# Optimal decisions on pension plans in the presence of financial literacy costs and income inequalities

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**Optimal Decisions on Pension Plans in the Presence of Financial Literacy Costs and Income Inequalities**

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Abstract

*Pension reforms are on the political agenda of many countries. Such reforms imply an increasing responsibility on individuals' side in building an efficient portfolio for retirement. In this paper we provide a model describing workers' choices on the allocation of retirement savings in presence of a) mandatory contribution; b) portfolio decision; c) financial literacy costs. In particular, we characterise the results both from a positive and normative standpoint, by highlighting the determinants of the individual's choice, with special focus on financial literacy costs and wage level inequalities and by characterizing the optimal contribution rate to mandatory complementary pension schemes.*

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**Keywords:** Financial literacy, Choice on pension Plans; Optimal portfolio composition, Income inequality.

## 1. Introduction

In recent decades pension systems of both developed and developing economies have been undergoing major reforms. Broadly speaking, such reforms have been introduced on account of considerations which descend from the "optimal portfolio theory": in presence of assets whose risks are not completely correlated, differentiation of a portfolio over different assets can generate a more efficient investment than investing in a single asset.

Typically, according to such reforms savings for retirement can be directed towards two different channels (or pillars): on the one hand, public sector managed systems, typically based on a PAYG financing mechanism, and, on the other hand, fully funded systems. The latter are either occupational or personal and are typically managed by private firms, although cases in which the State runs occupational schemes do exist, and can be either mandatory or not, while investing in a personal saving plan is a discretionary decision.

Since the two pillars provide different internal yields (usually the long run rate of growth of GDP in the case of PAYG systems, and the long run interest rate in the fully funded case) and different risks (demographic and political risks in the former case, market volatility and default risk in the latter case; see for example Nataraj and Shoven 2003 for a comparison of these risks), the majority of developed/developing countries are building up multi-pillar models for retirement savings, although this trend is occurring at different speeds and with different weights attached to each pillar, depending on social preferences, path dependency of policy decisions and economic development of each country (see Galasso 2006 for a discussion of the aims and the problems related to pension reforms in different countries).

Among the common features of the aforementioned reforms there is the possibility given to workers to choose how to invest a part of their retirement contributions between different opportunities. This is particularly true, for example, for Italy, Sweden and United Kingdom where a certain share of pension contributions can be moved between alternative schemes but also for

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Argentina and Peru where mixed private/public schemes are present and workers are called to choose between them. Even the US somehow falls in this category as the 401k plan can be considered a possible option in which to invest retirement savings. Moreover, several countries undergoing important pension reforms (for example Hungary, Poland and Uruguay) have given workers the possibility to choose between the old and new system. Finally, on the extreme edge, there is Mexico whose whole pension system is run by private companies so that workers have directly to choose between alternative pension funds. Hence, we can say that even if the alternatives provided to workers are different from country to country[1], there is a trend in recent reforms which entails increasing responsibility on the workers' side in building up an adequate portfolio for facing retirement needs.

In this respect, the issue of an adequate degree of financial literacy has been raised by several researchers, in that the choice of both the amount and the composition of the retirement-saving-portfolio implies the understanding and the evaluation of the different opportunities that are now offered. However, although several papers have produced a solid piece of evidence on the relevance of financial literacy in determining the "planning attitude" or the degree of farsightedness of individuals[2], to the best of our knowledge scarce effort has been put on the theoretical analysis of the effects that the cost of achieving such an adequate level of financial knowledge can play in determining workers' optimal decisions. A notable exception is represented by Jappelli and Padula (2010) where they show that financial literacy positively affects the rate of returns of investments in the financial markets and where a theory on optimal investment in financial literacy is provided.

Given the existence of a cost to achieve the necessary degree of financial literacy, it is worth to analyse the role of such a cost in determining the distribution of investment decisions in the population and how it interacts with the wage level of different individuals in driving the results.

In this paper, we aim at filling this gap. More precisely, the scope of this paper is threefold. First, to build a model unveiling the determinants of individuals' decision on consumption and retirement savings in presence of a) mandatory contribution rate; b) the opportunity of choosing among assets involving different interest rates and volatility; c) costs of financial literacy. Second, to assess what role the costs of financial literacy and wage/income levels have in driving the results of the retirement saving choice. Third, to provide a normative analysis of the optimal contribution rate and to draw some policy implications under the scenario depicted above. For the sake of simplicity our model focuses on the choice between two alternative plans only: a safe scheme (possibly run by the state) and a riskier scheme with higher expected returns and higher volatility; however, the framework can be extended to include more choice options without qualitatively changing our results.

Our paper has much to do with previous literature on overlapping generation models dealing with consumption and retirement saving in the presence of public pensions (for example, Samuelson 1958 and 1975 or Blake 2006 for a broader overview); although the presence of uncertain returns from pension schemes makes our case different from what has been examined in previous literature. In fact, some recent articles deal with pension systems where saving decisions

[1] Typically, the mandatory contribution the worker has to decide upon is relevant, although not huge: for example, in Italy it is 6.91% of gross wages, in Sweden 2.5% and in United Kingdom, on average 6%, however in some cases, like Mexico, it reaches about 13% . For more detailed data see OECD Private Pension Outlook (2008).

