Symmetrical Information and Credit Rationing: Graphical Demonstrations

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Symmetric Information and Credit Rationing: Some Graphical Demonstrations

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1. Introduction

Credit rationing describes a situation in which lenders do not raise interest rates to clear excess demand. Since the pioneering work of Stiglitz and Weiss (1981), most of the theoretical studies on credit rationing over the past two decades have assumed *ex ante* asymmetric information to be the underlying cause of credit rationing. Although theories of informational problem provide valuable insights into the economy, the current literature's partial treatment of this subject is not without cost. For instance, non-information-related factors that also cause credit rationing may be overlooked by practitioners and by government agencies when correcting the problem.

The purpose of this paper is to use a simple model to demonstrate that credit rationing can take place even if information regarding investment projects and managerial behavior is symmetric, both *ex ante* and *ex post*, between borrowers and lenders. In particular, this paper highlights problems in the U.S. Bankruptcy Code in explaining this phenomenon. It shows that the pro-debtor bankruptcy proceedings often entail substantial costs to lenders when borrowers file for bankruptcy (Weiss 1990, Longhofer 1997), and that such costs give rise to a nonlinear function of lenders' profits with respect to interest rates. The result of the paper indicate the existence of a role for government intervention in re-designing the Code.

The model presented in this paper also investigates the role of collateral in credit rationing. It shows that the use of collateral in debt contracts can usually mitigate rationing problems. However, the paper also demonstrates that with nontrivial bankruptcy costs, even a 100 percent collateral, in the sense of Bester (1985), does not necessarily eliminate all the rationing possibilities. This result differs from Bester (1985). Another result of the model is that riskier borrowers pledge *more* collateral, which is opposite to the predictions of many asymmetric-information-based theories (*e.g.*, Bester 1985, Chan and Kanatas 1985, Besanko and Thakor 1987). The existence of this positive relationship between collateral and credit risk draws empirical support from Berger and Udell (1990).

A feature of this paper is that we use a graphical approach to demonstrate credit rationing. Unlike most other studies in which credit rationing exists under sets of propositions and lemmas, such a graphical representation has the advantage of simplicity and clarity, and should be particular appealing to practitioners.

Current literature's interests in credit rationing is inspired by Stiglitz and Weiss (1981). In that paper the authors show that asymmetric information leads to adverse selection and moral hazard problems which, under certain circumstances, prohibit lenders to increase interest rates or collateral requirement when markets have excess demand. Bester (1985) shows in a similar framework that rationing can be avoided if lenders compete by choosing combinations of interest rates *and* collateral. Williamson (1986, 1987) shows that in addition to adverse selection and moral hazard problems, asymmetric information also causes costly monitoring which alone can give rise to credit rationing. Longhofer (1997) adds the element of absolute priority rule violations to Williamson's model, and shows it aggravates the rationing problem. Chan and Kanatas (1985) investigate the signaling role of collateral in loan contracts, and find that the amount of pledged collateral is positively related to the quality of borrowers. Besanko and Thakor (1987) also find such a positive relationship when collateral acts as a sorting device in credit markets. On the contrary to the theoretical predictions cited above, Berger and Udell (1990) find from a data set containing information on over one million business loans that higher amount of collateral is actually associated with riskier borrowers.

The model presented here can be seen as a generalized version of Williamson (1986, 1987) and Longhofer (1997). They are similar in that there is no *ex ante* asymmetric information, and credit rationing is attributed to bankruptcy costs. The source of cost in Williamson (1986, 1987) is from monitoring borrowers' investment return, which must be spent because of the assumed *ex post* moral hazard. A criticism, however, could be raised of whether monitoring costs, in practice, would be large enough to explain credit rationing. In this paper, bankruptcy costs are motivated by the design of the Code and the practice in the bankruptcy court. The substantial costs imposed on creditors through this channel are well established in the literature, and this should also be of more interests to practitioners. Another difference between this paper and those cited above is that we explicitly analyze the role of collateral in credit rationing. While in the absence of credit rationing collateral and interest rates may be substitutes in debt contracts, collateral assumes a very different role in the event of credit rationing because, unlike interest rates, an increase in collateral requirement does not increase the probability of default. This is discussed in section 3.1.

The rest of the paper is organized as follows. To motivate costly bankruptcy, Section 2 briefly discusses issues related to the Bankruptcy Code. Section 3.1 presents the model and Section 3.2 uses a graphical approach to analyze the model. Section 4 concludes the paper.

2. Why Bankruptcy is Costly to Lenders

There are plenty of studies in the finance literature on why creditors entail substantial costs when debtors file bankruptcy petitions. In addition to the legal and administrative expenses and the wasted managerial resources, a widely recognized factor is the failure of the bankruptcy courts to enforce the absolute priority rule (APR) (see for example Franks and Torous 1989 and Weiss 1993), which requires that (senior) creditors be repaid in full before shareholders get any payment.

