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Abstract

In a standard model of vertical differentiation, wage is assumed to determine the quality. Wage is also subject to bargaining. Increased bargaining power of the worker in the low quality firm reduces quality differential, and increases price competitiveness. The Opposite happens from a similar change in the high quality firm.

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Introduction

Most wage bargaining models of oligopoly assume quantity competition and homogenous product (Dowrick, 1989, Bughin, 1995, Kraft 1998). This note introduces wage bargaining in a model of vertical differentiation where firms compete in both quality and prices (Gabszewicz and Thisse, 1979; Shaked and Sutton, 1982). Wage, however, is not just a payment to the worker, but an input into the production of quality. We assume that the worker's skill or effort is the crucial determinant of quality, and this effort can be induced through higher wage payment. A recent paper by Gabszewicz and Turrini (2000) has shown that the worker's skill may play a decisive role in determining the product quality.

If higher wage leads to higher quality, the firm producing high quality would like to pay a higher wage than the worker's reservation level, whereas the firm producing the low quality variant would like to pay just the minimum. It is in the common interest of the firms to achieve the maximal differentiation leading to higher prices and profits. However, if the workers bargain over the wage, the resulting quality will be excessive relative to the firm's most preferred levels. But for the competitiveness of the market it is the quality differential, rather than the individual levels of the quality, that matters most. How the bargaining powers of the workers in the two firms affect the competitiveness, is the main interest of this paper.

It turns out that the two wages bear an asymmetric relation that has implications for the equilibrium quality differential and market competitiveness. From the point of view of the firm that chooses to produce high quality, two wages are strategic substitutes, while from the point of view of the other firm they are strategic complements. Two differently sloped wage reaction curves produce asymmetric comparative static results. Here, it may be noted that in standard quantity setting models, wages are generally strategic complements. In our model, an increase in the bargaining power of the worker in the low quality firm, reduces quality differential, increases price competitiveness, and reduces profit for both firms. In contrast, an increase in the bargaining power of the worker in the high quality firm increases quality differential, reduces price competitiveness, and increases profit of the low quality firm. These results may have some implication for the co-ordination problem that is endemic to the models of quality competition.

The basic model

There are two firms indexed 1 and 2 engaged in a game of quality choice followed by price competition for a product that is vertically differentiated. Let the lower quality of the product be denoted as q_L and the associated quantity demanded as X_L. Similarly, the higher quality of the product is denoted as q_H and its associated demand is X_H. A typical consumer's preference is: $U_i = q_i - (1/\gamma)\, p_i$, $i=L,H$ where the price of the i-th quality product is p_i and γ is a parameter that can be interpreted as the inverse of the marginal rate of substitution between income and quality (see Tirole, 1988, p.96).[1] There is a continuum of consumers who are uniformly distributed over an interval ($\underline{\gamma}$, $\bar{\gamma}$). Assume $\bar{\gamma} = 1 + \underline{\gamma}$; $0 < \underline{\gamma} < 1$. As is the case with such models, consumers will be segregated in terms of their purchase of the variety. There is a critical reservation value (γ^C, say) above which all consumers consume the high quality variety and below which consumers consume the low quality variety: $\gamma^C = (p_H - p_L)/(q_H - q_L)$. Further assume that the entire market is served[2].

Having chosen different qualities in the first stage, firms engage in price competition in the second stage. For simplicity, we assume that the cost of production is fixed. It can be

[1] γ is a reservation value for the consumer depending on his income. In particular, γ can be interpreted as the inverse of the marginal rate of substitution between income and quality: $\gamma = 1/U'(I)$ where I is the level of income. We assume U to be concave so that γ rises with the level of income.

Consumers can be thought of being uniformly distributed between income levels ($\underline{I}$, $\bar{I}$). Assuming that the reservation value of the person with the highest ($\bar{I}$) and lowest ($\underline{I}$) incomes are $\bar{\gamma}$ and $\underline{\gamma}$ respectively, the distribution can be stated in terms of the interval ($\underline{\gamma}$, $\bar{\gamma}$).