[2] The literature on financial literacy has stressed the importance of this factor in the financial decisions of individuals and in particular in saving behaviour and retirement decisions (for an overview of this issue see OECD 2005). Most of the works on this subject approach the issue from a behavioural perspective or through empirical analyses: Clark et al (2003) compare questionnaires about retirement goals filled by the same individuals before and after attending a financial seminar and notes how retirement decisions change after such an event. Lusardi and Mitchel (2009) performs an analysis with an American dataset containing detailed information of both the retirement planning decisions and financial knowledge of individuals and find a strong influence of the latter. Still on the American case, but with a different database, Lusardi and Mitchel (2011) analyse how the lack of financial literacy is particularly relevant in the decision of some categories of individuals while Fornero and Monticone (2011) explore this issue for Italy finding evidence that financial literacy is usually scarce and that the participation to pension plans is significantly influenced by it.

are taken under some degree of uncertainty: Demange (2009), for example, builds an overlapping generation model with macro shocks and assesses the political support by new generations to different rates of contribution and intergenerational risk sharing; D'Amato and Galasso (2010) use an overlapping generation model with shocks on aggregate production to compare the optimal and the politically feasible level of intergenerational risk sharing. Somehow differently, Maurer, Mitchell and Rogalla (2010) explore another form on uncertainty and perform an empirical analysis of the effect of uncertainty in labour income on the life-cycle portfolios. An interesting example of a work introducing volatility in the returns from the retirement savings is Gordon and Varian (1988): their work focuses on the intergenerational risk sharing but they explore a case where saving is fixed at an exogenous level and does not come from an optimization process[3]. However, none of the above focus on the saving/consumption decision in the presence of uncertain returns[4] and alternative pension schemes nor on how financial literacy and income levels shape the results.

As for pension systems and income inequality within the population, the usual link that is drawn is that a given pension system may affect the degree of income inequality: see for example Creedy (1994) and Benedict and Shaw (1994). Here however, we highlight the existence also of a reverse relationship, so that individuals with different income levels turn out to choose different pension schemes.

As for financial literacy, our theoretical model relies to some extent on the empirical evidence provided by Iyengar et al. (2004) where the authors, using data concerning the 401k plan participation in the US, show that providing individuals with too many choice options may lead to lower motivation towards the optimal choice. Moreover, the authors argue that workers' wide opportunity set can create a burden for workers that is related to the time lost in keeping track of the different options. In the light of these findings, in our model we allow for the presence of a cost that has to be paid in order to achieve an adequate level of financial literacy which is necessary to assess the more complex investment schemes. This cost has then to be paid to access a given scheme (possibly due to the time lost to fully understand it) and is related to the complexity of the scheme (so that it can be assumed to be zero for simpler/safer investment schemes).

Some of the results we provide are expected: expected rate of returns and volatility have an important role in the attractiveness of the schemes, and similarly, the risk aversion (propensity) of the individuals pushes the favour toward a safer (riskier) option. However, some results are less obvious: first, in line with the above cited empirical findings, the cost necessary to achieve an adequate degree of financial literacy can discourage workers to opt for more complex schemes, which can result into a failure of the reforms promoting investment diversification. We find that this is particularly true (and socially undesirable) for low income individuals which in fact turn out to be excluded from this option. Second, in presence of a too high rate of contribution also high income individuals may be discouraged in diversifying their portfolio through the investment into the riskier scheme. This is because the higher the income share devoted to the risky investment, the higher volatility of the portfolio and, hence, the lower the expected utility stemming from such an investment for risk adverse individuals. As a consequence, we show that a policy involving identical compulsory contribution rates in presence of heterogeneity in incomes would in fact split the population into two subgroups: middle-class individuals investing in more complex/risky schemes, and individuals at the tails of the income distribution investing in the safer scheme (although poorer and richer individuals are driven into such safe scheme by different reasons: the former by the financial literacy costs, the latter by a too high volatility of the investment).

Finally, in order to deal with such wage-dependency of the outcome at the aggregate level, we characterize the optimal (compulsory) contribution rate in presence of differences in income, and we show that it implies lump-sum contribution or, at least, contribution rates that are decreasing in

[3] Related to the Gordon and Varian (1988) work on intergenerational risk sharing is Veall (1986).

[4] Clearly, some classic works like Phelps (1962), Merton (1969) and Samuelson (1969) have analysed saving decisions under uncertain returns: however, they did not focus on the retirement savings, on fixed contribution rates nor on the possibility of choosing between alternative schemes

the wage level. However, while mitigating the problem, such a solution still leaves individuals at the bottom of income distribution (and in some cases also at the top of the distribution) still choosing the safe scheme due to the inaccessibility to an adequate level of financial literacy or to the excessive volatility. Hence, some direct policy implications are provided in order to deal with this problem.

The rest of the paper is organized as follows: in section 2 we build a model of optimal consumption and saving in the presence of a mandatory scheme with stochastic returns; in section 3 we allow for alternative pension schemes and we examine what determines the decision of workers; in section 4 we focus in particular on how the wage levels affect the decision; in section 5 we characterize the role of the mandatory contribution rate and we determine its optimal level; in section 6 we formulate some policy implications and, in section 7, we conclude.

## 2. Consumption and saving in the presence of mandatory complementary social security

The basic problem we want to analyse concerns the decisions of consumption and saving in a context where workers are forced to invest a share of their income in a complementary social security scheme which entails some uncertainty over the rate of returns.

We imagine a small closed economy where individuals live 2 periods. In the first period they work, receiving a wage $w$, and choose how much to consume and to save (at a safe rate $r$); in the second period they just consume what they have saved. In addition to voluntary savings, workers have to adhere to a complementary social security scheme $i$ (i.e. second pillar) where they must invest a share of their income. For the sake of simplicity, and without loss of generality, we omit the first pillar pension scheme. The exact rate of compulsory contribution $\gamma$ is fixed by law and the scheme $i$ yields stochastic returns drawn from a normal distribution, so that $r_i \sim N(\bar{r}_i, \sigma_i^2)$, where $\bar{r}_i$ and $\sigma_i^2$ are, respectively, mean and variance of the returns. Some schemes may display a zero variance so that they yield fixed returns and are equivalent to investing in a safe asset. In addition, workers have to pay a cost $C_i$ (which in some cases can be zero) to access the $i$ scheme, and this is due to the complexity of the mechanism governing the scheme and to the effort needed to keep track of the performance of the asset. This cost is clearly related to the financial literacy of workers and can be seen as the cost necessary to obtain the degree of financial literacy adequate to the full understanding of the scheme.