Violation of the APR can be viewed as a result of the pro-debtor U.S. Bankruptcy Code. For instance, the power to choose between liquidation and reorganization is conferred upon incumbent managers, and if reorganization is chosen, which is usually the case (Weiss 1990), managers have the exclusive right to propose the first reorganization plan. Since managers are in general in favor of shareholders, the proposed plan usually benefits shareholders at the expense of creditors. Although creditors can propose their own plan if the managers' plan is not accepted by all creditors and shareholders, the potential costs in terms of time and resources make the creditors' proposal a rarity in bankruptcy courts.[1] As a result, the violation of the APR is a rule rather than an exception.[2]

Even for secured debts, the situation is not much better. The Bankruptcy Code protects secured creditors' debts only up to the value of the collateral, and it is not unusual for repossessed collateral to be found in poor condition.[3]

The potential bankruptcy cost has a significant effect on the pricing of commercial loans. Scott and Smith (1986) investigate the marginal effects of the enactment of the Bankruptcy Reform Act of 1979, and find that the cost of loans was significantly affected by the incremental disadvantages. The authors conclude that "ex-ante bankruptcy costs are important in the pricing of loans" (Scott and Smith 1986, pp. 140).

A similar ex-ante bankruptcy cost is also found in personal loan markets. Gropp *et al.* (1995) investigate the effects of different exemption levels of personal bankruptcy across states, and find that the size of a state's bankruptcy exemption has a statistically significant effect on the probability that potential borrowers in the state are denied credit and/or discouraged from applying to borrow. In this context, the exemption level limits

[1]Weiss (1990) studies thirty-seven firms that filed for bankruptcy between 1979 and 1986, and finds that only in one of the cases did the creditors create a plan.
[2]In Franks and Torous' (1989) sample from 1970 through 1989, 21 out of 27 firms departed from the APR. Weiss (1990) finds such a violation in 27 of his 37 cases, and in Eberhart *et al.* sample, it occurs in 23 out of 24 cases.
[3]Businesses that are in financial distress often neglect routine maintenance and repairs of equipment and machinery. Hard use by uncaring operators also results in abnormally high depreciation. Therefore creditors may find the saleable value of repossessed collateral to be substantially less than anticipated (Behrens 1985).

the creditors' ability to recover loans upon default.

3. Credit Rationing with Costly Bankruptcy

3.1 The Model

Borrowers

Assume there are N firms endowed with non-investment assets A_i for firm i at the beginning of the period. Each firm is given a different investment opportunity which requires capital K $\forall i = 1, \ldots, N$, and has a return factor equal to R, $R \in [0, \infty)$, with a density function $f_i(R)$. Except for the different endowment A_i and density function f_i (the two are uncorrelated), firms are otherwise homogeneous. Firms can either use A_i to buy a risk-free asset with a return factor equal to $1+r_f$, or pledge part of the asset as collateral, C_i, to borrow K and carry out the investment; $C_i \leq A_i$ we assume further for simplicity that if the risky investment is chosen, assets not pledged for collateral cannot be used to buy risk-free assets. For example, A_i could be machines and equipment necessary for production, therefore firms can either liquidate the entire assets and invest the proceeds in security markets, or pledge equipment as collateral and borrow K from banks to purchase production materials. Without loss of generality, we normalize K to 1. Behaviors of firms are common knowledge to both sides.

The firm's payoff schedule for the risky investment is (drop subscripts if no confusion may arise):

$$\pi^F = \begin{cases} R - (1+r) & \text{if there is no filing for bankruptcy,} \\ -C & \text{if there is filing for bankruptcy.} \end{cases}$$

That is, upon bankruptcy the firm has to forfeit collateral C and the residual value of the investment R to lenders. Thus a firm would file for bankruptcy if and only if

$$-C > R-(1+r), \quad \text{or,} \quad R < 1+r\text{-}C.$$

In the case of a 100 percent collateral, i.e. $C=1+r$, the firm is indifferent between filing or not, and we assume there is a positive probability of filing. Two observations emerge from the bankruptcy condition. First, while from a lender's point of view raising interest rates can compensate for a project's risk *if* the project succeeds, higher interest rates also increase the probability of bankruptcy filing as the threshold $1+r\text{-}C$ becomes higher. This tradeoff has been pointed out in early finance literature and is especially elucidated by Kim (1978). Secondly, collateral discourages bankruptcy filing ($1+r\text{-}C$ becomes lower), because it increases the opportunity cost of bankruptcy.

