} + c_{2,s} = y_1 + \frac{y_2}{\phi_2} = Y_S \tag{10}$$

Hence, the difference in life-time per-adult equivalent consumption between the *Demographics* and the *Single* model is given by

$$C_D - C_S = C_D - Y_S = (y_2 - c_{2,D}) \left(\frac{\phi_2 - 1}{\phi_2} \right). \tag{11}$$

which proofs Result 2. Whenever $y_2 > c_{2,D}$, i.e. the household in the *Demographics* model is a borrower, the life-time per-adult equivalent consumption under the *Demographics* model is larger than under the *Single* model. The opposite is true for $y_2 < c_{2,D}$, i.e. when the household in the *Demographics* model is a saver.

The intuition for this result can be explained best with a concrete example. Assume that the household income is zero in the first period ($y_1 = 0$), and positive in the second period ($y_2 > 0$). In this case life-time per-adult equivalent income in the *Single* model is $\frac{y_2}{\phi_2}$ which by the budget constraint equals life-time per-adult equivalent consumption. In the *Demographics* model in turn, the household has the income y_2 available for consumption. For any utility function satisfying the Inada condition period one consumption will be positive such that $c_{2,D} < y_2$. Given that household size is one in period one, in the calculation of life-time per-adult equivalent consumption in the *Demographics* model only period two consumption is deflated by the equivalence scale. Since $c_{2,D} < y_2$, "less" in absolute terms is lost through the deflation by the equivalence scale in the calculation of life-time per-adult equivalent consumption in the *Demographics* model compared to the *Single* model.[4]

Essentially, Result 2 is the implication of a pure accounting exercise. The key driving force behind is that households can shift consumption between periods whereas income is predetermined, at least in any model with exogenous labor supply. If income and consumption allocations are not fully synchronized, then transforming household income to a per-adult equivalent drives a wedge between per-adult equivalent consumption in the *Demographics* model and adult equivalent income, and, as a direct consequence also, between per-adult equivalent consumption in the *Demographics* and *Single* model.

These differences in life-time per-adult equivalent consumption are also important in the presence of income heterogeneity. First, in the *Single* model the timing of income matters as it determines life-time per-adult equivalent income. Even for the same life-time household income

[4] More formally, for $y_1 = 0$ and $y_2 > 0$, $c_{2,D} < y_2$ implies that $C_D = y_2 - \frac{\phi_2 - 1}{\phi_2} c_{2,D} > y_2 - \frac{\phi_2 - 1}{\phi_2} y_2 = \frac{y_2}{\phi_2} = Y_S = C_S$.

$y_1^A + y_2^A = y_1^B + y_2^B$ but a different timing $\frac{y_1^A}{y_2^A} \neq \frac{y_1^B}{y_2^B}$ life-time per-adult equivalent incomes differ in the *Single* but not in the *Demographics* model. This implies an artificial inequality in life-time per-adult equivalent consumption in the *Single* model that is not present in the *Demographics* model. Second, it is straightforward to show that for heterogeneity in life-time household income $y_1^A + y_2^A \neq y_1^B + y_2^B$ but the same timing of income $\frac{y_1^A}{y_2^A} = \frac{y_1^B}{y_2^B}$, the implied inequality in life-time per-adult equivalent consumption between the *Single* and *Demographics* model is proportional to the differences in life-time per-adult equivalent consumption in the two models, i.e. $\frac{\text{Var}(C_S)}{\text{Var}(C_D)} = \left(\frac{C_S^A}{C_D^A}\right)^2$.

Note that the derivation and implications of Result 2 are completely independent of the relationship between N_2 and ϕ_2 which do however determine $c_{2,D}$ and thus, for a given y_1 and y_2, the relationship between the two per-adult equivalent consumption levels.

2.3 CRRA Preferences

In quantitative life-cycle models, CRRA preferences are the prevailing choice for the utility function. We now briefly discuss the role of the parameter of relative risk aversion in the *Demographics* model. For our simple setup given by Equations (3) and (4) we obtain closed form solutions for the optimal per-adult equivalent consumption allocation

$$c_{1,D} = \frac{1}{1 + \left(\frac{N_2}{\phi_2}\right)^{\frac{1}{\alpha}} \phi_2} \text{ and } \frac{c_{2,D}}{\phi_2} = \frac{\left(\frac{N_2}{\phi_2}\right)^{\frac{1}{\alpha}}}{1 + \left(\frac{N_2}{\phi_2}\right)^{\frac{1}{\alpha}} \phi_2}. \tag{12}$$

Since CRRA preferences are just a special case of the general utility function discussed before, we can see here again that it is only the ratio $\frac{N_2}{\phi_2}$ which determines the profile of the per-adult equivalent consumption. Larger values of α imply a flatter profile, as the lower intertemporal elasticity of substitution decreases the willingness to have differences in per-adult equivalent consumption between the two periods.

As long as $\phi_2 < N_2$ not only the per-adult equivalent but also the household consumption profile is increasing. This is also true if we consider multiplication factors lower than the actual household size for any value of $\alpha > 1$.[5]

[5]The exact condition under which the household consumption profile is increasing, independently whether this is

2.4 Summary

Two mechanisms introduce a bias into per-adult equivalent consumption predicted by the *Single* model relative to the *Demographics* model. First, the *Single* model ignores the change in the relative price of per-adult equivalent consumption induced by changes in household size in the presence of economies of scale. With CRRA preferences this channel looses importance as the coefficient of relative risk aversion increases or equivalently as the intertemporal elasticity of substitution decreases. Second, as an outcome of a pure accounting exercise, the *Single* model is generally associated with a different life-time per-adult equivalent consumption than in the *Demographics* model, which in the presence of income heterogeneity, feeds into inequality measures.

