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Kevin J. Boyle, Michael P. Welsh, and Richard C. Bishop

INTRODUCTION

In an excellent article from a recent issue of this journal, Sellar, Stoll, and Chavas (1985) make a technical error which causes them to misstate their closed-ended estimates of willingness to pay. The authors state that expected willingness to pay is calculated as

$$E(WTP) = \int_0^{X_{max}} [1 - F(x)]dx = X_{max} - \int_0^{X_{max}} F(x)dx \quad [1]$$

where x is a random variable (willingness to pay), $[1 - F(x)]$ represents the probability of answering yes to what the authors refer to as a closed-ended-contingent-valuation question, $F(x)$ represents the probability of a no response and is a cumulative distribution function (c.d.f.), and X_{max} corresponds to the highest closed-ended dollar offer used in the valuation exercise. Bishop, Heberlein, and Kealy (1983), in the first application of closed-ended-contingent-valuation questions, used this same formula to compute expected willingness to pay. The problem arises because equation [1] is not a correct statement of expected willingness to pay.

This issue is important for several reasons. First, the correct computation of closed-ended values is necessary for the authors to make appropriate comparisons of values across estimation methods. Second, correctly stated formulas are important to future researchers who may be reviewing previous work. Finally, the procedure that Sellar, Stoll, and Chavas used to compute expected values has some important implications with respect to the advantages and disadvantages of applying closed-ended questions in contingent-valuation studies. For discussions of the various techniques of asking contingent-valuation questions see: Sellar, Stoll and Chavas (1985); Smith, Desvousges, and Fisher (1986); and Boyle and Bishop (1988).

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A STATISTICALLY CORRECT EXPECTED VALUE

The expected value of any random variable is defined as

$$E(X) = \int_{-\infty}^{\infty} xf(x)dx \quad [2]$$

where $f(x)$ is a probability density function (p.d.f.) and $\partial F(x)/\partial x = f(x)$. Hanemann (1984) has shown that the expected value of any nonnegative random variable, such as willingness to pay, can be written as

$$E(X) = \int_0^{\infty} [1 - F(x)]dx. \quad [3]$$

This result comes from the fact that the ex-

The authors are, respectively, assistant professor of agricultural and resource economics and assistant professor of wildlife, University of Maine; assistant professor of agricultural economics, Oklahoma State University; and professor of agricultural economics, University of Wisconsin-Madison. They would like to thank Ralph Townsend for his comments on an earlier draft of this paper. All errors and omissions are the sole responsibility of the authors. This research is supported by the Agricultural Experiment Station, University of Maine, the Graduate School and the College of Agricultural and Life Sciences, University of Wisconsin-Madison. Maine Agricultural Experiment Station Publication #1210.

pected value of any arbitrary random variable can be expressed as

$$E(X) = \int_0^{\infty} [1 - F(x)]dx - \int_{-\infty}^{0} F(x)dx$$

where the second expression on the right-hand side is zero for nonnegative random variables (See Mood, Graybill, and Boes 1974). For equation [3] to define an expected value, the following properties must hold:

$$\lim_{x \to 0} F(x) = 0$$

and

$$\lim_{x \to \infty} F(x) = 1.$$

These conditions imply that the area below the corresponding p.d.f. is exactly equal to one.

When the range of integration is truncated at a point $X_{\max}$ which is less than infinity, then

$$\int_0^{X_{\max}} \max f(x)dx < 1$$

and

$$F(X_{\max}) < \lim_{x \to \infty} F(x) = 1.$$

Thus, a property of the c.d.f. is violated and

$$E(X) \neq \int_0^{X_{\max}} [1 - F(x)]dx. \qquad [4]$$

This result implies that equation [1] is not a correct statement of expected willingness to pay.

There are two procedures that can be used to calculate a proper expected value. First, rather than truncating the range of integration as shown in equation [1], the range of integration can be carried out to infinity as is done in equation [3]. Alternatively, there may be some very good reasons for truncating the range of integration that the researchers become aware of after examining their data set (Boyle and Bishop 1988). Such judgement decisions on the part of the researchers are often essential to the quality of the resulting value estimates (Smith, Desvousges, and Fisher 1986). If the range of integration is to be truncated, as is done in equation [1], it is necessary to normalize the estimated c.d.f. so that its statistical properties are not violated by the truncation process.

