Measuring and explaining economic inequality: An extension of the Gini coefficient.

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Running title: **On an extension of the Gini coefficient**

Abstract:

This paper proposes a new class of inequality indices based on the Gini's coefficient (or index). The properties of the indices are studied and in particular they are found to be regular, relative and satisfy the Pigou-Dalton transfer principle. A subgroup decomposition is performed and the method is found to be similar to the one used by Dagum [4, 5] when decomposing the Gini index. The theoretical results are illustrated by case studies, using actual Cameroonian data.

Keyword: Measuring inequality, Generalisation of the Gini index, Pigou-Dalton's transfer, Subgroup decomposition

JEL Classification : C43, D31, D63

1. Introduction

Research studies on the measurement of economic inequality are dominated by the Gini index (or coefficient) and the entropy family of indices. Many studies have been devoted to the properties of these two categories of indices. Since the early works of Gini [6] , the Gini index has been studied by several authors, nowadays it lends itself to axiomatic characterisation and at least to two kinds of generalisations [2, 12] . Its decomposition into sub-groups which previously was not very satisfactory has been improved by the recent works of Dagum [4, 5] who proposes a new approach for solving the problem. More recently, S.Mussard [7] proposed a simultaneous decomposition of the Gini index into sub-groups and sources of income etc.

The present study is in keeping with this area of research which it attempts to extend. We propose a family of inequality indices, denoted $I_G^{(\alpha)}$, which generalise the Gini index, and which intersects the entropy family through the coefficient of variation squared. We analyse the axiomatic properties of our class of indices and we show in particular, that, it is a class of relative, regular indices which satisfy the Pigou-Dalton transfer principle. We study the consequences of a transfer from a richer to a poorer individual and we show that the effect of such a transfer is maximal at a central value of the income distribution which we define. Next we show that $I_G^{(\alpha)}$ lends itself to decomposition into sub-groups. The decomposition proposed is a generalisation of Dagum's decomposition of the Gini index.

The remainder of the paper is organized as follows: In section 2, we present notations and preliminaries. In section 3 we define the index $I_G^{(\alpha)}$ and we analyze its properties. Decomposition of the proposed index into sub-groups is undertaken in section 4. Section 5 analyzes the particular case of α =2 corresponding to coefficient of variation squared which also belongs to the family of entropy indices. Finally, section 6 concludes the paper and section 7 is devoted to references.

2. Notations and Preliminaries

In this paper, $P = \{1,2,3,...,i,..n\}$ is a population of n members. X is a positive variable defined in P, and represents an income source distribution between the n members of P. We denote $x_1, x_2, x_3, \cdots, x_i, \cdots, x_n$, the values of X on the n members of P respectively. We assume that P is partitioned into K subpopulations $P_1, P_2, P_3, \cdots, P_h, \cdots, P_K$ with respectively $n_1, n_2, n_3, \cdots, n_h, \cdots, n_K$, $\left(\sum_{h=1}^{K} n_h = n\right)$ members. The value of X on member number i of P_h is written x_{hi}. The restriction of X in P_h is written X_h; $\mu(\mu_h)$ is the mean of X in P (in P_h) and $Var(X)$ ($Var(X_h)$) represents the variance of X in P (in P_h). Also, $CV^2(X)\left(CV^2(X_h)\right)$ is the square of the coefficient of variation of X in $P(in\ P_h)$:

$$CV^2(X) = \frac{Var(X)}{\mu^2} \text{ and } CV^2(X_h) = \frac{Var(X_h)}{\mu_h^2} .$$

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i \ ; \ \mu_h = \frac{1}{n_h}\sum_{i=1}^{n_h} x_{hi} \ ; \ \mu = \frac{1}{n}\sum_{h=1}^{K} n_h \mu_h \tag{1}$$

For any real number α, we define the following real functions:

$$D_\alpha(x) = \sum_{x_i \le x}(x - x_i)^\alpha - \sum_{x_i \ge x}(x - x_i)^\alpha = \sum_{x_i \le x}|x - x_i|^\alpha - \sum_{x_i \ge x}|x - x_i|^\alpha \tag{2}$$

And,

$$H_\alpha(x) = \sum_{x_i \le x}(x - x_i)^\alpha - \sum_{x_i \ge x}(x - x_i)^\alpha = \sum_{i=1}^{n}|x - x_i|^\alpha \tag{3}$$

where, $D_\alpha(x)$ represents the sum of differentials (to the power α) relative to x of the income less than x minus the sum of differentials relative to x of the incomes which are greater than it. $H_\alpha(x)$ represents the sum of differentials to the power α, relative to x of all the incomes of the population.

Properties of $D_\alpha(x)$ and $H_\alpha(x)$ and their relationships

Properties of $D_\alpha(x)$

(i) If $\alpha = 0$,

- $\forall x \in R,\ D_0(x) =$ (Number of x_i less or equal to x)-(Number of x_i greater or equal x)
- If we assume $x_1 < x_2 < x_3 < \ldots < x_n$,

$$D_0(x) = \begin{cases} -n & if\ \ x < x_1 \\ 2i - n & if\ \ x_i < x < x_{i+1} \\ (2i-1) - n & if\ \ x = x_i \\ n & if\ \ x > x_n \end{cases} \tag{4}$$

- D_0 is therefore an increasing step function; $D_0(x) = 0$ at the median of X:

  If n is an odd number, n=2p+1, the only point for which $D_0(x) = 0$ is noted M_0 and we have $M_0 = x_{P+1}$.

  If n is even, n=2p, for all x such that, $x_p < x < x_{p+1}$, $D_0(x) = 0$.