[2] . This assumption simplifies our demand functions. The demand functions for the two varieties are:

$$X_H = \int_{\gamma}^{\bar{\gamma}} d\gamma = \bar{\gamma} - \frac{p_H - p_L}{q_H - q_L} \qquad X_L = \int_{\underline{\gamma}}^{\gamma^c} d\gamma = \frac{p_H - p_L}{q_1 - q_2} - \underline{\gamma}$$

imagined that one single worker is hired and he is willing to supply variable hours of work according to the firm's demand, as long as a fixed wage is agreed upon. The analogy can be extended to a group of workers and a fixed wage bill, or any other non-labor fixed input as well. The wage is agreed upon in the first stage when the quality is also chosen.

It is straightforward to see that the equilibrium prices are[3]:

$p_H = \frac{1}{3}(2+\underline{\gamma})\Delta q$, $p_L = \frac{1}{3}(1-\underline{\gamma})\Delta q$ resulting in profits $\pi_H = m\Delta q - W_H$, $\pi_L = n\Delta q - W_L$ where $m = \frac{1}{9}(2+\underline{\gamma})^2$, $n = \frac{1}{9}(1-\underline{\gamma})^2$ and $\Delta q = q_H - q_L$..

Note that both the firms would prefer maximal differentiation as that would increase prices and profits. To what extent the differentiation can be enlarged will depend on the costs of choosing quality. Here we may be reminded that this game has generically two asymmetric Nash equilibria with firms implicitly coordinating on their asymmetric quality choice. If they end up with identical quality, their profits will be zero, a result of co-ordination failure.

Now moving on to the first stage, we propose that the wage paid to the worker matters for the quality produced. The product quality is a direct result of worker's superior efforts that can be induced through higher wages. This is a sort of efficiency wage. That worker's skill can be a crucial determinant of quality and that the link between the wage paid to the worker and the quality achieved, have been emphasized in a recent paper by Gabszewicz and Turrini (2000). While they have explicitly modeled the link between skill, tasks and quality, we take a reduced form (monotonic) relation between wage and the quality.

Let $(q_L, q_H) \in [\underline{q}, \bar{q}]$, and $q = q_L$, if $W \leq W^0$, $q = q_H$ if $W \geq W^1$, and $q = f(w)$, if $W^0 < W < W^1$, $f'(.) > 0$, $f''(.) < 0$. The technology of producing quality is same for both the firms. It is only the different levels of the wage that will give rise to different qualities. In reality, however, firm's choice of capital and R&D may also matter along with the workers' efforts. But to highlight the role of the worker's effort we make it the sole determinant of quality.

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[3]See, for example Motta(1993) for a full characterization of this stage.

Further, we assume that the worker bargains over the wage. The bargaining process is captured through Nash bargaining.

Wage bargaining

Suppose the worker in firm *i* (*i*=1,2) wants to maximize his net income $(W_i - \underline{W}_i)$ where $\underline{W}_i$ is his reservation wage. Let β_i be the bargaining power of the worker in firm *i*, and $(1-\beta_i)$ be the bargaining power of firm i, vis-à-vis its worker $(0 \le \beta_i \le 1)$ Each firm-worker pair bargains independently and simultaneously in the first stage of the game. The differences in the bargaining power and reservation wages of the two workers can be due to different institutional environments within which the firms operate.

Assuming firms' reservation profits to be zero, the bargaining problem in firm *i* is written as

$$\text{Max} \quad Z_i = \pi_i^{(1-\beta_i)} \left(W_i - \underline{W}_i \right)^{\beta_i} \tag{1}$$

At this stage we need to specify which firm is expected to produce the high quality variety and which firm the low quality variety. Suppose firm 1 will choose a quality level higher than that of firm 2. Both pairs will look forward to the second stage outcome and choose wages from the following equations given by the first order conditions of maximization:

$$\frac{\delta Z_i}{\delta W_i} = \left[(1-\beta_i) \left(W_i - \underline{W}_i \right) \frac{\delta \pi_i}{\delta W_i} + \beta_i \pi_i \right] = 0, \qquad i=1,2 \tag{2}$$

The second order condition for the high quality firm (firm 1) is easily satisfied by the concave *f(W)* function. But for the low quality firm we need to make an additional assumption. To ensure that it is satisfied at all β_1 and β_2, we impose a restriction on the *f(W)* function[4]:

