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INEQUALITY FACTOR DECOMPOSITION UNDER UNIFORM ADDITIONS PROPERTY WITH APPLICATIONS TO CAMEROONIAN RURAL DATA

By

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Abstract:

This paper proposes new factors decomposition methods for classical inequality indices such as Gini index, variance and squared coefficient of variations index. The approach consists in relaxing the normalization property in order to extend the natural decomposition to a decomposition rule which satisfies the uniform additions property. The regression-based method using the new formulations of components contributions is carried out. Empirical examples, using Cameroonian data, are provided to demonstrate the use of the procedure and to contrast our results to those based on Morduch and Sicular (2002) principle, especially in the case of the Gini index.

Keywords: Inequality factors decomposition, uniform additions property, regression-based decomposition, Cameroon.

JEL Classification: O15, C43, D63

1- Introduction

Recently, the social dimension as concerns development issues now has a central position in economic research. Thus, questions on inequality are in awareness. Problems on how to measure and explain inequality (among others, Morduch and Sicular 2002, Shorrocks 1980, Lerman and Yitzhaki 1985) as well as its effects on poverty and economic growth (among others, Galor and Zeira, 1993; Bourguignon 2004, Shorrocks and Van Der Hoeven 2004) are gradually ushering themselves on top of the agenda of most researchers in development Economics.

This rekindled interest on inequality issues re-put on the table the debate on how to analyse income inequality and its determinants. One of the option retained by researchers was to aggregate income inequality into an index, such as the Gini index, the variance, the squared coefficient of variation or the Theil-T index, then undertake a decomposition of this index by income sources such as salary, transfers, return on investment, etc. Firstly, these researchers proposed a relatively simple decomposition by income sources that turns out to be a functional representation linked the structure of the inequality index we are considering (Rao 1969, Fei et al. 1978, Pyatt et al.1980, Fields 1979). In 1982, Shorrocks, in a more general framework, proposed decomposition by income source that is independent of the considered inequality index. The Shorrocks method is based on a set of axioms. Shorrocks shows that, using six axiomatic properties, there exists a single procedure to evaluate the contributions of the various components of income to total inequality. This result is obtained in two phases. First, the author shows that there exist an infinite number of decomposition rules applicable to each inequality index. Second, Shorrocks introduces two particular axioms to obtain the uniqueness of the decomposition rule. The first of these two axioms is: *Normalization for equal factors distribution* which stipulates that: the contribution of all income sources having an equal distribution is zero. The second axiom states that, if total income is divided into two components of which one is a permutation of the other, then these two components must have the same inequality contribution to total income. This axiom is called the *two factors symmetry assumption*. Naturally, these two properties have caused a lot of debate.

In this paper, we focus our attention on the first axiom whose implications have been a subject of controversy between various authors who can be split into two groups. The first group, headed by Morduch and Sicular (2002), consists of authors who think that, the normalization axiom is undesirable, especially if the considered measure index is relative. They propose that, this axiom be replaced by the *uniform additions* principle (this principle states that the contribution of all positive income sources equally distributed be negative) This property pushes advocates of this group to simply reject all natural decomposition of indices that do not verify this principle. They propose that the Theil-T index be preferred to the Gini index just because the natural decomposition of the Theil index verifies the property of uniform addition while the Gini index does not (for details see Guang Hua Wang 2002).

In opposition to the first group, the second group of authors headed by F.A. Cowell and C.V. Fiolio (2006) observe that, there is no merit in analysing a decomposition procedure which satisfies the property of uniform additions. These authors justify the property of normalization in two ways. Firstly, they remark that it is more sensible that an equally distributed source of income contribute nothing in accounting for inequality. Secondly, they justify the pertinence of the property of normalization by the fact, this property enabled Shorrocks to obtain the unique decomposition of income sources which is independent of the considered inequality index.