3 Quantitative Model

By constructing a quantitative model, our aim is to test and evaluate the implications of our theoretical analysis with a simple, stripped-down version of a standard incomplete markets life-cycle model, which can then be compared to actual US data.

Households start their economic life in period t_0 with zero assets. During their working life until period t_w they receive a stochastic income y_t in every period. There is no labor supply choice. From period $t_w + 1$ onwards households are retired and have to live from their accumulated savings during working life. We abstract from pensions. Life ends with certainty at age T and households do not leave bequests and cannot die with debt. Households have access to a risk-free bond a which pays the interest rate r. Households can borrow up to the natural borrowing constraint, i.e., an age specific level of debt that they can repay for sure.

In the *Demographics* model household size changes over the life-cycle deterministically as in Attanasio, Banks, Meghir, and Weber (1999) and Gourinchas and Parker (2002) and is homogenous across all households. The maximization problem is given by

also true for the per-adult equivalent consumption profile, is given by

$$\alpha > 1 - \frac{\ln \delta_2}{\ln \phi_2}$$

where δ_2 equals the multiplication factor which we have set equal to household size [$\delta_2 = N_2$] whereas e.g. Fuchs-Schündeln (2008) uses $\delta_2 = \phi_2$.

$$\max_{\{a_{t+1}\}_{t=t0}^{T-1}} E_0 \sum_{t=t0}^{T} \beta^{t-t_0} N_t u\left(\frac{c_t}{\phi_t}\right) \quad \text{subject to} \tag{13}$$

$$c_t + a_{t+1} \leq (1+r)a_t + y_t \tag{14}$$

$$a_{t+1} \geq a_{\min,t}. \tag{15}$$

where ϕ is a function of household size and its composition ($N_{ad,t}$ and $N_{ch,t}$). The income process is given by:

$$\ln y_t = \varrho_t + \epsilon_t, \tag{16}$$

where ϱ_t is an age-dependent, exogenous experience profile and

$$\epsilon_t = \rho\epsilon_{t-1} + \varepsilon_t \text{ with } \varepsilon_t \sim N(0, \sigma^2). \tag{17}$$

The Euler equation to this problem is given by

$$\frac{N_t}{\phi_t} u'\left(\frac{c_t}{\phi_t}\right) = \beta(1+r)\frac{N_{t+1}}{\phi_{t+1}} E_t\left[u'\left(\frac{c_{t+1}}{\phi_{t+1}}\right)\right]. \tag{18}$$

The structure of the *Single* problem is very similar. Demographics do not affect the utility function while income y_t is deflated by household size through equivalence scales ϕ_t:

$$\max_{\{a_{t+1}\}_{t=t0}^{T-1}} E_0 \sum_{t=t0}^{T} \beta^{t-t_0} u\left(c_t\right) \quad \text{subject to} \tag{19}$$

$$c_t + a_{t+1} \leq (1+r)a_t + \frac{y_t}{\phi_t} \tag{20}$$

$$a_{t+1} \geq a_{\min,t}, \tag{21}$$

with y_t following the same process as described in Equations (16) and (17).

The Euler equation to this problem is given by

$$u'\left(c_t\right) = \beta(1+r)E_t\left[u'\left(c_{t+1}\right)\right]. \tag{22}$$

4 Quantitative Features of the Model

A model period is one year. Agents start life at age 25, retire when 65 and live with certainty until age 75. To maintain simplicity, agents receive no social security income when retired and interest rates are zero ($r = 0$). We set the CRRA coefficient α to 1.57, the same value used in Attanasio, Banks, Meghir, and Weber (1999). We then calibrate β such that household consumption is the same at age 25 and at age 75. By doing this, we allow each model to better accommodate the timing in the 'hump' of household consumption seen in US data (see next section). In practical terms, the range of calibrated β's across models is centered with very low dispersion around 0.98, a reasonable value for an annual model.

As for equivalence scales, we perform the analysis using both the OECD and the Nelson (1993) scales. The OECD scale has the lowest economies of scale while the opposite is true for the Nelson scale in a wide range of scales used in the literature. Each additional adult and child represent (resp.) 0.7 and 0.5 adult equivalents according to the OECD; for the Nelson case the numbers are 0.06, 0.1 and 0.07 for each additional adult, the first child and each subsequent children respectively. For example, a family of four (two adults and two children) is equivalent to 2.7 adults living alone according to the OECD scale; the number for the Nelson scale is 1.23. For explicit formulations of different equivalence scales used in the empirical consumption literature, see Table 1 in Fernández-Villaverde and Krueger (2007).

4.1 Income

We use data from the Current Population Survey, from 1984 to 2003. We use the March supplements for years 1985 to 2004, given that questions about income are retrospective. We use total wage income (deflated by CPI-U, leaving amounts in 2000 US dollars), and apply the tax formula of Gouveia and Strauss (1994) to get after-tax income.[6]

We construct total household income $W_{i\tau}$ for household i observed in year τ, as the sum of individual incomes in the household for all households with at least one full time/full year worker. The latter is defined as someone who worked more than 40 hours per week and more than 40 weeks per year and earned more than \$2 per hour. Then, we estimate the following regression:

[6]Wage income in the CPS is pre tax income.