(ii) If $\alpha > 0$

- D_α is continuous and differentiable (except at points x_1, x_2, x_3,…,x_n if $0 < \alpha < 1$) ; we have, $D_\alpha^{'}(x) = \alpha H_{\alpha-1}(x)$ **>0.**
- D_α is strictly increasing from $-\infty$ to $+\infty$, on R .Therefore, it exists a unique point noted M_α, for which $D_\alpha(M_\alpha) = 0$. $D_\alpha(x)$ is positive for any $x \geq M_\alpha$ and negative for any $x \leq M_\alpha$.
- In particular, $\forall x \in \mathrm{R}$, $D_1(x) = nx - n\mu$ and $M_1 = \mu$ = mean of X. (5)

(iii) If $\alpha < 0$

D_α is not defined at points $x_1 < x_2 < x_3 < \ldots < x_n$. It is continuous differentiable and strictly decreasing in each of the intervals $\left]x_i\,, x_{i+1}\right[$ where it varies from $+\infty$ to $-\infty$. In the interval $\left]x_i\,, x_{i+1}\right[$, $D_\alpha = 0$ at a unique point denoted e_{α_i} (i=1,2,…,n-1).

<u>Properties of</u> $H_\alpha(x)$

(i) For $\alpha \geq 1$, H_α is convex (strictly convex if $\alpha > 1$), decreases from $+\infty$ to $M_{\alpha-1}$ then increases from $M_{\alpha-1}$ to $+\infty$. In other word, $M_{\alpha-1}$ is the (unique if $\alpha > 1$) minimum for H_α.

(ii) For $0 < \alpha < 1$, H_α is concave in each of interval $\left[x_i\ , x_{i+1}\right]$, where it admits a maximum at $e_{\alpha_i - 1}$ (i=2,3,…,n) and a vertical tangent at each point x_i.

(iii) For $\alpha = 0$, H_α is constant and equal to n .

Relationship between $D_\alpha(x)$ and $H_\alpha(x)$

(i) $\forall \alpha > 1$, D_α and H_α are two continuous and differentiable functions , and we have,

$$D'_\alpha(x) = \alpha H_{\alpha-1}(x) \text{ and } H'_\alpha(x) = \alpha D_{\alpha-1}(x)$$

(ii) For any integer p greater than 1, and for any α >p, set

$$\alpha(\alpha-1)(\alpha-2)\ldots(\alpha-p+1) = A^P_\alpha$$

If $D^{(p)}_\alpha$ and $H^{(p)}_\alpha$ are the p[th] derivatives of D_α and H_α respectively, we have,

$$D^{(p)}_\alpha(x) = \begin{cases} A^p_\alpha D_{\alpha-p}(x) & if\ p\ is\ even \\ A^p_\alpha H_{\alpha-p}(x) & if\ p\ is\ odd \end{cases} \quad \text{and} \quad H^{(p)}_\alpha(x) = \begin{cases} A^p_\alpha H_{\alpha-p}(x) & if\ p\ is\ even \\ A^p_\alpha D_{\alpha-p}(x) & if\ p\ is\ odd \end{cases} \tag{6}$$

3. The Gini Index of Order α and Its Properties

Definition 1:

We denote the Gini index of order α $(\alpha > 0)$ of any positive distribution X in P , the function I^α_G, which is defined by,

$$I_G^{(\alpha)}(X) = \frac{1}{2n^2\mu^\alpha}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|x_i - x_j\right|^\alpha = \frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|y_i - y_j\right|^\alpha$$

$I_G^{(\alpha)}(X)$ is equal to half of the mean of differentials to the power α of the y_i $\left(y_i = \frac{x_i}{\mu}\right)$.

Lemma 1:

(i) If $\alpha=1$, $I_G^{(\alpha)}$ is equal to the standard Gini index I_G.

(ii) If $\alpha=2$, $I_G^{(\alpha)}$ is equal to the coefficient of variation squared CV^2.

Proof: It is obvious that $I_G^{(1)} = I_G$. We only need to show that $I_G^{(2)}(X) = CV^2(X)$.

Since $CV^2(X) = \frac{Var(X)}{\mu^2}$, it is therefore sufficient to show that

$Var(X) = \frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|x_i - x_j\right|^2$. Develop this last term to get,

$$\frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|x_i - x_j\right|^2 = \frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}(x_i - x_j)^2 = \frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(x_i^2 + x_j^2 - 2x_ix_j\right)$$

$$= \frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}x_i^2 + \frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}x_j^2 - \frac{2}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}x_ix_j$$

$$= \frac{n}{2n^2}\sum_{i=1}^{n}x_i^2 + \frac{n}{2n^2}\sum_{j=1}^{n}x_j^2 - \frac{2}{2n^2}\sum_{i=1}^{n}x_i\sum_{j=1}^{n}x_j$$

$$= \frac{1}{2n}\sum_{i=1}^{n}x_i^2 + \frac{1}{2n}\sum_{j=1}^{n}x_j^2 - \left(\frac{1}{n}\sum_{i=1}^{n}x_i\right)\left(\frac{1}{n}\sum_{j=1}^{n}x_j\right)$$

$$= \frac{1}{n}\sum_{i=1}^{n}x_i^2 - \mu^2 = Var(X) \qquad \square$$

While the literature tends to treat the Gini index and the entropy class of indices separately, the above lemma proves that there exist a link between the Gini index and the coefficient of variation squared which belongs to the entropy family.

3.1 Axiomatic Properties

Proposition 1:

The index $I_G^{(\alpha)}$ satisfies the following properties:

(i) Relative invariance or Homogeneity of zero degree:

$\forall \lambda > 0,\ I_G^{(\alpha)}(\lambda X) = I_G^{(\alpha)}(X)$

(ii) Normalization:

If X is an egalitarian distribution: $X = (x, x, x, ..., x)$ then $I_G^{(\alpha)}\left(X\right) = 0$

(iii) Symmetry or Anonymity:

For any permutation ρ in $P = \{1, 2, 3, \cdots, i, \cdots, n\}$, $I_G^{(\alpha)}\left(x_{\rho(1)}, x_{\rho(2)}, \cdots, x_{\rho(n)}\right) = I_G^{(\alpha)}\left(X\right)$.

(iv) Dalton's population principle:

$$I_G^{(\alpha)}\left(\underbrace{x_1, x_1, \cdots, x_1}_{m times}; \underbrace{x_2, x_2, \cdots, x_2}_{m times}; \cdots; \underbrace{x_n, x_n, \cdots, x_n}_{m times}\right) = I_G^{(\alpha)}\left(X\right)$$

Proof: Assertion (ii) being obvious, we only prove (i), (iii) and (iv).