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[4] It is required that $Z_1^{//}(W_1) < 0$ and $Z_2^{//}(W_2) < 0$ at least locally for optimal W_1 and W_2. $Z_1^{''}(W_1) = \frac{\delta \pi_1}{\delta W_1} + (1-\beta_1)\left(W_1 - \underline{W}_1 \right) \frac{\delta^2 \pi_1}{\delta W_1^2}$. It can be seen that $\frac{\delta \pi_1}{\delta W_1} < 0 \ \left(for \beta_1 > 0 \right)$ at the optimal W_1 and $\frac{\delta^2 \pi_1}{\delta W_1^2} = mf^{//}(W_1) < 0$. Thus given the concavity of the $f(.)$ function the second

Assumption 1: $f(W)$ is such that $-Wf''(W)/f'(W) < 1$.

In equation (2) we note that if $\beta_i=0$, firm 1 would choose ω^* that solves $\delta\pi_1/\delta W_1 = mf'(W_1)-1=0$, and firm 2 will set $W_2=\underline{W}_2$, as $\delta\pi_2/\delta W_2=-(nf'(W_2)+1) <0$. Thus, when the worker does not have any bargaining power, firms set wages independent of each other, and (ω^*,$\underline{W}_2$) gives the most profitable quality differentiation via the function $f(W)$.

But we are interested in the case of $\beta_i>0$. Two things become important: (I) the relative payoffs between the firm and the worker and (ii) the wage reaction curves.

The relative payoffs: Equation (2) shows how W_i is to be set to maintain a balance between the firm's and the worker's payoff, taking the wage in the other firm as given. We may rewrite (2) as

$$\pi_i/(W_i-\underline{W}_i) = \{(1-\beta)/\beta\}[-\delta\pi_i/\delta W_i], \qquad i=1,2 \qquad (2')$$

The ratio of the profit to net wages must be monotonically related to the ratio of the players' bargaining powers $\{(1-\beta)/\beta\}$. This is a property of the Nash bargaining solution. In the

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order condition is easily satisfied for firm 1. Similarly, for firm 2 ,

$Z''(W_2) = -(nf'(W_2)+1) - n(1-\beta_2)\left(W_2 - \underline{W}_2\right)f''(W_2)$. This is negative if

$$-\frac{1}{f'(W_2)}(1-\beta_2)(W_2-\underline{W}_2)f''(W_2) < 1 + \frac{1}{nf'(W_2)}$$

The right hand side is bounded below by 1. The left hand side varies from zero $(when \beta_2 = 1)$ to $\left(W_2 - \underline{W}_2\right)\dfrac{f''(W_2)}{f'(W_2)}$ $(when \beta_2 = 1)$. Let us write the highest value of the LHS expression as,

$$\left(\frac{W_2-\underline{W}_2}{W_2}\right)\cdot\left(\frac{W_2 . f''(W_2)}{f'(W_2)}\right)$$

Which is a product of two terms. The first term is always a fraction. The second term is the elasticity of the 'marginal quality' function. We simply assume that this elasticity is less than unity. With this assumption the LHS will always be less than the RHS.

standard bargaining models where wage is a transfer to the worker, $[-\delta\pi_i/\delta W_i]=1$, and the ratio of the profit to wages will be exactly equal to the $\{(1-\beta)/\beta\}$. However, in the present context where wage has efficiency effects, $[-\delta\pi_i/\delta W_i]\neq 1$, and this is a source of bias in the ratio of the payoffs. For firm 1 (producing high quality), $[-\delta\pi_1/\delta W_1]=1-mf'(W_1)$ is positive but strictly less than 1. This can be seen from (2). Unless $\delta\pi_1/\delta W_1=mf'(W_1)-1<0$, equation (2) will not be satisfied. On the other hand, for firm 2, $[-\delta\pi_2/\delta W_2]=nf'(w_2)+1>1$. If we call $[-\delta\pi_i/\delta W_i]$ as a bias factor, then it is less than 1 for firm 1, and greater than 1 for firm 2. This means that the rent allocation in firm 1 is biased in favor of the worker and in firm 2 is biased in favor of the firm.

Proposition 1: The profit wage ratio in firm 1 (producing high quality) is less than proportional, and in firm 2 (producing low quality) is more than proportional to the ratio of the bargaining powers of the firm and the worker.