Workers lifetime utility $U$ depends on the consumption in the first period $c_1$ and in the second period $c_2$ :

1) $U = U(c_1, c_2)$

where monotonicity and concavity on consumption is assumed. Workers choose how much to consume (and thus to save) in the first period aiming to obtain the highest expected utility (denoted as $E[U(c_1, c_2)]$), the problem they face is then

2) $$\begin{cases} \max\limits_{c_1} E[U(c_1, c_2)] \\ s.t.\ c_2 \sim N[(w - c_1 - C_i)(1+r) + \gamma w(\bar{r}_i - r), \gamma^2 w^2 \sigma_i^2]. \\ s.t.\ c_1, E(c_2) \geq 0 \end{cases}$$

The first constraint in the above equation implies that in the second period workers consume exactly (and only) what they have saved: since the returns of complementary scheme is stochastic also consumption in the second period is stochastic (and depends on the characteristics of the scheme $i$) and normally distributed. The second constraint simply implies that consumption cannot be negative.

From an analytical point of view, the above problem is not trivial as it requires the computation of the expected utility of a gamble (the actual realization of consumption in the second period, in our case). To obtain a closed form solution we assume that lifetime utility takes the following (CARA) form:

3) $U = -e^{-ac_1} - \rho e^{-ac_2}$

where $\rho \in (0,1]$ is the rate of time preferences and $a>0$ is a parameter measuring risk aversion. The expected utility under the scheme $i$ is then:

3a) $E(U_i) = -E\left(e^{-ac_1}\right) - \rho E\left(e^{-ac_{2,i}}\right)$.

If utility takes the above form and since consumption in the second period is normally distributed we can exploit a well known result[5] to reformulate (2) as

$$4) \quad \begin{cases} \max\limits_{c_1} - e^{-ac_1} - e^{-a\left(\bar{c}_2 - a\sigma_{c_2}^2\right)} \\ s.t.\ c_2 \sim N\left[(w - c_1 - C_i)(1+r) + \gamma w(\bar{r}_i - r), \gamma^2 w^2 \sigma_i^{\ 2}\right] \\ s.t.\ c_1, E(c_2) \geq 0 \end{cases}$$

where $\bar{c}_2$ and $\sigma_{c_2}^2$ are, respectively, mean and variance of consumption in the second period. The above yields the following solutions in terms of optimal consumption ($c_{1,i}^*$ and $c_{2,i}^*$):

5a) $c_{1,i}^* = \left[(1+d_i)wx - \dfrac{\log \rho x}{a}\right]\dfrac{1}{(1+x)}$

5b) $c_{2,i}^* \sim \left[\left((1+d_i)wx + \dfrac{\log \rho x}{a}\right)\dfrac{x}{(1+x)}, \gamma^2 w^2 \sigma_i^{\ 2}\right]$

5c) $E\left(U_i^*\right) = -(1+x)\left(xe^{ad_i w}\right)^{-\frac{x}{1+x}} \rho^{\frac{1}{1+x}}$

where $x=1+r$, $d_i = \gamma \dfrac{\bar{r}_i - r - \gamma a w \sigma_i^{\ 2}/2}{x} - \dfrac{C_i}{w}$. Eq. (5c) describes the indirect expected utility of a worker under the scheme $i$. The above equations refer to the inner solution of the maximisation problem and they arise as long as the following condition holds:

6) $w \geq \max\left[\dfrac{\log \rho x}{(1+d_i)ax}, -\dfrac{\log \rho x}{(1+d_i)ax}\right]$

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[5] A known result is that, given any stochastic variable $z_j$ distributed normally with mean $z$ and variance $\sigma_z^2$ we have $E(e^{-az_j}) = e^{-a\left(z - a\sigma_z^2\right)}$: see Varian (1993). Note that our approach is equivalent to the use of a mean/variance utility function as is done, among others, by D'Amato and Galasso (2010).

Since the analysis of the corner solutions is beyond the scope of the present paper, (for a study of this issue see Spataro and Corsini 2011), in the rest of the paper, for the sake of simplicity, we assume that the above condition holds true, so that the interiority of solutions is always satisfied.

## 3. Choosing between alternative schemes

We analyse now how workers behave when they are given the possibility to choose between different complementary social security schemes. In particular we imagine that workers can choose between two possible schemes: a safe scheme (either run by the State or by a pension fund) and a risky scheme (either run by a pension fund or by the very firms).

If workers are given the possibility to choose between two schemes they will typically proceed in two steps: first they will choose which scheme to adhere to and then they will choose consumption and saving so as to maximise their expected utility. As a solution strategy we solve their problem through backward induction: we start from the last step, where workers determine their indirect utility (through consumption and saving) in a given scheme *i* (which is given by (5c)) and then we go back to the first step where workers choose the scheme that yields the highest indirect utility.

### *3.1. Safe scheme*

We imagine that the safe scheme $S$ yields a certain return $r_S$ with zero variance. For simplicity, and without loss of generality, we assume that the returns of this scheme are the same as those on voluntary saving so that $r_S \sim N(r,0)$. Moreover, given the simplicity of this scheme, we assume that no costs have to be paid to access and understand it so that $C_S=0$. Given the characteristics in terms of returns and costs of this scheme we have that $d_S = 0$ and the solutions in Eq. (5a)-(5c) take the following form:

7a) $$c_{1,S}^* = \left( wx - \log\frac{\rho x}{a} \right)\frac{1}{(1+x)}$$

7b) $$c_{2,S}^* = \left( wx + \log\frac{\rho x}{a} \right)\frac{x}{(1+x)}$$

7c) $$E(U_S^*) = -(1+x)e^{-aw\frac{x}{1+x}}\rho^{\frac{1}{1+x}}.$$

Note that consumption in both periods is a deterministic variable.

