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22 May 2011

Online at <https://mpra.ub.uni-muenchen.de/31059/>

MPRA Paper No. 31059, posted 22 May 2011 23:13 UTC

Should Economists Listen to Educational Psychologists?

Some Economics of Student Motivation

Jocelyn Donze* and Trude Gunnes†

May 22, 2011

Abstract

This paper sheds light on the role of student motivation in the success of schooling. We develop a model in which a teacher engages in the management of student motivation through the choice of the classroom environment. We show that the teacher is able to motivate high-ability students, at least in the short run, by designing a competitive environment. For students with low ability, risk aversion, or when engaged in a long-term relationship, the teacher designs a classroom environment that is more focused on mastery and self-referenced standards. In doing so, the teacher helps to develop the intrinsic motivation of students and their capacity to overcome failures.

Keywords: Education, Student Achievement, Intrinsic and Extrinsic Motivation, Effort, Goal Theory.

JEL Codes: D03, I21

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“There are three things to remember about education. The first is motivation. The second is motivation. The third is motivation.” Terrel Bell, former Secretary of Education, the Reagan Administration.

1 Introduction

Most schools suffer from widely recognized gaps between their mandates - the high academic and social achievement of their students - and their performance. To a large extent, these gaps can be explained by what is often mentioned as the most important problem in education: A lack of student effort. Education economists have thus far explained the under provision of effort by students' myopic behavior, cognitive problems, as well as some adverse effects of different institutional factors such as the type of grading system and the level of educational standards.¹ What is often ignored in this literature, however, is what educational psychologists consider to be the primary determinant of students' effort, i.e. their motivation (see for example Wigfield, Eccles, Roeser, Schiefele, 2009). Educational psychologists regard motivation, defined as the force that moves students to do something, as a complex and multifaceted object to analyze. Motivation is mainly affected by four factors: the extrinsic valuation of students for the task to achieve, their intrinsic interest for learning, their self-concept of ability and their perception of control. A further complication is that these factors evolve over time and are context dependent. They change with students' various stages of development and previous academic performances. Educational psychologists also insist that teachers can alter motivational factors by designing the classroom environment (Ames, 1992). By promoting a competitive environment, a teacher helps to develop students' extrinsic valuing of achievement; by emphasizing the importance of understanding and mastering, the teacher reinforces students' intrinsic interest for learning.

We develop a model in this article to explore students' motivation and to study how teachers should engage in its management by choosing the classroom environment that best

¹We review the economic literature below.

matches students' motivational patterns. To the best of our knowledge, this paper is the first theoretical economic analysis of the interplay between the classroom environment, student motivation and achievement. We first study the management of motivation over the short run: For given characteristics of the students, what is the best way for a teacher to help them accomplish success? We next study the management of motivation over the long run: How can the teacher design a classroom environment to maintain students' motivation, most notably if they experience failure? To answer these questions, we use a principal-agent framework in which the agent (student) is endowed with both an extrinsic and intrinsic source of utility. To further incorporate the psychological context, we rely on the achievement goal theory emanating from educational psychology (Nicholls, 1984; Dweck, 1986; Ames, 1992). This theory explains students' motivation in terms of goals they hold when they engage in an academic task. Goals can be understood as a student's subjective representation of the purposes of the task, the way success is defined, and the role of effort and ability in achievement. They are posited to influence achievement through their impact on study strategies. The achievement goal literature considers three types of goals that can coexist within the same individual (Elliot, 1999): Students with a *mastery goal* focus on learning and understanding, developing new skills, and achieving a sense of mastery based on self-referenced standards. Central to this orientation is the student's belief that effort leads to mastery, and hence contentment. Students with a *performance goal* focus on the demonstration of academic talent; they want to obtain high grades or outperform other students. They hold the belief that performance strongly depends on ability. Students with an *avoidance goal* want to avoid exhibiting any incompetence in order to preserve their self-confidence. They tend to withdraw from challenging tasks, and not participate in classroom activities.

The teacher influences the personal goals adopted by the students through the choice of the classroom environment. A *mastery goal structure* refers to pedagogical practices that emphasize learning, understanding and personal improvement. A *performance goal structure* refers to practices which emphasize competition, grades, and rankings. The two previous structures can be seen as opposite ends of a continuum. A structure between these two

extremes is referred to as a *multiple goal structure*.

A considerable amount of evidence reveals that the goals adopted by a student shape their study behavior and academic achievements (see Anderman and Wolters (2006) for an overview). Mastery goals generate adaptive study behaviors such as effort, deep processing of the learning material, task enjoyment and persistence in the face of difficulties or failure. These goals favor a long-term investment in learning. Interestingly, the positive outcomes seem to appear regardless of the ability of the student (Elliot and Dweck, 1988). Nevertheless, empirical studies do not establish a direct link between mastery goals and academic achievement. This puzzling result has been explained by the fact that exams consist of multiple choice questionnaires, which may favor surface over deep learning. An alternative explanation states that mastery-oriented students pursue their own learning agenda and spend quite a bit of time on personally interesting material not relevant for the test (Senko and Miles, 2008). Performance goals generate effort and surface learning, and favor academic achievement, particularly for high-ability students. The positive relationship between performance goals and grades has been explained by the fact that students with performance goals seek to align their learning agenda with that of the teacher by carefully trying to identify the assessment criteria. Even so, several researchers have suggested that performance goals could also damage students' self-concept of ability when working hard does not lead to success (Covington and Omelich, 1979; Skaalvik, 1997). As a result, it could be more difficult for performance-goal students to preserve their level of engagement over the long run. Finally, avoidance goals are generally regarded as undesirable, and are related to poor educational outcomes such as self-handicapping behaviors, low effort and low grades.²

Not surprisingly, educational psychologists generally favor mastery goals or multiple goals as the best way to induce constructive behaviors for a wide range of students. In contrast, education economists have relied more on students' reason than emotions, thereby neglecting essential motivational factors. The main objective of this paper is to study whether educa-

²Self handicapping refers to a choice that prevents a student from feeling responsible for failure such as partying the night before an exam.

tion economics can learn from the lessons of educational psychology to better understand students' motivation.

To accomplish this, we develop a framework with two agents: a teacher and a representative student with a learning task to achieve. The student is endowed with an exogenous ability and initial goal orientation. The teacher chooses a classroom structure that can be more or less performance or mastery oriented. The student observes the structure chosen by the teacher, modifies its goal orientation, and exerts effort. There is a test at the end of the period to verify whether the learning task has been successfully accomplished. The grade is an increasing function of the student's ability and effort. We make two assumptions which structure the model:

- The teacher acts in a test-based accountability environment, and internalizes this environment by choosing a classroom structure that maximizes the student's grade.
- A classroom structure more oriented towards performance goals causes the student to align his or her learning agenda with that of the teacher. *Ceteris paribus*, this increases the efficiency of effort, thereby making it easier for the student to succeed in the test.

