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ABSTRACT

Operational effectiveness goes beyond efficiency while it incorporates exogenous variables, non-controllable by the service units. Effectiveness is a fundamental driver for the success of an operational unit within a competitive environment. In this context, we seek to identify the active units that meet both the high or technical efficiency and the perceived high quality criteria. We also aim to develop a roadmap for effectiveness for every operational unit and we consider the feasibility of the results produced by the effectiveness assessment process in the short run. The target values uncovered by comparative optimization techniques (e.g. Data Envelopment Analysis) for efficiency and effectiveness measurement generally have limited managerial implications due to production constraints, available resources, and legal status. This paper introduces a modified Quality-driven – Efficiency-adjusted Data Envelopment Analysis (MQE-DEA) model to assess effectiveness and provide a step-by-step path to achieve high quality and high efficiency in every operational unit under evaluation. The MQE-DEA model has particular applicability to the effectiveness assessment of homogenous service units in which an inverse relationship underlies the two dimensions of effectiveness embraced in this study (e.g. bank branches, restaurant chain stores, governmental one-stop-shops).

Keywords: Data Envelopment Analysis (DEA); context-dependent DEA; Effectiveness; Efficiency; Perceived Quality

1. Introduction

Efficiency-related optimization techniques (e.g. Data Envelopment Analysis, Stochastic Frontier Analysis) cannot ensure operational units' prosperity or even viability in a mature market in which market shares' variability is marginal, opportunities for profitability growth are limited and competition is intense. In such markets, operational units' strategy should incorporate, besides efficiency optimization, customer satisfaction or perceived quality. Thus, the use of the term effectiveness seems more appropriate than efficiency to describe a holistic approach to modern, customer-oriented, organizational strategic planning. On the contrary, remaining solely loyal to introversion, such as input-output transformation process optimization, without considering exogenous variables and while operating in mature markets leads to a loop that ends with the shrinkage of a unit or even its silence.

The scope of the present study is the development of an effectiveness assessment model that elaborates efficiency and perceived quality data for every sample operational unit. The new model relies on the Quality-driven – Efficiency-adjusted DEA (QE-DEA) method put forth by Zervopoulos and Palaskas (2011). In advance, it introduces a time-variance assumption in order to determine feasible short-term targets for the sample units that fail to meet high standards of efficiency and perceived quality. In this paper, efficiency and perceived quality are deemed the dimensions of effectiveness.

The QE-DEA model is applicable to assess comparative effectiveness in case an inverse relationship appears between the dimensions of effectiveness. It suggests a customer-oriented strategy for every operational unit, letting the customers be the drivers of the organizational operations. In this context, the need for organizational restructuring is quality-driven towards optimized efficiency and improved customer satisfaction. According to the QE-DEA model, qualified operational units are solely those that meet both high quality perception and high efficiency criteria.

In this paper, a modified Quality-driven –Efficiency-adjusted DEA (MQE-DEA) model is introduced, putting emphasis on output-orientation. The new model is time-sensitive, respecting the implications of time in restructuring the production process to achieve the greatest effectiveness. To be more precise, the targets yielded by applying the MQE-DEA model are feasible in the short run. Moreover, the benchmarks identified are “strong”. They not only are effective, but their production process is free of slacks, and they are the most referenced units by their disqualified counterparts to achieve effectiveness.

The second section of this study discusses the literature review on methodologies on which the MQE-DEA model is grounded. In the following section, the QE-DEA model is presented and in the fourth section, the mathematical underpinning of the MQE-DEA model is analyzed. In the fifth section, the MQE-DEA model is applied to data from governmental one-stop-shop agencies, called Citizen Service Centers (CSCs). Concluding remarks are presented in the last section of the paper.

2. Literature Review

The studies related to the MQE-DEA model’s development methods are discussed below in order to present the properties and weaknesses of the current techniques. The intent of this discussion is to provide an understanding of the effectiveness assessment field and the mathematical underpinnings.

2.1. Data Envelopment Analysis (DEA)

DEA is a deterministic comparative efficiency assessment method for homogeneous[1] operational units’ benchmarking. The best practice units identified by DEA are regarded as reference units for the remaining sample under evaluation. Consequently, DEA serves not only as a benchmarking technique but also as a tool to determine quantitative target input or output values to attain optimum efficiency.

[1] Homogeneous are those that engage common resources to produce common goods or services operating diverse production processes.