Table 1: Calibrated Parameters

ρ	σ	σ_0
0.9906	0.0189	0.1575

$$\log\left(\frac{W_{i\tau}}{\phi_{i\tau}}\right) = D_{i\tau}^{age}\varrho^{age} + X_{i\tau}\gamma + \epsilon_{i\tau} \tag{23}$$

where $\phi_{i\tau}$ is an equivalence scale, $D_{i\tau}^{age}$ represents a set of age dummies of the head of household, ϱ^{age} and γ are estimated coefficients and ε are estimation errors. Note that for the *Demographics* model we use household income for the estimation, i.e. $\phi_{i\tau} = 1 \ \forall \ i, \tau$. We also control for cohort effects and time effects by introducing birth year and year dummies in $X_{i\tau}$. [7]

From this estimation, we are interested in the regression coefficients associated with age dummies of the household head (experience profiles in the model). In our exercise below, we use smoothed profiles, which we show in Figure 1 for different choices of equivalence scales.

From the estimation residuals, we calibrate the income process in (17). Our calibration procedure is standard and follows Storesletten, Telmer, and Yaron (2004): we pick values of ρ and σ in order to minimize the square difference between the profile of observed cross-sectional variances of income and the simulated one (given the chosen parameters). We also pick values of σ_0, the standard deviation for the unconditional distribution of the first income shock ε_0 in order to match the cross sectional variance of income for our first age group (25 years old). We present these values in Table 1. We discretize this calibrated process using the Rouwenhorst method, using 20 points for the shock space. This methodology is specially suited for our case, given the high persistence of the process, see the discussion in Kopecky and Suen (2010).

To maintain full comparability with our simple theoretical model, we perform an *ex-post equivalization* procedure for the income process in the *Single* model: we use the same calibrated income

[7] Since year dummies are perfectly collinear with age and birth cohort dummies, we follow Fernández-Villaverde and Krueger (2007) and Aguiar and Hurst (2009) and include normalized year dummies instead, such that for each year τ

$$\sum_{\tau}\gamma_\tau = 0 \quad and \quad \sum_{\tau}\tau\gamma_\tau = 0$$

where $\{\gamma_\tau\}$ are the coefficients associated to these normalized year dummies. This procedure was initially proposed by Deaton and Paxson (1994). To compare life-cycle profiles across different cohorts/time periods, we normalize the estimated coefficients associated to age dummies by adding the effect of a particular cohort/time. More specifically, we picked the cohort corresponding to the median age observed at the last observed year.

Figure 1: Experience Profile for Households

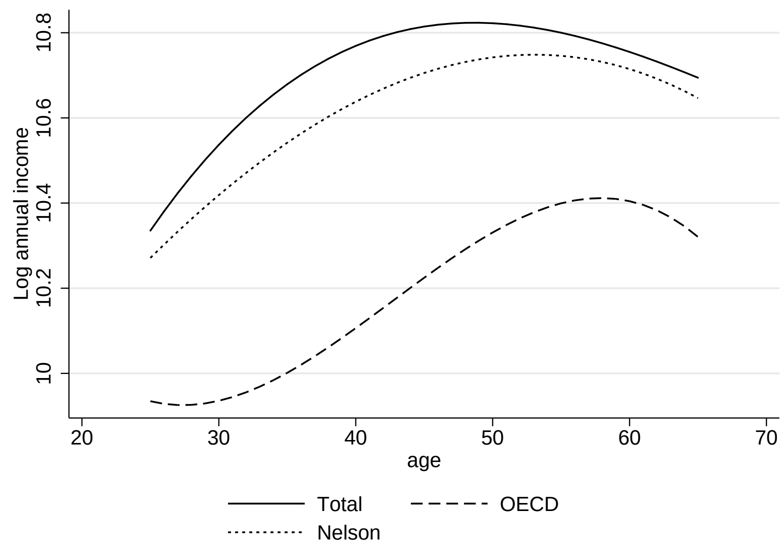
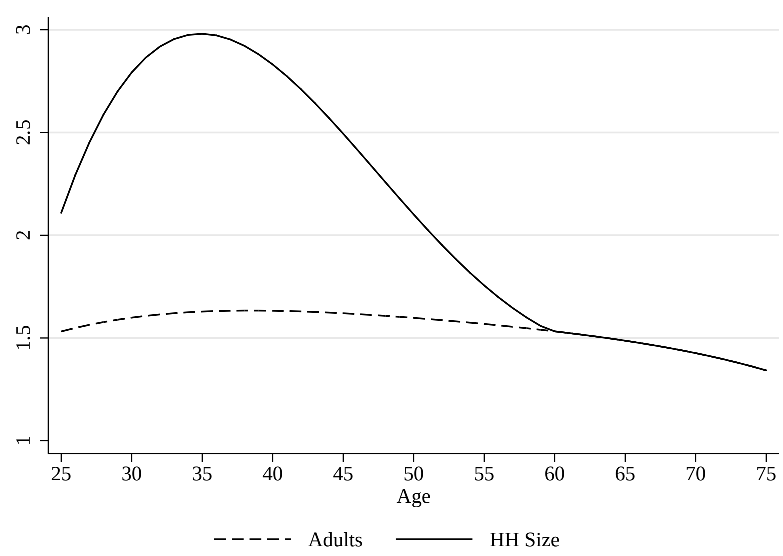


