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# Heterogeneous Expectations, Taylor Rules and the Merit of Monetary Policy Inertia<sup>☆,☆☆</sup>

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## Abstract

We present new results for the performance of Taylor rules in a New Keynesian model with heterogeneous expectations. Agents have either rational or adaptive expectations. We find that depending on the particular rule, expectational heterogeneity can create or increase the set of policies that leads to local explosiveness. This is a new level of destabilization compared to what is known. In addition, we demonstrate that policy inertia is an effective tool to safeguard the economy against local explosiveness. Thus, we provide a rationalization for central banks to adjust interest rates with notable inertia in response to shocks.

**JEL Classification:** E52, D84

*Keywords:* Monetary Policy, Taylor Rules, Heterogeneous Expectations,

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<sup>☆☆</sup>This paper is an excerpt of my manuscript “Heterogeneous Expectations and the Merit of Monetary Policy Inertia”.

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## 1. Motivation

Nowadays, central banks in the industrialized economies typically have a mandate to ensure price stability and in most countries to stabilize economic output. Their preferred policy instrument in many cases is the nominal interest rate. In the theoretical monetary literature it is often recommended that monetary policy should be rule-based. Therefore, monetary policy rules still appear to be a popular subject to study.

Advocates of rule-based monetary policy such as Clarida et al. (1999), Woodford (2003) and Galí (2008) among others provide theoretical justifications for the use of rules in the conduct of monetary policy. The core argument is that such rules may provide a nominal anchor for the economy, meaning that the central bank can control nominal variables such as inflation in a way that is beneficial for individual welfare. Controlling nominal variables is a desideratum of any monetary policy rule. The reason is that the common transversality conditions in macroeconomic models solely rule out explosions of real variables, but not of nominal variables. Recently Cochrane (2007) has reemphasized this issue.<sup>1</sup>

Our analysis focuses on simple monetary policy rules, also known as Taylor (1993)-type rules. A key characteristic of these rules is that the policy instrument of the central bank is a linear function of (expected) inflation and (expected) output gap. The monetary policy coefficients, which premultiply these two variables of interest, express the magnitude of response to deviations in the two variables from a certain policy target. In addition, such rules offer the advantage that

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<sup>1</sup>Note that the main point of Cochrane (2007) is a serious criticism of the theories that make the case for rule-based monetary policies in general. He has initialized a vivid debate on the benefit of conducting monetary policy by the help of rules in forward-looking economies that has been joined by McCallum (2009b). This debate is still in progress and is not the focus of this study.

a central bank can obviously relate its mandate to its policy instrument, which increases policy transparency.

Numerous variants of rules have been proposed and their dynamic properties have been assessed. Thus, it is quite astonishing that these assessments are commonly conducted under the assumption of homogeneous expectations of agents. Usually these studies embed the rules into a New Keynesian (NK) model, where it is routinely assumed that agents have homogeneous rational expectations (RE). Then authors ask, whether a specific rule can yield local determinacy, i.e. there exists a unique stationary rational expectations equilibrium (REE).<sup>2</sup> In addition, authors often conduct a robustness-check and assume that agents may not have RE but learn adaptively and ask whether the REE is expectational stable.<sup>3</sup>

A widely-cited analysis of Taylor-type rules is Bullard and Mitra (2002), who examine the rules with regard to determinacy and E-stability.<sup>4</sup> They find that the Taylor-type rules are relatively good tools to enforce determinacy and E-stability for a large fraction of the considered monetary policy parameter space. Most important, they find that a rule featuring contemporaneous expectations instead of current values yields the same results and is highly desirable. The latter due to the fact that a rule with contemporaneous expectations requires the central bank to have less information about actual economic conditions and therefore this rule is highly operational.<sup>5</sup>

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<sup>2</sup>Determinacy most importantly rules out undesirable evolutions of endogenous variables such as large fluctuations, see for example Woodford (1999, p.69).

<sup>3</sup>This approach is rigorously discussed in Evans and Honkapohja (2001). In this scenario, it is assumed that the homogeneous agents act as econometricians and forecast the future development of prices and other endogenous variables.

<sup>4</sup>When an equilibrium is denoted expectational stable it is also often denoted learnable or it is said to have the property of E-stability. These concepts are all closely related.

<sup>5</sup>Expectations in a monetary policy rule can be thought of as the central bank's forecast of a variable. It is obviously easier to use a forecast of a contemporaneous

Overall, the results of Bullard and Mitra (2002) suggest that responding more than one-for-one to inflation, i.e. sticking to the Taylor-principle<sup>6</sup>, and responding modestly to output gap deviations is a rather good policy independent of the particular rule. In the related analysis Bullard and Mitra (2007), the rules have the additional feature of policy inertia.<sup>7</sup> It turns out that policy inertia can make determinacy even more likely and in turn reduce the threats of local indeterminacy or explosiveness.<sup>8</sup>

A potential shortcoming of the aforementioned analyses is the fact that all assume homogeneity of agents in the economy, despite the fact that heterogeneity is a universal feature in reality. Heterogeneity, if captured by structural parameters, can have an impact on the dynamics of an economy and affect the dynamic properties of rules. We focus on heterogeneity of expectations in the economy. Agents form either RE or adaptive expectations. In particular, we focus on heterogeneous expectations in a NK model as elaborated in Branch and McGough (2009). We examine the consequences for local stability when the central bank conducts monetary policy by several simple rules. Thus, the analysis herein may be viewed as a kind of robustness-check for the numerical results of Bullard and Mitra (2002, 2007) mentioned above.

It is important to acknowledge that we are not the first to conduct that kind

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aggregate variable than to correctly observe it, as mentioned by Bullard and Mitra (2002, p.1112) and emphasized by McCallum (1999).

<sup>6</sup>Taylor (1993) suggests such a simple interest rate rule and assumes an inflation coefficient of 1.5, i.e. if inflation deviates from its target level, then the central bank should react with the nominal interest rate more than one-for-one, in this case one-and-a-half-for-one. In Taylor (1999) he denotes this suggestion from 1993 (with regard to the functional form) a "normative recommendation". In Taylor (1999) he explicitly advocates an inflation coefficient larger than one in such a policy rule. This policy stance towards inflation is denoted the "Taylor-principle" in the literature.

<sup>7</sup>Policy inertia denotes the modern central banks' practice to alter their policy instrument with remarkable inertia in response to economic shocks.

<sup>8</sup>Other noteworthy studies in the tradition of Bullard and Mitra (2002, 2007) are Preston (2005) and Duffy and Xiao (2009).

of analysis. Branch and McGough (2009, p.11ff.) analyze a forward-looking rule. They find that the presence of agents with *purely adaptive* expectations next to fully rational agents turns policies, which used to yield indeterminacy in the case of RE, into policies that yield determinacy (“Result 3”). Furthermore, the opposite is true if the non-rational agents have *extrapolative* expectations (“Result 4”).<sup>9</sup> In consequence, they conclude that purely adaptive expectations may have a stabilizing effect, whereas extrapolative expectations may have a destabilizing effect.<sup>10</sup> Please be aware that Branch and McGough (2009, p.10) themselves claim that they considered other rules: “... we also checked for robustness when monetary policy adopts rules that depend on lagged and contemporaneous data. The qualitative results presented below are robust to the particular form of the policy rule”. Unfortunately, no further reference is made to those alternative rules therein.

Overall, we think that a more detailed study of alternative Taylor-type rules in an economy with heterogeneous expectations is necessary and interesting, especially when one slightly increases the level of heterogeneity compared to Branch and McGough (2009, p.11ff.). Thus, we analyze the rules considered in Bullard and Mitra (2002, 2007). Our results confirm their results for some monetary policy rules, but not for all.

In fact, the rule featuring contemporaneous expectations remains the most desirable policy specification. There are three reasons for that. First, it does not

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<sup>9</sup>Be aware that in our context non-rational expectations are always adaptive in the sense that agents use past observations of an endogenous variable to forecast its future value. We distinguish *purely adaptive* and *extrapolative* expectations to make clear that the weight on the past observations is smaller than one in the former case and larger than one in the latter case.

<sup>10</sup>We suggest to stick to a different wording with regard to stability. More precisely, we suggest to stick to the mathematical perspective, where local explosiveness means instability, local determinacy means stability and local indeterminacy means too much stability and opens the door to extrinsic uncertainty.

require to measure current period aggregate variables and therefore is operational, as is a well-known. Second, given that the central bank sticks to the Taylor-principle and moderately feeds back to contemporaneous expectations about the output gap, such a rule renders the economy determinate for the whole parameter space under consideration. Finally, this result holds, no matter if the central bank is actually aware of the heterogeneity of expectations in the economy. In this sense, we shed new light on the question on how important it is, that the central bank is aware of the expectational heterogeneity when it makes its interest rate decisions based on forecasts.

Moreover, we detect new regions of local explosiveness. In consequence, purely adaptive expectations do not yield larger regions of determinacy in general, whereas extrapolative expectations yield larger regions of indeterminacy in general. This finding is at odds with the results of Branch and McGough (2009, p.11ff.). Strikingly, policy inertia increases the regions of determinacy remarkably. This confirms the results of Bullard and Mitra (2007). Thus, policy inertia remains a highly desirable ingredient of a simple monetary policy rule even in the case of expectational heterogeneity. This insight illustrates the merit of policy inertia.

The remainder of the paper is organized as follows. In Section 2 we briefly describe the economic model that is the subject of our study. We also explain how we numerically analyze the dynamic properties of rules and make some comments on our calibration. Section 3 contains the basic analysis of the dynamic properties of four simple monetary policy rules without and with policy inertia in a NK model with heterogeneous expectations. Finally, Section 4 concludes and points out directions for further research.

## 2. The Approach of the Analysis

Our analysis comprises the economic environment, the methodology of numerical analysis and the choice of calibration.

### 2.1. The Economic Environment

Building on Branch and McGough (2009), we consider a heterogeneous expectations reduced form NK economy. Within the non-policy block, the dynamic IS curve is given by

$$x_t = \widehat{E}_t\{x_{t+1}\} - \sigma^{-1} \left( i_t - \widehat{E}_t\{\pi_{t+1}\} \right) \quad (1)$$

and the *NK Phillips Curve* (NKPC) is given by

$$\pi_t = \beta \widehat{E}_t\{\pi_{t+1}\} + \lambda x_t. \quad (2)$$

In this model the aggregate output gap is denoted  $x_t$ . The variable  $i_t$  is the nominal interest rate set by the central bank and  $\pi_t$  is the rate of inflation. The parameter  $\sigma$  is the inverse of the inter-temporal elasticity of substitution of private consumption.  $\beta$  is the common discount factor and  $\lambda$  is typically a combination of additional structural parameters.