This particular method, relying on linear programming, makes no assumption on the underlying production function of each sample operational unit. Unlike its rival efficiency evaluation methods (e.g. Stochastic Frontier Analysis), DEA empirically estimates an optimum production function, or the best-practice frontier, formed by the reference units' input-output data.

DEA elaborates input and output data to calculate efficiency, at the same time taking into account the impact of returns to scale on the efficiency status of every sample operational unit. To be more precise, there are two main approaches in DEA literature regarding incorporation of the returns to scale. The seminal paper put forth by Charnes, Cooper and Rhodes (1978) assumes that constant returns to scale prevail upon all the sample operational units' production process. As a result, active units may be deemed inefficient merely because of their disagreement with the arbitrarily selected returns to scale. The second approach, developed by Banker, Charnes and Cooper (1984), called the BCC model, is more adaptive than the previous one while respecting the variation in returns to scale between the operational units production process. In this case, the efficiency results yielded and the best-practice frontier formed by the BCC-DEA model are more representative of the reality.

Another major breakdown of the DEA method is that the orientation of the analysis better fits the disposability of modifications on either input or output variables. It is common that restrictions applied to the units under assessment, such as availability of resources or protectionism (e.g. in public organizations) on one hand, and market maturity and intense competition on the other hand, become the drivers of the orientation selection by the operational units' policymakers. The input or output-oriented DEA aims to calculate the minimum input values (target inputs), holding the outputs fixed, or to determine the maximum output levels (target outputs), keeping the original inputs unaltered. For instance, in case DEA is applied to public organizations in which protectionism appears over the input variables, especially the number of employees occupied, an output-oriented analysis is more appropriate.

Translating the DEA comparative efficiency assessment concept into linear programming formulae respecting the returns to scale variance as well as the output orientation (BCC-DEA output), results:

$$
\begin{aligned}
& \gamma^{*} = \min \gamma \\
\text{subject to} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io} \qquad i = 1,\dots,m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq \gamma y_{ro} \qquad r = 1,\dots,s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0 \qquad\qquad j = 1,\dots,n
\end{aligned}
$$

where γ^* is the efficiency score of the oth operational unit, the subscript "o" denotes the sample operational unit currently assessed, x_{io} and y_{ro} stand for the ith input and the rth output of the oth unit respectively, and the *lambdas* (λ_j) represent the input and ouput non-negative weights.

2.2 Context-dependent DEA

The Context-dependent DEA method put forth by Seiford and Zhu (2003) is regarded as a rational adjustment on DEA outcomes. In other words, considering the feasibility of the target inputs or outputs suggested by DEA for the sample's inefficient units due to short-term restrictions, the Context-dependent DEA method introduces milestones towards attaining efficiency. Assuming that the best-practice frontier formed by the traditional DEA is a "global" reference set adopted by Context-dependent DEA as well, intermediate frontiers, or "local" reference sets, are specified by the latter method in order to define feasible short-term targets for the units that lack efficiency. As a result, the sample operational units are classified into multilayered efficiency frontiers.

In order to implement the sample partitioning concept, Seiford and Zhu (2003) consider an n-number operational units' sample with a dataset consisting of m inputs and s outputs. By assuming that Ω^l denotes the n-units and R^l the set of efficient and zero-slack units located on the "global" reference set, the remaining active units, the "weak" efficient or the inefficient units, are clustered around the $\Omega^{l+1} = \Omega^l - R^l \;\forall\; l=1,\dots,n$ sets ($R^l \cap R^{l'} = \emptyset$ and $R^l \cup R^{l'} = \Omega^l$).

Respecting the variable returns to scale orientation of the study, the clustering algorithm is written:

Step 1: Run the BCC-DEA model to identify the units that compose the "global" reference set (R^l)

Step 2: a. If $\Omega^{l+1} = \emptyset$, then stop.

b. Otherwise, eject the R^l units from the Ω^l set to obtain the $\Omega^{l+1} = \Omega^l - R^l$ subset and rerun the BCC-DEA model.

Step 3: Let $l=l+1$ and go back to Step 2 until $\Omega^{l+1} = \emptyset$.