The main objective of this paper is to show how the natural decomposition of some classical inequality indices can be modified to take into account the property of uniform additions. The goal here is not to side with either or the other group of authors as concerns the property of normalization. Our aim is to show that, by relaxing Shorrocks normalization axiom for a category of inequality indices, we obtain a family of decomposition which verify the property of uniform additions. In this analysis, we are particularly interested in inequality indices commonly used in most empirical analysis, such as, the Gini index, the squared coefficient of variation and the generally entropy family indices. Concerning the first two indices, we propose a parametric decomposition family with the parameter which is a real function of the total income. This parameter may be interpreted as the weight of the impact of the uniform additions on the decomposition approach. When the parameter equal zero, the procedure yields a decomposition rule (the natural decomposition) which satisfies the Normalization for equal factors distribution property. In other hand, a value of the parameter greater than zero leads to a decomposition rule satisfying the uniform addition property; with the sensitivity of the property in proportion to the magnitude of the parameter value chosen. Hence, our approach can be seen as the bridge between the proponents and opponents of the property of uniform additions. In the case of the squared coefficient of variation, the obtained decomposition family contains the decomposition rule proposed by Morduch and Sicular (2002).

Next, we carry the regression-based inequality decomposition method comparable to the Morduch and Sicular (2002) work. Naturally, this leads to new formula of estimated contributions of the regression components. We demonstrate our method with data for rural households in the Centre province of Cameroon which were collected in 2001 by the National Institute of Statistic.

The outline of the rest of the paper is as follows. Section 2 proposes definitions and main results. Section 3 is devoted to the regression-based decomposition. As to Section 4, the preceding results are applied to analyse the rural Cameroonian households' consumption inequality. The application is presented in two phases. First, the total households consumption is broken down into different consumption sources. We compute the contributions to total consumption inequality of these consumption sources. There, it is found that, the gap is not sensitive between decompositions rules satisfying the uniform additions property and those satisfying normalization property. Secondly, we introduce the regression-based with the constant term to decompose the total income into different determinants. We evaluate the contribution to the total income inequality of the different determinants, and the results are in contrast with the first case; the effect of the uniform additions property becomes more sensitive. Finally we conclude our study in Section 5.

2- Definitions and main results

The starting point is concerned with the inequality measure. We assume that the inequality is measured by a function $I(X)$ which is continuous and symmetric.

The main property can be stated as:

Definition 1: An inequality index $I(X)$ satisfies the property of uniform additions, if for any constant c > o, $I(X+ce)<I(X)$. Where $e=(1,1,\ldots,1)$ is the vector of ones.

The property of uniform additions says that, measure inequality should decrease if everyone in the population receives an equal transfer. This property has a direct analogism with respect to the factor decomposition. Suppose now that, there are K different income sources $X^1, X^2, \ldots, X^K$ so that $X=\sum_{k=1}^{K} X^k$ and for any individual $i (i=1,2,\ldots,n)$ $X_i=\sum_{k=1}^{K} X_i^k$. Let denote S^k (s^k) the absolute (proportional) contribution to the inequality $I(X)$ of the total income and attributed to the income source X^k.

Definition 2: A decomposition method for a given inequality index $I(X)$ satisfies the property of uniform additions if $s^k < 0$ ($s^k > 0$) when $X^k = ce$, where $e = (1,1,\ldots,1)$ is the vector of ones and c is a constant greater (less) than zero.

Roughly speaking, inequality decomposition fulfils the property of uniform additions if it gives strictly negative (positive) contribution to the whole inequality for any income source that is equally distributed and positive (negative). It is important to note that, satisfaction of uniform additions property for an inequality index does not necessary imply that any associated decomposition also satisfies the property as formulate in definition 2. This is particular the case for the most popular inequality indices such as Gini coefficient, the coefficient of variation squared and the General entropy family of indices.

We will focus on these inequality indices by examining if their classical decomposition fulfils the uniform additions property. An alternative decomposition method is proposed when this property is not satisfied. We will also restrict our attention to the most direct and commonly used decomposition rules for each index, usually considered as 'natural decomposition'. These rules impose (Shorrocks, 1982) on the inequality index to be written as weighted sum of total incomes:

$$I(X) = \sum_{i=1}^{n} a_i(X) x_i \qquad (1)$$

Note that the inequality indices mentioned above all satisfy this property.

The absolute contribution of the income source k to the total inequality $I(X)$ is simply:

$$S^k = \sum_{i=1}^{n} a_i(X) x_i^k \qquad (2)$$

It is often useful to consider the proportion of the total inequality contributed by different components. In this case, the proportional contribution of income source k to the overall inequality is :

$$s^k = \frac{\sum_{i=1}^{n} a_i(X) x_i^k}{I(X)} \qquad (3)$$

Of course, the method yields an exact decomposition; this means that, the sum of $S^k(k=1,2,\ldots,K)$ equal to $I(X)$ and the sum of the K proportional contributions equal to one.