### *3.2. Risky scheme*

Under the risky scheme workers obtain returns $r_R$ which are drawn from a distribution $r_R \sim N(\bar{r}_R, \sigma_R^2)$ where $\bar{r}_R$ and $\sigma_R^2$ are mean and variance respectively. Given the risky nature of this scheme we assume that $\bar{r}_R > r$: in fact if this is not true this scheme would be clearly unattractive and all (risky averse) workers would simply choose the safe scheme.

We also assume that in this scheme $C_R>0$, implying that a certain cost has to be paid to access it. This cost is partly a direct fixed cost (for example the fees a worker have to pay to attend a course on basic finance) and partly an opportunity cost (related to the time lost to understand and to keep track of the scheme). In particular we assume $C_R = F + f \cdot w$ where $F \geq 0$ is the fixed component and $f \cdot w \geq 0$ is the opportunity (variable) cost component which is then related to worker's wage/income.

This said, Eqs. (5a)-(5c) for the risky scheme become:

8a) $c^*_{1,R} = \left( w(1 + d_R)x - \log \frac{\rho x}{a} \right) \frac{1}{(1 + x)}$

8b) $c^*_{2,R} \sim \left\{ \left[ (1 + d_R)wx + \log \frac{\rho x}{a} \right] \frac{x}{(1 + x)}, \gamma^2 w^2 \sigma_R{}^2 \right\}$

8c) $E(U^*_R) = -(1 + x)e^{-a(1+d_R)w\frac{x}{1+x}} \rho^{\frac{1}{1+x}}$ .

where

9) $d_R = \gamma \frac{\bar{r} - r_s - \gamma a w \sigma_R{}^2 / 2}{x} - f - \frac{F}{w}$ .

*3.3. Incentive function*

We now have all the ingredients to compare the expected indirect utilities stemming from the two schemes. If we define the incentive to adhere to the risky scheme as $I = E(U^*_R) - E(U^*_S)$ then the value of *I* determines the choices of the individuals: in fact a worker will opt for the risky scheme whenever $I > 0$ and for the safe scheme for $I \leq 0$. We can easily compute the value of *I* from Eq. (3) and (6a):

10) $I = E(U_R) - U_S = (1 + x)\left(1 - e^{ad_R w}\right)^{-\frac{x}{1+x}} (xe)^{-\frac{x}{1+x}aw} \rho^{\frac{1}{1+x}}$ .

The above result allows us to provide the following proposition:

**PROPOSITION 1**
A worker chooses the risky (safe) scheme if and only if $d_R > (\leq) 0$ .
**PROOF**
*The maximization of expected utility implies that a worker chooses to adhere to the risky scheme if and only if* $I > 0$ *and thus, from Eq. (10), a worker adheres to it if and only* $(1 + x)\left(1 - e^{ad_R w}\right)^{-\frac{x}{1+x}} (xe)^{-\frac{x}{1+x}aw} \rho^{\frac{1}{1+x}} > 0$ . *Since the parameters a, w, x and* $\rho$ *are necessarily positive this inequality holds true for* $\left(1 - e^{ad_R w}\right) > 0$ *which is true if and only if* $d_R > 0$ . *An analogous reasoning can be brought forth for* $d_R \leq 0$ . ■

Proposition 1 states that the choice of workers depends only on $d_R$ and therefore we can study the determinants of the choice simply analysing the variable $d_R$ . In particular the following condition determines the choices of workers:

11) $I > 0 \Leftrightarrow d_R > 0 \Leftrightarrow -\frac{a\gamma^2\sigma^2}{2x} w^2 + \gamma\left(\frac{\bar{r} - r_s - f \cdot x / \gamma}{x}\right) \cdot w - F > 0$ .

Moreover we can assert the following which qualifies the shape of the incentive function:

**REMARK 1 TO PROPOSITION 1**

For any $\gamma > 0$, $d_R$ is an increasing function of $\bar{r} - r_s$ and a decreasing function of $a$, $\sigma^2$, $f$ and $F$.

**PROOF**

*By computing the derivatives of $d_R$ with respect to the relevant parameters, it is easy to see that* $\partial d_R / \partial(\bar{r} - r_s) = \gamma / x > 0$ ; $\partial d_R / \partial a = -\gamma^2 \cdot \sigma^2 \cdot w / x < 0$ ; $\partial d_R / \partial \sigma^2 = -\gamma^2 \cdot a \cdot w / x < 0$ ; $\partial d_R / \partial f = -1 < 0$ *and* $\partial d_R / \partial F = -1 / w < 0$.■

The results contained in Proposition 2 are quite intuitive: they state that workers are more likely to opt for the risky scheme (and thus to differentiate their portfolio) when the difference between the mean returns of the risky and of the safe scheme is higher, and less likely the more adverse to risk the individuals, the more volatile the returns of the risky scheme and the larger the costs to access the risky scheme. Note that the results in Remark 1 can be extended to the incentive function as a whole so that the value of *I* is increasing in $\bar{r} - r_s$ and decreasing in *a,* $\sigma^2$, *f,* and *F.*

Things are more complex as to the role that wage/income *w* and the rate of compulsory contribution $\gamma$ have on the choice of workers: these aspects appear to be crucial in the choice of workers and a detailed analysis of their role will be at the centre of the next sections.

## 4. The role of wages

A relevant element which affects the value of $d_R$ is the wage *w*, so that workers of different wage levels may prefer different schemes: this in turn may imply a partition of the population where only some categories of workers opt for diversification while others opt for the safe asset only. In particular *w* does not affect monotonically $d_R$ and in fact, as we can see from Eq. (11), the sign of $d_R$ is determined by a second order equation in *w*. This allows us to state the following proposition:

**PROPOSITION 2**

There exist two values of wages, $w_1$ and $w_2$, for which $d_R$=0. Then, for any $\gamma > 0$, for wage levels whithin the interval $(w_1, w_2)$ workers choose the risky scheme and for wage levels outside the interval workers choose the safe scheme.