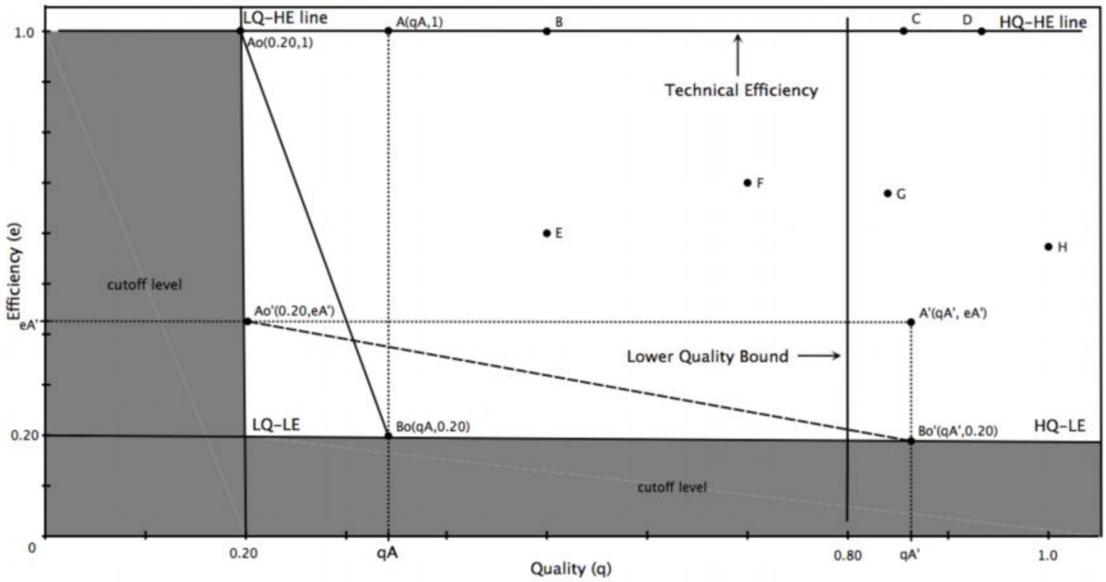
$\Omega^{l+1} = \emptyset$ is the stopping rule.

3. Quality-driven – Efficiency-adjusted DEA (QE-DEA)

The QE-DEA method developed by Zervopoulos and Palaskas (2011) is deemed a reverse approach to the mainstream effectiveness-based DEA measurements while it sets customer satisfaction as the core element in the operational units' strategic planning process. QE-DEA overcomes a weakness of the Quality-adjusted DEA (Q-DEA) model (Sherman and Zhu, 2006), and also tackles problems of the DEA method. To be more precise, Q-DEA suggests the removal from the comparative effectiveness assessment process of the units that merely meet the efficiency criterion. By interpreting the impact of this decision in a strategic setting, the removed unit is condemned to competitiveness "emasculation" while it is not allowed to identify its comparative weaknesses. Additionally, traditional DEA fails to specify best practice units that simultaneously meet high-efficiency and high-perceived quality standards when a trade-off underlies the dimensions of effectiveness. As a result, quality is handled mostly as an output variable, assuming that monotonicity prevails between inputs and outputs (Soteriou & Zenios, 1999; Chilingerian & Sherman, 1990; Bessent et al., 1984).

QE-DEA relies on the planar analysis of the QE-DEA model locating the sample operational units into four quadrants: a) high-perceived quality – high-efficiency (HQ-HE); b) low-perceived quality – high-efficiency (LQ-HE); c) low-perceived quality – low-efficiency (LQ-LE); and d) high-perceived quality – low-efficiency (HQ-LE) (Figure 1). Additionally, an active area for efficiency and perceived quality is the interval (0.2, 1]. The active area decision is based on the scaling into percentage of the five-point Likert scale format, applied for the customer satisfaction survey conducted in order to specify each sample unit's quality score, and on the work of Paradi et al. (2004) regarding the accuracy of the DEA results.

Figure 1. QE-DEA Concept Planar Analysis



Unlike the Q-DEA model, QE-DEA suggests replacement of the high-efficiency and low-quality units (i.e. unit A located on the LQ-HE segment) by the hypothetical counterparts (i.e. unit A' located on the HQ-LE segment) to emulate the original unit's perceived quality – efficiency symmetry. The latter model supports the underlying trade-off between the dimensions of effectiveness that appears in numerous markets such as restaurant chain stores and governmental one-stop-shops (De Bruijn, 2007; Sherman & Zhu, 2006).