By  $\widehat{E}_t\{z_{t+1}\}$  we denote the heterogeneous expectations operator for any aggregate variable  $z_{t+1}$  as specified in Branch and McGough (2009, p.3).<sup>11</sup> More

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<sup>11</sup>Please note that Branch and McGough (2009) make use of an “axiomatic approach” and impose some assumptions that may appear restrictive to other scholars, but are a necessity to achieve the aggregate equations (1) and (2). Briefly, the assumptions that may be regarded as critical are the specification of higher order beliefs and the assumption that wealth dynamics do not matter for the evolution of aggregate variables. For a detailed discussion of these issues we refer the reader to Branch and McGough (2009).

specifically, we stick to their assumption that the heterogeneous expectations operator for any aggregate variable  $z_t$  is given by

$$\widehat{E}_t\{z_{t+1}\} = \alpha E_t^1\{z_{t+1}\} + (1 - \alpha)E_t^2\{z_{t+1}\}.$$

Thereby  $\alpha \in [0, 1]$  is the share of agents that are rational and  $E_t^1\{z_{t+1}\} = E_t\{z_{t+1}\}$  is their RE operator. The fraction  $(1 - \alpha)$  is not fully rational in the sense that they form expectations by the forecasting model  $E_t^2\{z_{t+1}\} = \theta E_t^2\{z_t\} = \theta^2 z_{t-1}$ , where the parameter  $\theta$  governs the nature of the forecast that can either be *purely adaptive* ( $\theta < 1$ ) or *extrapolative* ( $\theta > 1$ ). With regard to aggregate expectations of endogenous variables it follows that

$$\widehat{E}_t\{x_{t+1}\} = \alpha E_t\{x_{t+1}\} + (1 - \alpha)\theta^2 x_{t-1}, \quad (3)$$

$$\widehat{E}_t\{\pi_{t+1}\} = \alpha E_t\{\pi_{t+1}\} + (1 - \alpha)\theta^2 \pi_{t-1} \quad (4)$$

holds.

In the subsequent analysis, we will close the model in each subsection with a different simple monetary policy rule and inspect its dynamic properties in the resulting system.

Inspection of (3) and (4) reveals that past values of aggregate endogenous variables can affect the aggregate demand and supply when RE and adaptive expectations coexist and therefore this model is self-referential. In consequence, monetary policy rules that perform well in pure RE models may not necessarily do so under heterogeneous expectations of this particular type.

## 2.2. The Method of Numerical Analysis

Our analysis is based on numerical methods. In particular, we calculate and visualize so-called regions of local determinacy, local indeterminacy and local explosiveness. These regions are plotted in a plane where the axes measure the monetary policy parameters. As mentioned before, we choose to do a numerical analysis as we deal with high dimensional economic systems. These systems do not always allow to provide analytical conditions under which a certain rule yields determinacy.

The aforementioned dynamic IS curve and NKPC together with a monetary policy rule will usually lead to a second-order stochastic difference system of the form

$$\mathbf{y}_t = \mathbf{A} E_t\{\mathbf{y}_{t+1}\} + \mathbf{C} \mathbf{y}_{t-1}, \quad (5)$$

where  $\mathbf{y}_t$  is a  $m \times 1$  vector of endogenous variables and matrices  $\mathbf{A}$  and  $\mathbf{C}$  are  $m \times m$  matrices. In order to analyse the dynamics of the system (5), one needs to calculate the eigenvalues, because the eigenvalues characterize the system dynamics. One can do so by following a solution procedure for the system (5) that, as a by-product, yields the eigenvalues of the system.

We may either make use of the solution method detailed in Blanchard and Kahn (1980) or the more general and robust purely numerical method proposed by Klein (2000). The advantage of the latter method is that it can cope with matrices  $\mathbf{A}$  and  $\mathbf{C}$  even if they are singular. Therefore, we rely on the latter method (as outlined in McCallum (2009a, p.13ff.)) for the subsequent analyses.

We consider solutions to the system (5) of the type

$$\mathbf{y}_t = \mathbf{\Lambda} \mathbf{y}_{t-1}, \quad (6)$$

where  $\mathbf{\Lambda}$  is a  $m \times m$  matrix. We consider (6) to be the *Perceived Law of Motion* (PLM). One period ahead, (6) is given by

$$E_t\{\mathbf{y}_{t+1}\} = \mathbf{\Lambda} \mathbf{y}_t = \mathbf{\Lambda}^2 \mathbf{y}_{t-1}. \quad (7)$$

Next, we plug (7) into the original system (5). The result is, what is labeled the *Actual Law of Motion* (ALM) of the economy

$$\mathbf{y}_t = \mathbf{A}[\mathbf{\Lambda}^2 \mathbf{y}_{t-1}] + \mathbf{C} \mathbf{y}_{t-1} = [\mathbf{A} \mathbf{\Lambda}^2 + \mathbf{C}] \mathbf{y}_{t-1}. \quad (8)$$

In a REE, the PLM has to coincide with the ALM. Formally this means that

$$\mathbf{\Lambda} \stackrel{!}{=} [\mathbf{A} \mathbf{\Lambda}^2 + \mathbf{C}] \quad (9)$$

has to hold.

Obviously, we can augment condition (9) by the matrix identity  $\mathbf{\Lambda} = \mathbf{\Lambda}$  and write the two of them as

$$\begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{\Lambda}^2 \\ \mathbf{\Lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & -\mathbf{C} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{\Lambda} \\ \mathbf{I} \end{bmatrix}, \quad (10)$$

or more compact as

$$\bar{\mathbf{A}} \begin{bmatrix} \Lambda^2 \\ \Lambda \end{bmatrix} = \bar{\mathbf{C}} \begin{bmatrix} \Lambda \\ \mathbf{I} \end{bmatrix}. \quad (11)$$

Matrices  $\bar{\mathbf{A}}$  and  $\bar{\mathbf{C}}$  are of dimension  $2m \times 2m$ .

Our ultimate goal are the so-called generalized eigenvalues (GEVs) of  $\bar{\mathbf{C}}$  with respect to  $\bar{\mathbf{A}}$  or equivalently the GEVs of the matrix pencil  $[\bar{\mathbf{C}} - \lambda\bar{\mathbf{A}}]$ . The approach of Klein (2000) utilizes the *Schur generalized decomposition theorem* which states that there exist some unitary  $2m \times 2m$  matrices  $\mathbf{Q}$  and  $\mathbf{Z}$  such that we can decompose matrices  $\bar{\mathbf{A}}$ ,  $\bar{\mathbf{C}}$  into the upper triangular  $2m \times 2m$  matrices  $\mathbf{T}$  and  $\mathbf{S}$  respectively, which is  $\mathbf{Q}\bar{\mathbf{C}}\mathbf{Z} = \mathbf{T}$  and  $\mathbf{Q}\bar{\mathbf{A}}\mathbf{Z} = \mathbf{S}$  respectively.

Furthermore, the GEVs of the matrix pencil  $[\bar{\mathbf{C}} - \lambda\bar{\mathbf{A}}]$  are defined as the ratio of the elements of the main diagonal of  $\mathbf{T}$  to the main diagonal of  $\mathbf{S}$ , i.e.  $\lambda_i = t_{ii}/s_{ii}$ . For our purposes, we calculate the GEVs for any combination of the monetary policy parameters. Next, we count the number of GEVs, whose moduli is inside or outside the unit circle for any combination of the monetary policy coefficients.

Precisely this information allows us to visualize regions of local determinacy, local indeterminacy or local explosiveness in the policy space as in Bullard and Mitra (2002). In particular, at any point in the policy space, where the number of GEVs whose moduli lie outside the unit circle equals the number of free variables, there is local *determinacy*. Next, when the number of GEVs whose moduli lie outside the unit circle is lower than the number of free variables we have local indeterminacy of some order.

The order is precisely the number by which the free variables exceed the number of GEVs whose moduli lie outside the unit circle. Thus, when the difference

is one, we denote it *Order 1 Indeterminacy*. This characterizes a situation with a system exhibiting a one dimensional continuum of stationary equilibria. When the difference is two, we label that *Order 2 Indeterminacy*. This denotes a situation with a system exhibiting a two dimensional continuum of equilibria and so on. Thereby we indicate “the number of independent sunspots required to specify the solution”, see Evans and McGough (2005, p.1816).

Finally, in a situation where the number of GEVs whose moduli lie outside the unit circle is larger than the number of free variables in the system, there is local *explosiveness*.<sup>12</sup>

### 2.3. The Calibration of the Economy

For our numerical analysis, we need to calibrate our model. We choose values for the structural parameters according to Table 1 below. A comparison of our

Parameter	Value	Source
$\alpha$	$\in \{1.00, 0.60\}$	-
$\beta$	0.99	-
$\lambda$	0.024	Bullard and Mitra (2002, p.1114)
$\sigma$	0.157	Bullard and Mitra (2002, p.1114)
$\theta$	$\in \{0.90, 1.10\}$	Branch and McGough (2009, p.11ff.)
$\varphi_\pi$	$\in [0.00, 2.00]$	Branch and McGough (2009, p.11ff.)
$\varphi_x$	$\in [0.00, 2.00]$	Branch and McGough (2009, p.11ff.)
$\varphi_i$	$\in \{0.00, 0.65\}$	Bullard and Mitra (2007, p.1188)

Table 1: Calibration of the economy.

choices to the ones of Bullard and Mitra (2002, p.1114) and Bullard and Mitra (2007, p.1182) reveals that these studies provide results for the same values of  $\beta$ ,  $\lambda$  and  $\sigma$ .

Moreover, the two studies cover the same parameter space with regard to the monetary policy coefficients of the simple rules  $\varphi_\pi$  and  $\varphi_x$  in Section 3 below.

<sup>12</sup>In our analysis we ignore the special case, where one or more moduli of the GEVs may lie on the unit circle.

Thus, there is a high degree of comparability of our results with these widely cited studies. Note that the choice of the monetary policy parameter  $\varphi_i$  is based on empirical evidence by Sack (1998).

Finally, please recall that our analysis considers expectational heterogeneity. Therefore we study the coexistence of rational and non-rational agents ( $\alpha \neq 1.00$ ) next to the base case of rational agents only ( $\alpha = 1.00$ ). In the former case the parameter  $\theta$  is in action. This parameter characterizes the type of non-rational expectations.