**PROOF**

*We know from Proposition 1 that workers choose the risky scheme if and only if $d_R$>0. From Eq. (11) we see that the sign of $d_R$ is determined by a second order equation in w with negative second order coefficient: therefore there are two values of w for which $d_R$=0 and $d_R$ is positive for values of w that are outside the interval whose boundaries are the roots of Eq. (11).* ■

In particular $w_1$ and $w_2$ take the following values:

$$12) \begin{cases} w_1 = \frac{\bar{r} - r_s - f \cdot x / \gamma - \sqrt{(\bar{r} - r_s - f \cdot x / \gamma)^2 - 2a\sigma^2 Fx}}{a\gamma\sigma^2} \\ w_2 = \frac{\bar{r} - r_s - f \cdot x / \gamma + \sqrt{(\bar{r} - r_s - f \cdot x / \gamma)^2 - 2a\sigma^2 Fx}}{a\gamma\sigma^2} \end{cases}.$$

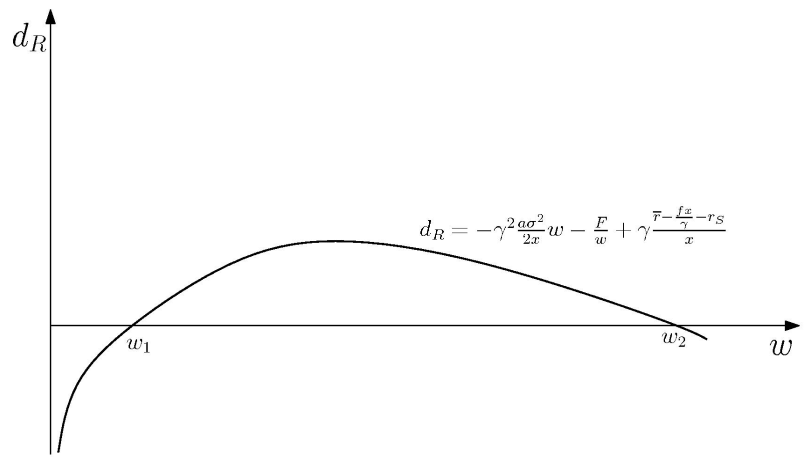
The above equation[6] implies that whenever $\bar{r}-r_s > \sqrt{2a\sigma^2 Fx}$ there exists a minimum rate of contribution ($\gamma^{MIN}$) which is necessary and sufficient for $w_1$ and $w_2$ to exist and be positive:

13) $\gamma^{MIN} = \frac{f \cdot x}{\bar{r}-r_s-\sqrt{2a\sigma^2 Fx}}$ .

On the contrary, when $\bar{r}-r_s \le \sqrt{2a\sigma^2 Fx}$ , the incentive is negative for any level of wages and contribution rate.

A graph can be helpful to understand the role of wages in the choice of workers: in Figure 1 we draw a curve depicting $d_R$ as a function of $w$ for a given contribution rate: when the curve is above zero, $d_R$ is positive and so is the incentive: workers choose the risky scheme; symmetrically, when the curve is below zero workers opt for the safe scheme.

**Figure 1:** The incentive to adhere to the risky scheme in relation to wages




A relevant economic feature of our results is the following: there exists a minimum wage threshold ($w_1$) below which workers do not choose to diversify their portfolio: the reason for this rests on the fact that when wages are low, the size of the investment ($\gamma w$) in the risky scheme is too low to compensate the costs to obtain the necessary degree of financial literacy. As for the second wage threshold ($w_2$) our analysis shows that above such level workers do not invest in the risky scheme either: in this case the reason relies on the fact that if the size of the investment is too high, the variance of expected consumption grows exponentially, producing a lower expected utility for risk averse individuals. We should bear in mind that in reality this model should be applied mostly to employees and, considering the moderate size of $\gamma$ adopted in reality, it is not certain that the higher threshold is ever met, still, at least from analytical point of view, we have to consider it.

To sum up, under a given level of compulsory contribution rate, we observe a partition of population between the two schemes which is determined by the wage levels of individuals: the middle class workers opt for the riskier scheme while the lower income class choose the safe

[6] The values $w_1$ and $w_2$ exist and are positive for: $(\bar{r}-r_s-f\cdot x/\gamma)^2 > 2a\sigma^2 Fx$ and $\bar{r}-r_s-f\cdot x/\gamma > 0$. When the former condition is not met the incentive cannot be positive for any level of wages, when the latter is not met, the incentive cannot be positive for any *positive* level of wages. These conditions have an economic interpretation: they tell us that in order for the risky scheme to be attractive, the difference between the mean returns of the two schemes has to make up for the costs due to the uncertainty of the risky scheme (which are related to *a*, $\gamma$ and $\sigma^2$) and to the entry costs requirements (which are related to *F*, *f* and *x*).