In order to apply the properties of the output-oriented QE-DEA model the following algorithm has been developed:

Step 1: Apply output-oriented BCC-DEA in order to identify sample operational units' efficiency scores

Step 2: If LQ-HE units = Ø, then stop.

Otherwise, prior to modification of the actual LQ-HE units into hypothetical HQ-LE, determine the trade-off between the dimensions of effectiveness for every LQ HE unit.

Next, calculate the outputs of the hypothetical units holding the inputs fixed and return to Step 1.

In case LQ-HE units = Ø, the output-oriented QE-DEA model coincides with the traditional output-oriented BCC-DEA model.

The aim of the QE-DEA model is the identification of the effective operational units, namely the units classified in the HQ-HE segment and the development of a roadmap for the disqualified units to attain effectiveness.

4. Output-oriented MQE-DEA

The input-oriented modified Quality-driven – Efficiency-adjusted DEA (MQE-DEA) model put forth by Brissimis and Zervopoulos (2011) is altered substantially in order to comply with the output-oriented application of the QE-DEA model in conjunction with the context-dependent DEA. The combination of the two methods results a feasible short-term effectiveness assessment framework for service organizations that seek to maximize the outputs produced and the perceived by the customers quality of service provided holding the inputs engaged fixed. Like the original QE-DEA model, its modified version has enhanced applicability in case inverse relationship underlies efficiency and perceived quality.

The time-awareness of the MQE-DEA model in identifying best-practice solutions for the operational units under assessment strengthens its managerial implication. In other words, the target output calculated for the ineffective or the "weak" effective units respect the difficulties in increasing the market share of these particular units, increasing their customer based and their revenues in the short-run.

A major classification is applied by the MQE-DEA model to the sample units distinguishing the "strong" from the "weak" effective units and the ineffective ones. "Strong" effective operational units are those composing the "global" effectiveness frontier. "Weak" effective units are deemed those that merely meet the high-perceived quality and high-efficiency standards while slacks appear in their production process. As a result, they have limited impact on either the remaining "weak" effective units or the ineffective ones. The more the operational units under assessment comply with the MQE-DEA standards, the higher level reference set they are located.

The properties of the output-oriented MQE-DEA model relax on a three-stage algorithm:

Step 1: Apply the output-oriented BCC-DEA model for specifying sample operational units efficiency scores.

Step 2: If LQ-HE sample units = Ø, then run the output-oriented Context-dependent DEA algorithm and stop.
Otherwise, determine the trade-off between the dimensions of effectiveness and define the hypothetical operational units.
Next, specify the outputs of the hypothetical units holding the inputs fixed.

Step 3: Replace the actual units by their hypothetical counterparts and run the output-oriented Context-dependent DEA algorithm.

The planar analysis of the output-oriented MQE-DEA model is common with that of the QE-DEA model presented in section 3 in Figure 1. The quality score for every sample unit is the average customer satisfaction rating reported during the fieldwork research. A five-point Likert scale response format has been used for the customer satisfaction ratings that is easily transformed into percentages in order to be symmetrical to the scale used for the efficiency scores.

For applying the properties of the output-oriented MQE-DEA algorithm, we initially develop a formula that secures the perceived quality-efficiency symmetry of the actual LQ-HE units to the hypothetical ones:

$$\frac{(A_0B_0)}{(A_0'B_0')}=\frac{(q_A-0.20)(0.20-1)}{(q_A'-0.20)(0.20-e_A')} \tag{1}$$

The distance function formula $(AB)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ introduced in formula (1) results:

$$\frac{\sqrt{(q_A-0.20)^2+(0.20-1)^2}}{\sqrt{(q_A'-0.20)^2+(0.20-e_A')^2}}=\frac{(q_A-0.20)(0.20-1)}{(q_A'-0.20)(0.20-e_A')} \tag{2}$$

A generalized formula is presented below in which the researcher or the policymaker may decide for the cut-off points and consequently for the active area of the efficiency and perceived quality.

$$\frac{\sqrt{(q_A-q_0)^2+(e_0-1)^2}}{\sqrt{(q_A'-q_0)^2+(e_0-e_A')^2}}=\frac{(q_A-q_0)(e_0-1)}{(q_A'-q_0)(e_0-e_A')} \tag{3}$$