(i) $I_G^{(\alpha)}\left(\lambda X\right) = \dfrac{1}{2n^2(\lambda\mu)^\alpha}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|\lambda x_i - \lambda x_j\right|^\alpha = \dfrac{\lambda^\alpha}{2n^2\lambda^\alpha\mu^\alpha}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|x_i - x_j\right|^\alpha = I_G^{(\alpha)}\left(X\right)$

(iii)

$$I_G^{(\alpha)}\left(x_{\rho(1)}, x_{\rho(2)}, \cdots, x_{\rho(n)}\right) = \frac{1}{2n^2\mu^\alpha}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|x_{\rho(i)} - x_{\rho(j)}\right|^\alpha = \frac{1}{2n^2\mu^\alpha}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|x_i - x_j\right|^\alpha = I_G^{(\alpha)}\left(X\right).$$

(iv) $$I_G^{(\alpha)}\left(\underbrace{x_1, x_1, \cdots, x_1}_{m\ times}; \underbrace{x_2, x_2, \cdots, x_2}_{m\ times}; \cdots; \underbrace{x_n, x_n, \cdots, x_n}_{m\ times}\right) = \frac{1}{2(nm)^2\mu^\alpha}\sum_{k=1}^{nm}\sum_{l=1}^{nm}\left|x_k - x_l\right|^\alpha$$

$$= \frac{m^2}{2(nm)^2\mu^\alpha}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|x_i - x_j\right|^\alpha = I_G^{(\alpha)}(X)$$

□

Proposition 2:

For $\alpha \geq 1$, $I_G^{(\alpha)}$ satisfies the Pigou-Dalton transfer principle and is therefore a relative, regular index.

Proof: For $\alpha = 1$, $I_G^{(\alpha)}$ is equal to Gini coefficient and thus satisfies Pigou-Dalton transfer principle. For $\alpha > 1$, the social welfare function associated with $I_G^{(\alpha)}(X)$ is,

$$W_\alpha(X) = -\, I_G^{(\alpha)}(X) = \frac{-1}{2n^2\mu^\alpha}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|x_i - x_j\right|^\alpha = \frac{-1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|y_i - y_j\right|^\alpha$$

where $y_i = \dfrac{x_i}{\mu}$ is the relative income of the individual i. Denote $Y = (y_1, y_2, y_3, \cdots, y_n)$ the distribution of relative income corresponding to X. This function may be written as the sum of individual appreciation,

$$W_\alpha(Y) = \sum_{i=1}^{n} u_\alpha(y_i) \text{ where } u_\alpha(y) = \frac{-1}{2n^2}\sum_{j=1}^{n}\left|y - y_j\right|^\alpha = \frac{-1}{2n^2} H_\alpha(y) \qquad (7)$$

And H_α is defined as in (3).

From Eq. (2), (3) and (6), we deduce that,

If $\alpha > 1$, the derivative of u_α is:

$u'_\alpha(y) = \dfrac{-\alpha}{2n^2}\left(\sum_{y_i \leq y}(y - y_i)^{\alpha-1} - \sum_{y_i \geq y}(y_i - y)^{\alpha-1}\right) = \dfrac{-\alpha}{2n^2} D_{\alpha-1}(y)$. And it follows that (see paragraph *Properties of* $D_\alpha(x)$*; (ii)*) u'_α is strictly decreasing, u_α is thus concave and consequently $I_G^{(\alpha)}$ satisfies the Pigou-Dalton transfer principle □

Remark:

(i) In economic terms, the value of $u_\alpha(y_i)$ corresponds to the utility[1] associated with income y_i and the value of $W_\alpha(Y)$ to the social utility associated with the distribution of incomes $(y_1, y_2, y_3, \cdots, y_n)$.

[1] We note that an utility function is defined up to an increasing monotonic transformation.

(ii) If $\alpha < 1$, $I_G^{(\alpha)}$ does not satisfy the Pigou-Dalton transfer principle although some transfers may reduce the value of $I_G^{(\alpha)}$. It is for instance the text book case:

X = 23 ,45 ,67 ,43.5 ,123, 78, 45, 89, 213, 90, 23, 45, 67, 43.5, 123, 78, 45, 89, 213, 90 and $\alpha = 0.3$; for which we have $I_G^{(0.3)}(X)=$ 0 .368. When individual 2 transfer 10 units to individual 1, the index increases to 0.37201. When individual 5 transfers 23 units to individual 7, the index decreases to 0.3674.

From now in the rest of paper, we assume that $\alpha \geq 1$.

Corollary 1:

The maximum value of $I_G^{(\alpha)}$, for $\alpha \geq 1$, is equal to $\frac{(n-1)}{n} n^{\alpha-1}$. This value is obtained with the perfect inegalitarian X distribution where only one individual holds the entire resource.

Proof : The fact that the maximum value of $I_G^{(\alpha)}(X)$ can be obtained with the perfectly unequal distribution X_e is a direct consequence of The Pigou-Dalton transfer principle. If r represents the individual who holds the entire resource in X_e and x the total resource held by r, then:

$$I_G^{(\alpha)}(X_e) = \frac{1}{2n^2\mu^\alpha} \sum_{i=1}^{n}\sum_{j=1}^{n} \left|x_i - x_j\right|^\alpha = \frac{1}{2n^2\left(\frac{x}{n}\right)^\alpha} \left(\sum_{j=1}^{n} \left|x_r - x_j\right|^\alpha + \sum_{i \neq r}^{n} \left|x_i - x_r\right|^\alpha \right)$$

$$= \frac{1}{2n^2\left(\frac{x}{n}\right)^\alpha} \left((n-1)x^\alpha + (n-1)x^\alpha \right) = \frac{n^\alpha (n-1)}{n^2} = \frac{n-1}{n} n^{\alpha-1} \qquad \square$$

This result shows in particular that, there is no upper limit for inequality; it depends on the size of the population and the parameter α. If α >1 and n exceeds 10, the upper value is greater than 1. However, it is interesting to note that :

$J_G^{(\alpha)}(X) = \frac{I_G^{(\alpha)}(X)}{(n-1)n^{\alpha-2}} = \frac{1}{2(n-1)n^\alpha \mu^\alpha} \sum_{i=1}^{n}\sum_{j=1}^{n} \left|x_i - x_j\right|^\alpha$, which is obtained from $I_G^{(\alpha)}$ by normalization , takes on its values in the interval $[0 , 1]$.