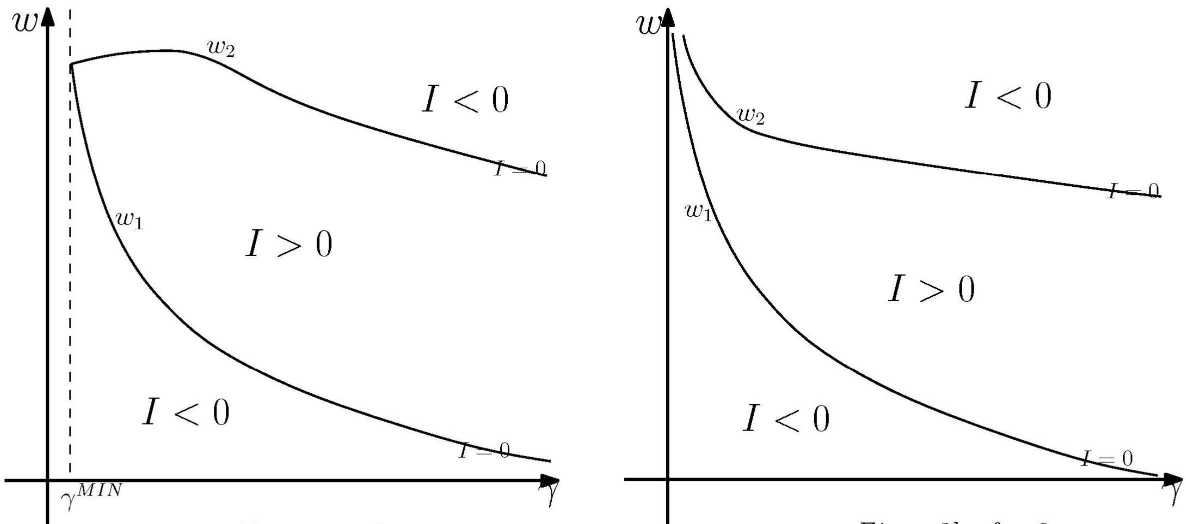
(because of relatively too high financial literacy costs) as well as the higher income class (because of too high volatility of their investment). The exclusion of the low income class from the risky scheme might be problematic from a social point of view as it does not descend strictly from their preferences (as in the case of the high income class) but rather by the relatively too high cost necessary to obtain the necessary degree of financial literacy to access the risky scheme, so that their freedom of choice is somehow constrained by their scarce resources.

## 5 The rate of compulsory contribution and its optimal level

Previous section considered the value of $\gamma$ as fixed at a certain level: here first we analyse how $\gamma$ affect the incentive and then we examine what happens when $\gamma$ can be chosen optimally by a benevolent policy maker.

Preliminarily, it is worth recalling that Eq. (9), combined with Proposition 1, states that $\gamma$ affects the choice of individual only through $d_R$. This said, we can see from Eq. (7) that the effect of $\gamma$ on $d_R$ is not monotonic. Some interesting results are obtained if we consider the effect of $\gamma$ conjunctly with *w*. In particular, starting from Proposition 2 we can see that Eq. (12) defines two curves (described from the equations for $w_1$ and $w_2$) which determine the couples $(w, \gamma)$ for which $d_R$ (and thus *I*) is equal to zero: those curves determine the intervals in terms of *w* and $\gamma$ for which workers adhere to the risky scheme and their shapes differ depending on whether $f>0$ or $f=0$. We depict the curves[7] for the two cases in Figures 2a and 2b.

Figure 2: **Sign of the incentive as function of the contribution rate and wage**




$Figure 3a : f > 0$ $\qquad$ $Figure 3b : f = 0$

For both cases, the area between the two curves determines the values of $\gamma$ and *w* for which workers choose the risky scheme while, on the contrary, outside those curves workers invest in the safe scheme. Basically, in order to induce the adhesion to the risky scheme the value of $\gamma$ should neither be too small nor too large. There is a clear economic interpretation to this result: on the one hand the compulsory rate of contribution cannot be too small, otherwise the resulting investment would be too small to cover the entry costs; on the other hand, a too large $\gamma$ would generate too high volatility in the returns, making the risky scheme unattractive. Note that from Eq. (13) we

[7] Note that the two curves go to zero for $\gamma \to \infty$. In addition, for $f>0$, the two curves cross at $\gamma = \gamma^{MIN}$ and $w_1$ is monotonically decreasing while $w_2$ has a maximum. The formal analysis concerning the shapes of these curves are available upon request to the authors.

know that for *f>0* there exists a lower bound for the rate of contribution ( $\gamma^{MIN}$ in the figure) below which no worker opts for the risky scheme: this is the main difference with the case of *f=0*.

The above considerations lead the way to the fact that could exist a level of compulsory contribution which maximises the value of $d_R$ and is optimal in promoting the adhesion to the risky scheme. In fact in the next proposition we provide formally the existence and the level of such optimal level of $\gamma$:

**PROPOSITION 3**

There exists a value $\gamma^*$ of the compulsory rate of contribution which is optimal in promoting the adhesion to the risky scheme. Such value maximises the value of $d_R$ and is given by $\gamma^* = \frac{\bar{r}-r_s}{2aw\sigma^2}$.

**PROOF**

*The adhesion to the risky scheme occurs whenever I>0 and I is an increasing function of* $d_R$ *and is only affected by* $\gamma$ *through* $d_R$. *Therefore I reaches its maximum with respect to* $\gamma$ *whenever* $\gamma = \gamma^* = \arg\max_{\gamma} d_R$. *Since* $\partial d_R / \partial\gamma = \frac{\bar{r}-r_s-2aw\gamma\sigma^2}{x}$ *it descends that* $\gamma^* = \frac{\bar{r}-r_s}{2aw\sigma^2}$. ■

Note that the value $\gamma^*$ maximises the utility for workers choosing the risky scheme and leaves unaffected those opting for the safe scheme (in fact $d_R$ does not enter Eq. (7c)): therefore it can be considered to all extent optimal for the (expected) utility maximization of all workers.

Several comments to Proposition 3 are worth doing. First of all it has relevant policy implications: in fact it allows us to determine the exact rate that a policy maker should set in the attempt to promote adhesions to the risky scheme. The optimal rate is decreasing in wages and we can assert:

**REMARK 1 TO PROPOSITION 3**

The incentive to adhere to the risky scheme is maximum when the compulsory contribution takes the form of a flat contribution which is the same for all level of wages and that is equal to $\frac{\bar{r}-r_s}{2a\sigma^2}$.

Second, even if setting $\gamma = \gamma^*$ does indeed promote adhesions to the risky scheme, it does not necessarily succeed in inducing individuals to do so: in fact, there are values of the parameters for which $d_R(\gamma^*) < 0$. In other words, while $\gamma^*$ always succeeds in maximising $d_R$ it does not guarantee the incentive *I* to be positive. In particular, it is easy to see from Eq. (9) that $d_R(\gamma^*) > 0$ if and only if

14) $\left[\frac{3}{8}\frac{(\bar{r}-r_s)^2}{a\sigma^2 x} - F\right]\frac{1}{w} > f$.