The expected profit from the risky investment is thus:

$$E(\pi^F) = -\int_0^{1+r-C} Cf(R)dR + \int_{1+r-C}^{\infty}(R-1-r)f(R)dR \qquad (1)$$

Using the first order conditions of (1), it is straightforward to show that, other things being equal, a firm would always prefer a lower r and a lower C. Lastly, we note that, after taking into account the option of risk-free investment, the *net* profit of the risky investment is

$$E(\pi^F) - A \times r_f, \qquad (2)$$

where is $A \times r_f$ is the risk-free return had the firm chosen to buy risk-free assets. A firm is willing to engage in risky investment only if (2) ≥ 0.

Lenders

Lenders also face two investment opportunities; they can buy the risk-free asset or lend money to firms at the interest rate r_i for firm i. The payoff schedule in relation to the lending is:

$$\pi^L = \begin{cases} (1+r) & \text{if the firm does not file for bankruptcy,} \\ R+C-B & \text{if the firm files for bankruptcy,} \end{cases}$$

where B is the bankruptcy cost. If B=0, it implies lenders can costlessly take over the investment residual and collateral. If B=R, it implies creditors can only secure the debt up to the value of the collateral; this is the assumption used by Stiglitz and Weiss (1992). In this case we can assume the investment residual is spent on bankruptcy proceedings and distributed among the many diversified creditors and shareholders. Without loss of generality, we assume B is a linear function of R, $B = \beta + \alpha R$, and we let $\alpha>0$ and $\beta>0$ for non-zero bankruptcy costs.

After taking into account the opportunity cost of risk-free investment, lenders' profit of lending is:

$$\int_0^{1+r-C} ((1-\alpha)R + C - \beta) f(R) dR + \int_{1+r-C}^{\infty} (1+r) f(R) dR - (1+r_f). \qquad (3)$$

The first order condition of (3) with respect to r is:

$$\begin{aligned} &[(1-\alpha)(1+r-C)+C-\beta]f(1+r-C)-(1+r)f(1+r-C)+\int_{1+r-C}^{\infty} f(R)dR \\ &= [-\alpha(1+r-C)-\beta]f(1+r-C)+\int_{1+r-C}^{\infty} f(R)dR \end{aligned} \qquad (4)$$

The first two terms before the equal sign are, respectively, the increased expected loss and the decreased expected gain arising from the higher probability of default, and the third term is the increased payoff if the investment succeeds. With this equation we have

Proposition 1 *If there is no ex ante bankruptcy cost to lenders, excess demand in the market can always be cleared by raising interest rates.*

This is seen by having $\alpha=0$ and $\beta=0$ in the first order condition, and thus we have lenders' profit as a positive function of interest rates. In such case lenders can match any excess demand in the market by a higher interest rate; the higher risk of default is always justified. The other side of the coin is that with nontrivial bankruptcy cost ($\alpha>0$, $\beta>0$), lenders' profit could be decreasing in interest rates. Thus

Proposition 2 *With nontrivial bankruptcy costs, lenders might not be able to increase interest rates to clear market's excess demand.*

This is true even if the loan is 100 percent collateralized, i.e. $C=1+r$. This is different from Bester (1985)in which firms can choose combinations of $\{r, C\}$ along the line of $C=1+r$ without decreasing expected returns. The key difference here is that, in the event of default, lenders cannot fully recover the loan with $C = 1+r$ if bankruptcy cost is nonzero. In fact, even with $C=1+r+B = 1+r+\beta+\alpha R$ the first order condition in (4) is not guaranteed to be positive. The result of Bester (1985) can be treated as a special case in our model with β=*0* and *C=1+r*.

Lastly, we assume the credit market is competitive, therefore we impose a zero profit condition to lenders. This condition is not necessary for the model, but it simplifies the analysis.

$$\int_0^{1+r-C}\left((1-\alpha)R+C-\beta\right)f(R)dR+\int_{1+r-C}^{\infty}(1+r)f(R)dR-\left(1+r_f\right)=0 \qquad (5)$$

Equilibrium

Since lenders know the type of each borrower, a separating equilibrium is possible. For firm *i*, lenders can offer a menu of contracts $\{r_i^j, C_i^j\}$ that satisfy (5), and the firm chooses the one that yields the highest expected profit according to (1), provided that (2)≥0 is satisfied.[4]

3.2 Some Examples

In this section, we present a graphical analysis of credit rationing based on (2) and (5). Let us assume $f_i(R)$ is a log-normal function with mean and standard deviation equal to μ_i and α_i, respectively. The parameter set that we vary in the following discussion is {μ, σ, A, α, β, r_f}, with each parameter being defined as before. We also make sure that

[4] Or the lender can simply provide the only contract that he knows the borrower would accept. This is possible because the borrower's information is public.

in all the examples $\mu_i > 1+r_f$, so that the investments considered are socially desirable.