An interesting feature of the model is that despite these two "procompetitive" assumptions, promoting a mastery-oriented goal structure will nevertheless be the optimal policy for the teacher in many different circumstances. We first consider a static framework in which the student is only endowed with two goals: performance and mastery. Consistently with the findings of goal-theory literature, we show that choosing a mastery-oriented goal structure permits the teacher to induce a level of student effort independent of ability: by focusing on mastery, the teacher avoids relying too strongly on the ability-performance connection related to a pure performance goal. The consequence of this is that the teacher chooses a mastery-oriented classroom structure for low- or intermediate-ability students. For a high-ability student, the teacher chooses a performance goal structure. In doing so, the teacher induces an efficient effort and thus a high grade.

We consider three variants of the static model. In the first variant, we assume that

the student can obtain an exogenous level of utility by choosing not to exert any effort, which corresponds to introducing an avoidance goal for the student. The introduction of the participation constraint causes the teacher to choose a structure more oriented towards mastery goals if the student's ability is not too high. In our framework, the teacher's unique objective is to maximize the test result so that the teacher chooses a structure that is too performance-oriented compared to the student's aspirations. To prevent the student from adopting the avoidance goal, the teacher must align his or her objectives more closely with those of the student by increasing the mastery goal structure. In the second variant, we study the case in which the student is risk adverse. Risk aversion signifies that the student dislikes not having complete control over the test result. We show that for a given classroom structure, a higher risk aversion causes a student with a low or intermediate ability to exert less effort in order to reduce the variance of the outcome, even if the probability to fail the test increases. At equilibrium, the teacher chooses a structure more oriented towards mastery goals than under risk neutrality. By doing so, the teacher favors the student's intrinsic motivation, which is not affected by the risk, thus remotivating effort. In the third variant, we introduce a cost difference between the different classroom structures.

The choice of the classroom structure also affects the way a student reacts if he or she fails the test. To study the management of student motivation over the long run, we introduce a dynamic (two-period) version of the model. We suppose that failing the test in the first period negatively affects the probability of succeeding in the second period. We also assume that failing reduces the student's intrinsic interest for the task, *unless* the teacher has initially chosen a classroom structure sufficiently oriented towards mastery goals. We show that if the teacher is sufficiently patient, a mastery-oriented structure is chosen in the first period, even if it makes the student a bit less successful in this period. In doing so, the teacher is able to keep the student on track in the second period, even after a failure.

This article proceeds as follows: Section 2 yields an overview of the related economic literature. Section 3 presents the static framework, whereas section 4 presents the dynamic version. Section 5 concludes.

2 Related Literature

This article relates to the new microeconomics of education that considers students' effort as the most important input to education production. This literature studies strategic interactions between teachers and students. For example, Correa and Gruver (1987) and De Fraja et al. (2010) consider the case in which effort levels provided by the various participants in the education process are strategic substitutes. Bishop (1994) relates the classroom to a classic prisoner dilemma situation in which students pressure each other not to study because of being graded on a curve (i.e. a relative grading system) while teachers are pressured to pass students in order to keep failure rates low. In an empirical study, Bonesrønning (2004) shows that hard grading leads to improved achievements. However, he argues that hard grading is less likely to occur in an environment with a competition among schools. Akerlof and Kranton (2002) focus on how students care about their social position in school, and how they try to fit in with their peers. They state that it is rational for students such as "burnouts" to provide a low effort when this corresponds to the ideal amount of their social identity group.

Much of the existing literature focuses on the *level* of effort rather than on the *source* or the *efficiency* of effort. In addition, the effort-performance relationship is static and does not vary after a failure. Our model attempts to capture these elements: We study how different types of motivation stem from the goal orientation of the student and affect the level and efficiency of effort. We also take into account that students might feel incompetent after a failure.

This article further relates to the literature on accountability systems and educational standards. Even though these recent reforms have been anticipated to ensure that all students - regardless of any existing disadvantage - benefit from significant achievement gains in school, the results have been mixed: beside their positive effects, higher standards may also discourage the marginal student who finds it better to drop out of school (Costrell, 1994; Betts, 1998). The same goes for test-based accountability systems which are put in place as an attempt to extract more effort from teachers. Nonetheless, they may trigger teachers to

game the system and induce a change in the way teachers distribute their effort among students of different abilities (Hanushek and Raymond, 2002; Neal and Schanzenbach, 2010).³ Therefore, accountability systems often increase the scores among students in the middle of the achievement distribution but not among the least academically advantaged students.

Our model states that a sufficiently patient teacher can succeed in motivating low-ability students, therefore establishing a sound accountability. This stands in sharp contrast to short-term strategies such as preemptively retaining students from taking the test or substituting away from low-stakes subjects.

This article is also associated with the literature developed by psychologists and pursued by economists dealing with intrinsic motivation. For over three decades, researchers in psychology have debated as to whether external incentive programs inhibit the agent's intrinsic motivation and performance, which is a phenomenon referred to as the crowding out effect (Fehr and Falk 2008; Frey and Jegen, 2001; Ryan and Deci, 2000). For a long time, economists have primarily considered the positive relationship between external incentives and outcomes. Recently, however, researchers in contract theory have studied how contracts should be modified in order to take both explicit and implicit incentives into account (Kreps, 1997; Bénabou and Tirole, 2003). Notably, this literature identifies conditions under which extrinsic rewards are harmful instead of beneficial. Along these dimensions, we study how the student's motivational pattern depends on the choice of the principal. In our model, however, the principal's choice also affects the efficiency of effort.

³Teachers may for instance increase the use of special education placements, “teach for the test” and substitute away from low-stakes subjects (Jacob, 2002).

3 Static Management of Student Motivation

We consider a model with a teacher (she) and a representative student (he) who interact during one period. There is complete information. First, we consider a framework in which the student has two possible achievement goals, performance and mastery, and is risk neutral regarding the test result. Thereafter, we extend our analysis to include an avoidance goal before finally dealing with a risk-adverse student.

3.1 The Model with Performance and Mastery Goals

The student. He has knowledge to acquire. There is a test at the end of the period to verify whether the knowledge has been acquired or not.⁴ The student can either pass or fail the test. The student has a (cognitive) ability, $\theta \in [0, 1]$, and he exerts some effort $e \in [0, 1]$. Having a higher ability and/or exerting a higher effort increase the probability of passing the test. We denote X as the random variable equal to 1 when the student is successful and 0 otherwise. The student is endowed with a goal orientation.

The teacher. She chooses a classroom structure, $s \in [0, 1]$. A low s means that the teacher favors a mastery goal structure, while a high s means that the teacher favors a performance goal structure. The choice of the classroom structure alters the student's goal orientation. We assume that the teaching costs are not affected by the choice of the classroom structure. We will relax this assumption thereafter.