Considering first the variance (I_{VAR}) and the coefficient of variation squared (I_{CV}), their natural decomposition rules are respectively:

$$I_{VAR}(X) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)x_i \qquad \text{for the variance} \qquad (4)$$

and

$$I_{CV}(X) = \frac{1}{n\mu^2}\sum_{i=1}^{n}(x_i - \mu)x_i \qquad \text{for the square of coefficient of variation.} \qquad (5)$$

The proportional contributions associated to the two indices are identical since the μ^2 term cancel in the case of the coefficient of variation:

$$s_{VAR}^k = s_{CV}^k = \frac{\sum_{i=1}^{n}(x_i - \mu)x_i^r}{\sum_{i=1}^{n}(x_i - \mu)x_i} = \frac{\sum_{i=1}^{n}(x_i - \mu)x_i^r}{nVar(X)} = \frac{Cov(X, X^k)}{Var(X)} \qquad (6)$$

This implies that the proportional contribution of any income source $X^k = \mu^k(1,1,\ldots,1)$ which is equal distributed is

$$s_{VAR}^k = s_{CV}^k = \frac{Cov(X, X^k)}{Var(X)} = \frac{Cov(X, \mu^k e)}{Var(X)} = 0. \qquad (7)$$

This clearly shows that, the natural decompositions of the variance and the coefficient of variation squared violate the uniform additions property.

For an alternative decomposition which satisfies the property, we need to rewrite the expressions (3) or (4) of $I_{VAR}(X)$ or $I_{CV}(X)$ as follow:

$$I_{VAR}(X) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)x_i = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)\left(1 + \frac{\alpha(X)}{x_i}\right)x_i \qquad (8)$$

and

$$I_{CV}(X) = \frac{1}{n\mu^2}\sum_{i=1}^{n}(x_i - \mu)x_i = \frac{1}{n\mu^2}\sum_{i=1}^{n}(x_i - \mu)\left(1 + \frac{\alpha(X)}{x_i}\right)x_i \qquad (9)$$

Where $\alpha : R^n \to R$ is a continuous and positive function; α can be interpreted as a parameter of the decomposition method. Note that the case $\alpha(X) = 0$ corresponds to the natural decomposition.

In the same spirit as above, the corresponding proportional contributions of the income source k to the overall inequality are now expressed as:

$$s_{VAR}^{k} = s_{CV}^{k} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)\left(1 + \frac{\alpha(X)}{x_i}\right)x_i^k}{Var(X)} = \frac{Cov\left(X,\ X^k + \alpha(X)\frac{X^k}{X}\right)}{Var(X)} \tag{10}$$

This alternative decomposition satisfies the property of uniform additions: Suppose that $x_i > o$ and that $x_i^k = \mu^k > o$ for all i, then,

$$s_{VAR}^{k} = s_{CV}^{k} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)\left(1 + \frac{\alpha(X)}{x_i}\right)x_i^k}{Var(X)} = \frac{\frac{\alpha(X)}{n}\sum_{i=1}^{n}\left(\frac{x_i - \mu}{x_i}\right)\mu^k}{Var(X)} = \frac{\mu^k \alpha(X)\left(n - \mu\sum_{i=1}^{n}\frac{1}{x_i}\right)}{nVar(X)}$$

$$= \frac{\mu^k \mu\alpha(X)\left(\frac{1}{\mu} - \frac{1}{n}\sum_{i=1}^{n}\frac{1}{x_i}\right)}{Var(X)} < 0.$$

An interesting observation that can be made here is that the alternative proportional contribution rule given in Eq.10 can be related to the former Shorrocks natural decomposition given in Eq 6 as:

$$s_{VAR}^{k} = s_{CV}^{k} = \frac{Cov\left(X,\ X^k + \alpha(X)\frac{X^k}{X}\right)}{Var(X)} = \frac{Cov(X, X^k)}{Var(X)} + \frac{Cov\left(X, \alpha(X)\frac{X^k}{X}\right)}{Var(X)} \tag{11}$$

This noticeably shows that, the alternative decomposition method corresponds to the natural decomposition rule and a corrective term. This term, which for the factor X^k equal to $Cov\left(X, \alpha(X)\frac{X^k}{X}\right)$, takes into account the link between the total income X and the share of the factor X^k.