Contracts with No Collateral

We first consider cases in which no collateral ($C_i = 0$) is used in debt contracts.

Figure 1 illustrates a case of *no* credit rationing. It expresses a lender's and two randomly-chosen borrowers' net profits as a function of interest rates. The parameter values used in the figure are {μ=1.50, σ=0.25, A=0.5, α=0.85, β=0.1, r_f=0.07}, for Figure 1(a), and μ is changed to 1.48 for Figure 1(b). That is, we assume the investment has a 50% and 48% return, respectively, with a standard deviation equal to 0.25, where the borrower's asset (0.5) is half that of the total borrowing, and the risk-free asset's return is 7%.

As shown, the lender's profit function is concave, which is essential in deriving credit rationing results. In Figure 1(a), the set of contracts satisfying the zero-profit condition is $\Pi = \{\triangleq, \doteq\}$with interest rates $\{r_i^{\triangleq}\ r_i^{\doteq}\}$ respectively; and $r_i^{\triangleq} < r_i^{\doteq}$. Because the firm becomes strictly worse off with regard to the rates, contract $\triangleq$ will be the optimal choice. It is straightforward to see that, as the investment becomes riskier, the lender's profit curve moves downward, and the optimal contracts move to the right with higher interest rates. If we rank firms according to their optimal contract rates with firm i=1 having the lowest rate, then $r_i^{\triangleq}$ is the *prime rate*: the rate that lenders charge to the most credit worthy customers.

If there is a sufficient number of firms and investments are continuous, there must exist a firm $i = \bar{h}$ for which $r_{\bar{h}}^{A} = r_{\bar{h}}^{B}$. This is shown in Figure 1(b), and is due to the concavity of lender's net profit function. Therefore for firm $i = \bar{h} + 1$, $\prod_{\bar{h}+1} = \varnothing$, and there is no market rate for the firm. And yet the firm's profit is still positive at $r_{\bar{h}}^{A}$. The firm is thus *credit rationed*: it is denied credit although it is willing to pay interest at $r_{\bar{h}}^{A}$.

Figure 2 shows such cases. Using Figure 1(a) as the benchmark, Figure 2 shows that rationing can arise if the mean of the expected return is lower (2(a)), the uncertainty, *i.e.*, standard deviation is higher (2(b)), or the risk-free return is higher (2(c)).

Rationing arises because lenders cannot always compensate for the increased risk by raising interest rates; a higher rate yields a higher return when the loan is paid off, but it also makes bankruptcy more likely. This trade-off gives rise to an *interest ceiling* (*i.e.* $r_{\bar{h}}^{A}$) beyond which increases in the interest rate would reduce the lender's overall return. On the other hand, because of the different payoff functions of the borrowers, there is still demand for credit at such a rate. The excess demand has thus to be rationed at the interest ceiling.

Contracts with Collateral

We show that the use of collateral can mitigate, but not necessarily eliminate, credit rationing. This result differs from Bester (1985) in which credit rationing can be completely eliminated by the choice of interest rates and collateral configuration.

Figure 3 shows a lender's (3(a)) and a borrower's (3(b)) iso-profit curves, starting from profit being equal to zero, as functions of *r* and *C*. The values of the parameters are the same as in Figure 2(a), except that we let A =1 in these figures.

The nonlinearity of the lender's iso-profit curves with respect to *r* and *C* is highly visible, and it is check-mark shaped. The rising part of the "check" indicates that a lender is willing to accept a risky borrower's offer of higher interest rates only if collateral is available. Collateral could compensate for the risk because it discourages borrowers from filing for bankruptcy. Borrowers are better off if they move toward the southwest, and would generally prefer less collateral if interest rates were higher.

To see how the optimal contract is determined when collateral is available, we plot the lender's and the borrower's zero-profit loci in Figure 4,using the parameters of Figure 2(a). The optimal contract must be on the lender's zero-profit loci and within the borrower's positive profit region, which is the shaded area. Given the characteristics of the borrower's iso-profit curves, the optimal contract should be around the lower-left corner of the lender's zero-profit curve.

Credit rationing in Figure 2(a) is resolved with collateral as shown in Figure 4. Collateral, however, does not solve all problems. One obvious possibility is that borrowers may not have enough collateral in terms of what is required by lenders. This is shown in Figure 5(a). In this example, the borrower is credit rationed according to the definition, because it is willing to pay an interest rate up to 80% but is still denied credit.[5]

Another case in which collateral does not amend credit rationing is shown in Figure 5(b) which is characterized by a low expected return and a high variance. The stripe area indicates fully collateralized contracts ($C>1+r$) that yield non-negative profit to lenders. In this case, even a fully collateralized contract does not help. This is because when the opportunity cost of borrowing and lending (*i.e.* r_f) is high and there is also a high probability that the investment is not going to succeed (and hence the bankruptcy cost would be imposed), it is difficult to find a contract that will mutually benefit the two parties. Nevertheless, if one only looks at the interest rate dimension, there are still grounds for credit rationing.