Please also notice that Branch and McGough (2009, p.11ff.) we allow for a higher degree of heterogeneity as we choose  $\alpha \in \{1.00, 0.60\}$  in our analysis. This choice is motivated by the evidence for heterogeneous expectations among agents in micro data that corresponds to  $\alpha = 0.60$ , see Branch (2004).

### 3. Dynamic Properties with Taylor-type Rules

Herein, we carry out a numerical investigation of the dynamic consequences of simple monetary policy rules without and with policy inertia. These are linear rules that condition the central bank's instrument rate on the rate of inflation and the output gap which shall reflect the central bank's mandate. We also consider policy inertia in the analysis to capture the tendency of central banks to gradually alter their policy instrument.

#### 3.1. Monetary Policy Rule with Contemporaneous Data

Assume, as in Bullard and Mitra (2002, sec. 3.1.) that the central bank feeds back to contemporaneous data on inflation and the output gap.<sup>13</sup> Such a rule

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<sup>13</sup>Be aware that each simple rule considered herein may have some advantages and shortcomings with regard to measurement issues etc. that are not related to the dynamic properties. For a discussion of these issues, we refer the interested reader to Bullard and Mitra (2002) or McCallum (1999).

may be of the functional form

$$i_t = \varphi_\pi \pi_t + \varphi_x x_t + \varphi_i i_{t-1}. \quad (12)$$

For the moment, we ignore policy inertia, i.e. we set  $\varphi_i = 0.00$ . We can plug this version of (12) into (1), combine the latter with (2) and get a system as (5) with the vector  $\mathbf{y}_t = [x_t, \pi_t]'$  and system matrices

$$\mathbf{A} = \frac{\alpha}{(\sigma + \varphi_x + \lambda\varphi_\pi)} \begin{bmatrix} \sigma & 1 - \beta\varphi_\pi \\ \sigma\lambda & \lambda + \beta(\sigma + \varphi_x) \end{bmatrix} \quad (13)$$

and

$$\mathbf{C} = \frac{(1 - \alpha)\theta^2}{(\sigma + \varphi_x + \lambda\varphi_\pi)} \begin{bmatrix} \sigma & 1 - \beta\varphi_\pi \\ \sigma\lambda & \lambda + \beta(\sigma + \varphi_x) \end{bmatrix}. \quad (14)$$

Please be aware that with RE only ( $\alpha = 1.00$ ) the matrix  $\mathbf{C}$  is a matrix of zeros and we are exactly in the case considered by Bullard and Mitra (2002, p.1115). In consequence, all the analytical proofs therein hold, both, with respect to determinacy and E-stability.

Now, we compare the case of homogeneous RE ( $\alpha = 1.00$ ) to the case of heterogeneous expectations ( $\alpha = 0.60$ ), where non-rational expectations are either purely adaptive ( $\theta = 0.90$ ) or extrapolative ( $\theta = 1.10$ ). Consider the numerical illustration in Figure 1.<sup>14</sup>

For the beginning, realize that Panel 1(a) is nothing but an extract of Bullard

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<sup>14</sup>Please note that in all figures below that plot regions the color-code is as follows: *red* regions label *Order 2 Indeterminacy*, *blue* regions label *Order 1 Indeterminacy*, *green* regions label *Determinacy* and *yellow* regions label *Local Explosiveness*. The horizontal axis measures the policy coefficient  $\varphi_\pi$  and the vertical axis measures the policy coefficient  $\varphi_x$ .

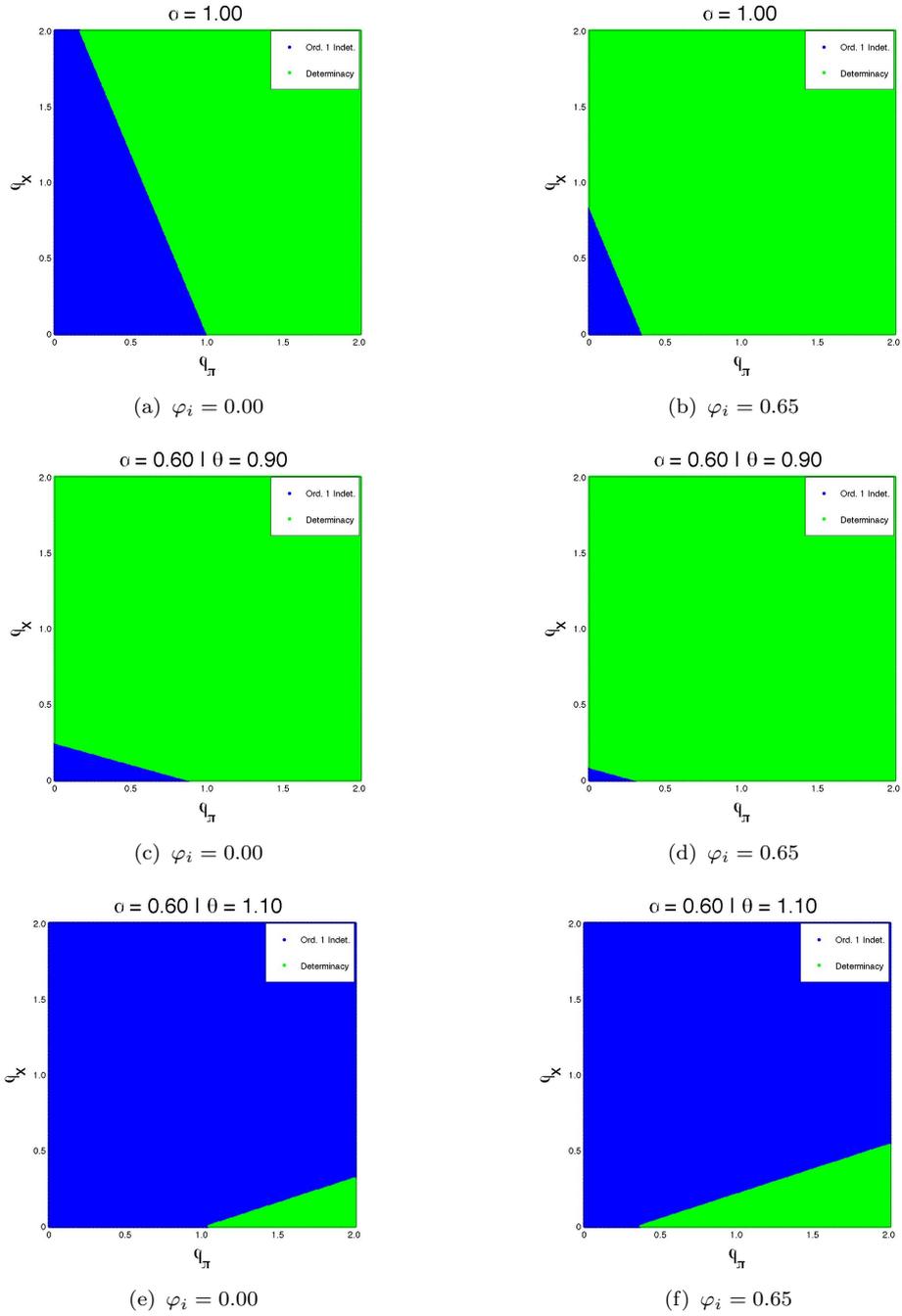


Figure 1: Regions of (in-)determinacy and explosiveness for the rule with feedback on contemporaneous data. The right column contains the results for this rule with policy inertia.

and Mitra (2002, Fig.1, p.1117) and restates their numerical result with regard to determinacy. We observe that a large share of the policy space yields determinacy and the Taylor-principle yields determinacy throughout the parameter space.<sup>15</sup>

Furthermore, inspection of the differences between Panels 1(c) and 1(e) indicates two results. In case of contemporaneous data in the policy rule, where next to RE, purely adaptive expectations ( $\theta = 0.90$ ) exist, the Taylor-principle still yields determinacy in the whole parameter space, whereas this is not true in the case of extrapolative expectations ( $\theta = 1.10$ ).

Next, the region of determinacy increases relatively to the region of indeterminacy for the case of purely adaptive expectations, whereas the reverse is true for the case of extrapolative expectations. Put differently, policies that used to lead to indeterminacy under homogeneous RE yield determinacy in the presence of purely adaptive expectations and the opposite is true in the presence of extrapolative expectations. This has been observed by Branch and McGough (2009, p.11) for a forward-looking monetary policy rule (as we will discuss in Section 3.3) and we can confirm that observation herein for a policy rule with contemporaneous data.<sup>16</sup>

Now, consider the case with policy inertia, i.e.  $\varphi_i = 0.65$  in (12). We can combine this version of (12) and (1) with (2) and get a system as (5) with the

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<sup>15</sup>Please note that we discuss our results in the light of the Taylor-principle as it appears to be a quite robust phenomenon that sticking to this principle yields determinacy. But be aware that this principle is not an exact and general condition (see Bullard and Mitra (2002)).

<sup>16</sup>Surely it would be of interest to have exact conditions that explain the influence of  $\alpha$  on stability. In this particular case, it requires to study a quartic function and our current research is concerned exactly with this issue.

vector  $\mathbf{y}_t = [x_t, \pi_t, i_t]'$  and matrices

$$\mathbf{A} = \frac{\alpha}{(\sigma + \varphi_x + \lambda\varphi_\pi)} \begin{bmatrix} \sigma & 1 - \beta\varphi_\pi & 0 \\ \sigma\lambda & \lambda + \beta(\sigma + \varphi_x) & 0 \\ \sigma(\varphi_x + \varphi_\pi\lambda) & \varphi_x + \varphi_\pi(\lambda + \beta\sigma) & 0 \end{bmatrix} \quad (15)$$

and

$$\mathbf{C} = \frac{1}{(\sigma + \varphi_x + \lambda\varphi_\pi)} \times \begin{bmatrix} (1 - \alpha)\theta^2\sigma & (1 - \alpha)\theta^2(1 - \beta\varphi_\pi) & -\varphi_i \\ (1 - \alpha)\theta^2\sigma\lambda & (1 - \alpha)\theta^2(\lambda + \beta(\sigma + \varphi_x)) & -\lambda\varphi_i \\ (1 - \alpha)\theta^2\sigma(\varphi_x + \varphi_\pi\lambda) & (1 - \alpha)\theta^2(\varphi_x + \varphi_\pi(\lambda + \beta\sigma)) & \sigma\varphi_i \end{bmatrix}. \quad (16)$$

When  $\alpha = 1.00$  we are in the case of homogeneous RE. Numerical results are presented in the right column of Figure 1. First, compare Panel 1(b) to Panel 1(a), the case without policy inertia. One can observe that in an economy with homogeneous RE the set of policies  $\{\varphi_\pi, \varphi_x\}$  that yield determinacy increases.<sup>17</sup>

This is a result that was also reported by Bullard and Mitra (2007) for policy rules that we will study in Sections 3.2 and 3.3 below. A comparison of Panel 1(d) to Panel 1(c) as well as Panel 1(f) to Panel 1(e) reveals that this pattern of observation is robust to heterogeneous expectations. It holds independent of the nature of the expectations of non-rational agents. Moreover, the Taylor-principle appears to be an appropriate policy recommendation in the case of homogeneous RE as well as in the case where the non-rational agents have purely adaptive expectations. Unfortunately this is not generally true, when non-rational agents

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<sup>17</sup>Sensitivity analyses with parameter  $\varphi_i$  suggest that the larger the policy inertia, the larger the regions of determinacy throughout most of the cases in this study.

have extrapolative expectations.