Figure 2: Profiles for Household Size and Composition



Note: Both figures are constructed using data from the CPS, 1983-2003.

profiles and shocks in both the *Demographics* and *Single* models (the calibration in Figure 1 and Table 1) and then feed the per-adult equivalent experience profiles to the *Single* model. Besides making the quantitative model more comparable to the theoretical model, this approach maintains the same shock structure across considered equivalence scales, making the comparison of biases more direct, since no extra 'noise' is being introduced by different volatility parameters. This would be the case for an alternative approach, or an *ex-ante equivalization*: estimating Equation (23) with a particular equivalence scale $\phi_{i\tau}$, resulting in different age profiles and calibrated income shocks for the *Single* model. Since the income has already been turned into per-adult equivalents *ex-ante*, there is no need to do so in budget constraint (20).[8]

4.2 Family Structure

We use the March supplements of the CPS for years 1984 to 2003.[9] For each household, we count the number of adults (individuals age 17+) and the number of children: individuals age 16 or less who are identified as being the "child" of an adult in the household. We restrict our sample to consider households with at most 2 adults and 4 children. We compute two separate profiles: one for number of adults and one for number of children. As above, we run dummy regressions to extract life-cycle profiles, where the considered age is that of the head (irrespective of gender) and control for cohort and year effects. After extracting these life-cycle profiles, we smooth them using a cubic polynomial in age, and restrict the number of children to zero after age 60. The results of this procedure are in Figure 2.

As in Attanasio, Banks, Meghir, and Weber (1999) and Gourinchas and Parker (2002), the number for adults and children in the household over the life-cycle are not integers. The transformation into adult equivalents using the OECD scale is trivial; for the Nelson scale, we use

$$\phi(N_{ad}, N_{ch}) = 1 + 0.06(N_{ad} - 1) + 0.1\min\{1, N_{ch}\} + 0.07\max\{0, N_{ch} - 1\}.$$

A similar adjustment would need to be done with all other equivalence scales that distinguish between the order of additional children.

[8] In our computations below, we do not find major differences between the *ex-post* or *ex-ante equivalization* strategies, so we show results only for the former. The results of these exercises are available on request.

[9] Since we are only interested in the average household size and composition by age rather than the evolution over the life-cycle for an individual, we use the CPS instead of the PSID because of the substantially larger sample size.

Figure 3: Household Consumption Relative to Age 25

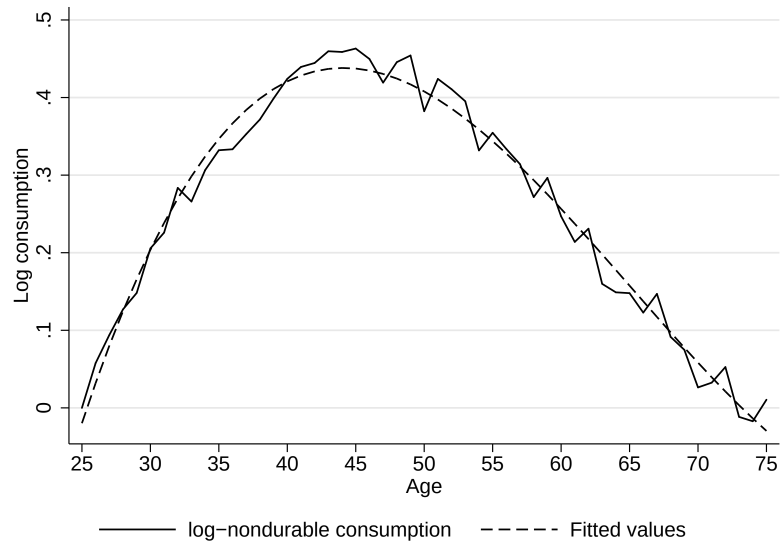
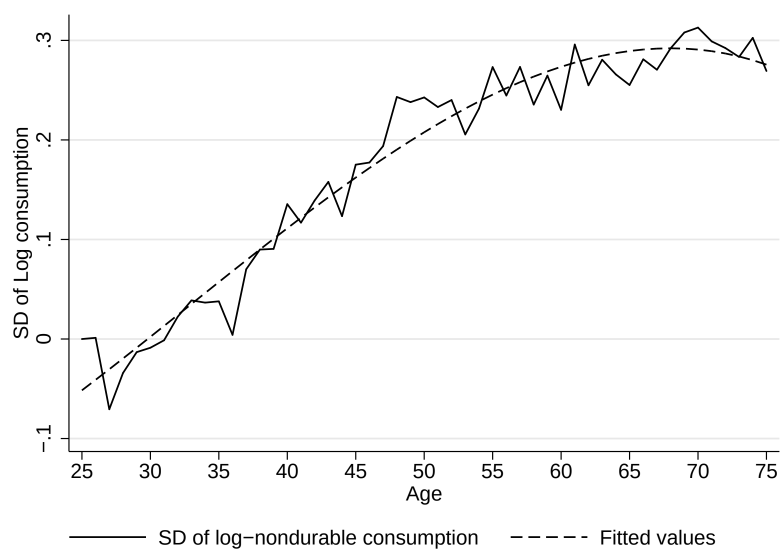


Figure 4: Standard Deviation of log consumption



Note: Data from the CEX, 1983-2003. Our consumption measure refers to nondurables, without housing. Age refers to the age of the male head of household.