Collateral and Credit Risk

Many of the models based on asymmetric information predict that safer borrowers pledge more collateral (Bester 1985, Chan and Kanatas 1985, Besanko and Thakor 1987). The explanation is that safer borrowers reveal their type by choosing high collateral contracts. This prediction, however, is contrary to the "conventional wisdom" (Berger and Udell 1990, p. 21) and is later found by the cited authors to be inconsistent with empirical data, which indicates that *riskier* borrowers pledge more collateral.

We show that the model presented in this paper implies the existence of a positive

[5] Implicitly we assume, of course, that the highest market rate is below 80%.

relationship between credit risk and collateral. To this end, we use the parameters of Figure 1(a) as the benchmark, and change the values of μ and σ toward the risky end. For each case, we pick up the optimal contract of $\{r, C\}$ using the numerical information similar to that used to plot Figure 4.[6] The results are given in Figure 6. Figure 6 shows that pledged collateral is higher for investment projects with a smaller expected return and a higher variance of the return.

4. Conclusion

This paper graphically demonstrates that a costly bankruptcy alone can cause credit rationing even if information is symmetric between borrowers and lenders. If bankruptcy costs are minimal, *i.e.* lenders can costlessly confiscate the borrower's collateral and investment residual upon bankruptcy, credit rationing would not occur because lenders could always raise interest rates to justify credit risk. With nontrivial bankruptcy costs, however, the lenders' profit function is concave throughout the relevant range of interest rates, and there exists an interest ceiling beyond which further raising the interest rate would decrease profit. We argue that a pro-debtor Bankruptcy Code is a major reason for costly bankruptcies, and thus market efficiency can be improved through the government's intervention in re-designing bankruptcy laws.

Collateral is found to play an important role in such a setting. The use of collateral

[6] To prevent the indeterminacy of optimal collateral, we rule out the vertical part of the lender's zero-profit loci as shown in Figure 4.

can mitigate problems associated with credit rationing because it compensates lenders in the event of bankruptcy and also reduces the probability of default. Collateral, however, does not eliminate all credit rationing possibilities. We also find that the amount of pledged collateral is positively related to credit risk, which is consistent with the empirical finding of Berger and Udell (1990).

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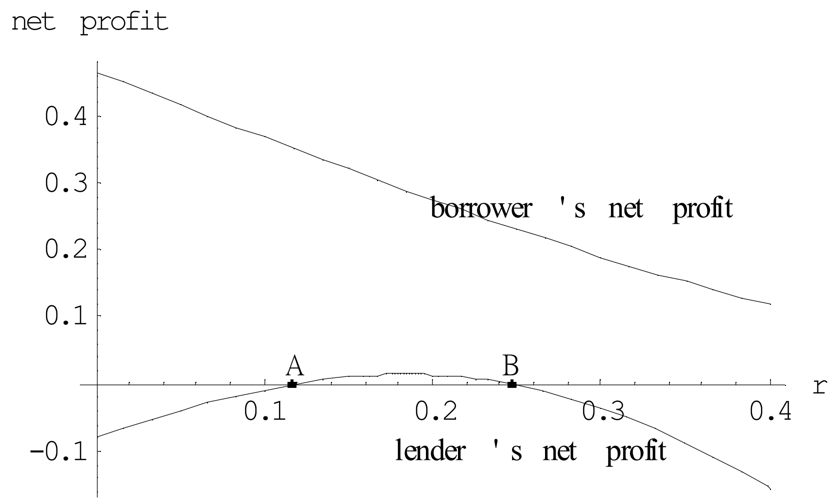
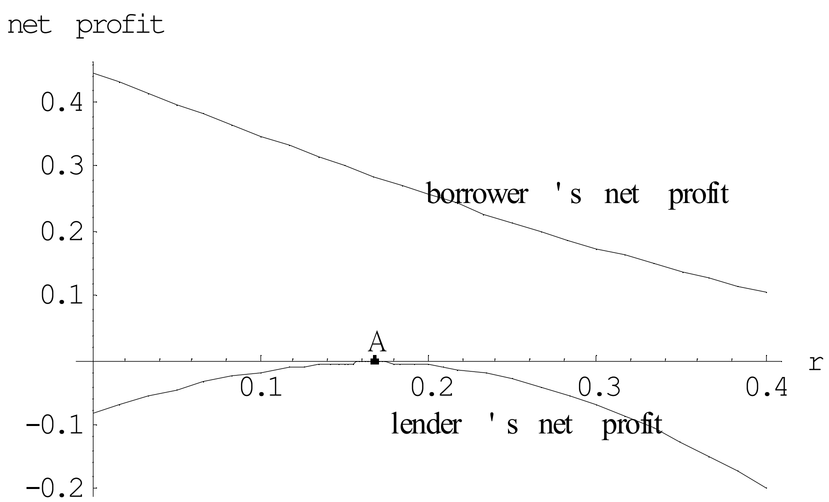
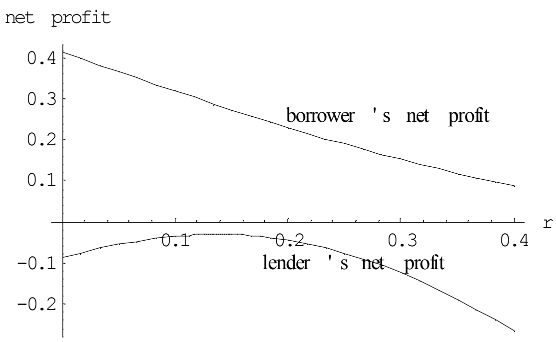