### 3.2. Monetary Policy Rule with Lagged Data

Next we assume, as in Bullard and Mitra (2002, sec. 3.2.) that the central bank feeds back to lagged data on inflation and the output gap, i.e.

$$i_t = \varphi_\pi \pi_{t-1} + \varphi_x x_{t-1} + \varphi_i i_{t-1}. \quad (17)$$

Notice that for the beginning we ignore policy inertia in the rule and set  $\varphi_i = 0.00$ . We combine this version of (17) with (1) and (2) in order to get a system as (5) with the vector  $\mathbf{y}_t = [x_t, \pi_t]'$  and matrices<sup>18</sup>

$$\mathbf{A} = \alpha \begin{bmatrix} 1 & \sigma^{-1} \\ \lambda & \lambda\sigma^{-1} + \beta \end{bmatrix} \quad (18)$$

and

$$\mathbf{C} = \begin{bmatrix} (1 - \alpha)\theta^2 - \varphi_x \sigma^{-1} & \sigma^{-1}[(1 - \alpha)\theta^2 - \varphi_\pi] \\ \lambda[(1 - \alpha)\theta^2 - \varphi_x \sigma^{-1}] & (1 - \alpha)\theta^2 \beta + \lambda\sigma^{-1}[(1 - \alpha)\theta^2 - \varphi_\pi] \end{bmatrix}. \quad (19)$$

In the case when expectations are completely rational ( $\alpha = 1.00$ ) matrix  $\mathbf{C}$  is a matrix of zeros. Then, we are exactly in the case of Bullard and Mitra (2002, p.1118) and their results hold.

When we turn to the numerical results in Figure 2, inspection of Panel 2(a) makes clear that it is just an extract of Bullard and Mitra (2002, Fig.2, p.1120).

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<sup>18</sup>Note that Bullard and Mitra (2002, sec. 3.2.) forward (17) by one period and then combine it with (1) and (2) in order to get a system as (5) with  $\mathbf{y}_t = [x_t, \pi_t, i_t]'$  for the derivation of the set of sufficient conditions and the related formal proof. Our analysis is purely numerical, and for the sake of simplicity, we eliminate as much variables as we can. The numerical results appear to be equivalent.

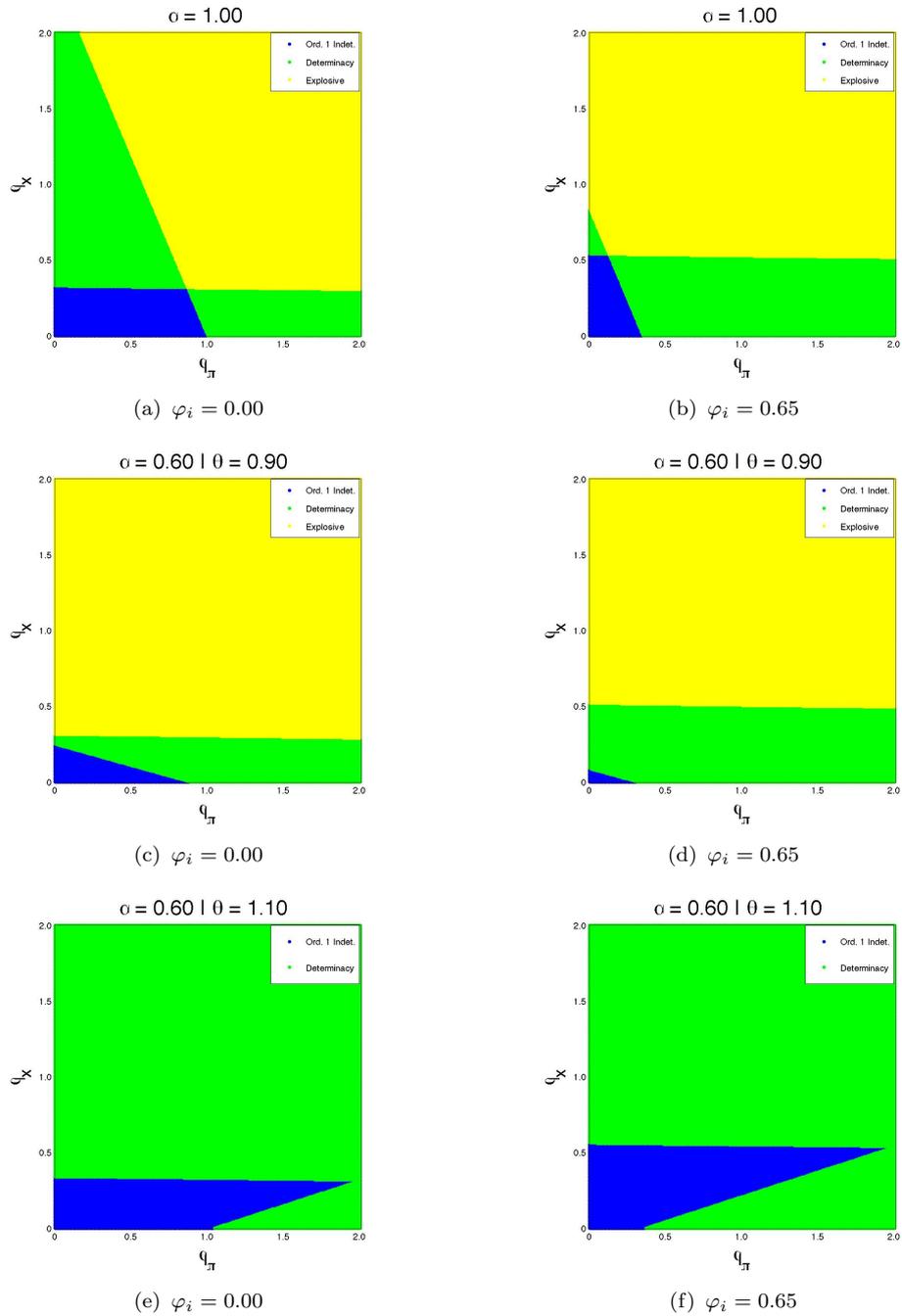


Figure 2: Regions of (in-)determinacy and explosiveness for the rule with feedback on lagged data. The right column contains the results for this rule with policy inertia.

We find regions of determinacy, indeterminacy and local explosiveness. In addition, the Taylor-principle only yields determinacy in case of modest feedback to output gap deviations.

Next, in Panel 2(c) we observe that when non-rational agents are present and have purely adaptive expectations ( $\theta = 0.90$ ), both the regions of determinacy and indeterminacy of order one become smaller and the region of explosiveness increases. Note from Panel 2(e) that if the non-rational agents have extrapolative expectations ( $\theta = 1.10$ ), then the reverse is true. The regions of determinacy and indeterminacy of order one increase but local explosiveness is no longer present.

If we regard local explosiveness as a serious threat, then one cannot conclude that the presence of purely adaptive expectations is favourable to stability and the presence of extrapolative expectations is not. Thus, our findings for the rule with lagged data are at odds with the results in Branch and McGough (2009, p.11ff.).

Finally, there are two additional observations. First, sticking to the Taylor-principle is not a good policy in general, as it cannot rule out regions of indeterminacy or local explosiveness. Second, a policy that exclusively feeds back to the output gap ( $\varphi_x \neq 0, \varphi_\pi = 0$ ) has the potential to yield determinacy, which is a rather unusual observation.

Now, we assume that the central bank favours policy inertia, which is similar to the rule studied in Bullard and Mitra (2007, p.1183ff.). We set  $\varphi_i = 0.65$ . This version of rule (17) together with equations (1) and (2) can be written as a

system (5) with a vector  $\mathbf{y}_t = [x_t, \pi_t, i_t]'$  and matrices<sup>19</sup>

$$\mathbf{A} = \frac{1}{(\varphi_x + \varphi_\pi \lambda - \varphi_i \sigma)} \begin{bmatrix} -\alpha \varphi_i \sigma & -\alpha(\varphi_\pi \beta + \varphi_i) & 1 \\ -\alpha \varphi_i \sigma \lambda & -\alpha[\varphi_i(\sigma \beta + \lambda) - \varphi_x \beta] & \lambda \\ \alpha \sigma(\varphi_x + \varphi_\pi \lambda) & \alpha[\varphi_x + \varphi_\pi(\sigma \beta + \lambda)] & -\sigma \end{bmatrix} \quad (20)$$

and

$$\mathbf{C} = \frac{(1 - \alpha)\theta^2}{(\varphi_x + \varphi_\pi \lambda - \varphi_i \sigma)} \begin{bmatrix} -\varphi_i \sigma & -(\varphi_\pi \beta + \varphi_i) & 0 \\ -\varphi_i \sigma \lambda & -[\varphi_i(\sigma \beta + \lambda) - \varphi_x \beta] & 0 \\ \sigma(\varphi_x + \varphi_\pi \lambda) & [\varphi_x + \varphi_\pi(\sigma \beta + \lambda)] & 0 \end{bmatrix}. \quad (21)$$

Note it is an easy task to verify that for the case of homogeneous RE ( $\alpha = 1.00$ ), we are exactly in the case of Bullard and Mitra (2007, p.1183ff.) and their results apply.

We present our numerical results in the right column of Figure 2 below. From comparison of Panel 2(b) to 2(a) it is hard to tell if the region of determinacy really increases in the case of policy inertia in an economy with homogeneous RE.<sup>20</sup> Furthermore, comparisons of Panel 2(d) to Panel 2(c) as well as Panel 2(f) to Panel 2(e) indicate that policy inertia does not improve the dynamic properties with regard to determinacy in general. It is only true for the case of purely adaptive expectations.