5 Results

We compare the performance of each model (*Single* and *Demographics*) against evidence on household consumption from the survey of Consumer Expenditures (CEX) for the years 1984 to 2003.[10] We use the definition of nondurables in Aguiar and Hurst (2009), which consists of household expenditures not including housing services.

From the CEX we extract life-cycle profiles of consumption in a similar way as we do for income profiles: we estimate a regression with age dummies controlling for both cohort and time effects. Figure 3 shows the coefficients associated to the age dummies in the regression with the log of nondurable consumption as a dependent variable, along a fitted cubic polynomial in age. As in Fernández-Villaverde and Krueger (2007) and Aguiar and Hurst (2009), we normalize the consumption profile with respect to that of age 25. The figure shows the well known 'hump' shape of household consumption and the fact that the level of consumption at age 25 is almost the same as the one at age 75 (we use this fact to calibrate β in the model, as explained above). The hump achieves its peak around age 45, some 10 years after the peak in household size (see Figure 2).

Our measure for lifetime inequality is the standard deviation of log consumption at each age. This is depicted in Figure 4, which shows an increasing dispersion over the life-cycle. Again, we show differences with respect to age 25 and a smoothed series. In what follows, we will compare these smoothed lines (normalized also to be zero at age 25) with model predictions for both averages and inequality.

5.1 Household Consumption

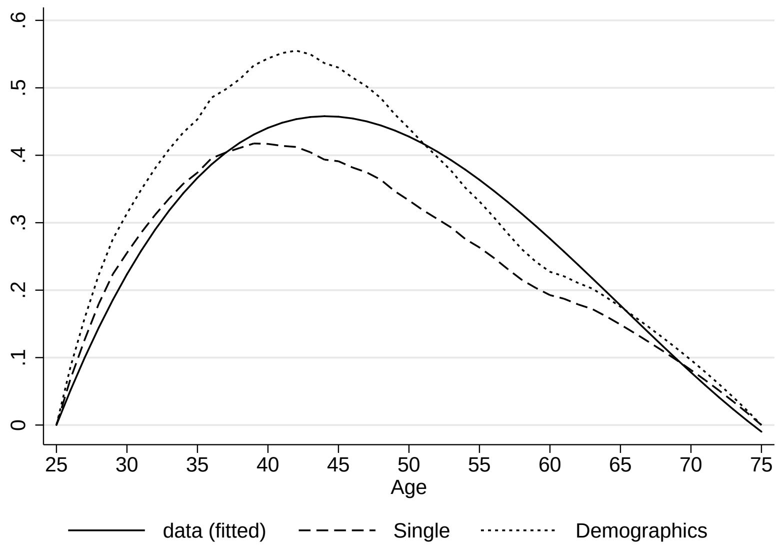
For each model, we simulate fifty thousand life-cycles and produce age specific statistics to be compared with the data. In this section we concentrate on household consumption, since it makes figures of life-cycle consumption easily comparable across models, as opposed to the alternative of showing per-adult equivalent profiles (in that case, the empirical profile from US data would be different depending on the equivalence scale used[11]).

For the *Single* model, we compute its predictions for single agents and then we aggregate those

[10] As in Fernández-Villaverde and Krueger (2007), we ignore years 1982 and 1983 due to methodological differences in the survey.

[11] In Figure 9 (see the Appendix), we show per-adult equivalent consumption when different equivalence scales are considered.

Figure 5: Household Consumption, Model vs. Data, OECD scale

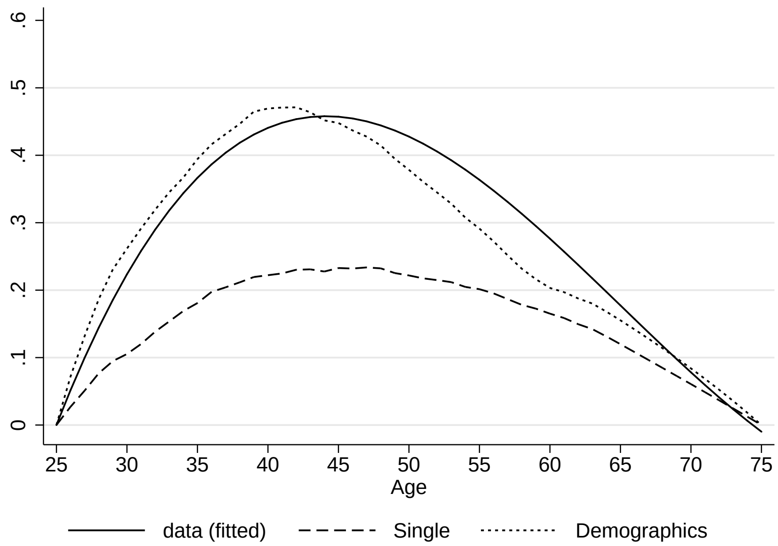


Note: Calibration of β (discount factor) implies a value of 0.9889 and 0.9834 for the *Single* and *Demographics* models respectively.

using the profile for household size and composition in Figure 2 in conjunction with the appropriate equivalence scale (i.e., $c^h = c^s\phi(N)$ where c^h is household consumption and c^s is consumption predicted from the single model). The *Demographics* model produces predictions for household consumption directly, so we make no further adjustments. Below we present figures with results of our exercises.