(a) Borrower *j*'s optimal contract is ≙. The figure is drawn with {$\mu = 1.50$, $\sigma = 0.25$, $A = 0.5$, $\alpha = 0.85$, $\beta = 0.1$, $r_f = 0.07$}.

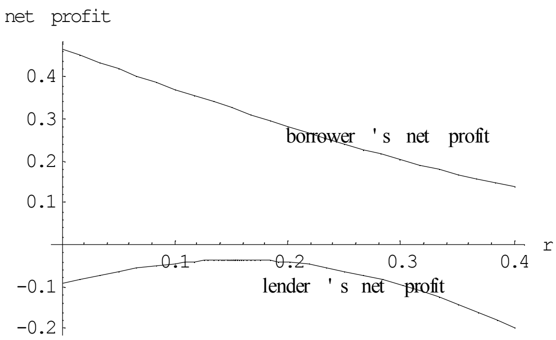


(b) Borrower $\overline{h}$'s optimal contract is ≙. The figure is drawn with {$\mu = 1.48$, $\sigma = 0.25$, $A = 0.5$, $\alpha = 0.85$, $\beta = 0.1$, $r_f = 0.07$}.

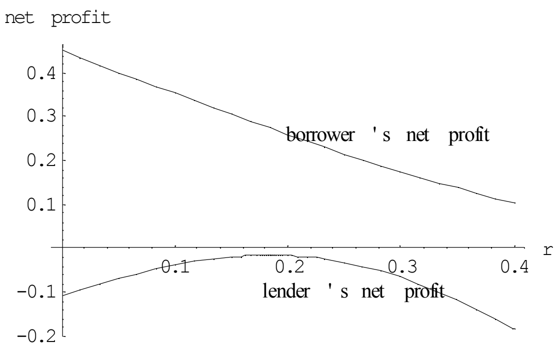
Figure 1: Cases With No Credit Rationing



(a) Lower Mean of the Expected Return. The figure is drawn with $\{\mu = 1.45, \sigma = 0.25, A = 0.5, \alpha = 0.85, \beta=0.1, r_f = 0.07\}$.

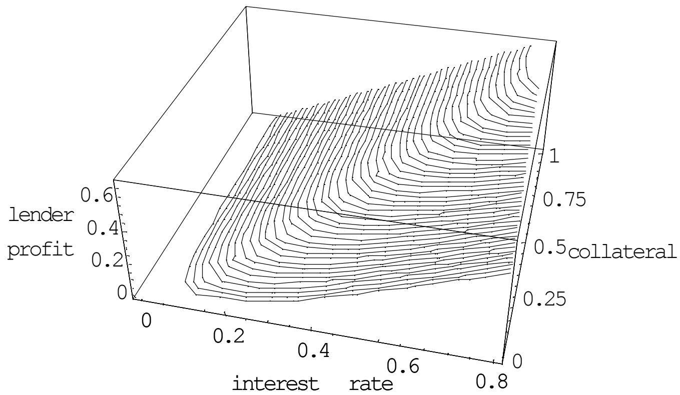


(b) Higher Variance of the Expected Return. The figure is drawn with $\{\mu = 1.50, \sigma = 0.30, A = 0.5, \alpha = 0.85, \beta=0.1, r_f = 0.07\}$.