In addition, with policy inertia the Taylor-principle is no suitable policy recommendation for a lagged data rule in general. Sticking to that principle cannot

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<sup>19</sup>As in Bullard and Mitra (2007, p.1183ff.) we forward the rule by one period, before we build the system.

<sup>20</sup>Note that Bullard and Mitra (2007, p.1183ff.) attribute a beneficial role to policy inertia as the region that yields both determinate and E-stable outcomes increase with policy inertia.

rule out indeterminacy or local explosiveness universally.

### 3.3. Forward-Looking Monetary Policy Rule

This section basically recapitulates the numerical analysis of Branch and McGough (2009, p.11ff.). We do so for completeness on the one hand and on the other hand because our calibration is slightly different, i.e.  $\alpha \in \{1.00, 0.60\}$ . We choose the latter in order to highlight the fact that heterogeneous expectations might cause local explosiveness in this specific setting. This is an observation possibly overlooked by Branch and McGough (2009, p.11ff.).

Thus, similar as in Bullard and Mitra (2002, sec. 3.3.) or Branch and McGough (2009, p.11ff.) we assume that central bank feeds back to RE on period  $t + 1$  inflation and the output gap, i.e.

$$i_t = \varphi_\pi E_t\{\pi_{t+1}\} + \varphi_x E_t\{x_{t+1}\} + \varphi_i i_{t-1}. \quad (22)$$

One could also think of the expectations in the rule (22) as the central bank's forecast of the aggregate variables based on its period  $t$  information set. For the time being, we assume that there is no policy inertia, i.e.  $\varphi_i = 0$ .

We recast (22), (1) and (2) as our standardform (5) with a vector  $\mathbf{y}_t = [x_t, \pi_t]'$  and matrices

$$\mathbf{A} = \begin{bmatrix} \alpha - \sigma^{-1}\varphi_x & \sigma^{-1}(\alpha - \varphi_\pi) \\ \lambda(\alpha - \sigma^{-1}\varphi_x) & \alpha\beta + \lambda\sigma^{-1}(\alpha - \varphi_\pi) \end{bmatrix} \quad (23)$$

and

$$\mathbf{C} = (1 - \alpha)\theta^2 \begin{bmatrix} 1 & \sigma^{-1} \\ \lambda & (\beta + \lambda\sigma^{-1}) \end{bmatrix}. \quad (24)$$

Note that for the case of RE only ( $\alpha = 1.00$ ) the matrix  $\mathbf{C}$  is a matrix of zeros. In this case all the analytical proofs with respect to determinacy and E-stability in Bullard and Mitra (2002, p.1121) hold.

Next, consider the visualization of numerical results in Figure 3 below. Panel 3(a) is an exact reproduction of “north-west” panel in Branch and McGough (2009, Fig.1, p.12) which is an extract of Bullard and Mitra (2002, Fig.3, p.1123). The difference is that in the latter study, there is no distinction between indeterminacy of different orders and for that reason labels in Panel 1(a) are different compared to the latter.<sup>21</sup>

In Panel 1(a) we observe regions of indeterminacy of order 1 and order 2 next to regions of determinacy. In addition, it is obvious that the Taylor-principle does not hold in general, but only for modest feedback to output gap deviations.

Next, Panels 3(c) and 3(e) make clear that in presence of heterogeneous agents, regions of explosiveness may arise. Interestingly, these regions seem to originate and expand from an area around  $(\varphi_\pi \approx 1, \varphi_x = 0)$  with decreasing  $\alpha$ , the fraction of non-rational agents. As a consequence, sticking too close to the Taylor-principle under this rule might turn out to be a rather dangerous policy in an economy with heterogeneous expectations. As a matter of fact, such a policy could trigger explosive paths of the price level under the rule (22) without policy inertia.

Our findings for this particular rule make clear that the results in Branch and McGough (2009, p.11ff.) are heavily dependent on the fraction of non-rational

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<sup>21</sup>If one compares the two figures Branch and McGough (2009, Fig.1, p.12) and Bullard and Mitra (2002, Fig.3, p.1123), one realizes that regions of indeterminacy of order one, are found to be E-stable and regions of indeterminacy of order two, are found to be E-unstable by Bullard and Mitra (2002, p.1121ff.). From our perspective, it would be interesting to examine, whether or not there is a link between the concepts of E-stability and indeterminacy of some order.

agents. For our choice of expectational heterogeneity ( $\alpha = 0.60$ ) explosive regions emerge for both the case of purely adaptive expectations and the case of extrapolative expectations. Therefore, one cannot claim that the former type of adaptive expectations may improve the dynamic properties with regard to determinacy in general, whereas for the latter type the opposite is true.

Finally, note from Panel 3(c) that in the presence of purely adaptive expectations policies that solely feed back to output gap deviations ( $\varphi_x \neq 0, \varphi_\pi = 0$ ) again have the potential to yield determinacy.

Let us get back to rule (22) and assume that central bank attaches importance to policy inertia as in Bullard and Mitra (2007, p.1184ff.). Then the system to analyze (5) has matrices

$$\mathbf{A} = \begin{bmatrix} \alpha - \sigma^{-1}\varphi_x & \sigma^{-1}(\alpha - \varphi_\pi) & 0 \\ \lambda(\alpha - \sigma^{-1}\varphi_x) & \alpha\beta + \lambda\sigma^{-1}(\alpha - \varphi_\pi) & 0 \\ \varphi_x & \varphi_\pi & 0 \end{bmatrix} \quad (25)$$

and

$$\mathbf{C} = \begin{bmatrix} (1 - \alpha)\theta^2 & (1 - \alpha)\theta^2\sigma^{-1} & -\varphi_i\sigma^{-1} \\ (1 - \alpha)\theta^2\lambda & (1 - \alpha)\theta^2(\beta + \lambda\sigma^{-1}) & -\varphi_i\sigma^{-1}\lambda \\ 0 & 0 & \varphi_i \end{bmatrix} \quad (26)$$

corresponding to a vector  $\mathbf{y}_t = [x_t, \pi_t, i_t]'$ .

If there are only fully rational agents ( $\alpha = 1.00$ ), we are exactly in the case of Bullard and Mitra (2007, p.1184ff.). Hence their results with respect to determinacy and E-stability hold. The numerical results are illustrated in the right column of Figure 3.

By comparing Panel 3(b) to Panel 3(a) we find that in the case of homogeneous

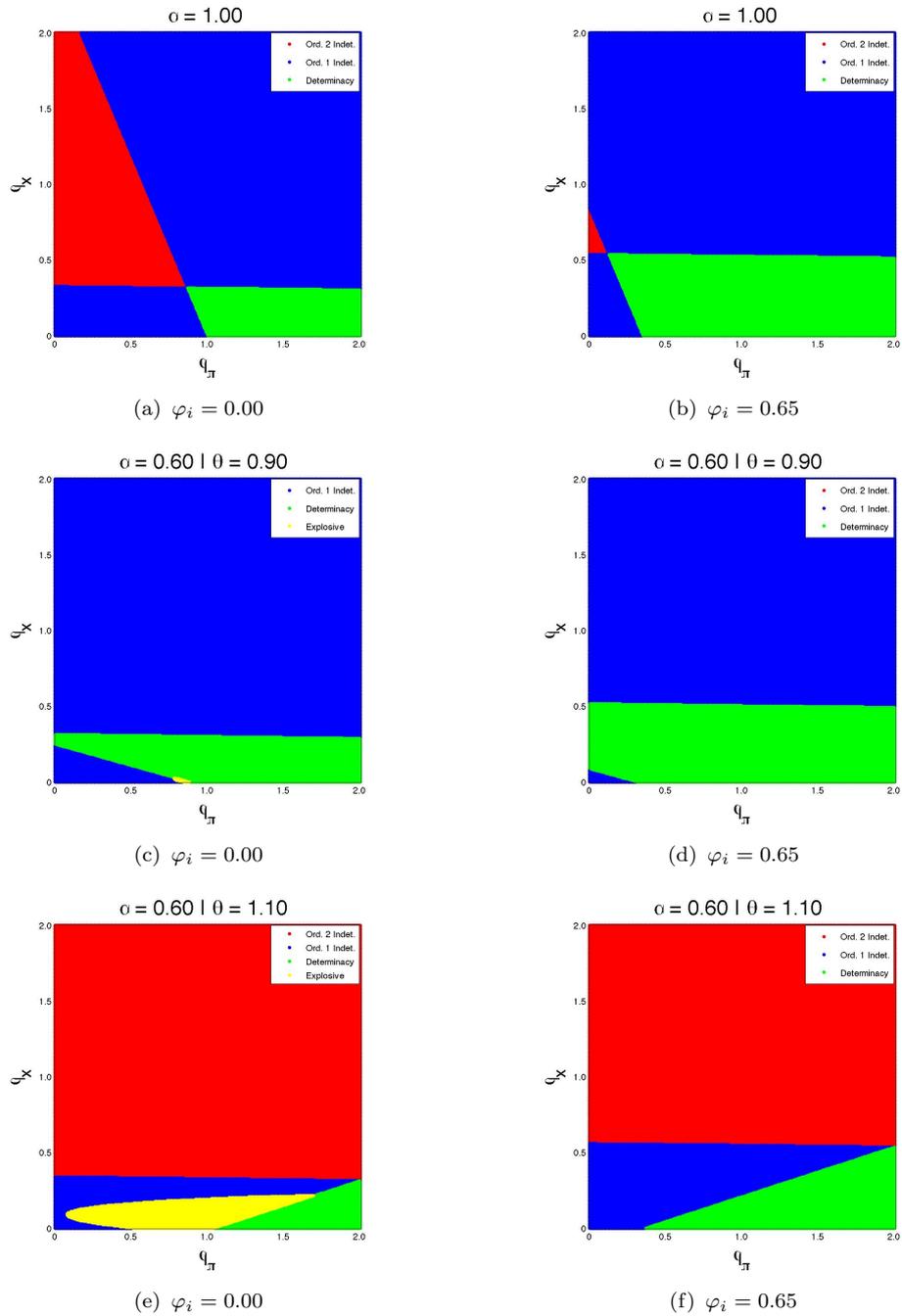


Figure 3: Regions of (in-)determinacy and explosiveness for the rule with feedback on expectations of period  $t + 1$  values. The right column contains the results for this rule with policy inertia.