Figure 5 shows the results for our exercise using the OECD equivalence scale. In the figure we compare the predictions for the *Single* and *Demographics* models versus the data. A striking feature is the fact that our very simple quantitative framework is able to capture very well the hump shaped profile of household consumption as seen in the data, hinting at the importance of family size and composition in explaining the facts extracted from the CEX. This is basically the same result as in Attanasio, Banks, Meghir, and Weber (1999). However, the fact that the quantitative model lacks several mechanisms usually assumed in the literature (e.g. a realistic social security system, transitory versus permanent income shocks, survival probabilities, a longer lifespan, among others) hints that we might be attributing too much protagonism to household size. Although interesting, the interaction between household size effects and these other modeling alternatives are beyond the scope of this paper.

Figure 6: Household Consumption, Model vs. Data, Nelson scale



Note: Calibration of β (discount factor) implies a value of 0.9826 and 0.9809 for the *Single* and *Demographics* models respectively.

Our analysis confirms the result which we derived in the theoretical section: whether the model incorporates economies of scale inside the problem of the agent (as in the *Demographics* model) or uses equivalence scales only as an accounting device (the *Single* case) matters for the size of the predicted 'hump' in household consumption. In the particular case of OECD scales, we see that both models are relatively close, with the *Single* model better predicting consumption up to age 40 and the *Demographics* model, for ages 50 and older. We have to underscore once more the subtle but important distinction between models: the *Demographics* model predicts higher household consumption when household size is bigger because agents in that model are optimally choosing consumption, given the price incentives introduced by changing economies of scale. In the *Single* model, agents ignore these price effects. However, in the latter we get still a hump, since single agents track income earlier in life (given the calibrated β) and decrease consumption in retirement, producing a non-demography related hump, to which consumption of additional equivalent adults at certain points in the life-cycle are added to produce total household consumption. This can be seen more clearly by comparing Figure 5 and Figure 10 (in the Appendix), where we present life-cycle profiles of per-adult equivalent consumption.

Figure 6 shows the exercise when we use the Nelson equivalence scales. In this case, the

Table 2: Absolute percentage deviations of life-cycle consumption profiles with respect to US data

	Mean	Max
Single - OECD	4.46	10.28
Demographics - OECD	4.87	10.34
Single - Nelson	12.82	23.02
Demographics - Nelson	3.32	8.15

Note: The table shows the Mean and Maximum percentage differences between the predictions of the *Single* and *Demographics* models for the profile of life-cycle household consumption, compared to the profile for US data from the CEX

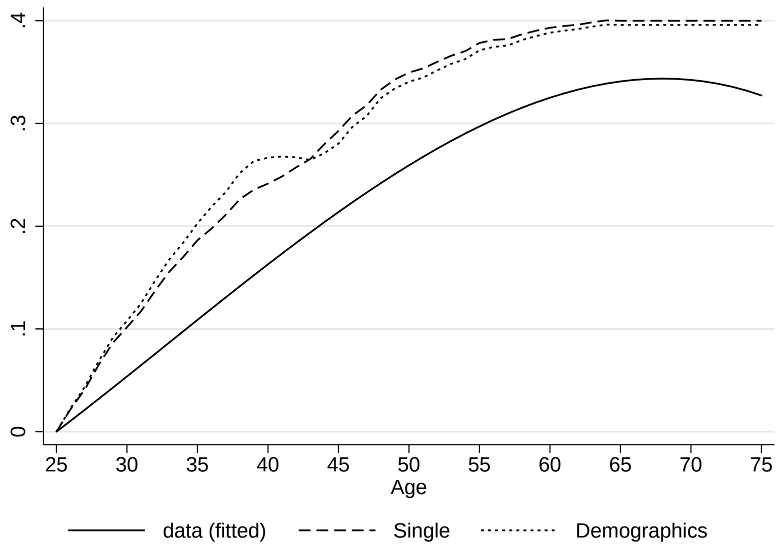
discrepancy between models is substantial, with the *Single* model vastly under predicting the size of the 'hump' in consumption over the life-cycle. On the other hand, the *Demographics* model stays on target, providing the best overall prediction across all models. The same result can be deduced by comparing the per-adult equivalent profile from Figure 11 in the Appendix.

The size of the gap between model predictions is related to the amount of economies of scale implied by the equivalence scales. As we discussed earlier, the OECD scale implies very little economies of scale inside the household, as opposed to what the Nelson scale prescribes.[12] The intuition is the following: if economies of scale are non-existent, living in a multi-person household does not entail any gains nor savings in terms of public consumption. Mathematically, this is represented by $\phi(N) = N$, and from equation (7), we know that agents in the *Demographics* model face the same relative prices as single individuals and the profiles of household consumption coincide across models. On the other hand, if economies of scale in consumption are sizeable (i.e., $\phi(N) \ll N$) and the incentives to consume when household size is bigger are high (see again the Euler equation in (7)). To be more specific, in Table 2 we present the absolute difference between the predictions from both models for the normalized life-cycle profiles of consumption versus the same profile for US data. Since these predictions are in logs, these differences can be interpreted as percentage deviations.

The main message from this section is that the single agent approach is bound to introduce a heavy bias in terms of predictions for household consumption if economies of scale of living together are high: if the gains for individuals of sharing a home are high, then we loose important

[12]From before, a household composed of two adults and two children is equivalent to 2.7 adults living alone according to the OECD scale, but just 1.23 adults living alone.

Figure 7: Standard Deviation of log household consumption, OECD scale



information from modeling all agents as if they were bachelor/single households.