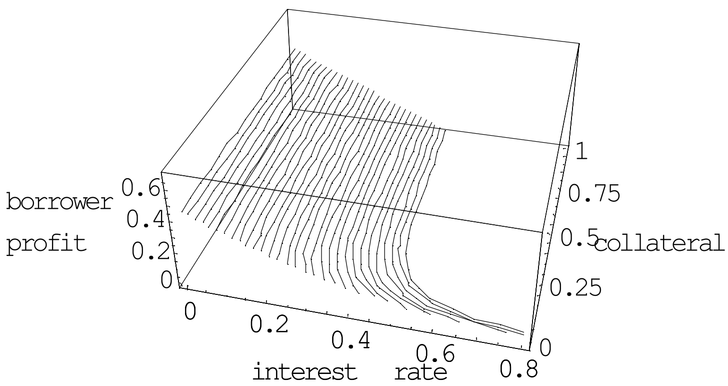


(c) Higher Risk-Free Rates. The figure is drawn with $\{\mu = 1.50, \sigma = 0.25, A = 0.5, \alpha = 0.85, \beta=0.1, r_f = 0.10\}$.

Figure 2: Cases With Credit Rationing



(a) A lender's iso-profit curves. The figure is drawn with {μ = 1.45, σ = 0.25, A = 1, α = 0.85, β=0.1, r_f = 0.07}.



(b) A borrower's iso-profit curves. The figure is drawn with {μ = 1.45, σ = 0.25, A = 1, α = 0.85, β=0.1, r_f = 0.07}.

Figure 3: Visualizing Profit Functions

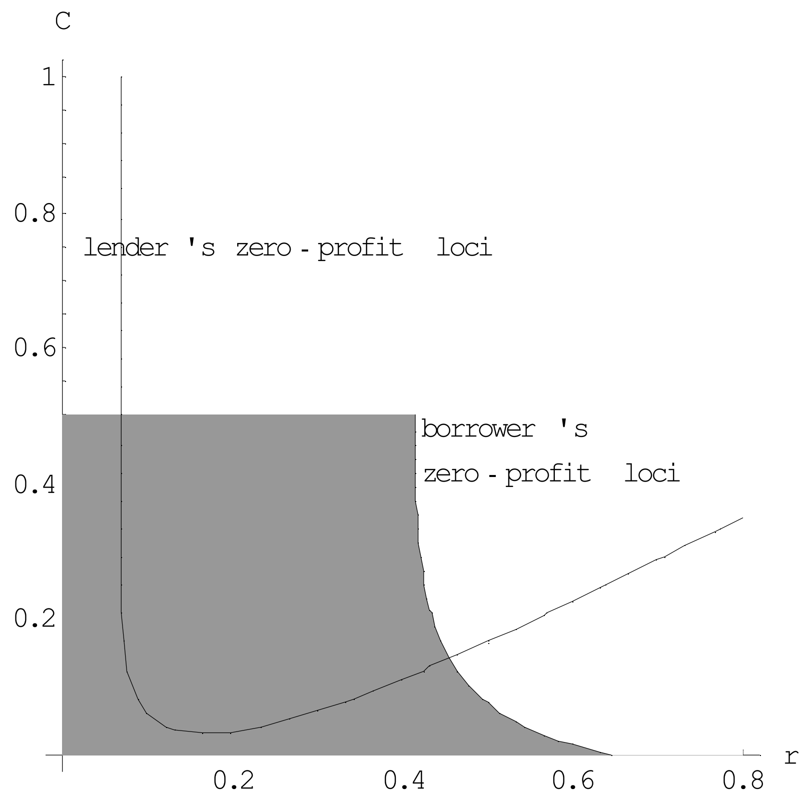
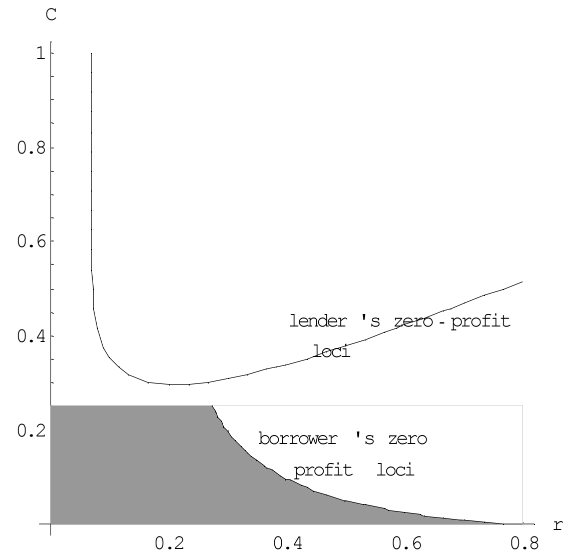
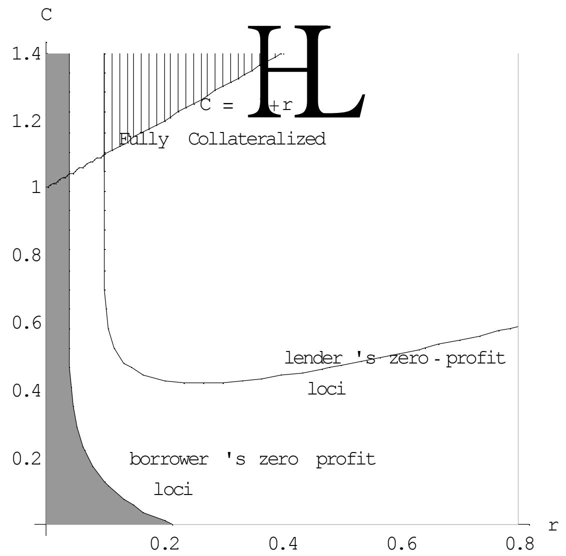