This condition is strictly necessary for the existence of a positive $\gamma$ that induces the adhesion to the risky scheme and, when this condition is not met, workers necessarily choose the safe scheme. Finally, from the above condition we see that, for *f>0*, workers above a certain wage will necessarily choose the safe scheme so that we can formulate the following remark:

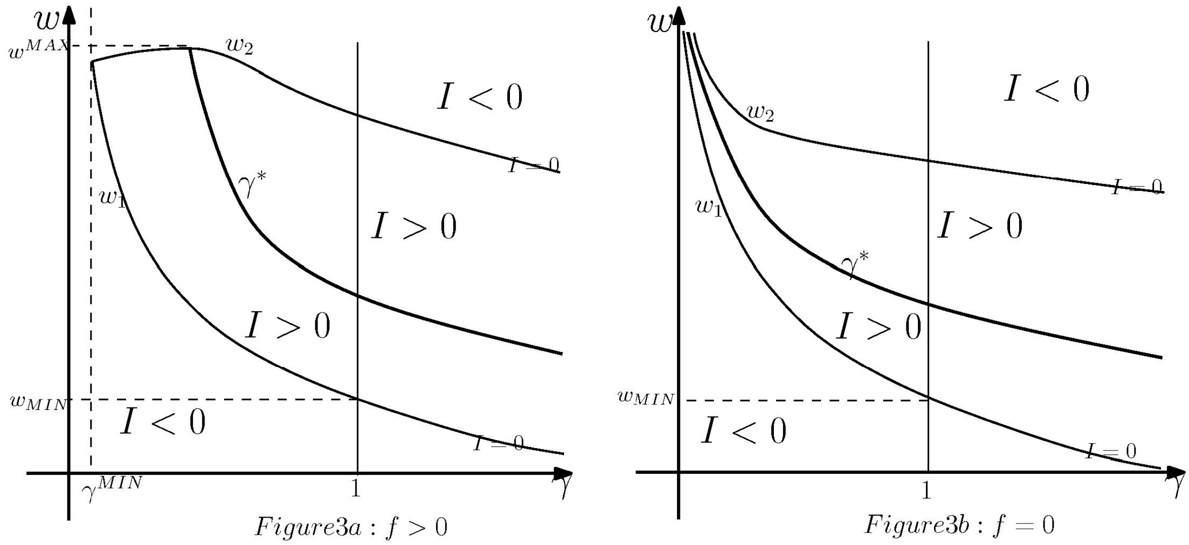
**REMARK *2* TO PROPOSITION 3**

Even when the contribution rate is set at its optimal rate, workers whose wage is above a certain threshold choose the safe scheme. The value $w^{MAX}$ of that threshold is $\left[\frac{3}{8}\frac{(\bar{r}-r_s)^2}{a\sigma^2 x} - F\right]\frac{1}{f}$; for the case *f=0* the value of this threshold goes to infinity so that it is not binding.

Third, we should bear in mind that, for its very nature, the rate of compulsory contribution cannot be higher than 1 and therefore there could be situations for which a policy maker could not actually set $\gamma = \gamma^*$ : in those cases $\gamma = 1$ would be the second best solution to promote diversification though it might not be sufficient to guarantee adhesions.

We can summarize our findings by depicting in a graph the optimal rate of compulsory contribution: in the figures below we represent the curve $\gamma^*$ compared to the curves[8] $w_1$ and $w_2$ and to the constraint $\gamma < 1$.

Figure 2: **The optimal rate of contribution**




From the pictures above, first, we can see that the $\gamma^*$ locus is always above the $w_1$ locus and it is always below $w_2$ only for $f$=0. Second, we observe that below a certain wage ($w_{MIN}$ in the figure) only a $\gamma$ above 1 would induce a worker to choose the risky scheme. This value of wages (which is obtained inserting $\gamma = 1$ in the equation of $w_1$) defines a threshold: workers whose wage is below $w_{MIN}$ cannot be induced to opt for the risky scheme through the adoption of an adequate (feasible) rate of compulsory contribution and other forms of incentives should be implemented. Hence we can provide the following:

**REMARK 3 TO PROPOSITION 3**

For workers below a certain wage, there is no feasible contribution rate able to promote their adhesion to the risky scheme. The value $w_{MIN}$ of that threshold is $\frac{\bar{r}-r_s-f\cdot x-\sqrt{(\bar{r}-r_s-f\cdot x)^2-2a\sigma^2 Fx}}{a\sigma^2}$.

We can note an asymmetry with respect to Remark 2: in fact, the lower threshold for wages *does not* disappear for the case $f$=0.

It is worth to recall that setting the rate of compulsory contribution at its optimal level would induce only positive effects because, as we already said, it improves the expected utility of all workers. However, this is only true if workers are not constrained on their first period consumption, so that the conditions to obtain inner solutions are met: when this is not the case, there might be situations in which setting $\gamma^*$ would produce corner solutions and, therefore, it might actually

[8] Note that the curve $\gamma^*$ is necessarily above curve $w_1$ and, for $f$>0, it meets curve $w_2$ at its vertex. Moreover both curves are drawn under the assumption that $\frac{3}{8}\frac{(\bar{r}-r_s)^2}{2a\sigma^2 x} > F$ because, when this condition is not met, the problem becomes trivial and the incentive is always negative.

reduce utilities: however, the analysis of this case is beyond the scope of the present work (but see Spataro and Corsini (2011) for a preliminary discussion of these aspects).

A final comment is worth doing: to all extent the optimal level of contribution would also be the solution that each individual would choose if he was given the possibility to autonomously set the rate of contribution. Here we do not want to enter the discussion on whether the centralized or decentralized solution is preferable, however, one should consider that there could be some problems associated to the latter as we could observe moral hazard behaviours and a rise in the financial literacy costs related to the decision which would now entails even more complexity.