Thus, if $Cov\left(X, \alpha(X)\frac{X^k}{X}\right) > 0$, the contribution of the source k obtained by the alternative decomposition is greater than its contribution in the natural decomposition rule and inversely. It is easy to check that an increase in the value of $\alpha(X)$ may increase the gap between the two methods.

If the income source X^k is such that $\frac{X^k}{X}$ is not correlated with the total income X (it is particularly the case where X^k is proportional to X or if X^k is very light relatively to X so that $\frac{X^k}{X}$ is close to zero), the two methods will practically attribute the same amount of contribution to the income sources X^k. In contrary, if X^k is evenly distributed (of course it is the case where X^k is constant) with a large total income share, the effect of the second term will be perceptive, and the gap between the two decompositions rules may be sensitive in proportion to the magnitude of $\alpha(X)$.

For $\alpha(X)=1$, the proportional contribution rule becomes:

$$s_{VAR}^k = s_{CV}^k = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)\left(1+\frac{1}{x_i}\right)x_i^k}{Var(X)} = \frac{Cov\left(X,\ X^k+\frac{X^k}{X}\right)}{Var(X)} \tag{12}$$

On the other hand, if $\alpha(X)=\frac{1}{n}\sum_{i=1}^{n}x_i=\mu$, the proportional contribution rule becomes:

$$s_{VAR}^k = s_{CV}^k = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)\left(1+\frac{\mu}{x_i}\right)x_i^k}{Var(X)} = \frac{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i^2-\mu^2}{x_i}\right)x_i^k}{Var(X)} = \frac{Cov\left(X,\ X^k+\mu\frac{X^k}{X}\right)}{Var(X)} \tag{13}$$

Note that, in this case, the proportional contribution rule coincides with the one proposed by Morduch and Sicular (2002).

Consider now the Gini coefficient (I_G). If the total income is ordered and individuals are indexed by their total income rank so that $x_1 \le x_2 \le,\ldots,\le x_n$, the Gini index can be written:

$$I_G(X) = \frac{2}{n^2\mu}\sum_{i=1}^{n}\left(i-\frac{n+1}{2}\right)x_i = \frac{2}{n\mu}Cov(Rank, X) \tag{14}$$

The natural decomposition of the Gini coefficient is then expressed as:

$$S_G^k = \frac{2}{n^2\mu}\sum_{i=1}^{n}\left(i-\frac{n+1}{2}\right)x_i^k \tag{15}$$

And the corresponding proportional contribution rule is simply:

$$s_G^k = \frac{\frac{2}{n^2\mu}\sum_{i=1}^{n}\left(i-\frac{n+1}{2}\right)x_i^k}{I_G(X)} = \frac{Cov\left(Rank,\ X^k\right)}{Cov(Rank, X)} \tag{16}$$

If the income source k is equally distributed and $x_i^k = \mu^k$ for all i,

$$s_G^k = \frac{\frac{2}{n^2\mu}\sum_{i=1}^{n}\left(i-\frac{n+1}{2}\right)\mu^k}{I_G(X)} = \frac{\frac{2\mu^k}{n^2\mu}\sum_{i=1}^{n}\left(i-\frac{n+1}{2}\right)}{I_G(X)} = 0. \quad \text{since } \frac{1}{n}\sum_{i=1}^{n} i = \frac{n+1}{2} \tag{17}$$

And so, just like the variance and the coefficient of variation squared, the natural decomposition of Gini coefficient violates the property of uniform additions.

Applying the same principle as in the case of the variance will lead to an alternative decomposition rule of the Gini coefficient.

$$I_G(X) = \frac{2}{n^2\mu}\sum_{i=1}^{n}\left(i-\frac{n+1}{2}\right)x_i = \frac{2}{n^2\mu}\sum_{i=1}^{n}\left(i-\frac{n+1}{2}\right)\left(1+\frac{\alpha(X)}{x_i}\right)x_i \tag{18}$$

The corresponding proportional contribution of the source k to the overall inequality is now expressed as:

$$s_G^k = \frac{\frac{2}{n^2\mu}\sum_{i=1}^{n}\left(i-\frac{n+1}{2}\right)\left(1+\frac{\alpha(X)}{x_i}\right)x_i^k}{I_G(X)} = \frac{Cov\left(Rank,\ X^k+\alpha(X)\frac{X^k}{X}\right)}{Cov(Rank, X)} \tag{19}$$

$$= \frac{Cov(Rank,\ X^k)}{Cov(Rank, X)} + \frac{Cov\left(Rank,\ \alpha(X)\frac{X^k}{X}\right)}{Cov(Rank, X)} \tag{19b}$$

The second term in Eq.19b gauges the link between the ranks in the total income and the share of the income source X^k.