NORMALIZATION PROCEDURE

A normalized p.d.f. is defined as

$$f_n(z) = \begin{cases} Kf(z) & \text{if } 0 \leq z \leq X_{\max} \\ 0 & \text{if } \quad z > X_{\max} \end{cases} \qquad [5]$$

where z is the truncated random variable and K is a constant of normalization. Using the fact that

$$\int_0^{X_{\max}} f_n(z)dz = \int_0^{X_{\max}} Kf(z)dx = 1 \quad \text{if } 0 \leq z \leq X_{\max}$$

we can solve for the constant of normalization as

$$K = 1/F(X_{\max}).$$

Therefore,

$$f_n(z) = \begin{cases} f(z)/F(X_{\max}) & \text{if } 0 \leq z \leq X_{\max} \\ 0 & \text{if } \quad z > X_{\max} \end{cases} \qquad [6]$$

and the corresponding c.d.f. is derived as

$$F_n(z) = \int_0^z f_n(u)du = F(z)/F(X_{\max}) \quad \text{if } 0 \leq z \leq X_{\max}. \qquad [7]$$

As a result,

$$F_n(z) = \begin{cases} F(z)/F(X_{\max}) & \text{if } 0 \leq z \leq X_{\max} \\ 0 & \text{if } \quad z > X_{\max} \end{cases} \qquad [8]$$

and

$$E(Z) = \int_0^{X\max} [1 - F(z)/F(X_{\max})]dz. \qquad [9]$$

Thus, the expected value of the truncated random variable is derived from a normalized distribution where the properties of the c.d.f. are not violated. In addition, the expected value, as stated in equation [9], is derived from a modification of the original untruncated distribution, no additional estimation is necessary.

IMPLICATIONS

First, if Sellar, Stoll, and Chavas (1985) did not adjust the estimated c.d.f.'s, then expected willingness to pay should have been computed by integrating the estimated c.d.f.'s from zero to infinity, as shown by equation [3]. For this case, the closed-ended estimates reported by the authors are under-statements of value, i.e., the mass in the upper tail of the distribution is neglected.

Second, if the authors really intended to truncate the range of integration, then they should have normalized the estimated c.d.f.'s prior to computing expected values. For this case, the reported closed-ended estimates are over statements of value. That is, we show that $F(X_{max})$ is less than one and the normalized c.d.f. (see equation [8]) is shifted upwards relative to the unnormalized c.d.f. by a factor of $[1/F(X_{max})]$. This results in the area of integration in equation [9] being smaller than the area of integration in equation [1] and, consequently, the normalized distribution yields a smaller expected value. The difference between the normalized and unnormalized expected values decreases as $F(X_{max})$ approaches one, i.e., $[1/F(X_{max})]$ approaches one.

Third, the results presented here hold regardless of the actual distribution under consideration. That is, these results are taken from general statistical properties and hold regardless of whether a logit model or probit model, or any other continuous distribution, is used to analyze closed-ended-contingent-valuation responses.

The fourth, and final, implication is that truncating the range of integration is tantamount to arguing that problems exist with respect to the estimated c.d.f. A continuous distribution on the interval $(0, \infty)$ should not be used. There are two obvious problems. First, values do not really exist over this range. Second, the upper tail of the estimated c.d.f. does not assymptotically approach one as fast as expected. Welsh (1986) has found that this problem can occur in reverse for the lower tail of the distribution, i.e., for the estimated distribution $\lim_{x \to 0} F(x) > 0$. The nonnegativity assumption is violated. In simple English, the estimated c.d.f.'s can have fat tails.

AN ALTERNATIVE

In the face of a fat tails problem, the median of the estimated distribution can be used as an alternative welfare measure, an approach Hanemann (1984) advocates. The median is desirable from an empirical perspective because it is relatively robust with respect to marginal changes in the shape of an estimated distribution. The expected value, in contrast, can change dramatically with even a very slight change in the estimated distribution. However, as Hanemann states, ". . . the choice among [the median or the expected value] entails a value judgement as to the appropriate method of conducting welfare evaluations."