Wage reaction curves: Another important aspect of the bargaining solution is that W_1 is no longer independent of W_2. They will be determined from the two wage reaction functions given by (2). The wage reaction curve of firm 1 when it produces high quality, denoted as $\Phi_H^1(W_2,\beta_1)$, implicitly solves

$$Z_1^{/}(W_1;W_2,\beta_1)=(1-\beta_1)\left(W_1-\underline{W}_1\right)(mf'(W_1)-1)+\beta_1(mf(W_1)-mf(W_2)-W_1)=0$$

The wage reaction curve of firm 2 when it produces low quality, denoted as $\Phi_L^2(W_1,\beta_2)$ implicitly solves,

$$Z_2^{/}(W_2;W_1;\beta_2)=-(1-\beta_2)\left(W_2-\underline{W}_2\right)(nf'(W_2)+1)+\beta_2(nf(W_1)-nf(W_2)-W_2)=0$$

In other words,

$$Z_1^{/}(\Phi_H^1(.);W_2;\beta_1)=0 \qquad (3)$$

$$Z_2^{/}(\Phi_L^2(.);W_1,\beta_2)=0 \qquad (4)$$

from which we obtain:

$$\frac{\delta\Phi_H^{/}}{\delta W_2}=-\frac{1}{Z_1^{//}}\cdot\frac{\delta Z_1^{/}}{\delta W_2}=\frac{\beta_1 mf'(W_2)}{Z_1^{//}}\quad <0 \qquad \text{as } Z_1^{//}(\cdot)<0$$

$$\frac{\delta\Phi_L^2}{\delta W_1} = -\frac{1}{Z_2^{//}} \cdot \frac{\delta Z_2^{/}}{\delta W_1} = -\frac{\beta_2 n f^{/}(W_1)}{Z_2^{//}} > 0 \qquad \text{as } Z_2^{//} < 0.$$

The wage reaction curve of firm 1 (high quality) is downward sloping, while the wage reaction curve of firm 2 (low quality) is upward sloping. This is true if $\beta_1, \beta_2 > 0$, as is evident from the above expression. Thus, from the point of view of the high quality firm wages are strategic substitutes, while from the point of view of the low quality firm they are strategic complements. In models of quantity competition wages are typically strategic complements and that aspect is retained for the low quality firm because its primary aim is to choose the lowest quality and hence to lower the wage. For the high quality firm, the interest is exactly opposite. It wants to offer a higher wage and raise the quality.

The intersection of the two reaction curves gives the equilibrium wages and qualities. However, a mild condition is needed to ensure that equilibrium always exists for all $\beta_1, \beta_2 > 0$. Relegating the discussion of it in the footnote, we move to the comparative static analysis[5] [6].

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5. Note that the domain of $\Phi_H^1(W_2;\beta_1)$ is $\left[\underset{-}{W}_2, \hat{W}_2\right]$ for all β_1 where $\hat{W}_2$ is implicitly defined by $\pi_1\left(\omega^*, \hat{W}_2\right) = 0$. Then $\hat{W}_2$ is the highest level of W_2 that firm 1 needs to consider. $\hat{W}_2$ is also the intercept of the Φ_H^1 curve, when $W_1=\omega^*$, the lowest wage firm 1 will offer. Since Φ_H^1 is downward sloping and Φ_L^2 is upward sloping, we need to assume that the uppermost Φ_L^2 curve must start from a W_2 strictly less than $\hat{W}_2$. As $\frac{\delta\Phi_L^2}{\delta\beta_2} > 0$, the uppermost Φ_L^1 curve must correspond to $\beta_2 = 1$ in which case $\pi_2 = 0$. When $W_1=\omega^*$, the uppermost $\Phi_L^2(W_1)$ curve has an intercept W_2^+. If $W_2^+ = \Phi_L^1(\omega^*, \beta_2 = 1)$ dose not exceed $\hat{W}_2$, then for all $\beta_1, \beta_2 \in [0,1]$, the reaction curves will intersect and we have equilibrium.

Proposition 2: (a) With an increase in β_1, both W_1 and W_2 increase, and the quality differential (q_1-q_2) also rises leading to higher prices. On the other hand, with an increase in β_2, W_2 increases but W_1 falls, and therefore the quality differential is reduced and prices fall. (b) With an increase in β_2, π_1 and π_2 both fall, whereas with an increase in β_1, only π_1 will fall, but π_2 will increase.