Test result. We assume that the result is equal to

$$X = \begin{cases} 1 & \text{with probability } \theta es \\ 0 & \text{with probability } 1 - \theta es \end{cases} \quad (1)$$

The probability of passing the test is increasing in the student's ability, θ , and effort, e . It is also increasing in s , i.e. when the teacher chooses a classroom structure more oriented

⁴We suppose throughout the paper that the test result is a perfect indicator of the acquisition of knowledge.

towards performance goals. When the teacher stresses performance, the student becomes more attentive to her demands so that the efficiency of effort and the probability of passing the test increase.

Payoffs. We assume the student is risk neutral. Following Kreps (1997), Akerlof and Kranton (2002) and Bénabou and Tirole (2003), among others, we assume that the student has two sources of utility: extrinsic and intrinsic, which correspond to performance and mastery goals, respectively. We use the following separable utility function:

$$\theta es + \gamma e(1 - s) - 0.5e^2 \tag{2}$$

where $\gamma \in [0, 2]$. The first term represents the extrinsic satisfaction associated with the task. We take it equal to the expected value of the test result, $E(X)$. The second term represents the intrinsic satisfaction from acquiring knowledge, i.e. the joy of learning, and is increasing in effort. It is also decreasing in s : the student's intrinsic satisfaction diminishes as the teacher becomes more oriented towards a performance goal structure. Parameter γ reflects the (relative) propensity for intrinsic motivation. When γ is equal to 0, the student has a pure performance goal: only the extrinsic motivation matters. When γ is equal to 2, mastery goals become the preeminent goal orientation of the student. We will show that in this case, the student exerts the maximum level of effort even when his ability is nil. The third term is the cost to exert effort.

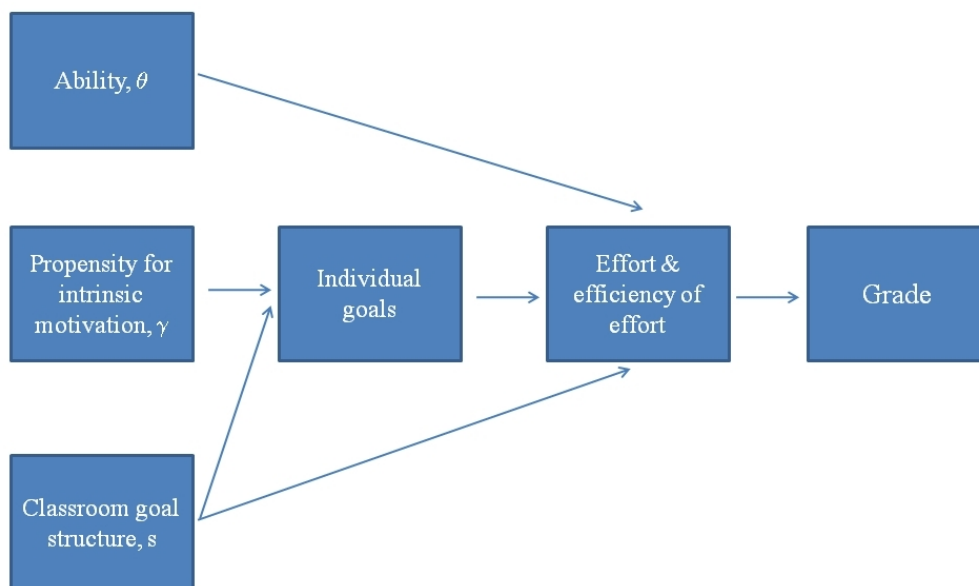
The teacher is risk neutral. We assume that she acts in an accountability environment and that her payoff is equal to the expected value of the test result, $E(X)$.

Timing of the game.

- The teacher chooses a classroom structure, $s \in [0, 1]$.
- The student observes s and exerts effort $e \in [0, 1]$.
- The student takes the test and obtains a result X .

The structure of the model is represented in Figure 1.

Figure 1: Structure of the Model



3.2 The Motivational Equilibria

We consider the maximization problem of the student for a given classroom structure s .

Solving

$$e^*(s) = \arg \max_{e \in [0,1]} \theta es + \gamma e(1-s) - 0.5e^2 \quad (3)$$

yields

$$e^*(s) = \min \{ \gamma + (\theta - \gamma)s, 1 \} \quad (4)$$

When $\theta > \gamma$, the student is characterized by possessing a high ability and/or his extrinsic interest for the task is relatively more pronounced than his intrinsic interest. In this case, effort increases as the teacher chooses a more performance-oriented classroom structure. When $\theta < \gamma$, effort increases as the teacher chooses a more mastery-oriented classroom structure. Note that when $\gamma > 1$, we are necessarily in the latter case.

We now consider the maximization problem of the teacher. We have

$$s^* = \arg \max_{s \in [0,1]} \theta s e^*(s) \quad (5)$$

The solution is

$$s^* = \begin{cases} \frac{1}{2} \frac{\gamma}{\gamma - \theta} & \text{if } \theta \leq \gamma/2 \\ 1 & \text{if } \theta \geq \gamma/2 \end{cases} \quad (6)$$

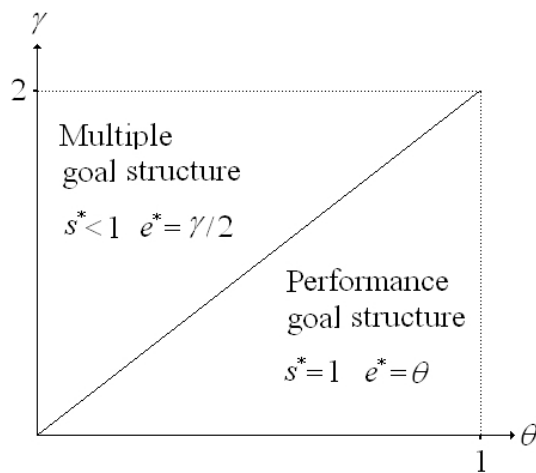
The equilibrium is fully described by expressions (4) and (6). At equilibrium, the student's effort, $e^*(s^*)$ is equal to $\gamma/2$ when $\theta \leq \gamma/2$ and equal to θ when $\theta \geq \gamma/2$. We sum up the results in the following proposition.