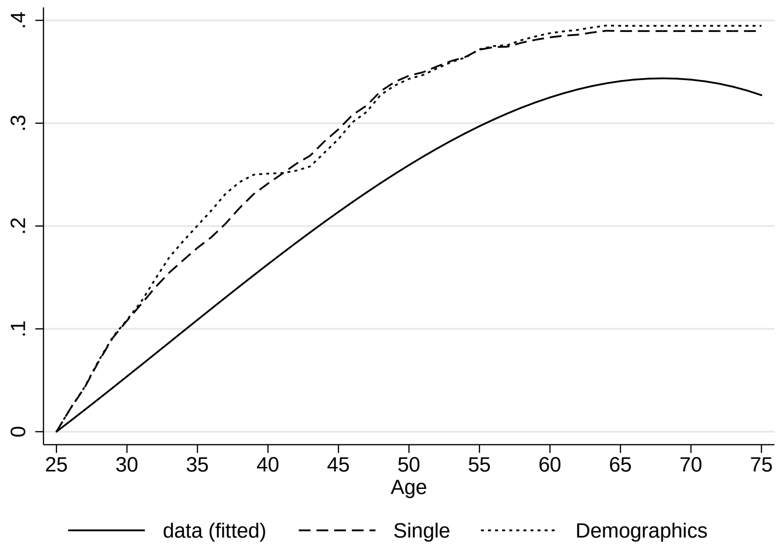
Result 2 of our theoretical model showed that as an implication of a pure accounting exercise the discounted life-time per-adult equivalent consumption levels differ between the *Single* and *Demographics* model if household consumption and income in the *Demographics* model are not fully synchronized. Our calculations show that under the proposed parameterization of the model, present value of lifetime (per-adult equivalent) consumption is higher in the *Demographics* than in the *Single* model. The numbers are 2.4% when the OECD scale is used, versus 0.5% when it's the Nelson scale. The difference is smaller for the Nelson Scale because household consumption and income are more synchronized over the life-cycle.

5.2 Consumption Inequality

In this section we compare the predictions with respect to life-cycle inequality. As before, we consider the simulated sample of household consumption, from where we calculate the standard deviation of log consumption at each age in the simulated life-cycles. The results are presented in Figure 7 and Figure 8.

From the figures we see that both models are able to replicate increasing consumption inequality and capture the relative increase in this measure over the life-cycle, although both models overstate

Figure 8: Standard Deviation of log household consumption, Nelson scale



this increase. We also see that the predictions of both the *Single* and *Demographics* model are very close to each other, with no model having a clear edge in terms of closeness to the data. Around ages 35 to 40, and for both equivalence scales, the *Demographics* model has a small departure from the *Single* model, which is around the timing of the peak in household size.

6 Discussion: identifying the *Demographics* model

In this section we use estimates from Attanasio, Banks, Meghir, and Weber (1999) in order to identify under which equivalence scale, the *Demographics* model is closest to the data. In other words, we perform a simple test of how much economies of scale there exist. First, we take the preferences used in Attanasio, Banks, Meghir, and Weber (1999):

$$u(c, N_{ad}, N_{ch}) = \exp(\zeta_1 N_{ad} + \zeta_2 N_{ch}) \frac{c^{1-\alpha}}{1-\alpha}, \tag{24}$$

Given this utility function and the parameters ζ_1, ζ_2 and α obtained in that paper from an Euler equation estimation (0.71, 0.34 and 1.57 respectively), we can make a simple comparison between (24) and (5) (for given equivalence scale ϕ and assuming CRRA preferences):

Table 3: Empirical values for RHS of Equation (26)

N_{ad}	N_{ch}	N	OECD	NAS	HHS	DOC	LM	Nelson
2	0	2	0.75	0.77	0.86	0.88	0.98	0.98
2	1	3	0.60	0.63	0.70	0.73	0.82	0.87
2	2	4	0.56	0.60	0.66	0.66	0.80	0.88
2	3	5	0.57	0.63	0.67	0.67	0.82	0.95

Note: Our considered equivalence scales are constructed respectively by the Organization for Economic Cooperation and Development (OECD), the National Academy of Sciences (NAS), the Department of Health and Human Services (HHS), the Department of Commerce (DOC), Lazear and Michael (1980) and Nelson (1993).

$$\exp(\xi_1 N_{ad} + \xi_2 N_{ch}) = \frac{N_{ad} + N_{ch}}{\phi(N_{ad}, N_{ch})^{1-\alpha}} \qquad \forall\ N_{ad}, N_{ch}. \tag{25}$$

For $N_{ad} = 1$ and $N_{ch} = 0$, our setup implies $\delta = \phi = 1$ whereas the preference parameter in Attanasio, Banks, Meghir, and Weber (1999) is $\exp(\zeta_1)$. We therefore normalize the utility function (24) by this number and rearrange (25) to obtain

$$1 = \frac{\exp(\zeta_1[N_{ad} - 1] + \zeta_2 N_{ch})\phi(N_{ad}, N_{ch})^{1-\alpha}}{N_{ad} + N_{ch}}, \tag{26}$$

an expression which does not necessarily hold empirically. In Table 3 we show the values for the right hand side of (26) for the array of equivalence scales considered by Fernández-Villaverde and Krueger (2007) and for different household arrangements, in terms of number of adults and children present

The equivalence scales are displayed in increasing order of implied economies of scale: as discussed earlier, the OECD scale shows the smallest while the Nelson scale, the highest. Comparing across columns, we see that under the latter, the empirical value of the right hand side of (26) is closest to one, hence, the condition in that equation is most likely to hold. This independent evidence validates our proposed *Demographics* model when economies of scale are high (as implied by the Nelson scale). As we showed above, this creates the biggest differences with the predictions of the *Single* approach since sharing a household matters quite a lot for consumption.