## 6. Policy implications for the promotion of adhesions to complementary pensions.

Clearly, the aim of the paper is not to discuss further the pros and cons of the current pensions reforms process. However, our model allows us to provide some clear policy implications that can be implemented should the policy maker realize that the results of the reforms and in particular the possibility for individuals to opt for a riskier scheme are driven not only by individuals' preference (i.e. risk aversion) but also by differences in the wage levels and by the lack of financial literacy.

We already discussed in previous section that a government should set the compulsory contribution at its optimal level which, as we saw from Proposition 3 and Remark 1, would imply a flat contribution or, at least, contribution rates that are decreasing in wages. This strategy has the advantage of not directly costing resources to the government but has the partial drawback of failing in promoting the adhesion of workers whose wages are below or above certain thresholds. Those threshold are, respectively:

15a) $$w_{MIN} = \frac{\bar{r} - r_s - f \cdot x - \sqrt{(\bar{r} - r_s - f \cdot x)^2 - 2a\sigma^2 Fx}}{a\sigma^2}$$

15b) $$w^{MAX} = \left[\frac{3}{8}\frac{(\bar{r} - r_s)^2}{a\sigma^2 x} - F\right]\frac{1}{f}$$

These thresholds depend, among other things, on $\bar{r}$ , $F$ and $f$ and in particular $w_{MIN}$ is a negative function of $\bar{r}$ and a positive function of $F$ and $f$ while for $w^{MAX}$ the opposite is true.

If government aims at promoting adhesion to the risky scheme also for workers whose wage falls outside these boundaries it would have to intervene in a more direct manner and aim at decreasing the value of $w_{MIN}$ and at increasing the value of $w_{MAX}$ . We can think of at least three possible ways to reach this: 1) fiscal incentives for workers opting for the risky scheme; 2) a lump sum benefit for workers opting for the risky scheme; 3) information campaigns and/or campaigns aimed at increasing financial literacy. These three actions can be implemented either for all workers or only for those categories that are in the need of incentives to adhere: clearly the latter solution requires fewer resources but might involve some consequences on the fairness of the intervention. Here below we discuss the details and the consequences of the three different actions.

**Fiscal incentives** This form of intervention consists in a reduction on taxes on returns of the risky asset. Therefore, in our model, it is equivalent to an increase in $\bar{r}$ and would produce a decrease of the threshold $w_{MIN}$ and an increase of the threshold $w_{MAX}$ (in fact, from Eqs. (15a) and (15b) we have $\partial w_{MIN} / \partial \bar{r} < 0$ and $\partial w^{MAX} / \partial \bar{r} > 0$ ). A drawback of this policy is that even if brought to the extreme consequences of a complete removal of taxes from the risky scheme, it might not be enough to promote the adhesions of all workers (that is, it might be not enough to make the thresholds disappear).

**Flat benefit** This form of action consists in a flat bonus given to workers that opt for the risky scheme. Given the way we formalized the model this is, to all extent, equivalent to a reduction of $F$. Therefore it would decrease the value $w_{MIN}$ and increase the value of $w^{MAX}$. The main advantage is that if the benefit is large enough (in particular if it is equal to $F$), the lower threshold disappears, so that all low income workers, once the contribution rate is at its optimal level, switch. On the contrary, however, there is no lump sum benefit that is large enough to produce the disappearing of the higher threshold.

**Information Campaign** This form of action consists in the dissemination of detailed information about the working of the risky scheme and, more in general, on the mechanism behind social security and the computation of pension benefits. Also the promotion of courses about basic financial concepts and aimed at increasing the degree of financial literacy falls into this category. According to our model, better information and financial literacy reduce the costs related to accessing the risky scheme so that both $F$ and $f$ decrease. Consequentially, $w_{MIN}$ would decrease and $w^{MAX}$ would increase. The main advantage of this action is that, in principle, it could promote the disappearing of both the lower threshold (if it is able to bring $F$ to zero) and higher threshold (to the extent that it brings $f$ close to zero). However, the exact effects and costs of this action are difficult to assess, as they depend on the effectiveness of spreading information and increasing financial literacy in such manner as to bring down $F$ and $f$. Another partial drawback is that it might be difficult to confine the information campaigns only to workers that are below or above the two thresholds so that in the end a broader audience should be chosen as target, with this possibly leading to higher costs or less effectiveness of the policy.

## 7. Conclusions

In this paper we tackle the issue of retirement saving both from a positive and normative point of view. At the heart of this paper there is the study of the new trend in pension systems which is providing more freedom of choice for workers among different portfolio options. The other side of the coin of such increased freedom to choose is higher individual responsibility and higher request/costs for financial literacy.

Here we analyse the mechanisms behind the retirement saving choice and we build a model able to represent this process and assess its determinants in presence of stochastic returns and financial literacy costs. More precisely, our analysis highlights how the choice is influenced, among other things, by two relevant factors: the level of wages and the rate of the mandatory contribution. In particular we show that at the aggregate level a partition of the population in adhesions to pension schemes, according to the income level, may emerge. As for the contribution rate level, we show that, on one hand, a too large rate of mandatory contribution might be detrimental to the riskier scheme because it would generate a too volatile (and thus unattractive) investment. On the other hand, a too small rate would produce an investment equally small so that the scheme with higher literacy costs would necessarily become unattractive, as the size of the investment would be too small to even repay the costs necessary to access it. These considerations imply that it might be necessary to strike a balance between a too small and too high rate of contribution and, from the perspective of a benevolent authority, there might be an “optimal rate” of contribution: in the paper we prove the existence of the latter and we show that it should imply lump sum contribution to the complementary social security. Our analysis suggests that the choice of individuals may be driven not only by their personal preferences (i.e. risk aversion) but also by their income level and by the lack of financial literacy: since this may reduce the freedom of choice of individuals a benevolent authority might desire to intervene to correct this and we use our model to provide some policy measures to cope with these issues.

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