Proposition 1 *At equilibrium, the teacher chooses a performance goal structure when the student's ability is higher than half the propensity for intrinsic motivation. Otherwise, the teacher chooses a multiple goal structure. As a result of this, she induces an effort that is independent of the student's ability.*

The equilibria are represented in Figure 2 and the equilibrium path and payoffs are described in Table 1. When the ability θ is higher than the propensity for intrinsic motivation, γ , both the effort and its efficiency increase as the teacher chooses a structure more oriented towards performance. Thus, the teacher chooses a performance goal structure, $s^* = 1$. When θ is between $\gamma/2$ and γ , the teacher still chooses $s^* = 1$. In this case, the student's effort would be higher if the classroom structure was more oriented towards mastery goals. However, this mastery-induced effort would be less efficient and the probability of the student passing the test would decrease. When θ is below $\gamma/2$, the teacher chooses a classroom structure more oriented towards mastery goals: $s^* = \gamma/(2\gamma - 2\theta)$. In our framework s^* is always larger than $1/2$: A pure mastery goal structure would nullify the efficiency of the effort. In fact, at equilibrium, the teacher chooses a multiple goal structure that combines performance and mastery goals. In doing so, she induces an effort level $e^*(s^*) = \gamma/2$, independent of the student's ability θ . For the teacher, the benefit of breaking the ability-effort connection is higher than the loss coming from the reduced efficiency of this effort. Note that the case in which a multiple goal structure is optimal ($\theta \leq \gamma/2$) also applies to a high-ability student when his propensity for intrinsic motivation is very pronounced (close to 2).

Our results are broadly consistent with empirical findings in educational psychology literature. Elliott and Dweck (1988) have found that performance goals enhance the effort

Figure 2: Equilibria in the static framework



and achievement for students with high cognitive abilities. Other researchers have found that mastery goals induce positive patterns of learning regardless of the actual or perceived students' ability (Nicholls, 1984; Bandura and Dweck, 1985; Elliott and Dweck 1988).

	s^*	$e^*(s^*)$	U^{t^*}	U^{p^*}
If $\theta \geq \gamma/2$	1	θ	θ^2	$\frac{\theta^2}{2}$
If $\theta \leq \gamma/2$	$\frac{1}{2} \frac{\gamma}{\gamma-\theta}$	$\frac{\gamma}{2}$	$\frac{1}{4} \frac{\gamma^2 \theta}{\gamma-\theta}$	$\frac{\gamma^2}{8}$

Table 1: equilibrium payoffs

We now consider three extensions of the static model : 1) the teacher needs to take into account a student's participation constraint, which represents an avoidance goal; 2) the student is risk averse; and 3) there is a cost difference between performance and mastery classroom structures.

3.3 Extensions: Participation Constraint, Risk Aversion, and Cost Difference

Participation constraint and avoidance goal. We assume that the student agrees to participate only if he obtains a utility no smaller than some level \underline{U}^s . We interpret \underline{U}^s

as the utility that the student obtains by adopting an avoidance goal, that is deliberately deciding not to participate in classroom activities. In this case, failure is certain although the student's self-worth is not damaged. We take $\gamma \leq 1$ and we assume that $\gamma^2/8 < \underline{U}^s \leq \gamma^2/2$.⁵ For a given classroom structure s , the expression (4) of the student's effort becomes $e^*(s) = \theta s + \gamma(1 - s)$. The associated student's utility is $U^p = 0.5(\theta s + \gamma(1 - s))^2$. The teacher maximizes the expectation of the grade, $\theta e^*(s)s$, subject to the participation constraint $U^p \geq \underline{U}^p$.

If $\theta \leq \gamma/2$, the utility of the student in the unconstrained world, $\gamma^2/8$, is below the reservation utility, \underline{U}^p . Therefore, the teacher needs to align her preferences with the student's in order to prevent avoidance. To do so, she diminishes s^* from the unconstrained level, $\frac{1}{2} \frac{\gamma}{\gamma - \theta}$, down to the level s_p^* satisfying $0.5(\theta s^* + \gamma(1 - s^*))^2 = \underline{U}^p$, that is, $s_p^* = \frac{\gamma - \sqrt{2\underline{U}^p}}{\gamma - \theta}$. If $\gamma/2 \leq \theta \leq \gamma$, the teacher diminishes s^* from the unconstrained level, 1, down to the level $s_p^* = \min \left\{ 1, \frac{\gamma - \sqrt{2\underline{U}^p}}{\gamma - \theta} \right\}$. If $\theta \geq \gamma$, the teacher chooses $s_p^* = 1$. The associated student utility is $\theta^2/2$, which is larger than $\gamma^2/2$: The participation constraint is satisfied. The results are summarized in the following proposition.

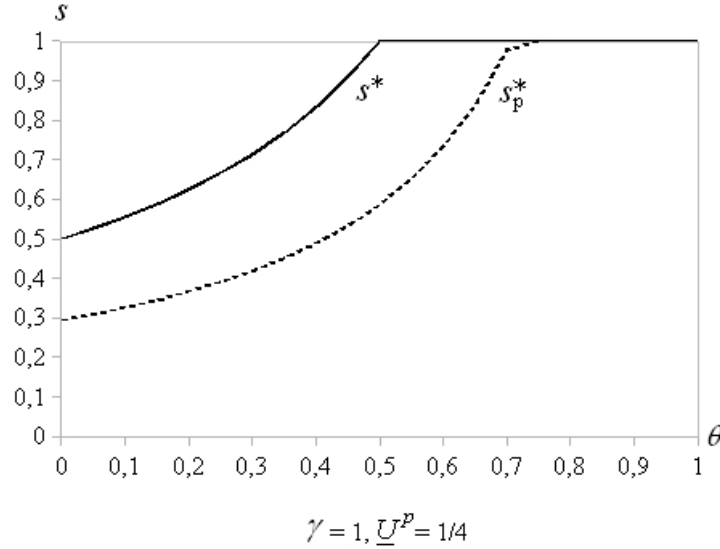
Proposition 2 *Introducing a participation constraint for the student causes the teacher to increase the mastery goal structure when the student's ability is low or intermediate in order to prevent him from adopting an avoidance goal.*

We compare the classroom structure both with and without the participation constraint in Figure 3 for $\gamma = 1$ and $\underline{U}^p = 1/4$. For a given classroom structure, a small level of ability increases the student's inclination to adopt an avoidance behavior to preserve his self-esteem. By increasing the mastery goal structure, the teacher in this case can develop the intrinsic satisfaction of the student, thus keeping him away from avoidance.

⁵Condition $\underline{U}^p > \gamma^2/8$ guarantees that the reservation utility is sufficiently high so that the participation constraint will play in some cases. Condition $\underline{U}^p \leq \gamma^2/2$ guarantees that the avoidance goal is not too desirable for the student. Condition $\gamma \leq 1$ guarantees that $\theta f + \gamma(1 - f)$ is smaller than 1 for any θ and f and permits to avoid taxonomy.

Our result is consistent with empirical findings from the achievement goal literature. For example, Turner, Midgley, Meyer, Gheen, Anderman, Kang, and Patrick (2002) have observed that a mastery goal structure is negatively associated with avoidance and self-handicapping behaviors.