7 Conclusions

In this paper we suggest a simple framework to understand the sources of bias when single agent models are used to make predictions for aggregate consumption or consumption related variables, such as aggregate welfare.

Our proposed *Demographics* model acknowledges that economies of scale in household consumption (measured by equivalence scales, which are widely used in the quantitative literature) have an effect on the agent's perceived relative prices of consumption over the life-cycle when family size and composition change. This is in stark contrast to the common practice of simulating a single agent model, where these induced price effects are ignored. Hence, we find that economies of scale in consumption inside the household are positively related to the bias introduced by the single agent approach, given that in such a case the price effects are stronger.

We also perform a quantitative exercise and find that the single agent approach underestimates the effect of family size on consumption over the life-cycle for mean consumption profiles but not for inequality.

Finally, using estimates from Attanasio, Banks, Meghir, and Weber (1999), we ask which equivalence scale (or amount of economies of scale in the household) makes our model closest to the empirical facts. We find that scales implying very high economies of scale do the job, which in turn, suggest the need to move away from single agent models to understand life-cycle consumption.

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Appendix

Additional Figures

Figure 9: Log of Nondurable Consumption, Relative to Age 25

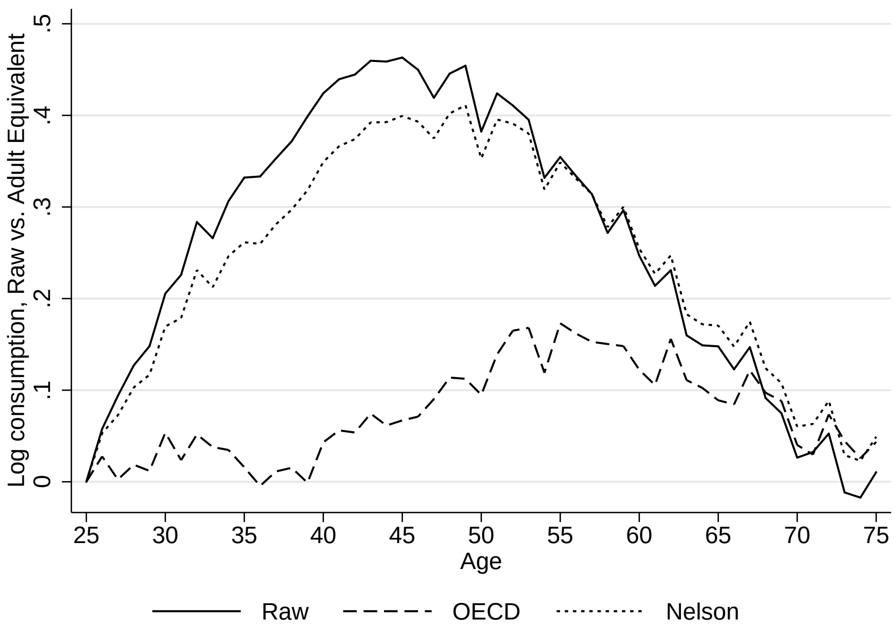


Figure 10: Adult Equivalent Consumption Relative to Age 25, OECD scale

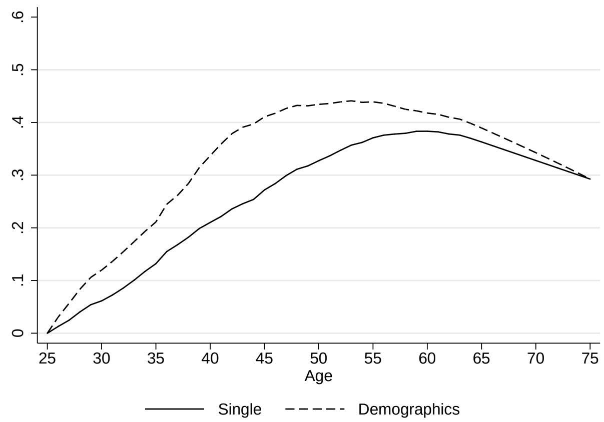
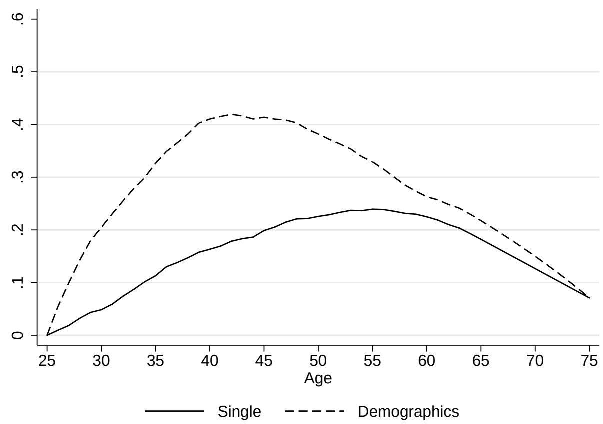


Figure 11: Adult Equivalent Consumption Relative to Age 25, Nelson scale



Note: Model output in adult equivalent terms. The *Single* case shows directly the predictions from the respective model. In the *Demographics* case, predicted consumption is deflated by the corresponding equivalence scale (OECD vs. Nelson).