RE the region of determinacy increases. This pattern remains stable for the case of heterogeneous expectations, independent of the nature of expectations of non-rational agents as Panels 3(d) and 3(f) reveal. Most notably, policy inertia eliminates regions of local explosiveness in the case of heterogeneous expectations. Moreover, the Taylor-principle does not hold in general as in the case without policy inertia.

A priori, it is not clear, why the central bank should feedback to RE of aggregate variables. It may simply do so, because it assumes a pure RE model of the economy. Alternatively, as Branch and McGough (2009, p.9) propose, one could assume that the central bank is aware of the exact nature of heterogeneous expectations and conditions its instrument on these expectations, which is

$$i_t = \varphi_\pi \widehat{E}_t\{\pi_{t+1}\} + \varphi_x \widehat{E}_t\{x_{t+1}\} + \varphi_i i_{t-1}. \quad (27)$$

From our perspective, this appears to be a strong assumption in practice. We presume that tracking the exact shares  $\alpha$  of agents with different types of expectations demands a non-negligible effort from central banks. Moreover, the central bank needs to determine the nature of adaptive expectations  $\theta$ . This may come at large information costs. Nevertheless, it is of interest, whether or not the potential benefit of such a rule could justify the costs.

As before, we start with rule (27) without considering policy inertia ( $\varphi_i = 0$ ). This leads to a system with a vector  $\mathbf{y}_t = [x_t, \pi_t]'$  and matrices

$$\mathbf{A} = \alpha \begin{bmatrix} 1 - \sigma^{-1}\varphi_x & \sigma^{-1}(1 - \varphi_\pi) \\ \lambda(1 - \sigma^{-1}\varphi_x) & \beta + \lambda\sigma^{-1}(1 - \varphi_\pi) \end{bmatrix} \quad (28)$$

and

$$\mathbf{C} = (1 - \alpha)\theta^2 \begin{bmatrix} 1 - \sigma^{-1}\varphi_x & \sigma^{-1}(1 - \varphi_\pi) \\ \lambda(1 - \sigma^{-1}\varphi_x) & \beta + \lambda\sigma^{-1}(1 - \varphi_\pi) \end{bmatrix}. \quad (29)$$

Obviously we end up in the case of Bullard and Mitra (2002, p.1121) if we set  $\alpha = 1.00$ . In this case, all the analytical proofs with respect to determinacy and E-stability therein hold.

Our numerical results are outlined in the left column of Figure 4 below. Panel 4(a) does coincide with Panel 3(a) by construction. But how do things change once expectational heterogeneity is in place? We observe that the locally explosive regions in Panels 3(c) and 3(e) are not longer present in Panels 4(c) and Panel 4(e). Thus, it is evident that when the central bank makes use of a monetary policy rule featuring feedback on heterogeneous expectations, it may at least be able to rule out explosive paths of nominal variables. With regard to indeterminacy the results for rules (22) and (27) appear to be observationally equivalent in the absence of policy inertia.

Now, we may again ask how policy inertia in rule (27) affects the dynamics. Then, the system (5) with vector  $\mathbf{y}_t = [x_t, \pi_t, i_t]'$  has matrices

$$\mathbf{A} = \alpha \begin{bmatrix} 1 - \sigma^{-1}\varphi_x & \sigma^{-1}(1 - \varphi_\pi) & 0 \\ \lambda(1 - \sigma^{-1}\varphi_x) & \beta + \lambda\sigma^{-1}(1 - \varphi_\pi) & 0 \\ \varphi_x & \varphi_\pi & 0 \end{bmatrix} \quad (30)$$

and

$$\mathbf{C} = \begin{bmatrix} (1 - \alpha)\theta^2(1 - \sigma^{-1}\varphi_x) & (1 - \alpha)\theta^2\sigma^{-1}(1 - \varphi_\pi) & -\varphi_i\sigma^{-1} \\ (1 - \alpha)\theta^2[\lambda(1 - \sigma^{-1}\varphi_x)] & (1 - \alpha)\theta^2[\beta + \lambda\sigma^{-1}(1 - \varphi_\pi)] & -\varphi_i\sigma^{-1}\lambda \\ (1 - \alpha)\theta^2\varphi_x & (1 - \alpha)\theta^2\varphi_\pi & \varphi_i \end{bmatrix}. \quad (31)$$

Results are displayed in the right column of Figure 4. Panels 4(b), 4(d) and 4(f) reveal that at least qualitatively the results do not change compared to the situation, where the central bank is not aware of expectational heterogeneity.

The results in this subsection suggest that if a forward-looking rule is in place there are two ways of ruling out local explosiveness. One way is to track the exact nature of expectations as is done by rule (27). The second way is to simply add policy inertia to rule (22). The latter option is less costly with regard to information and may therefore be preferred by central banks that implement a forward-looking instrument rule. This is de facto another merit of policy inertia.

#### 3.4. Monetary Policy Rule with Contemporaneous Expectations

The last simple rule we are going to consider is the one in which the central bank feeds back to contemporaneous expectations on inflation and the output gap as in Bullard and Mitra (2002, sec. 3.4.), i.e.

$$i_t = \varphi_\pi E_t\{\pi_t\} + \varphi_x E_t\{x_t\} + \varphi_i i_{t-1}. \quad (32)$$

One can motivate such a rule by the fact that real time data of aggregate variables usually are not available for central bankers or only with high imprecision. Thus, it may be far more realistic to assume that the policy makers feed back to their RE forecast of period  $t$  variables, rather than actual contemporaneous data.

In such a situation, the information set of the central bank contains observa-

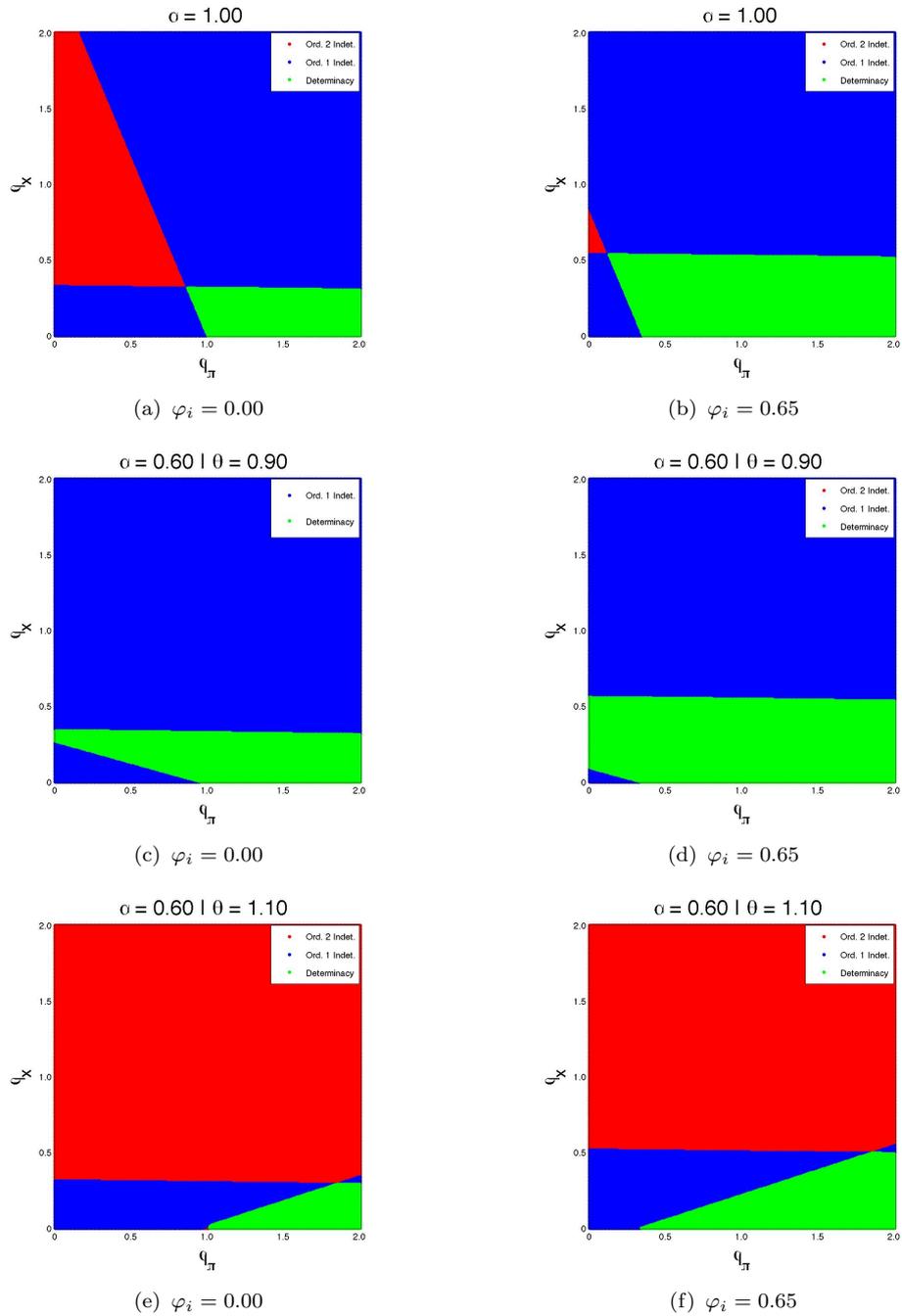


Figure 4: Regions of (in-)determinacy and explosiveness for the rule with feedback on heterogeneous expectations of period  $t + 1$  values. The right column contains the results for this rule with policy inertia.

tions up to period  $t - 1$ . In order to ensure symmetry in information sets, we follow Bullard and Mitra (2002, sec. 3.4.) and assume that policy makers as well as agents in the economy form expectations with an information set as of period  $t - 1$ .<sup>22</sup> Otherwise private sector agents would observe more data than the central bank. Thus, our economy now evolves according to

$$x_t = \widehat{E}_{t-1}\{x_{t+1}\} - \sigma^{-1} \left( i_t - \widehat{E}_{t-1}\{\pi_{t+1}\} \right) \quad (33)$$

and

$$\pi_t = \beta \widehat{E}_{t-1}\{\pi_{t+1}\} + \lambda x_t. \quad (34)$$

The average expectations of aggregate variables are now given by

$$\widehat{E}_{t-1}\{x_{t+1}\} = \alpha E_{t-1}\{x_{t+1}\} + (1 - \alpha)\theta^2 x_{t-1} \quad (35)$$

$$\widehat{E}_{t-1}\{\pi_{t+1}\} = \alpha E_{t-1}\{\pi_{t+1}\} + (1 - \alpha)\theta^2 \pi_{t-1} \quad (36)$$

instead of (3) and (4). Finally, (32) is transformed to

$$i_t = \varphi_\pi E_{t-1}\{\pi_t\} + \varphi_x E_{t-1}\{x_t\} + \varphi_i i_{t-1}. \quad (37)$$