Figure 3: Choice of classroom structure, with and without a participation constraint



Risk aversion and perception of control. We return to the case with no participation constraint. We now assume that the student is risk averse whereas the teacher is still risk neutral. We use a mean-variance framework: For a given expectation of the grade, the student’s extrinsic satisfaction diminishes as the variance of the result increases. Risk aversion expresses the student’s concern for having an incomplete control over the test result. The student’s utility takes the following shape

$$E(X) - rV(X) + \gamma e(1 - s) - 0.5e^2$$

where $0 \leq r < 1$ is a measure of risk aversion and $V(X) = \theta es(1 - \theta es)$. At equilibrium, student’s effort is

$$e_r^*(s) = \min \left\{ 1, \frac{\theta(1 - r)s + \gamma(1 - s)}{1 - 2r\theta^2 s^2} \right\} \quad (7)$$

Interestingly, a more risk adverse student does not always provide a higher level of effort. A sufficient condition for $\frac{\partial e_r^*}{\partial r} < 0$ is $\theta < \frac{\sqrt{2}}{2}$: effort is a decreasing function of risk aversion

for students with a low or intermediate ability. In fact, exerting less effort permits these students to diminish the risk/variance, even if the probability of succeeding decreases. We also have $\frac{\partial e_r^*}{\partial r} > 0$ if $\theta > \frac{\sqrt{2}}{2}$ and $s = 1$: Under a performance goal structure, high-ability students exert more effort as risk aversion increases in order to reduce risk.⁶ Solving the teacher's maximization problem yields

$$s_r^* = \min \left\{ 1, \frac{\gamma - \theta(1-r) - \sqrt{(\gamma - \theta(1-r))^2 - 2r\gamma^2\theta^2}}{2r\gamma\theta^2} \right\} \quad (8)$$

One can verify that when $\gamma \leq \sqrt{2}$, we have $s_r^* \leq s^*$ for any θ and any r , where s^* is given by expression (6). The inequality is strict for low levels of ability. The student's risk aversion makes the teacher choose a structure more oriented toward mastery goals because she then diminishes the importance of risk for the student, therefore eliciting more effort. The policy s_r^* is illustrated in the left part of Figure 4 for $r = 1/2$ and $\gamma = 1$.

When $\gamma \geq \sqrt{2}$, we have $s_r^* < s^*$ for $\theta < 2\frac{\gamma}{\gamma^2+2}$ and any r , and $s_r^* \geq s^*$ for $\theta \geq 2\frac{\gamma}{\gamma^2+2}$ and any r . In this case, the teacher chooses a classroom structure more oriented toward performance goals for intermediate- and high-ability levels. Putting more pressure on these students is efficient as they then exert more effort to better control the test result. Note that this teacher behavior is not possible when γ is smaller than $\sqrt{2}$, because the teacher already has a performance structure as an optimal strategy for intermediate and high ability students. The structure s_r^* is illustrated in the right part of Figure 4 for $r = 0.5$ and $\gamma = 1.8$.

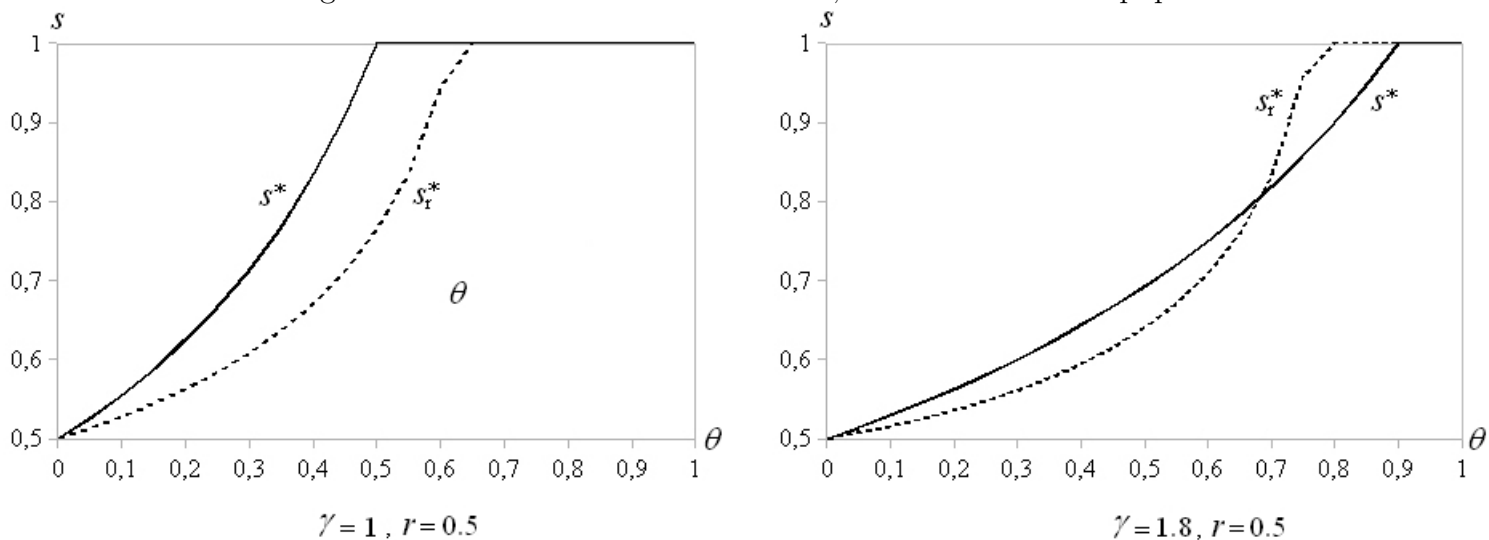
The results are summarized in the following proposition.

Proposition 3 *A higher student risk aversion makes the teacher increase the mastery goal structure when the student's ability is low or intermediate.*

Skinner et al. (1998) show that children who believe teachers are supportive and care about their progress develop a more positive sense of control over their outcomes. They are

⁶These results relate to the insurance literature on self protection in which a risk-adverse agent can increase the probability of success by exerting a costly self-protection activity (e.g. effort). Contrary to what intuition suggests, in this literature a more risk adverse agent does not necessarily exert more effort (Dionne and Eeckhoudt 1985; Jullien, Salanié and Salanié; 1999).

Figure 4: Choice of classroom structure, with a risk-averse pupil



less anxious and perform better academically. This is consistent with our prediction that when faced with a risk-averse student, the teacher increases the mastery goal structure to help reduce the student's perception of risk.