Proof: See Appendix.

An improvement in the bargaining strength of the low quality firm (firm 2) will reduce the equilibrium wage in the high quality firm, but increase its own wage. This directly reduces the quality differential, increases price competitiveness and reduces the profit of the high quality firm, despite its first order fall in its wage. The asymmetric wage effects are entirely due to the strategic substituteness perceived by the high quality firm. On the other hand, an improvement in the bargaining strength of the worker in the high quality firm will have familiar symmetric effects on the wages as seen in models of strategic complementary wages. Although wages rise in both firms, the net result will be a further improvement in the quality differential, and less competition. But only the low quality firm will benefit from the increased differential, and the high quality firm will lose.

We illustrate our results in Figure 1. In the lower half of the quadrant (where $W_1 > W_2$) we depict the equilibrium where firm 1 produces high quality and firm 2 produces low quality. The upper half (where $W_2 > W_1$) shows the other equilibrium where firms switch their places. To make our presentation a bit more concrete, take a particular form of the quality function: $q_i = 2b_i\sqrt{W_i}$.

The wage reaction curve of firm 1 (when it produces high quality) is given by R^1_H and that of firm 2 by R^2_L. R^1_H starts from a height of $W_2^{\wedge}$ and ends at a point in between ω^* and $W_1^{\wedge}$. ω^*

[6] Needless to say that there is another equilibrium where firm 2 produces the high quality good and firm 1 is the producer of low quality, keeping the same workers in their respective firms with unchanged bargaining powers. The characterization of the equilibrium is analogous and therefore, can be skipped.

is the firm's most preferred wage and $W_1^{\wedge}$ is the worker's most preferred wage. For $q_i = 2b_i\sqrt{W_i}$ it is straight forward to see that ω* = m^2b^2, $W_2^{\wedge}$= ω*/4 and $W_1^{\wedge}$=ω*[1+√{1−2√($\underline{W}_2$/ω*)}]^2.[7]

In the case of both the worker and firm having positive bargaining powers the reaction curve must be a weighted average of the two extreme relations. This is precisely shown by R^1_H. It starts from the same height as $W_2^{\wedge}$, but ends at a W1 greater than ω* but less than W1^.

The reaction curve of firm 2 is given by the upward sloping curve R^2_L. The uppermost R^2_L starts from a point like W_2^+. In our example, W_2^+= ω*(n/m)^2[-1+√{1+(2m/n)}]^2 .[8] It can be shown that $W_2^+ \leq W_1^{\wedge}$.[9] Hence the reaction curves always cross each other. With this alignment of the reaction curves, the comparative static results on wages become obvious.

Conclusion

Finally, to conclude we note that even though the firms are symmetric, they may face workers who may differ in terms of bargaining power. Then the two asymmetric equilibria will not be mirror images of each other. This gives rise to the possibility that one of them may Pareto dominate the other. If so, then the co-ordination problem may become less severe. Suppose firm 1 faces an extremely strong worker (β_1 close to 1), and firm 2 has a weak worker (β_2 close to zero). Now with this configuration if firm 1 produces high quality and firm 2 low quality, the joint profit is likely to be much higher than if they did otherwise. Firm 1 will always get very little profit, but firm 2's profit will be substantially higher in the

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[7] $W_1^{\wedge}$ solves the equation $\pi_1(W_1,W_2,\beta_1=1)=0$, or $k_1 2b_1\sqrt{W_1} - 2b_2k_1\sqrt{W_2} - W_1 = 0$. For any given W_2,

$W_1^{\wedge}(W_2)$= ω*[1+√{1−2√(W_2/ω*)}]^2

[8] W_2^+ solves the equation $\pi_2(W_1,W_2,\beta_2=1)=0$, or 2nb(√$W_1$-√$W_2$)-$W_2$=0. For any given W_1, $W_2^+(W_1)$ = ω*(n/m)^2[-1+√{1+2(√W_1)/(nb)}]^2.