Figure 4: Collateral Prevents Credit Rationing. The shaded area is the borrower's positive profit region. The figure is drawn with $\{\mu = 1.45, \sigma = 0.25, A = 0.5, \alpha = 0.85, \beta = 0.1, r_f = 0.07\}$.

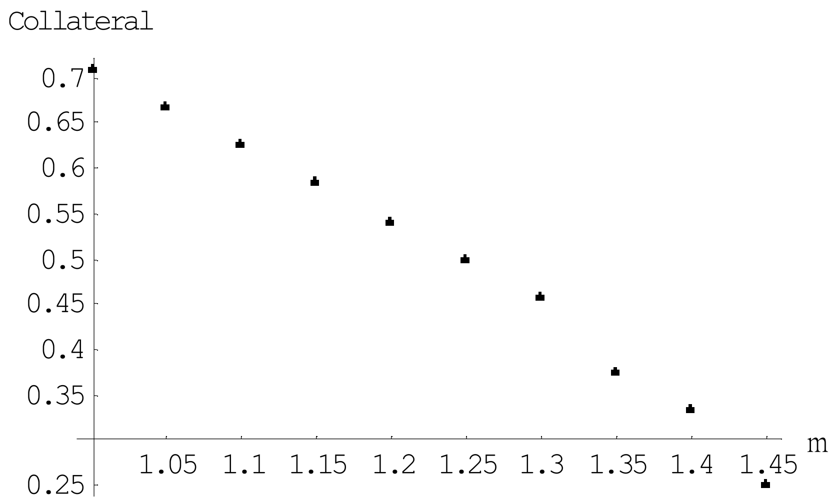


(a) Insufficient Collateral. The shaded area is the borrower's positive profit region. The figure is drawn with $\{\mu = 1.25, \sigma = 0.35, A = 0.25, \alpha = 0.85, \beta = 0.1, r_f = 0.07\}$.

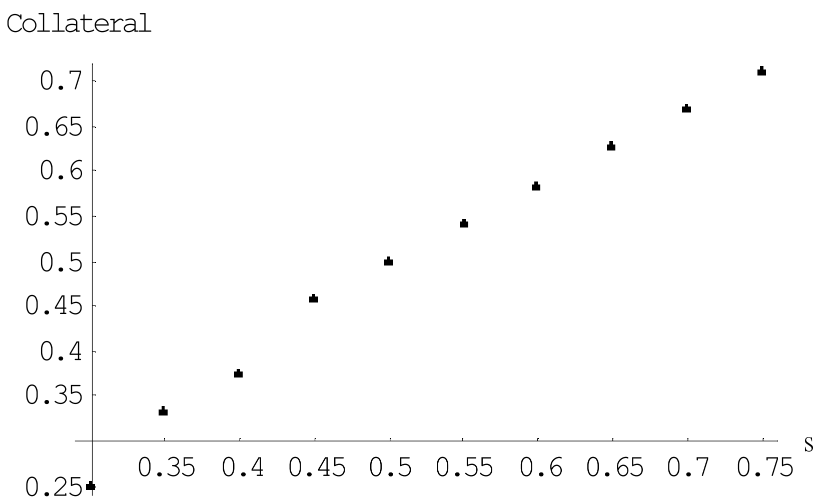


(b) 100% Collateral Does Not Amend Credit Rationing. The shaded area is the borrower's positive profit region , and the strip area indicates fully collateralized contracts that yield non-negative profit to lenders. The figure is drawn with $\{\mu = 1.18, \sigma = 0.4, A = 1, \alpha = 0.85, \beta = 0.1, r_f = 0.10\}$.

Figure 5: Credit Rationing with Collateral Contracts.



.(a) Lower Expected Returns are Associated with Higher Collateral. Other parameters used in this figure are $\{\sigma = 0.25, A = 0.5,\ \alpha = 0.85,\ r_f = 0.07\}$.



(b) Higher Standard Deviations are Associated with Higher Collateral. Other parameters used in this figure are $\{\mu = 1.50,\ A = 0.5,\ \alpha = 0.85,\ r_f = 0.07\}$.

Figure 6: Collateral vs. Credit Risk