We can rewrite the resulting system (33)-(37) as

$$\mathbf{A}_0 \mathbf{s}_t = \mathbf{A}_1 E_{t-1}\{\mathbf{s}_t\} + \mathbf{A}_2 E_{t-1}\{\mathbf{s}_{t+1}\} + \mathbf{A}_3 \mathbf{s}_{t-1}, \quad (38)$$

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<sup>22</sup>From our understanding the assumptions in Branch and McGough (2009, sec. 2.1.) are general enough to allow for a change in the timing of expectations.

where  $\mathbf{s}_t = [x_t, \pi_t]'$  is a  $p \times 1$  vector and matrices are given by

$$\mathbf{A}_0 = \begin{bmatrix} 1 & 0 \\ -\lambda & 1 \end{bmatrix}, \quad \mathbf{A}_1 = \begin{bmatrix} -\varphi_x \sigma^{-1} & -\varphi_\pi \sigma^{-1} \\ 0 & 0 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} \alpha & \sigma^{-1} \alpha \\ 0 & \beta \alpha \end{bmatrix}$$

and

$$\mathbf{A}_3 = \begin{bmatrix} (1 - \alpha)\theta^2 & \sigma^{-1}(1 - \alpha)\theta^2 \\ 0 & \beta(1 - \alpha)\theta^2 \end{bmatrix}.$$

The system (38) does not directly match our standard form (5). Nevertheless, we can utilize the approach of Binder and Pesaran (1999, p.140ff.) as (38) matches their general multivariate structural RE model

$$\sum_{i=0}^{n_1} \sum_{j=0}^{n_2} \mathbf{M}_{ij} E(\mathbf{s}_{t+j-i} | \boldsymbol{\Omega}_{t-i}) = \mathbf{0}, \quad (39)$$

where the matrices  $\mathbf{M}_{ij}$  are of dimension  $p \times p$  and the vectors  $\mathbf{s}_{t+j-i}$  are of dimension  $p \times 1$ .  $\boldsymbol{\Omega}_{t-i}$  denotes the non-decreasing information set. In our case, it is convenient to consider two lags  $n_1 = 2$  and two leads  $n_2 = 2$ , thus

$$\begin{aligned} \mathbf{0} &= \mathbf{M}_{00} \mathbf{s}_t + \mathbf{M}_{01} E_t\{\mathbf{s}_{t+1}\} + \mathbf{M}_{02} E_t\{\mathbf{s}_{t+2}\} + \mathbf{M}_{10} \mathbf{s}_{t-1} + \mathbf{M}_{20} \mathbf{s}_{t-2} \\ &\quad + \mathbf{M}_{11} E_{t-1}\{\mathbf{s}_t\} + \mathbf{M}_{21} E_{t-2}\{\mathbf{s}_{t-1}\} \\ &\quad + \mathbf{M}_{12} E_{t-1}\{\mathbf{s}_{t+1}\} + \mathbf{M}_{22} E_{t-2}\{\mathbf{s}_t\}. \end{aligned} \quad (40)$$

Note that  $\mathbf{M}_{00} = -\mathbf{A}_0$ ,  $\mathbf{M}_{10} = \mathbf{A}_3$ ,  $\mathbf{M}_{11} = \mathbf{A}_1$ ,  $\mathbf{M}_{12} = \mathbf{A}_2$  and  $\mathbf{0}_2 = \mathbf{M}_{01} =$

$\mathbf{M}_{02} = \mathbf{M}_{20} = \mathbf{M}_{21} = \mathbf{M}_{22}$ . Next, we can recast the latter expression as

$$\begin{aligned} \mathbf{0} = & \begin{bmatrix} \mathbf{M}_{00} & \mathbf{M}_{01} & \mathbf{M}_{02} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{s}_t \\ E_t \mathbf{s}_{t+1} \\ E_t \mathbf{s}_{t+2} \end{bmatrix} + \begin{bmatrix} \mathbf{M}_{10} & \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{s}_{t-1} \\ E_{t-1} \mathbf{s}_t \\ E_{t-1} \mathbf{s}_{t+1} \end{bmatrix} \\ & + \begin{bmatrix} \mathbf{M}_{20} & \mathbf{M}_{21} & \mathbf{M}_{22} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{s}_{t-2} \\ E_{t-2} \mathbf{s}_{t-1} \\ E_{t-2} \mathbf{s}_t \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} E_t \mathbf{s}_{t+1} \\ E_t \mathbf{s}_{t+2} \\ E_t \mathbf{s}_{t+3} \end{bmatrix} \end{aligned}$$

or with  $\mathbf{z}_t = [\mathbf{s}'_t, E_t \mathbf{s}'_{t+1}, E_t \mathbf{s}'_{t+2}]'$  more compact as

$$\mathbf{0} = \mathbf{\Gamma}_0 \mathbf{z}_t + \mathbf{\Gamma}_1 \mathbf{z}_{t-1} + \mathbf{\Gamma}_2 \mathbf{z}_{t-2} + \mathbf{\Gamma}_{-1} E_t \mathbf{z}_{t+1}. \quad (41)$$

Now, we can rewrite equation (41) as

$$\mathbf{0} = \begin{bmatrix} \mathbf{\Gamma}_0 & \mathbf{\Gamma}_1 \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{z}_t \\ \mathbf{z}_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{\Gamma}_2 \\ -\mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{z}_{t-1} \\ \mathbf{z}_{t-2} \end{bmatrix} + \begin{bmatrix} \mathbf{\Gamma}_{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} E_t \mathbf{z}_{t+1} \\ \mathbf{z}_t \end{bmatrix},$$

or by defining  $\mathbf{y}_t = [\mathbf{z}'_t, \mathbf{z}'_{t-1}]'$  more compactly as a second-order stochastic difference system, which in general can be written as

$$\begin{aligned} \mathbf{\Lambda}_0 \mathbf{y}_t &= -\mathbf{\Lambda}_{-1} E_t \{\mathbf{y}_{t+1}\} - \mathbf{\Lambda}_1 \mathbf{y}_{t-1} \\ \mathbf{y}_t &= -\mathbf{\Lambda}_0^{-1} \mathbf{\Lambda}_{-1} E_t \{\mathbf{y}_{t+1}\} - \mathbf{\Lambda}_0^{-1} \mathbf{\Lambda}_1 \mathbf{y}_{t-1} \\ \mathbf{y}_t &= \mathbf{A} E_t \{\mathbf{y}_{t+1}\} + \mathbf{C} \mathbf{y}_{t-1}. \end{aligned} \quad (42)$$

This is the same as our standard form (5).<sup>23</sup>

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<sup>23</sup> $\mathbf{\Lambda}_0$  is non-singular and invertible as matrices  $\mathbf{\Gamma}_0$  and  $\mathbf{\Lambda}_0$  are non-singular. We omit matrices  $\mathbf{A}$  and  $\mathbf{C}$  as they are both of dimension  $12 \times 12$  in this case.

The numerical results appear to be observationally similar to the left column of Figure 1 above for the rule with contemporaneous actual data (12). Loosely speaking, it has desirable properties with regard to determinacy under heterogeneous expectations.<sup>24</sup>

This is good news for the central bank. The interest rate rule depending on contemporaneous expectations (32) does only require data up to period  $t - 1$ , as mentioned above. Therefore, it is easier to implement compared to the contemporaneous data rule (12) and still yields similar results. Consequently, rule (32) is preferable to rule (12) even in an economy of heterogeneous expectations and not only in an economy of homogeneous RE as argued by Bullard and Mitra (2002, p.1108).

Next, we would like to consider the effect of policy inertia in rule (32), i.e.  $\varphi_i = 0.65$ . Similar steps as detailed above yield a system

$$\mathbf{A}_0 \mathbf{s}_t = \mathbf{A}_1 E_{t-1}\{\mathbf{s}_t\} + \mathbf{A}_2 E_{t-1}\{\mathbf{s}_{t+1}\} + \mathbf{A}_3 \mathbf{s}_{t-1}, \quad (43)$$

where  $\mathbf{s}_t = [x_t, \pi_t, i_t]'$  and matrices are given by

$$\mathbf{A}_0 = \begin{bmatrix} 1 & 0 & \sigma^{-1} \\ -\lambda & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{A}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \varphi_x & \varphi_\pi & 0 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} \alpha & \sigma^{-1}\alpha & 0 \\ 0 & \beta\alpha & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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<sup>24</sup>For the analysis we may also replace expected values by their actual counterparts in (38) as is done by Bullard and Mitra (2002, p.1123ff.). We understand the latter approach as a kind of shortcut. Then it is easy to verify that the matrices for the case of contemporaneous data rule and contemporaneous expectations rule coincide and that for  $\alpha = 1$  we are in the same case as in Bullard and Mitra (2002, p.1123ff.). Then all the analytical proofs with respect to determinacy and E-stability therein hold. We choose to analyze the system in a rigorous way as we are not aware of the argument behind “shortcut” of Bullard and Mitra (2002, p.1123ff.).

and

$$\mathbf{A}_3 = \begin{bmatrix} (1 - \alpha)\theta^2 & \sigma^{-1}(1 - \alpha)\theta^2 & 0 \\ 0 & \beta(1 - \alpha)\theta^2 & 0 \\ 0 & 0 & \varphi_i \end{bmatrix}.$$

Once more we make use of (39) and the subsequent steps outlined above to bring the system (43) into our standard form (5).<sup>25</sup> We find that the numerical results are the same as in the right column of Figure 4 for the contemporaneous data rule. Nevertheless, once more we would like to emphasize that the contemporaneous expectations rule (32) is preferable compared to the contemporaneous data rule (12) as it is operational.

Next, one could again assume that the central bank is aware of the heterogeneous expectations as in Section 3.3 above. Then the central bank sets the nominal interest rate not according to (32) but according to

$$i_t = \varphi_\pi \widehat{E}_{t-1}\{\pi_t\} + \varphi_x \widehat{E}_{t-1}\{x_t\} + \varphi_i i_{t-1}. \quad (44)$$

Also note that, given the assumptions in Branch and McGough (2009, p.3), we have

$$\widehat{E}_{t-1}\{x_t\} = \alpha E_{t-1}\{x_t\} + (1 - \alpha)\theta x_{t-1}, \quad (45)$$

$$\widehat{E}_{t-1}\{\pi_t\} = \alpha E_{t-1}\{\pi_t\} + (1 - \alpha)\theta \pi_{t-1}. \quad (46)$$

For the moment, we omit policy inertia, i.e.  $\varphi_i = 0$ . We can rewrite the system

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<sup>25</sup>Again we omit matrices  $\mathbf{A}$  and  $\mathbf{C}$  as they are both of dimension  $18 \times 18$  in this case.