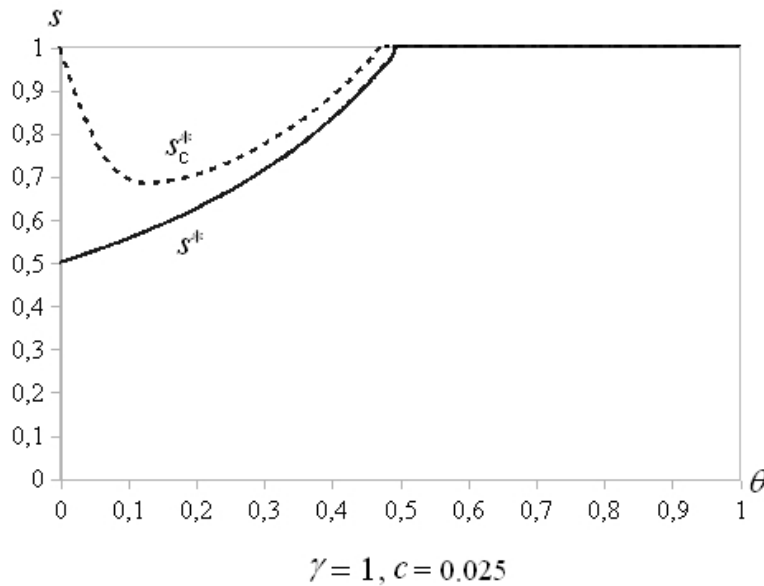
Cost differences between classroom structures. Thus far, we have assumed that changing the classroom structure does not modify the costs related to teaching. This explains why costs do not appear in the utility function of the teacher. Nevertheless, it seems reasonable to think that mastery oriented classroom structures require more involvement, more attention and more effort from the teacher. In other words, it is more costly to design an adequate mastery structure than a performance structure. We take into account the cost difference between structures by rewriting the teacher's objective function as $\theta es - c(1 - s)$ with $0 < c \leq \gamma^2/8$. The teaching cost is higher as the teacher increases the mastery classroom structure.⁷ The optimal student's effort is $e^*(s) = \min \{\gamma + (\theta - \gamma)s, 1\}$. Solving $\max_{s \in [0,1]} \theta e^*(s)s - c(1 - s)$ gives

$$s_c^* = \begin{cases} 1 & \text{if } \theta \leq \frac{\gamma - \sqrt{\gamma^2 - 8c}}{4} \\ \frac{c/\theta + \gamma}{2(\gamma - \theta)} & \text{if } \frac{\gamma - \sqrt{\gamma^2 - 8c}}{4} \leq \theta \leq \frac{\gamma + \sqrt{\gamma^2 - 8c}}{4} \\ 1 & \text{if } \theta \geq \frac{\gamma + \sqrt{\gamma^2 - 8c}}{4} \end{cases}$$

⁷One could replace the 1 in $c(1 - f)$ by any positive number higher than 1 without altering the result.

It is represented in Figure 5 for $\gamma = 1$ and $c = 0.025$. Not surprisingly, the teacher chooses a more performance oriented structure than when there is no cost difference: $s_c^* \geq s^*$. More interestingly, a performance classroom structure becomes the optimal policy for (very) low-ability students. A multiple goal structure would induce a higher effort for these students. Yet from the teacher's point of view, the increase in performance would not compensate for the increase in cost related to the multiple structure. Hence, the teacher is better off promoting a performance goal structure.

Figure 5: Classroom structures under different cost structures



Up until now, we have focused on how a teacher, by choosing the classroom structure, can accompany the student towards the test to help facilitate success. In other words, we have dealt with the management of motivation over the short run. However, the choice of the classroom structure also affects the way a student reacts to the test result, most notably when he has failed. To study the management of student motivation over the long run, we introduce a dynamic (two-period) version of the model.

4 Dynamic Management of Student Motivation

We consider a twice repeated version of the static model. We assume that a failure in the first period affects the student's attitude towards schooling in two ways: First, the probability to succeed in the test of the second period decreases. This assumption may come from the cumulative nature of knowledge, or alternatively, the failure may damage the student's self-confidence so that succeeding in the subsequent test becomes harder. Second, we assume that the failure negatively affects student's intrinsic motivation in the second period unless the teacher chooses a classroom structure that is sufficiently mastery oriented in the first period.⁸ Hence, we treat the design of a mastery-oriented structure as a long-run investment in the student's intrinsic motivation. Within this framework, the teacher faces a trade-off between promoting high grades in the short run through a performance structure, or permitting the student's to overcome a potential failure by implementing a mastery structure.

4.1 The Dynamic Model

There are two periods denoted by $t = 1, 2$. To concentrate on the dynamic issues, we focus on a student with a balanced motivational propensity, $\gamma_1 = 1$, and we consider the range of ability levels for which a pure performance structure is optimal in the static framework, $\theta \geq 1/2$.

The student. At period t , the student exerts an effort e_t . We denote by X_t the random variable equal to 1 if the test in period t is successful and 0 otherwise.

The teacher. She chooses a classroom structure $s_t \in [0, 1]$. As before, a higher s_t means that the structure is more performance-goal oriented. A smaller s_t means that the structure is more mastery-goal oriented. The teacher's payoff function for period t only depends on

⁸In educational psychology, there is a vast body of empirical evidence supporting the idea that, after a failure, performance-goal oriented students report more negative self-related thoughts and less interest for the learning task than mastery-goal oriented students (for example Dweck and Legett, 1988).

the expected test result in this period, $E(X_t|h_t)$, where h_t is the history of the game at the beginning of period t .

Test result. For period one, we take

$$X_1 = \begin{cases} 1 & \text{with probability } \theta e_1 s_1 \\ 0 & \text{with probability } 1 - \theta e_1 s_1 \end{cases} \quad (9)$$

We assume that the probability is unchanged after a success but negatively affected after a failure: If the realized value of X_1 , is equal to 1, then

$$X_2 = \begin{cases} 1 & \text{with probability } \theta e_2 s_2 \\ 0 & \text{with probability } 1 - \theta e_2 s_2 \end{cases} \quad (10)$$

However if the realized value of X_1 is equal to 0 then

$$X_2 = \begin{cases} 1 & \text{with probability } \frac{\theta e_2 s_2}{2} \\ 0 & \text{with probability } 1 - \frac{\theta e_2 s_2}{2} \end{cases} \quad (11)$$

Propensity for intrinsic motivation. We assume that the intrinsic motivation is unaffected after a success: $\gamma_2(1) = 1$. Nonetheless, after a failure in period one, the propensity for intrinsic motivation in period two depends on the classroom structure chosen by the teacher in the first period. We have:

$$\gamma_2(0) = \begin{cases} 1 & \text{if } s_1 \leq \hat{s} \\ 0 & \text{if } s_1 > \hat{s} \end{cases}$$

for a given $\hat{s} < 1$. In other words the teacher can preserve the intrinsic motivation of the student in period two after a failure by choosing a classroom structure sufficiently oriented towards the mastery goals in the first period. Otherwise, the intrinsic motivation vanishes.

Payoffs. The student is risk neutral. His expected payoff in period t after the history h_t is

$$E(X_t|h_t) + \gamma_t(h_t)e_t(1 - s_t) - 0.5e_t^2 \quad (12)$$

We have $h_1 = \emptyset$, $h_2 \in \{0, 1\}$ and by assumption, $\gamma_1(h_1) = 1$. We assume that the total payoff of the student is the discounted sum of his per-period payoffs. Let δ^p denote his discount factor.