This new decomposition satisfies the property of uniform additions. To see it, suppose that $x_i > o$ and that $x_i^k = \mu^k > o$ for all i, then,

if i_0 is the unique integer number such that $i_0 \le \frac{n+1}{2}$ and $i_0 + 1 > \frac{n+1}{2}$,

$$s_G^k = \frac{\frac{2}{n^2\mu}\sum_{i=1}^{n}\left(i-\frac{n+1}{2}\right)\left(1+\frac{\alpha(X)}{x_i}\right)x_i^k}{I_G(X)} = \frac{\frac{2\mu^k}{n^2\mu}\sum_{i=1}^{n}\left(i-\frac{n+1}{2}\right)\left(1+\frac{\alpha(X)}{x_i}\right)}{I_G(X)} =$$

$$\frac{\frac{2\mu^k}{n^2\mu}\sum_{i=1}^{n}\left(i-\frac{n+1}{2}\right)\frac{\alpha(X)}{x_i}}{I_G(X)} = \frac{\frac{2\mu^k}{n^2\mu}\sum_{i=1}^{i_0}\left(i-\frac{n+1}{2}\right)\frac{\alpha(X)}{x_i}}{I_G(X)} + \frac{\frac{2\mu^k}{n^2\mu}\sum_{i=i_0+1}^{n}\left(i-\frac{n+1}{2}\right)\frac{\alpha(X)}{x_i}}{I_G(X)}$$

$$< \frac{\frac{2\mu^k}{n^2\mu}\sum_{i=1}^{i_0}\left(i-\frac{n+1}{2}\right)\frac{\alpha(X)}{x_{i_0}}}{I_G(X)} + \frac{\frac{2\mu^k}{n^2\mu}\sum_{i=i_0+1}^{n}\left(i-\frac{n+1}{2}\right)\frac{\alpha(X)}{x_{i_0}}}{I_G(X)} = \frac{\frac{2\mu^k\alpha(X)}{n^2\mu x_{i_0}}\sum_{i=1}^{n}\left(i-\frac{n+1}{2}\right)}{I_G(X)} = 0.$$

Hence, the same approach has been used to define the alternative decomposition of I_{VAR}, I_{CV} and I_G. A natural question that arises at this stage is: Do there exist a similarity between these indices? The answer is yes, because, the three indices can be written in the form

$I(X) = \sum_{i=1}^{n} a_X(x_i)x_i$ with $\sum_{i=1}^{n} a_X(x_i) = 0$ and where $a_X : R \to R$ is a strictly increasing function. And the following proposition states that, the alternative approach leads to a case where income source decomposition satisfies the uniform additions property.

Proposition 1:
Assume that, the inequality index is a continuous and symmetric function $I(X)$ *which can be put in the form:* $I(X) = \sum_{i=1}^{n} a_X(x_i)x_i$ *for any income source* X *,*
with $\sum_{i=1}^{n} a_X(x_i) = 0$ *and where* $a_X : R \to R$ *is a strictly increasing function.*
Then,
$S^k = \sum_{i=1}^{n} a_X(x_i)\left(1+\frac{\alpha(X)}{x_i}\right)x_i^k$ *yields an exact factors decomposition which satisfies the uniform additions property.*

Proof: By hypothesis, $I(X) = \sum_{i=1}^{n} a_X(x_i)x_i$, $\sum_{i=1}^{n} a_X(x_i) = 0$ and a_X is strictly increasing.

$$\sum_{k=1}^{K} S^k = \sum_{i=1}^{n} a_X(x_i)\left(1+\frac{\alpha(X)}{x_i}\right)\sum_{k=1}^{K} x_i^k = \sum_{i=1}^{n} a_X(x_i)x_i + \alpha(X)\sum_{i=1}^{n} a_X(x_i) = \sum_{i=1}^{n} a_X(x_i)x_i = I(X)$$

And the decomposition is exact.