By conducting the appropriate calculations [Appendix – Section 1], a generalized formula for hypothetical efficiency scores (e_A') determination is revealed:

$$e_A'=e_0+\sqrt{\frac{[(q_A-q_0)^2(e_0-1)^2](q_A'-q_0)^2}{[(q_A-q_0)^2+(e_0-1)^2](q_A'-q_0)^2-(q_A-q_0)^2(e_0-1)^2}} \tag{4}$$

The second phase of the MQE-DEA algebraic analysis for the calculation of the hypothetical outputs is based on the efficiency ratio (Charnes et al., 1978):

$$\mathrm{e}=\frac{\sum_{r=1}^{s}u_r y_r}{\sum_{i=1}^{m}v_i x_i} \tag{5}$$

where: e = efficiency score

y_r = amount of output $r \;\; \forall\; r = 1,...,s$

u_r = weight assigned to output r

x_i = amount of input $i \;\; \forall\; i = 1,...,m$

v_i = weight assigned to input i

Let efficiency score be equal to unity

$$1 = \frac{\sum_{r=1}^{s} u_r y_r}{\sum_{i=1}^{m} v_i x_i} \tag{6}$$

or,

$$\sum_{i=1}^{m} v_i x_i = \sum_{r=1}^{s} u_r y_r \tag{7}$$

Towards the determination of the hypothetical output while the inputs are fixed and the hypothetical efficiency score is known from formula (4), we alter substantially formula (5).

$$e' = \frac{\sum_{r=1}^{s} u_r y_r'}{\sum_{i=1}^{m} v_i x_i}, \text{ where } e' \neq e \text{ and } y_r' \neq y_r \tag{8}$$

Following the calculations presented in Section 2 of the Appendices, the output values of the hypothetical units are specified.

$$\left.\begin{array}{l} y_1' = e' y_1 \\ y_2' = e' y_2 \\ \dots \\ y_s' = e' y_s \end{array}\right\} \tag{9}$$

5. Conclusion and Further Research

This paper introduces a “rational” short-term effectiveness assessment methodology that is particularly applicable in case an inverse relationship appears between the dimensions of effectiveness. The developed MQE-DEA model is output-oriented in order to relax input disposability restrictions, such as protectionism over the resources engaged, which commonly are experienced in public organizations.

The aim of the MQE-DEA model is the identification of the comparative maximum outputs which should be produced by every non-best-practice unit, holding the input levels fixed while taking into account, at the same time, the provision of high quality standards based on customers' or citizens' perception. To select a best-practice operational unit, all the high efficiency, high quality and zero-slack production process criteria should be met. Although best-practice units are perceived solely as the "global" reference units, the MQE-DEA model sets local reference units that act as intermediate or short-term targets for the lower effectiveness level units. By introducing short-term targets, a customized step-by-step path for improvement can be traced for every disqualified sample unit based on comparative assessment.

The output-oriented MQE-DEA model could be extended by applying its properties in cases in which non-discretionary output variables exist. Further analysis is needed in multi-dimensional settings where more than two variables determine effectiveness. Another field which could be explored is the generalization of the model yielding results that take into account population and not sample data.

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APPENDIX

Section 1

Equation (3) can be rewritten as:

$$\frac{(q_A - q_0)^2 + (e_0 - 1)^2}{(q_A' - q_0)^2 + (e_0 - e_A')^2} = \frac{(q_A - q_0)^2 (e_0 - 1)^2}{(q_A' - q_0)^2 (e_0 - e_A')^2}$$

$$[(q_A - q_0)^2 + (e_0 - 1)^2](q_A' - q_0)^2 (e_0 - e_A')^2 = [(q_A' - q_0)^2 + (e_0 - e_A')^2](q_A - q_0)^2 (e_0 - 1)^2$$

Let
$$c_1 = [(q_A - q_0)^2 + (e_0 - 1)^2]$$

and
$$c_2 = (q_A - q_0)^2 (e_0 - 1)^2$$

Then
$$c_1 (q_A' - q_0)^2 (e_0 - e_A')^2 = [(q_A' - q_0)^2 + (e_0 - e_A')^2] c_2$$