The teacher's expected payoff in period t is $E(X_t|h_t)$. We assume that the total payoff of the teacher is the discounted sum of her per-period payoffs. Let δ^t denote her discount factor.

Timing of the game and strategies. At each period $t = 1, 2$,

- The teacher chooses a classroom structure $s_t \in [0, 1]$.
- The student observes s_t and exerts an effort level $e_t \in [0, 1]$.
- The teacher and the student observe the realized value of X_t .

Strategies are $s_1, s_2(1), s_2(0)$ for the teacher and $e_1, e_2(1, s_2)$ and $e_2(0, s_2)$ for the student.

4.2 The Subgame Perfect Equilibrium

We solve the second period (sub)game following a classroom structure s_1 and an effort e_1 . We determine student's effort and teacher's choice of classroom structure conditionally to the test result of period one.

Suppose the student is successful, $X_1 = 1$, then we have

$$e_2^*(1, s_2) = (\theta - 1)s_2 + 1 \quad (13)$$

and

$$s_2^*(1) = 1 \quad (14)$$

This follows from expressions (4) and (6) and $\theta \geq \gamma_1/2 = 1/2$. After a success in period one, the probability of the student passing the test is sufficiently high to justify that the teacher chooses a performance goal structure in period two. From Table 1, we know that at equilibrium, the (expected) payoff for the student in period two is $U_2^{p*}(1) = \theta^2/2$ and the expected payoff for the teacher is $U_2^{t*}(1) = \theta^2$.

Suppose the student fails the test in period one, $X_1 = 0$. His probability of success in period two decreases. We consider two cases:

- (i) $s_1 > \hat{s}$: the teacher favors a performance goal structure in period one. In this case, the intrinsic motivation of the student totally vanishes after the failure: $\gamma_2(0) = 0$. The student

chooses an effort level $e_2^*(0, s_2) = \theta s_2/2$ and the teacher chooses the classroom structure $s_2^*(0) = 1$. In period two, a performance structure is the best way to motivate a student who has lost his intrinsic interest for learning. At equilibrium, the payoff for the student in period two is $U_2^{p*}(0) = \theta^2/8$ and the payoff for the teacher is $U_2^{t*}(0) = \theta^2/4$.

(ii) $s_1 \leq \widehat{s}$: in period one the teacher favors a classroom structure sufficiently oriented towards mastery goals in order for the student to preserve his intrinsic motivation after the failure: $\gamma_2(0) = 1$. The student chooses an effort level $e_2^*(0, s_2) = (\theta/2 - 1)s_2 + 1$ and the teacher chooses the classroom structure $s_2^*(0) = \frac{1}{2} \frac{1}{1-\theta/2}$. In this case, by establishing a multiple goal structure in period two, the teacher can build on the preserved intrinsic motivation of the student to induce effort. The equilibrium payoff for the student in period two is $U_2^{p*}(0) = 1/8$. The payoff for the teacher is $U_2^{t*}(0) = \frac{1}{4} \frac{\theta/2}{1-\theta/2}$.

We solve period one knowing $e_2^*(\cdot)$ and $s_2^*(\cdot)$. For a given classroom structure s_1 , the student maximizes:

$$\theta e_1 s_1 + e_1(1 - s_1) - \frac{1}{2} e_1^2 + (\theta e_1 s_1) \delta^p U_2^{p*}(1) + (1 - \theta e_1 s_1) \delta^p U_2^{p*}(0) \quad (15)$$

We obtain:

$$e_1^*(s_1) = \begin{cases} 1 + (\theta - 1)s_1 + (\theta s_1) \delta^p \left(\frac{\theta^2}{2} - \frac{\theta^2}{8} \right) & \text{if } s_1 > \widehat{s} \\ 1 + (\theta - 1)s_1 + (\theta s_1) \delta^p \left(\frac{\theta^2}{2} - \frac{1}{8} \right) & \text{if } s_1 \leq \widehat{s} \end{cases} \quad (16)$$

In period one, the student provides more effort in the dynamic model than in the static framework. This supplementary effort is higher for a more patient student. Indeed, the existence of the second period extends the benefits of being successful in the first period, as the student's capacity to succeed the second test depends on his initial performance.

The teacher chooses s_1^* to maximize the discounted sum of her per-period payoffs:

$$\theta e_1^*(s_1) s_1 + (\theta e_1^*(s_1) s_1) \times \delta^t U_2^{t*}(1) + (1 - \theta e_1^*(s_1) s_1) \times \delta^t U_2^{t*}(0) \quad (17)$$

where $e_1^*(s_1)$ is given by (16). Two policies are potentially optimal: $s_1^* = 1$ and $s_1^* = \widehat{s}$.

The total expected payoff of the teacher when she chooses $s_1^* = 1$ is

$$\theta\left(\theta + \frac{3}{8}\delta^p\theta^3\right)\left(1 + \frac{3}{4}\delta^t\theta^2\right) + \delta^t\frac{\theta^2}{4} \quad (18)$$

The total expected payoff when she chooses $s_1^* = \hat{s}$ is

$$\left(\theta(\theta\hat{s} + 1 - \hat{s} + \theta\hat{s}\delta^p\frac{4\theta^2 - 1}{8})\hat{s}\right)\left(1 + \delta^t\left(\theta^2 - \frac{1}{4}\frac{\theta}{2 - \theta}\right)\right) + \frac{1}{4}\delta^t\frac{\theta}{2 - \theta} \quad (19)$$

We denote by $\tilde{s}(\theta)$ the particular value of \hat{s} that equalizes (18) and (19). The determinant of the corresponding second degree equation is

$$\Delta = 1 - 4\left(1 - \theta - \theta\frac{4\theta^2 - 1}{8}\delta^p\right)(\theta + \eta)$$

with

$$\eta = -\frac{8\delta^t(1 - \theta^2)(1 - \theta)^2 - 3\delta^p(4 + 3\delta^t\theta^2)(2 - \theta)\theta^3}{32(2 - \theta) + \delta^t(32\theta^2(2 - \theta) - 8\theta)}$$

We first consider the case where the discount factor of the student is nil, $\delta^p = 0$. Here, the determinant Δ is positive.⁹ We find

$$\tilde{s}(\theta) = \frac{1 - \sqrt{\Delta}}{2(1 - \theta)}$$

When $\tilde{s}(\theta)$ is below (above) \hat{s} , the structure \hat{s} yields the teacher a higher (lower) payoff than $\tilde{s}(\theta)$; as a result, the teacher prefers the structure $s_1 = \hat{s}$ ($s_1 = 1$) than the structure $s_1 = 1$ ($s_1 = \hat{s}$). Consequently, the optimal classroom structure chosen by the teacher in period one for a given ability θ and structure \hat{s} is

$$s_1^* = \begin{cases} 1 & \text{if } \hat{s} < \tilde{s}(\theta) \\ \hat{s} & \text{if } \hat{s} \geq \tilde{s}(\theta) \end{cases}$$