Corollary 2 :

If $\alpha \geq 1$, the variation $dI_G^{(\alpha)}(Y)$ of the index, consecutive to an infinitesimal transfer dh from a rich j to a poor i, implies a decrease in the index equal to:

$$dI_G^{(\alpha)}(Y) = \frac{\alpha dh}{2n^2}\left(D_{\alpha-1}(y_i) - D_{\alpha-1}(y_j)\right)$$

Where D_α is the function defined in (2)

Proof : Simply write, $dI_G^{(\alpha)}(Y) = dh\left(\frac{\partial I_G^{(\alpha)}(Y)}{\partial y_i} - \frac{\partial I_G^{(\alpha)}(Y)}{\partial y_j}\right) = dh\left(u_\alpha'(y_j) - u_\alpha'(y_i)\right)$ where $u_\alpha(y)$ is defined in (7)

$$= \frac{\alpha dh}{2n^2}\left(D_{\alpha-1}(y_i) - D_{\alpha-1}(y_j)\right) \qquad \square$$

Consequence of a transfer

The result of corollary 2, though given at the nearest increasing monotonic transformation, is interesting since it allows to study the behaviour of $dI_G^{(\alpha)}(Y)$ as a function of incomes y_i and y_j. Here we give the particular cases for $\alpha = 1, 2$ and $\alpha \geq 3$.

(i) If $\alpha = 1$,

$$dI_G^{(\alpha)}(Y) = \frac{dh}{2n^2}\left(D_0(y_i) - D_0(y_j)\right) = \frac{dh}{2n^2}\left[\left(2rank(y_i) - n - 1\right) - \left(\left(2rank(y_j) - n - 1\right)\right)\right]$$

$$= dh\frac{rank(y_i) - rank(y_j)}{n^2}$$

$$= \frac{dh}{n^2}(i - j) \quad \text{if} \quad y_1 < y_2 < \cdots < y_n$$

$dI_G^{(\alpha)}(Y)$ depends on the rank of individuals and not on their incomes: the index gives the same importance to the inequality among the poor as well as among the rich. This is a well-known result concerning the Gini coefficient.

(ii) If $\alpha = 2$, $dI_G^{(\alpha)}(Y) = \frac{2dh}{2n^2}\left(D_1(y_i) - D_1(y_j)\right)$ and by using formula (5) ,

$$= \frac{dh}{n^2}\left[\left(ny_i - n\right) - \left(ny_j - n\right)\right] = \frac{dh}{n}\left(y_i - y_j\right)$$

Again we find that, for the coefficient of variation squared, the decrease is independent of the income level of individuals, but depends only on the differential between these incomes: this index therefore gives the same importance to inequality among the poor as well as among the rich.

(iii) If $\alpha \geq 3$, then $\alpha - 2 \geq 1$ and we know (see paragraph 2 ; *Properties of* $H_\alpha(x)$ *; (i)*)

that $H_{\alpha-2}$ is convex and admits a minimum $M_{\alpha-3}$. Consequently, the second derivative of u_α, which is equal to $u_\alpha^{''}(y) = -\dfrac{\alpha(\alpha-1)}{2n^2} H_{\alpha-2}(y)$ is concave, and admits a maximum at $M_{\alpha-3}$. This means that the index gives more importance to inequality among individuals who have an income close to the 'central' value $M_{\alpha-3}$; the most importance is given to individuals who have income equal $M_{\alpha-3}$.The index gives less importance to inequality among poor as to that among the rich. The reason to qualify $M_{\alpha-3}$ as a central value could be justified by noting that, if $\alpha = 3$, $M_{\alpha-3}$ is the median (see Eq. (4)) population income and if $\alpha = 4$, $M_{\alpha-3}$ is the average income of the population (see Eq. (5)).

Proposition 3:

For any distribution X, one and only one of the following properties is verified:

(i) $I_G^{(\alpha)}(X)$ is a decreasing function of α which tends towards a real constant when α tends towards $+\infty$

(ii) There exist an α_0 for which we have: $\alpha > \alpha' \geq \alpha_0 \Rightarrow I_G^{(\alpha)}(X) > I_G^{(\alpha')}(X)$; in this case $I_G^{(\alpha)}(X)$ tends towards $+\infty$ when α tends towards $+\infty$.

Proof: Consider the distribution X and all the possible relative differentials $\left|\dfrac{x_i - x_j}{\mu}\right|$ i=1,2,…,n ; j=1,2,…,n.

Represent by $a_1, a_2, \ldots, a_p$ those of the differentials which are strictly greater than 0 and smaller or equal to 1, and by $b_1, b_2, \ldots, b_q$ the differentials which are strictly greater than 1. It is obvious that:

$$I_G^{(\alpha)}(X) = f(\alpha) = \frac{1}{2n^2}\left(\sum_{k=1}^{p} a_k^\alpha + \sum_{k=1}^{q} b_k^\alpha\right).$$

The first and second derivative of f are respectively:

$$f'(\alpha) = \frac{1}{2n^2}\left(\sum_{k=1}^{p} \ln(a_k) a_k^\alpha + \sum_{k=1}^{q} \ln(b_k) b_k^\alpha\right) \text{et } f''(\alpha) = \frac{1}{2n^2}\left(\sum_{k=1}^{p} \ln^2(a_k) a_k^\alpha + \sum_{k=1}^{q} \ln^2(b_k) b_k^\alpha\right).$$

This expression proves that f'' is strictly positive and consequently f' is strictly increasing in the interval $[0\,;+\infty[$.