$$c_1 (q_A' - q_0)^2 (e_0 - e_A')^2 = c_2 (q_A' - q_0)^2 + c_2 (e_0 - e_A')^2$$

$$c_1 (q_A' - q_0)^2 (e_0 - e_A')^2 - c_2 (e_0 - e_A')^2 = c_2 (q_A' - q_0)^2$$

$$(e_0 - e_A')^2 [c_1 (q_A' - q_0)^2 - c_2] = c_2 (q_A' - q_0)^2$$

$$(e_0 - e_A')^2 = \frac{c_2 (q_A' - q_0)^2}{c_1 (q_A' - q_0)^2 - c_2}$$

$$|e_0 - e_A'| = \sqrt{\frac{c_2 (q_A' - q_0)^2}{c_1 (q_A' - q_0)^2 - c_2}}$$

$$e_0 - e_A' = +\sqrt{\frac{c_2 (q_A' - q_0)^2}{c_1 (q_A' - q_0)^2 - c_2}} \quad \text{or} \quad e_0 - e_A' = -\sqrt{\frac{c_2 (q_A' - q_0)^2}{c_1 (q_A' - q_0)^2 - c_2}}$$

The first critical value:

$$e_0 - e_A' = +\sqrt{\frac{c_2 (q_A' - q_0)^2}{c_1 (q_A' - q_0)^2 - c_2}}$$

$$e_A' = e_0 - \sqrt{\frac{c_2 (q_A' - q_0)^2}{c_1 (q_A' - q_0)^2 - c_2}}$$

is rejected because the condition: $e_A' > e_0$ is not satisfied.

On the contrary, the alternative critical value:

$$e_0 - e_A' \qquad -\sqrt{\frac{c_2(q_A' - q_0)^2}{c_1(q_A' - q_0)^2 - c_2}}$$

$$e_A' = e_0 + \sqrt{\frac{c_2(q_A' - q_0)^2}{c_1(q_A' - q_0)^2 - c_2}}$$

is accepted, because the condition: $e_A' > e_0$ is satisfied.

The generalized formula is the following:

$$e_A' = e_0 + \sqrt{\frac{[(q_A - q_0)^2(e_0 - 1)^2](q_A' - q_0)^2}{[(q_A - q_0)^2 + (e_0 - 1)^2](q_A' - q_0)^2 - (q_A - q_0)^2(e_0 - 1)^2}} \tag{4}$$

Section 2

Equation (8) can be expressed in matrix form:

$$e' = \frac{[y_1, y_2, ..., y_s]\begin{bmatrix} u_1 \\ u_2 \\ ... \\ u_s \end{bmatrix}}{[x_1', x_2', ..., x_m']\begin{bmatrix} v_1 \\ v_2 \\ ... \\ v_m \end{bmatrix}}$$ (multiplying both sides by $\frac{1}{e'}$, where $e' \neq 0$)

$$\frac{e'}{e'} = \frac{[y_1, y_2, ..., y_s]\begin{bmatrix} u_1 \\ u_2 \\ ... \\ u_s \end{bmatrix}}{e'[x_1', x_2', ..., x_m']\begin{bmatrix} v_1 \\ v_2 \\ ... \\ v_m \end{bmatrix}}$$

$$1 = \frac{[y_1, y_2, ..., y_s]\begin{bmatrix} u_1 \\ u_2 \\ ... \\ u_s \end{bmatrix}}{[e'x_1', e'x_2', ..., e'x_m']\begin{bmatrix} v_1 \\ v_2 \\ ... \\ v_m \end{bmatrix}} \tag{8a}$$

Introducing (8a) to equation (7):

$$1 = \frac{[x_1, x_2, ..., x_m] \begin{bmatrix} v_1 \\ v_2 \\ ... \\ v_m \end{bmatrix}}{[e'x_1', e'x_2', ..., e'x_m'] \begin{bmatrix} v_1 \\ v_2 \\ ... \\ v_m \end{bmatrix}}$$

$$[e'x_1', e'x_2', ..., e'x_m'] \begin{bmatrix} v_1 \\ v_2 \\ ... \\ v_m \end{bmatrix} = [x_1, x_2, ..., x_m] \begin{bmatrix} v_1 \\ v_2 \\ ... \\ v_m \end{bmatrix} \qquad \text{(8b)}$$

Equation (8b) leads to the input adjustment formula:

$$\left.\begin{array}{l} x_1 = e'x_1' \\ x_2 = e'x_2' \\ ... \\ x_m = e'x_m' \end{array}\right\} \text{ or } \left.\begin{array}{l} x_1' = \dfrac{1}{e'} x_1 \\ x_2' = \dfrac{1}{e'} x_2 \\ ... \\ x_m' = \dfrac{1}{e'} x_m \end{array}\right\}$$