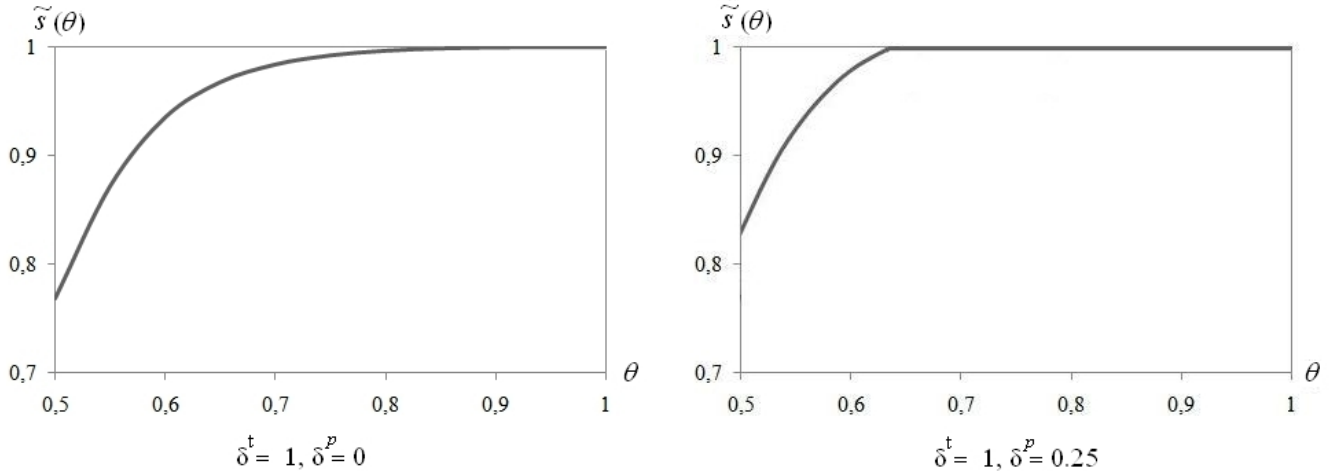
The function $\tilde{s}(\theta)$ is represented in figure 5 for $\delta^t = 1$. The lower the ability, θ , the larger the area in which the teacher chooses the multiple goal structure, \hat{s} , in the first period. By promoting both mastery and performance goals, the teacher accepts that the student performs less well in the first period in order for him to be able to overcome a possible failure. The choice of the multiple goal structure, \hat{s} , is however less appropriate for a high-ability

⁹This comes from the fact that $\eta < 0$ implies $4(1 - \theta - \theta\frac{4\theta^2 - 1}{8}\delta^p)(\theta + \eta) < 4(1 - \theta)(\theta) \leq 1$.

student for two reasons: First, it induces a significant decrease in the expected grade in the first period compared to the situation with a performance structure. Second, the probability of passing the test is larger for a high-ability student, thereby reducing the teacher’s benefit for developing the student’s failure tolerance.

Note that $\tilde{s}(\theta)$ increases as δ^t decreases and that $\tilde{s}(\theta) = 1$ for any θ when $\delta^t = 0$.¹⁰ As the teacher becomes less forward looking, he is less willing to sacrifice the student’s performance in the first period and hence to develop his failure tolerance.

Figure 6: The threshold values



We now study the effect of increasing δ^p starting from zero for a given positive value of δ^t . We have

$$\tilde{s}(\theta) = \frac{1 - \sqrt{\Delta}}{2(1 - \theta - \theta\delta^p\frac{4\theta^2-1}{8})}$$

One can verify that $\tilde{s}(\theta)$ is increasing in δ^p .¹¹ A more patient student exerts a higher level of effort in period one in order to successfully enter period two. Nevertheless, the extra effort is smaller when the structure is more mastery oriented in period one, because the student is then more “insured” against failure. For this reason, developing a multiple goal structure in the first period becomes less interesting for the teacher because it diminishes the student’s incentives to exert effort in this period. One can even verify that $\tilde{s}(\theta)$ does not exist when

¹⁰When δ^t increases, γ decreases. In turn, Δ increases and $\underline{f}(\theta)$ decreases.

¹¹Note that η is increasing in δ^s and that Δ is positive as long as $\eta \leq 0$.

δ^p is above 0.56. In this case the teacher chooses a pure performance goal structure and we are back to the results of the static case. We sum up the results in the following proposition.

Proposition 4 *In a dynamic context, the teacher, if sufficiently patient, chooses a first-period goal structure more mastery-oriented than in the static case, if the student is not too patient. This choice of structure permits the teacher to develop the failure tolerance of the student at the cost of a short run decrease in performance.*

This result corresponds to the idea of the achievement goal literature which states that by choosing a multiple goal structure, the teacher uses performance to spur an efficient effort in the short run and mastery to increase student's failure tolerance in the long run (Ames, 1992; Barron and Harackiewicz, 2001). However, the previous proposition also shows that multiple goal structures are less effective when students are more patient. Individuals' time preferences for the future tend to increase during childhood and adolescence, thus mastery goals are likely to be more useful for younger students in order to develop their long-term motivation. Hence, a student's stage of development may be an important variable moderating the effects of goals on motivation and performance.

5 Conclusion

This article studies the microeconomic foundations of student motivation in schools. Motivation is important to understand, as it is the underlying mechanism of students' effort and an influential factor in their performance. We focus on the use of the correct pedagogical policy, or more precisely the correct classroom structure as an important instrument for balancing the intrinsic and extrinsic motivational factors to help keep the students on track.

The model indicates that extrinsic rewards such as grades work well for high-ability students. Even so, for low- or intermediate-ability students, educators cannot solely rely on extrinsic rewards to foster performance, even in an accountability context. Faced with such students, the teacher should choose a classroom structure more oriented towards mastery. In doing so, the teacher is able to : (i) induce effort by developing student interest for the task, (ii) hinder these students from adopting an avoidance behavior and keep them participating in the classroom activities, (iii) motivate these students when they are risk averse by focusing less on the test result, and (iv) develop their capacity to overcome failure by maintaining their self-confidence over time.

These results suggest that in many cases economists should listen to educational psychologists: Psychologists have long advocated mastery goals or multiple goals to secure a stable motivation for a wide range of students in the educational system. On the other hand, economists have often neglected some of the students' motivational factors by only promoting pro-competitive learning environments. This could result in situations where only a small range of students are motivated to exert effort. Our work is a first step in clarifying the various circumstances under which the different types of classroom structures are beneficial to students. It could be interesting to extend the model to include informational asymmetries between the teacher and the student, or to go beyond the representative student by considering a classroom with heterogeneous students.

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