- If there are no differentials strictly greater than 1, then all the differentials fall between 0 and 1 and f' is strictly negative since it increases from $\frac{1}{2n^2}\sum_{k=1}^{p}\ln(a_k)$ to 0. In this case the function $f(\alpha)$ is strictly decreasing and assertion 1) of the proposition is verified.
- If on the other hand, there exist differentials which are strictly greater than 1, the function f' increases from $f'(0) = B = \frac{1}{2n^2}\left(\sum_{k=1}^{p}\ln(a_k) + \sum_{k=1}^{q}\ln(b_k)\right)$ to $+\infty$. If $B \geq 0$, f' is positive and f is strictly increasing. By taking $\alpha_0 = 1$, assertion (ii) of the proposition is verified. If $B < 0$, In accordance with the intermediate value theorem, there will exist a unique real r which nullifies the function f' and by taking $\alpha_0 = Max(r\,,1)$, assertion (ii) of the proposition is verified. □

3.2 Economic Interpretation and Choice of the Parameter α

The value of the index $I_G^{(\alpha)}(X)$ is defined as the mean of the relative differentials $\left|\frac{x_i - x_j}{\mu}\right|^{\alpha}$.

Now some of differentials $\left|\frac{x_i - x_j}{\mu}\right|$ may be smaller or equal to 1whereas others are strictly greater than 1. Taking the power of these differentials has the effect of amplifying them in case they are greater than 1 and reducing them in case they are less than 1. It results from this that, relative to the Gini index, the large differentials will contribute more to the final value of the index, while the differentials inferior to 1 will have their contribution reduced. From this standpoint, we may say that parameter α plays the judge by giving bonuses to small differentials (those which are less than 1) and sanctions to large differentials (those which are greater than 1). Since this phenomenon of bonus-sanction takes on increasing significance with the value of α, the problem of choosing the appropriate value of α will emerge. As in the case of the family of entropy indices, this problem strictly speaking, does not have a solution. In practice, economists simply prefer the first integer values (1 or 2) of parameter β of the entropy. In the case of the class of indices $I_G^{(\alpha)}, \alpha = 1$ or 2 correspond to the Gini index or to the square of the coefficient of variation which are among the indices widely used by practicians. Moreover in the case of $I_G^{(\alpha)}$, an approach for solving the problem of choosing parameter α may be proposed from the proposition 3 above. In effect, in the light of this proposition, income distributions are partitioned into two categories; the first one of which is made up of variables X which all have differentials $\left|\frac{x_i - x_j}{\mu}\right|$ less than or equal to 1 and the second with variables X having at least one differential $\left|\frac{x_i - x_j}{\mu}\right|$ greater than 1:

- If income distribution X is in the first category i.e X is not very inegalitarian so that all the relative differentials relative to their mean are less than or equal to 1, then $I_G^{\alpha}(X)$ will be a decreasing function of α which tends torward a real constant as α tends torward infinity. In this case we will choose α =1 in order not to have a very low value index and in order not to completely cancel the contribution of the very small differentials to the final value of $I_G^{\alpha}(X)$.
- If income distribution X is in the second category, this means that there exist at least two individuals whose differentials relative to the mean of their incomes is strictly greater than 1: $\exists x_i, x_j \left|\frac{x_i - x_j}{\mu}\right| > 1$ then $I_G^{\alpha}(X)$ tends toward infinity as α tends toward infinity and according to proposition 3, there will exist α_0 for which $I_G^{\alpha}(X)$ will become an increasing function of α : $\alpha_1 > \alpha_2 \geq \alpha_0 \Rightarrow I_G^{\alpha_1}(X) > I_G^{\alpha_2}(X)$

hence, α, for $\alpha \geq \alpha_0$, will be interpreted as a parameter of aversion to inequality, and it seems natural to choose $\alpha = \alpha_0$ (or close to α_0).This choice is also justified by the fact that before $\alpha_0, I_G^{\alpha}(X)$ is a decreasing function of α, and after α_0, the contribution of the large differentials, to the final value of the index, start being exceedingly amplified. To determine α_0, we may proceed by using an exact algorithm or groping by progressively increasing the value of α; in this later case we will reach α_0 as quickly as the large differentials, notably those which are greater than 1 will be relatively more important in number or in value. But if the small differentials are prevalent, α_0 will be large and the procedure might appear long; fortunately in practice and above all in developing countries most of the distributions studies are very inegalitarian and the large differentials are frequent and important in terms of value; in general we get α_0 close to 1 or 2 .

Case study 1: Student expenditures

During a study on the behaviour of students in school, their weekly expenditures were recorded. We consider here the amount of expenditures by the poorest 50 students.

______________________[INSERT TABLE 1 AROUND HERE]____________________

Here, we observe the fact that, to limit oneself to the poorest students has helped obtain a relatively not very inegalitarian distribution. It presents very frequent small differentials and infrequent and non significant (in term of value, $\frac{range}{\mu} = 1.246$) large differentials ; implying that the index decreases down to the value α_0=5 then starts increasing (slowly) toward infinity. In this case we could take $\alpha = 5$ or 6.

Case study 2: Inequality of food expenditures among Cameroonian households working in the formal sector

The ECAMII-2001 database is used. This is a household survey carried out by Cameroon's National Institute of Statistics. Here we consider households whose heads work in the formal sector, i.e. in an officially registered business, and who pay taxes regularly. We have thus retained 1070 households and the results are the following:

$I_G^{(1)}=0{,}34762 \qquad I_G^{(2)}=0{,}87247 \qquad I_G^{(3)}=8{,}41573 \qquad I_G^{(3.5)}=32{,}17541 \qquad I_G^{(4)}=128{,}52584$

Which show that the index starts to increase from the value of α_0=1 and the amplification of the large differentials is significantly felt when the value of α reaches 3. In this case, we can pick up α=1 or 2

4. Decomposition into Sub-Groups

Since the pioneer works of Bourguignon [1], Shorrocks [9, 10, 11] and Cowell [2], decomposability into subgroups (or sub-populations) constitutes one of the most required properties of an inequality index. We show that the $I_G^{(\alpha)}$ index lends itself to decomposition into sub-groups. The decomposition proposed is a generalisation of Dagum's [4, 5] decomposition of the Gini index. First, we present decomposition into two components: The within-groups component and the gross between-groups component. The latter is expressed in the form of effective inequalities between pairs of sub-populations rather than in terms of a simple difference between the means as is the case in the decomposition of many inequality indices. Next, we obtain a decomposition into three components by splitting up the gross between-groups component into two sub-components of which the first is called the net between-groups component, and the second, the transvariational[2] (or overlapping) between-group component.