[9] By comparing the two expressions we see that the required inequality is satisfied if $4n \leq m$, or $4(1-\gamma)^2 \leq (2+\gamma)^2$. The right hand side is always increasing in γ and the left hand side is decreasing in γ. At $\gamma=0$, the RHS is minimum and equal to the maximum value of the LHS. Since $\gamma \in [0,1)$, we can say that our condition is satisfied for the entire range of γ.

first equilibrium. The second equilibrium is worse for both. If the first equilibrium Pareto dominates the second one, then firms may be able to co-ordinate on it. In other words, if a firm has a stronger union, it is better suited to be the producer of the high quality variety-- a hypothesis that can be empirically examined.

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Appendix

Proof of Proposition 2:

a) Suppose there exists a pair W_1^*, W_2^* such that (3) and (4) are satisfied. That is

$$Z_1^{/}\left(W_1(\beta_1,\beta_2),W_2(\beta_1,\beta_2);\beta_1\right)=0$$

$$Z_2^{/}\left(W_2(\beta_1,\beta_2);W_1(\beta,\beta_{21});\beta_2\right)=0$$

By varying β_1, ceteris paribus, we obtain

$$Z_1^{//}\frac{\delta W_1^*}{\delta\beta_1}+\frac{\delta Z_1^{/}}{\delta W_2}\cdot\frac{\delta W_2^*}{\delta\beta_1}+\frac{\delta Z_1^{/}}{\delta\beta_1}=0$$

$$\frac{\delta Z_2^{/}}{\delta W_1}\cdot\frac{\delta W_1^*}{\delta \beta_1}+Z_2^{//}\frac{\delta W_2}{\delta \beta_1}=0$$

Applying the Cramer's rule it can be shown that

$$\frac{\delta W_1^*}{\delta \beta_1}=\frac{-Z_2^{//}\frac{\delta Z_1^{/}}{\delta \beta_1}}{D} \qquad \frac{\delta W_2^*}{\delta \beta_1}=\frac{\frac{\delta Z_2^{/}}{\delta W_1}\cdot\frac{\delta Z_1^{/}}{\delta \beta_1}}{D}$$

where D= $Z_1^{//}.Z_2^{//}-\frac{\delta Z_1^{/}}{\delta W_2}.\frac{\delta Z_2^{/}}{\delta W_1}>0$ for stability. Note that

$$\frac{\delta Z_1^{/}}{\delta \beta_1}=\pi_1-\left(W_1-\underset{-}{W_1}\right)\left(mf^{/}(W_1)-1\right)>0 \text{ as } mf^{/}(W_1)-1<0 \text{ and } \frac{\delta Z_2^{/}}{\delta W_1}=\beta_2 nf^{/}(W_1)>0.$$

Thus we ascertain $\frac{\delta W_1^*}{\delta \beta_1}>0$, $\frac{\delta W_2^*}{\delta \beta_1}>0$.

Next consider a ceteris paribus variation in β_2. Proceeding as before we derive

$$\frac{\delta W_1^*}{\delta \beta_2}=\frac{\frac{\delta Z_2^{'}}{\delta \beta_2}\frac{\delta Z_1^{/}}{\delta W_2}}{D} \qquad \frac{\delta W_2^*}{\delta \beta_2}=\frac{-Z_1^{//}\frac{\delta Z_1^{/}}{\delta \beta_2}}{D} \qquad \text{(D>0)}$$

Note, $\frac{\delta Z_2^{/}}{\delta \beta_2}=\left(W_2-\underset{-}{W}\right)\left(nf^{/}(W_2)+1\right)+\pi_2>0$

$\frac{\delta Z_1^{/}}{\delta W_2}=-\beta_1 mf^{/}(W_2)<0$ and therefore $\frac{\delta W_1^*}{\delta \beta_2}<0$, whereas $\frac{\delta W_2^*}{\delta \beta_2}>0$ as $Z_2^{//}<0$

Next consider the quality differential: $q_H-q_L=f(W_1)-f(W_2)$

It is straightforward to see, $\frac{\delta(q_H-q_L)}{\delta \beta_2}=f^{/}(W_1^*)\frac{\delta W_1^*}{\delta \beta_2}-f^{/}(W_2^*)\frac{\delta W_2^*}{\delta \beta_2}<0$ unambiguously.

But the sign of $\frac{\delta(q_H-q_L)}{\delta \beta_1}=f^{/}(W_1^*)\frac{\delta W_1^*}{\delta \beta_1}-f^{/}(W_2^*)\frac{\delta W_2^*}{\delta \beta_1}$ is not obvious.