(33)-(36) and (44)-(46) as

$$\mathbf{A}_0 \mathbf{s}_t = \mathbf{A}_1 E_{t-1}\{\mathbf{s}_t\} + \mathbf{A}_2 E_{t-1}\{\mathbf{s}_{t+1}\} + \mathbf{A}_3 \mathbf{s}_{t-1}, \quad (47)$$

where the vector of variables is  $\mathbf{s}_t = [x_t, \pi_t]'$  and the system matrices are given by

$$\mathbf{A}_0 = \begin{bmatrix} 1 & 0 \\ -\lambda & 1 \end{bmatrix}, \quad \mathbf{A}_1 = \begin{bmatrix} -\varphi_x \sigma^{-1} \alpha & -\varphi_\pi \sigma^{-1} \alpha \\ 0 & 0 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} \alpha & \sigma^{-1} \alpha \\ 0 & \beta \alpha \end{bmatrix}$$

and

$$\mathbf{A}_3 = \begin{bmatrix} (1 - \alpha)\theta(\theta - \varphi_x \sigma^{-1}) & \sigma^{-1}(1 - \alpha)\theta(\theta - \varphi_x) \\ 0 & \beta(1 - \alpha)\theta^2 \end{bmatrix}.$$

Again we use the general form (39) and the subsequent steps outlined above to bring the system into our standard form (5).<sup>26</sup>

The numerical results are illustrated in the left column of Figure 5. It appears that the numerical results look similar to the ones for the contemporaneous data rule in Figure 1 in Section 3.1 above. Therefore, they are also observationally similar to the results for the contemporaneous expectations rule (37) with feedback to homogeneous RE. This makes clear that it does not make a difference whether or not the central bank is aware of expectational heterogeneity in case of the contemporaneous expectations rule. This is true at least for the parameter space considered herein.

Finally, we study the impact of policy inertia in rule (44) on the dynamics, i.e.  $\varphi_i = 0.65$ . With assumptions (45)-(46) we can derive a system similar to

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<sup>26</sup>Again we omit matrices  $\mathbf{A}$  and  $\mathbf{C}$  as they are both of dimension  $12 \times 12$ .

(43) with matrices

$$\mathbf{A}_0 = \begin{bmatrix} 1 & 0 & \sigma^{-1} \\ -\lambda & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{A}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \varphi_x \alpha & \varphi_\pi \alpha & 0 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} \alpha & \sigma^{-1} \alpha & 0 \\ 0 & \beta \alpha & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and

$$\mathbf{A}_3 = \begin{bmatrix} (1 - \alpha)\theta^2 & \sigma^{-1}(1 - \alpha)\theta^2 & 0 \\ 0 & \beta(1 - \alpha)\theta^2 & 0 \\ \varphi_x(1 - \alpha)\theta & \varphi_\pi(1 - \alpha)\theta & \varphi_i \end{bmatrix}.$$

Again we can bring this version of (43) into our standard form (5).<sup>27</sup>

The numerical results are illustrated in the right column of Figure 5. It appears that the numerical results look similar to the ones obtained for the contemporaneous data rule (12) above. Therefore, they are also similar to the results for the rule (37) with policy inertia.

Thus, for the rule that depends on contemporaneous expectations, it does not make a qualitative difference whether the central bank tracks heterogeneous expectations or not.

Furthermore, these results again indicate that in an economy with expectational heterogeneity the central bank can still choose a rule that is easier to implement, i.e. the rule that depends on contemporaneous expectations. It will not encounter a disadvantage with regard to determinacy compared to the rule that depends on contemporaneous data.

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<sup>27</sup>Once more we omit matrices  $\mathbf{A}$  and  $\mathbf{C}$  as they are both of dimension  $18 \times 18$  in this case.

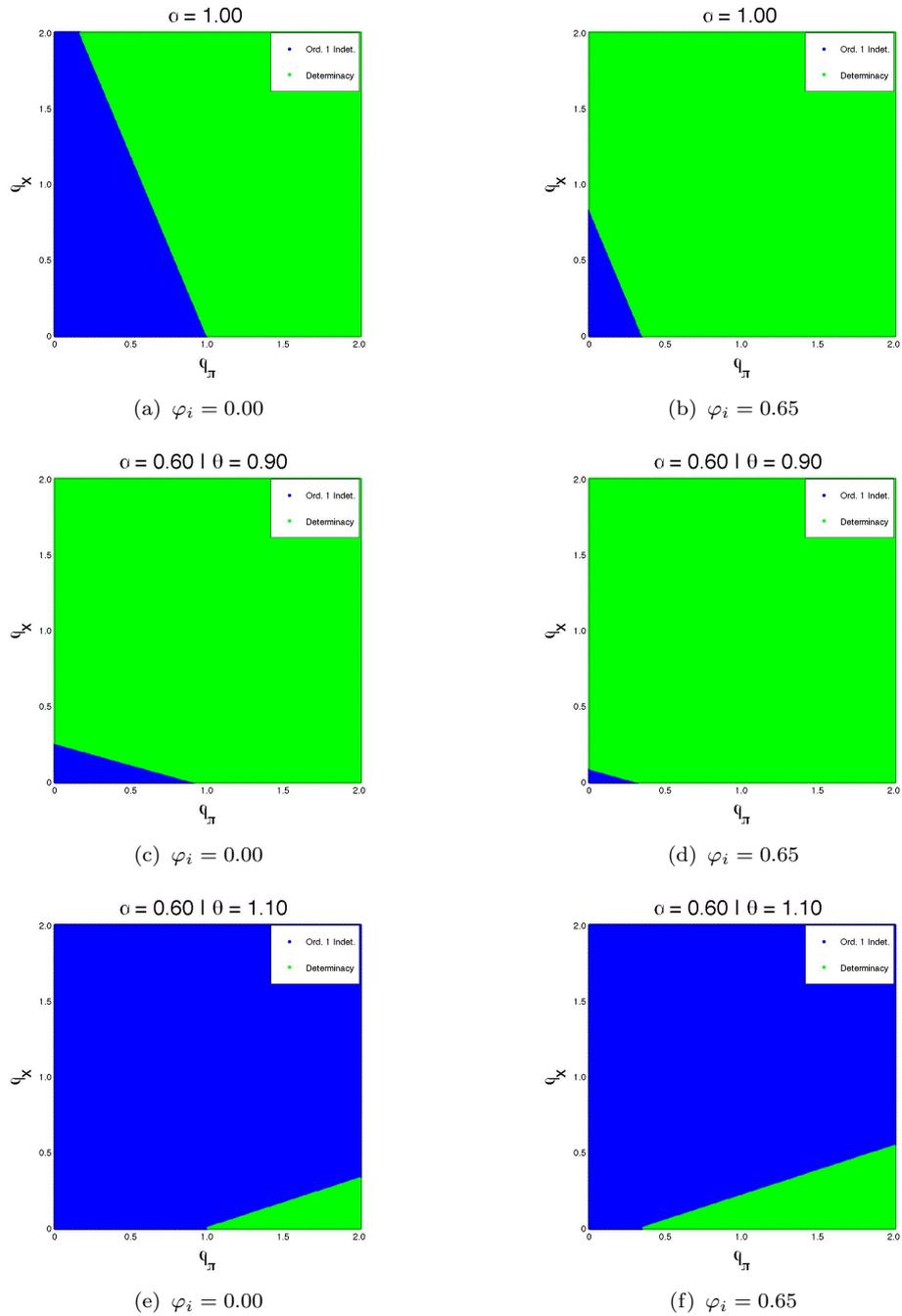


Figure 5: Regions of (in-)determinacy and explosiveness for the rule with feedback on heterogeneous expectations of period  $t$  values. The right column contains the results for this rule with policy inertia.

## 4. Conclusion

In our analysis of Taylor-type rules, we find that in an economy with heterogeneous expectations the contemporaneous expectations rule is more desirable than other often discussed rules. This is due to the fact that this policy prescription rules out explosiveness and does not require to track individuals' expectations. Furthermore, under this rule the Taylor-principle holds for a large share of the parameter space. If there is a moderate feedback to the output gap, it can hold in general.

Moreover, this paper also demonstrates that in case of a forward-looking rule that feeds back on purely RE, the economy may exhibit regions of local explosiveness depending on the degree of expectational heterogeneity. Interestingly these regions occur in the area, in which the central bank would fight inflation expectations moderately by more than one-for-one, i.e. sticking to the Taylor-principle. This is a new level of destabilization compared to what is known in the literature. Once the central bank is aware of the nature of expectations in the economy and feeds back to heterogeneous expectations, it is able to rule out local explosiveness. More generally, our analysis illustrates that rules that depend on forecasts can be improved by tracking the nature of expectations and applying this information to the central banks forecast.

Most importantly, policy inertia can improve the properties of the rules. We observe for almost all rules that with an increasing level of policy inertia the regions of determinacy appear to increase at the expense of regions of local indeterminacy and explosiveness. This holds no matter whether expectations in the economy are homogeneous RE or heterogeneous. Overall, this confirms the findings of Bullard and Mitra (2007) in the case of heterogeneous expectations. Policy inertia is a merit of simple monetary policy rules.

Policy recommendations in the light of our results are as follows. A central bank that prefers a simple rule may conduct monetary policy by a rule that depends on contemporaneous expectations with policy inertia. Furthermore, it may stick to the Taylor-principle in the sense that it feeds back to contemporaneous inflation expectations more than one-for-one combine this stance with moderate feedback to contemporaneous expectations about the output gap. A reasonable degree of policy inertia may then ensure stable prices.

This analysis focused on the set-up with coexistence of two types of expectations formation. Clearly, one may study a NK model with heterogeneous expectations that allows for coexistence for three or more different types of expectations. Such a study may serve to robustify our findings.

Alternatively, replacing one of the types of expectations considered herein could serve the purpose of a higher degree of robustness in a similar way. Branch and McGough (2009, p.14) mention this idea.

Future research may also aim to study Taylor rules in larger scale versions (e.g. capital accumulation or monetary and fiscal policy interactions) of the NK model with heterogeneous expectations.

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