Assume that the population is partitioned into sub-populations $P_k\ (k=1,2,\cdots,K)$ of size n_k and X_k is the restriction of X in P_k. For any subpopulation P_k, we set: $f_k=\frac{n_k}{n}$ and $s_k(\alpha)=\frac{n_k}{n}\left(\frac{\mu_k}{\mu}\right)^{\alpha}$. We then define for any couple of sub-populations P_h *and* P_k, the average difference of Gini of order α :

[2] ' transvariational' comes from 'transvariazione' which is the term used by C. Gini in 1916.

$$\Delta_{hk}(\alpha) = E|X_h - X_k|^{\alpha} = \frac{1}{n_h n_k}\sum_{i=1}^{n_h}\sum_{j=1}^{n_k}|x_{hi} - x_{kj}|^{\alpha}$$

And we introduce the inequality index between the subpopulation P_h *and* P_k:

$G_{hk}(\alpha) = \dfrac{\Delta_{hk}(\alpha)}{\mu_h^{\alpha} + \mu_k^{\alpha}}$.

We have in particular: $G_{hh}(\alpha) = \dfrac{\Delta_{hh}(\alpha)}{2\mu_h^{\alpha}} = \dfrac{1}{2n_h^2\mu_h^{\alpha}}\sum_{i=1}^{n_h}\sum_{j=1}^{n_h}|x_{hi} - x_{hj}|^{\alpha} = I_G^{\alpha}(X_h)$

Definition 2:

The gross economic wealth noted d_{hk}, is defined between two subpopulations P_h *and* P_k such that $\mu_h > \mu_k$: d_{hk} is the mean of the difference $(x_{hi} - x_{kj})$ for each income x_{hi} of a member in P_h greater than income x_{kj} of a member in P_k.

$$d_{hk} = \int_0^{+\infty} dF_h(y)\int_0^y |y-x|\ dF_k(x) = \frac{1}{n_h n_k}\sum_{\substack{i=1 \\ x_{hi} > x_{kj}}}^{n_h}\sum_{j=1}^{n_k}|x_{hi} - x_{kj}| \leq \Delta_{hk}$$

where $\Delta_{hk} = E|X_h - X_k| = \dfrac{1}{n_h n_k}\sum_{i=1}^{n_h}\sum_{j=1}^{n_k}|x_{hi} - x_{kj}| = \Delta_{hk}(1)$

Following Dagum, we set $p_{hk} = \Delta_{hk} - d_{hk}$ if $\mu_h > \mu_k$. p_{hk} corresponds to the transvariational component.

Definition 3:

The net economic wealth between two subpopulation P_h and P_k such that $\mu_h > \mu_k$: is defined by the difference $d_{hk} - p_{hk} > 0$; and the relative economic difference between two such subpopulations is given by:

$$D_{hk} = \frac{d_{hk} - p_{hk}}{\Delta_{hk}(1)}$$

It is clear that, $\Delta_{hk}(\alpha), G_{hk}(\alpha)$ and D_{hk} define symmetric matrices and it is well known (see Dagum [4, 5]) that D_{hk} is a distance on the set of distributions X_h which is null if and only if there is perfect overlapping between distributions and $0 \le D_{hk} \le 1$.

Proposition 4:

(i) For any $\alpha > 0$, the index $I_G^{(\alpha)}$ is decomposable into two components as follows :

$$I_G^{\alpha}(X) = \sum_{h=1}^{K} f_h s_h(\alpha) I_G^{(\alpha)}(X_h) + \sum_{h=2}^{K}\sum_{k=1}^{h-1} G_{hk}(\alpha)\left(f_k s_h(\alpha) + f_h s_k(\alpha)\right) \quad = I_{G_W}^{(\alpha)} + I_{G_B}^{(\alpha)}$$

(ii) For any $\alpha > 0$, the index $I_G^{(\alpha)}$ is Dagum decomposable into three components:

$$I_G^{\alpha}(X) = \sum_{h=1}^{K} p_h s_h(\alpha) G_{hh}(\alpha) + \sum_{h=2}^{K}\sum_{k=1}^{h-1} G_{hk}(\alpha) D_{hk}\left(f_k s_h(\alpha) + f_h s_k(\alpha)\right)$$

$$+ \sum_{h=2}^{K}\sum_{k=1}^{h-1} G_{hk}(\alpha)(1 - D_{hk})\left(f_k s_h(\alpha) + f_h s_k(\alpha)\right) \qquad = I_{G_W}^{(\alpha)} + I_{G_{BN}}^{(\alpha)} + I_{G_{BT}}^{(\alpha)}$$

Proof:

(i) Decomposition into two components

$$I_G^{(\alpha)}(X) = \frac{1}{2n^2\mu^\alpha}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|x_i - x_j\right|^\alpha = \frac{1}{2n^2\mu^\alpha}\sum_{h=1}^{K}\sum_{k=1}^{K}\sum_{i=1}^{n_h}\sum_{j=1}^{n_k}\left|x_{hi} - x_{kj}\right|^\alpha$$

$$= \frac{1}{2n^2\mu^\alpha}\sum_{h=1}^{K}\sum_{k=1}^{K} n_h n_k \Delta_{hk}(\alpha) = \frac{1}{2n^2\mu^\alpha}\sum_{h=1}^{K}\sum_{k=1}^{K}\frac{n_h n_k \Delta_{hk}(\alpha)}{\left(\mu_h^\alpha + \mu_k^\alpha\right)}\left(\mu_h^\alpha + \mu_k^\alpha\right)$$

$$= \frac{1}{2n^2\mu^\alpha}\sum_{h=1}^{K}\sum_{k=1}^{K} G_{hk}(\alpha) n_h n_k \left(\mu_h^\alpha + \mu_k^\alpha\right)$$

$$= \sum_{h=1}^{K}\left(\frac{n_h}{n}\right)^2\left(\frac{\mu_h}{\mu}\right)^\alpha G_{kk}(\alpha) + \frac{1}{2n^2\mu^\alpha}\sum_{1=h\neq}^{K}\sum_{k=1}^{K} G_{hk}(\alpha) n_h n_k\left(\mu_h^\alpha + \mu_k^\alpha\right)$$