However, when we substitute the relevant expressions of $\frac{\delta W_1^*}{\delta \beta_1}$ and $\frac{\delta W_2^*}{\delta \beta_2}$, we observe that

$$\frac{\delta(q_H-q_L)}{\delta \beta_1}=\frac{\delta Z_1^{/}}{\delta \beta_1}.\quad\frac{f^{/}(W_1^*)}{D}\quad\left[-Z_2^{//}-f^{/}(W_2^*)\beta_2 n\right]$$

The sign of the bracketed term determines the sign of $\frac{\delta(q_H - q_L)}{\delta\beta_1}$, as the terms outside the bracket are all positive. Further expanding the bracketed term, we observe

$$-Z_2^{//} - f^{/}(W_2)\beta_2 n = nf^{/}(W_2)(1-\beta_2)\left[1 + \frac{(W_2 - \underset{-}{W})}{W_2} \cdot \frac{W_2 f^{//}(W_2)}{f^{/}(W_2)}\right] + 1$$

By Assumption 1, the bracketed term is positive. Hence, $\frac{\delta(q_H - q_L)}{\delta\beta_1} > 0$.

The effects on the price are obvious from the fact that prices will rise (or fall) with an increase (or decrease) in the quality differential.

b) The comparative static effects of β_i on π_i are straightforward.

$$\frac{\delta\pi_2}{\delta\beta_2} = n\frac{\delta(q_1 - q_2)}{\delta\beta_2} - \frac{\delta W_2}{\delta\beta_2} < 0 \text{ since } \frac{\delta(q_1 - q_2)}{\delta\beta_2} < 0 \text{ and } \frac{\delta W_2}{\delta\beta_2} > 0$$

Similarly, $\frac{\delta\pi_1}{\delta\beta_1} = \left([mf^{/}(W_1) - 1]\frac{\delta W_1}{\delta\beta_1}\right) - mf^{/}(W_2)\frac{\delta W_2}{\delta\beta_1} < 0$ because $mf^{/}(W_1) - 1 < 0$ and $\frac{\delta W_1}{\delta\beta_1}$ and $\frac{\delta W_2}{\delta\beta_1}$ are both positive.

Next consider $\frac{\delta\pi_1}{\delta\beta_2} = (mf^{/}(W_1) - 1)\frac{\delta W_1}{\delta\beta_2} - mf^{/}(W_2)\frac{\delta W_2}{\delta\beta_2}$.

In this case the first term is positive as $\frac{\delta W_1}{\delta\beta_2} < 0$, while the second term is negative.

However, after substituting the relevant expressions, the above relation becomes

$$\frac{\delta\pi_1}{\delta\beta_2} = (1-\beta_1)\cdot\frac{\delta Z_2^{/}}{\delta\beta_2}\cdot\frac{mf^{/}(W_2)}{D}\left[(mf^{/}(W_1) - 1) + \left(W_1 - \underset{-}{W_1}\right)mf^{//}(W_1)\right]$$

The terms outside the bracket are all positive, but the ones inside the bracket are negative. Hence $\frac{\delta\pi_1}{\delta\beta_2} < 0$

Now consider $\frac{\delta\pi_2}{\delta\beta_1} = nf^{/}(W_1)\frac{\delta W_1}{\delta\beta_1} - (nf^{/}(W_2) + 1)\frac{\delta W_2}{\delta\beta_1}$

Which also appears to be ambiguous. Again we substitute the relevant expressions, and arrive at $\dfrac{\delta\pi_2}{\delta\beta_1} = (1-\beta_2)\dfrac{\delta Z_1^{/}}{\delta\beta_1} \cdot \dfrac{nf^{/}(W_1)}{D}\left[\left\{\left(\dfrac{f^{//}(W_2)}{f^{/}(W_2)} \cdot W_2\right) \cdot \dfrac{\left(W_2 - W_{\underline{2}}\right)}{W_2} + 1\right\} nf^{/}(W_2) + 1\right]$

Note that the terms outside the bracket are positive. Inside the bracket, the first term is a sum of a negative term and a positive term. However, by our Assumption 1 (or $Z_2^{//} < 0$ for all β_2) it can be ascertained that the positive term will dominate, and the entire expression will be positive. Thus, $\dfrac{\delta\pi_2}{\delta\beta_1} > 0$.