We would argue that the median has an undesirable feature in that it does not fully reflect the values of individuals who have the most to gain or lose, as the case may be, from a proposed policy. If, for example, an estimated distribution were skewed toward high values, the median would be less than the expected value. This is a relationship that we have often observed in contingent-valuation studies regardless of the type of contingent-valuation question employed (Boyle and Bishop 1988). The use of a logistic model to analyze responses to closed-ended-contingent-valuation questions, as was done by Sellar, Stoll, and Chavas, specifically allows for the possibility that an estimated distribution may be skewed toward higher values (Maddala 1984). To use the median as the welfare measure may negate this flexibility of the model.

If the distribution of values is skewed in reality, the median is not a correct welfare measure in that it does not adequately reflect the mass in the tails of the distribution. The question that requires further consideration, and which appears to have motivated Hanemann to advocate the use of the median as a welfare measure, is the determination whether the mass in the tails of estimated distributions is real or an artifact of sampling and estimation procedures.

AN EXTENSION

We believe that continuous distributions on the interval $(0, \infty)$ are appropriate for analyzing responses to closed-ended valuation questions and expected values should be derived using the formula presented in equation [3]. If possible, the range of integration should not be truncated. Our experience is that fat tails, which motivate truncating the range of integration, arise because of the selection of a range and distribution of dollar offers. More precisely, the tails of the estimated distribution are artifacts of the range of dollar values for which observations exist.

The goal, then, is to determine an optimal sampling procedure for obtaining the best estimate of the c.d.f. over its entire range. Much of the conceptual development has focused on obtaining estimates of fixed percentiles of the distribution (James, James, and Westenberger 1984; Kershaw 1985; McLeish and Tosh 1983). For contingent-valuation studies we are concerned with the entire range of the estimated distribution since expected values are computed by integrating the area under the curve defined by $[1 - F(x)]$.

Although we have not been able to derive an optimal sampling strategy for the selection of dollar offers in closed-ended valuation applications to date, we have found a sampling procedure which appears to minimize the potential for the estimated c.d.f.'s having fat tails (Bishop, et al. 1987; Boyle 1985); at least in our applications thus far. This sampling procedure is known as the "method of complementary random numbers" (Ehrenfeld and Ben-Tuvia 1962; Hillier and Lieberman 1980; Kleijnen 1975). This procedure obtains a preliminary estimate of the distribution of values. This can be done in a well-designed pretest survey in which respondents are asked to state a specific value rather than simply answering yes or no to proposed dollar amounts. Pretest valuation responses are used to construct an empirical c.d.f. that is used to specify the closed-ended offers for the final survey.

Closed-ended offers are developed in a four-step process. First, given a sample size of N, $N/2$ random numbers (probabilities, p_i's) are generated from a uniform distribution on the interval (0, 1). Second, an additional $N/2$ probabilities (q_i's) are derived as

$$q_i = 1 - p_i, \qquad \forall\, i.$$

This computation gives the researcher N probability data points, $N/2$ randomly selected p_i's and $N/2$ calculated q_i's. Third, the probabilities (p_i's and q_i's) are converted to dollar offers using the empirical c.d.f. of values derived from the pretest survey data, and the dollar offers are rounded to even dollar amounts. Finally, the dollar offeres are randomly assigned to surveys.

The process of selecting the p_i's and q_i's constitutes the method of complementary random numbers; for each p_i there is a corresponding q_i. The use of the empirical distribution employs prior information about the distribution of values to set the range and distribution of closed-ended offers. This process insures that the selected observations are balanced between the tails of the distribution, and clusters the majority of the offers around the median.

CONCLUSION

Our comment clarifies a point with respect to the calculation of expected values and suggests a sampling procedure that may help to improve future applications of closed-ended-contingent-valuation questions. The important message is that more consideration needs to be given to the design of closed-ended valuation questions, especially with respect to the selection of dollar offers.

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