$$= \sum_{h=1}^{K}\left(\frac{n_h}{n}\right)^2\left(\frac{\mu_h}{\mu}\right)^\alpha G_{hh}(\alpha) + \sum_{1=h>}^{K}\sum_{k=1}^{K} G_{hk}(\alpha)\frac{n_h}{n}\frac{n_k}{n}\left(\frac{\mu_h^\alpha}{\mu^\alpha} + \frac{\mu_k^\alpha}{\mu^\alpha}\right)$$

$$= \sum_{h=1}^{K}\left(\frac{n_h}{n}\right)^2\left(\frac{\mu_h}{\mu}\right)^\alpha G_{hh}(\alpha) + \sum_{h=2}^{K}\sum_{k=1}^{h-1} G_{hk}(\alpha)\frac{n_h}{n}\frac{n_k}{n}\left(\frac{\mu_h^\alpha}{\mu^\alpha} + \frac{\mu_k^\alpha}{\mu^\alpha}\right)$$

$$=\sum_{h=1}^{K} f_h s_h(\alpha) I_G^{(\alpha)}(X_h)+\sum_{h=2}^{K}\sum_{k=1}^{h-1} G_{hk}(\alpha)\left(f_k s_h(\alpha)+f_h s_k(\alpha)\right)$$

(ii)Decomposition into three components

$$=\sum_{h=1}^{K} f_h s_h(\alpha) G_{hh}(\alpha)+\sum_{h=2}^{K}\sum_{k=1}^{h-1} G_{hk}(\alpha)\left(f_k s_h(\alpha)+f_h s_k(\alpha)\right)\left(D_{hk}+1-D_{hk}\right)$$

$$=\sum_{h=1}^{K} f_h s_h(\alpha) G_{hh}(\alpha)+\sum_{h=2}^{K}\sum_{k=1}^{h-1} G_{hk}(\alpha) D_{hk}\left(f_k s_h(\alpha)+f_h s_k(\alpha)\right)$$

$$+\sum_{h=2}^{K}\sum_{k=1}^{h-1} G_{hk}(\alpha)\left(1-D_{hk}\right)\left(f_k s_h(\alpha)+f_h s_k(\alpha)\right)$$

$$= I_{G_W}^{(\alpha)}+I_{G_{BN}}^{(\alpha)}+I_{G_{BT}}^{(\alpha)}$$

where: $I_{G_W}^{(\alpha)}=\sum_{h=1}^{K} f_h s_h(\alpha) G_{hh}(\alpha)$; $I_{G_{BN}}^{(\alpha)}=\sum_{h=2}^{K}\sum_{k=1}^{h-1} G_{hk}(\alpha) D_{hk}\left(f_k s_h(\alpha)+f_h s_k(\alpha)\right)$ and

$$I_{G_{BT}}^{(\alpha)}=\sum_{h=2}^{K}\sum_{k=1}^{h-1} G_{hk}(\alpha)\left(1-D_{hk}\right)\left(f_k s_h(\alpha)+f_h s_k(\alpha)\right) \qquad \square$$

$I_{G_W}^{(\alpha)}$ is the contribution of the within subgroup inequality to the overall inequality. $I_{G_{BN}}^{(\alpha)}$ is the net contribution of the between subgroups inequality to the overall inequality. $I_{G_{BT}}^{(\alpha)}$ measures the contribution to the overall inequality, of the inequality coming from the transvariation between the subgroup pairs. Transvariation measures inequalities between subpopulations P_h and P_k considering only the overlapping section of their distributions X_h and X_k. High value of $I_{G_{BT}}^{(\alpha)}$ therefore means that X in general overlaps from one subpopulation to another and the intensities of the overlapping sections are important in the subpopulations. If the means of the K subpopulations are all the same, (it is the case when their distributions coincide) there is perfect overlapping and no net inequality; as consequence, the term $I_{G_{BN}}^{(\alpha)}$ is null and $I_{G_B}^{(\alpha)}=I_{G_{BT}}^{(\alpha)}$.

Case study 3: Decomposition of food expenditures inequality among Cameroonian households working in the formal sector

Again, we use the ECAMII-2001 data base, already used in case study 2, for formal sector workers. We have thus retained 1070 households and subdivide them according to area of residence (1=urban, 2=semi-urban and 3=rural).

We retain α =2 for analysis.

(i) Decomposition into two components

The matrix $\Delta_{hk}(\alpha)$

$$\Delta(\alpha)=\begin{pmatrix} 1630345809587.31 & 1538408420799.14 & 1085860142372.5 \\ 1538408420799.14 & 1438876452397.78 & 1029423218826.49 \\ 1085860142372.5 & 1029423218826.49 & 375429406898.964 \end{pmatrix}$$

The matrix $G_{hk}(\alpha)$

$$G(\alpha)=\begin{pmatrix} 0.9467 & 0.8360 & 0.8547 \\ 0.8360 & 0.7347 & 0.7413 \\ 0.8547 & 0.7413 & 0.4585 \end{pmatrix}$$

It gives unweighted inequalities between the different subgroups; it therefore allows for an evaluation of the impact of weighting on the final components of inequality.

________________[INSERT TABLE 2 AROUND HERE]__________________

(ii) Decomposition into three components

We do not reconsider the intra group component because it remains unchanged.

The matrix d_{hk}

$$d=\begin{pmatrix} 324457.1136 & 355541.9822 & 439988.6017 \\ 355541.9822 & 321475.4443 & 480625.5644 \\ 439988.6017 & 480625.5644 & 2301106.8412 \end{pmatrix}$$

The matrix p_{hk}

$$p=\begin{pmatrix} 324457.1136 & 293919.8244 & 151939.0825 \\ 293919.8244 & 321475.4442 & 130953.8726 \\ 151939.0825 & 130953.8726 & 230106.8412 \end{pmatrix}$$

The matrix of distances D_{hk}

$$D = \begin{pmatrix} 0 & 0.0949 & 0.4866 \\ 0.0949 & 0 & 0.5718 \\ 0.4866 & 0.5718 & 0 \end{pmatrix}$$

____________________[INSERT TABLE 3 AROUND HERE]____________________

We observe that the net total inequality between residence areas (0.08943) is relatively less pronounced than transvariational inequality (0.34932) i.e. the inequality arising from overlapping. It is worth noting that this last value arises largely $\left(\frac{0.29371}{0.34932} = 84\%\right)$ from overlapping between the amounts of households'expenditures in urban areas and those residing in semi-urban areas.

5. A Particular Case for α =2

When $\alpha = 2$ we know that $I_G^{(\alpha)}(X) = CV^2(X)$, and all the preceding shows that this index lends itself to a decomposition other than its classical decomposition. A comparison of both of these decompositions allows us in this particular case, to carry an evaluation of the contributions of sub-population to the between groups component of $I_G^{(2)}$.

Corollary 3:

The index of coefficient of variation squared lends itself to a Dagum type decomposition into two components, then into three components as follows :

$$CV^2(X) = \sum_{h=1}^{K} \left(\frac{n_h}{n}\right)^2 \left(\frac{\mu_h}{\mu}\right)^2 G_{hh}(2) + \sum_{h=2}^{K}\sum_{k=1}^{h-1} \frac{n_h}{n}\frac{n_k}{n}\frac{\mu_h^2 + \mu_k^2}{\mu^2} G_{hk}(2) = CV_W^2 + CV_B^2 \quad (8)$$

$$CV^2(X) = \sum_{h=1}^{K} \left(\frac{n_h}{n}\right)^2 \left(\frac{\mu_h}{\mu}\right)^2 G_{hh}(2) + \sum_{h=2}^{K}\sum_{k=1}^{h-1} \frac{n_h}{n}\frac{n_k}{n}\frac{\mu_h^2 + \mu_k^2}{\mu^2} D_{hk} G_{hk}(2) +$$
$$\sum_{h=2}^{K}\sum_{k=1}^{h-1} \frac{n_h}{n}\frac{n_k}{n}\frac{\mu_h^2 + \mu_k^2}{\mu^2} (1 - D_{hk}) G_{hk}(2) = CV_W^2 + CV_{BN}^2 + CV_{BN}^2 \quad (9)$$

By equating formula (9) of CV^2 index to the one derived by considering the classical decomposition of the variance (mean of variances + variance of means), we find a new expression for CV_B^2 which allows for an evaluation of the contribution of each subgroup to the between-group component.

Corollary 4:

(i) The between-groups component of formula (8) may be written as :

$$CV_B^2 = \sum_{h=1}^{K} \frac{n_h}{n}\left[\left(\frac{\mu_h}{\mu}-1\right)^2 + \left(1-\frac{n_h}{n}\right)\left(\frac{\mu_h}{\mu}\right)^2 G_{hh}(2)\right] \tag{10}$$

(ii) In the Dagum decomposition of the CV^2 index, the contribution of sub-population P_h to the between-groups component is:

$$CV_B^2(P_h) = \frac{n_h}{n}\left[\left(\frac{\mu_h}{\mu}-1\right)^2 + \left(1-\frac{n_h}{n}\right)\left(\frac{\mu_h}{\mu}\right)^2 G_{hh}(2)\right] \tag{11}$$

From Eq. (10) or (11) we can derive two lessons:

(i) If the means of subgroups coincide, (for example, if their distributions are all identical) the contribution of each subgroup to the gross between groups component is not null, but is proportional to its within group index and to its size.

(ii) The gross between group index, and consequently the total CV^2 index, are increasing functions of within group indices, which means, in particular, that this decomposition satisfies the Shorrocks [11] subgroup consistency property.

We have applied the above results to evaluate the contributions of each area of residence to the expenditure inequalities of the 1070 households (see case studies 2 and 3), and they are given below:

____________________[INSERT TABLE 4AROUND HERE]____________________

It emerges from the above results that the urban areas are the most inegalitarian. In fact they contribute up to 82.71% to within group inequality and 52.65% to between groups inequality. Urban areas account for up to 67.60% of the total inequality level in this sector in Cameroon.

6. Conclusions

The class of indices we have proposed generalises the Gini coefficient. These indices possess most of the most important axiomatic properties actually required for a good inequality index. It thus presents other possibilities for measuring and explaining inequality appropriately. It creates a link between the Gini index and the entropy family of indices, since it also contains the coefficient of variation squared. Nevertheless, others properties as income source decomposition have to be studied.

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Tableau 1 : Determination of α_0

α	1	2	3	4	5	5.5	6	6.5	7	8	8.75	9	10	20
$I_G^{\alpha}(X)$	0.176	0.100	0.073	0.063	0.062	0 .064	0.067	0.072	0.078	0.09	0.111	0.118	0.153	3.860

Source : Calculated by the author from a survey carried out by the NGO Humanus-Cameroun, 2000.

Table2: Contribution to the within and to the between groups components

Contribution of the groups to the within groups component			**Contribution of pairs of sub-groups to the between groups component.**		
Urban	0.35877	82.71%		Semi Urban	Rural
Semi Urban	0.07273	16.77%	Urban	0.32451	0.07253
Rural	0.00223	0.52%	Rural	0.03569	-
Total	0.43373	100%	**Total**	0.43873	

Table 3 : Contribution of pairs of subgroups to the net and to the transvariational between groups component

$\alpha = 2$	**Contribution to the net between groups component**		**Contribution to the between groups transvariational** component	
	Semi Urban	Rural	Semi Urban	Rural
Urban	0.0308	0.03822	0.29371	0.04033
Semi Urban	-	0.02041	-	0.01528
Total	0.08943		0.34932	

Table 4: Contribution of sub-groups to the within-groups component and between groups component

$\alpha = 2$	**Contribution to the within groups component**		**Contribution to the between group component**		**Total**	
Group 1= Urban	0.35877	82.71%	0.231	52.65%	0.58977	67.60%
Group 2=Semi urban	0.07273	16.77%	0.17852	40.69%	0.25125	28.80%
Group 3= Rural	0.00223	0.52%	0.02922	6.66%	0.03145	3.60%
Total	0.43373	100%	0.43874	100%	0.87247	100%