Suppose now that the income source k is equally distributed and $x_i^k = \mu^k > o$

$$S^k = \sum_{i=1}^{n} a_X(x_i)\left(1+\frac{\alpha(X)}{x_i}\right)x_i^k = \mu^k \sum_{i=1}^{n} a_X(x_i)\left(1+\frac{\alpha(X)}{x_i}\right) = \mu^k\alpha(X)\sum_{i=1}^{n} a_X(x_i)\frac{1}{x_i}$$

Since $I(X)$ is symmetric, we suppose that individuals are numbered so that $x_1 \le x_2 \le, \ldots, \le x_n$.

$\sum_{i=1}^{n} a_X(x_i) = 0$, implies there exists at least one i such that $a_X(x_i) \leq 0$.

On other side, $a_X(x)$ is different from the null function, and there exist at least one i such that $a_X(x_i) > 0$.

Considerer $i^-(X) = \{i / 1 \leq i \leq n,\ a_X(x_i) \leq 0\}$ and $i^+(X) = \{i / 1 \leq i \leq n, a_X(x_i) > 0\}$

For $i_0 = Max\, i^-(X)$, it is easy to verify that :

1) $i_0 \in i^-(X)$ and 2) $i_0 + 1 = Min\, i^+(X) \in i^+(X)$

Therefore,

$$S^k = \mu^k \alpha(X) \sum_{i=1}^{n} a_X(x_i) \frac{1}{x_i} = \mu^k \alpha(X) \sum_{i=1}^{i_0} a_X(x_i) \frac{1}{x_i} + \mu^k \alpha(X) \sum_{i=i_0+1}^{n} a_X(x_i) \frac{1}{x_i}$$

$$< \mu^k \alpha(X) \sum_{i=1}^{i_0} a_X(x_i) \frac{1}{x_{i_0}} + \mu^k \alpha(X) \sum_{i=i_0+1}^{n} a_X(x_i) \frac{1}{x_{i_0}} = \frac{\mu^k \alpha(X)}{x_{i_0}} \sum_{i=1}^{n} a_X(x_i) = 0. \quad \blacksquare$$

Another attractive attempt consists of studying the case of the Generalized Entropy family of indices (I_θ). According to Cowell (1980), these indices can be written:

$$I_\theta(X) = \frac{1}{\theta^2 - \theta} \left[\frac{1}{n} \sum_{i=1}^{n} \left(\frac{x_i}{\mu} \right)^\theta - 1 \right] \qquad \forall \theta \neq 0,1,$$

$$I_1(X) = \frac{1}{n} \sum_{i=1}^{n} \frac{x_i}{\mu} \ln\left(\frac{x_i}{\mu} \right) \qquad\qquad I_0(X) = \frac{1}{n} \sum_{i=1}^{n} \ln\left(\frac{\mu}{x_i} \right)$$

Applying the natural decomposition principle leads to rewrite $I_\theta(X)$:

$$I_\theta(X) = \frac{1}{\theta^2 - \theta} \left[\frac{1}{n} \sum_{i=1}^{n} \left(\frac{\left(\frac{x_i}{\mu} \right)^\theta - 1}{x_i} \right) x_i \right] \qquad \forall \theta \neq 0,1 \tag{20}$$

Therefore the associated contribution rule is:

$$S_\theta^k = \frac{1}{\theta^2 - \theta} \left[\frac{1}{n} \sum_{i=1}^{n} \left(\frac{\left(\frac{x_i}{\mu} \right)^\theta - 1}{x_i} \right) x_i^k \right] \qquad \forall \theta \neq 0,1 \tag{21}$$

When the source k is constant: $x_i^k = \mu^k > 0$ for all i,

$$S_\theta^k = \frac{\mu^k}{\theta^2-\theta}\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{\left(\frac{x_i}{\mu}\right)^\theta - 1}{x_i}\right)\right] = \frac{\mu^k}{\theta^2-\theta}\left[\frac{1}{n\mu}\sum_{i=1}^{n}\left(\frac{x_i}{\mu}\right)^{\theta-1} - \frac{1}{n}\sum_{i=1}^{n}\frac{1}{x_i}\right]$$

$$< \frac{\mu^k}{(\theta^2-\theta)\mu}\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\mu}\right)^{\theta-1} - 1\right] < 0 \text{ if } \quad 1 < \theta \leq 2$$

Thus, for $1 \leq \theta \leq 2$, the natural decomposition principle satisfies the uniform additions property. For the limit case $\theta = 1$, which corresponds to the Theil-T index, the uniform additions property is directly satisfied by the natural decomposition since the logarithm function is concave. Note that, for $\theta = 2$, $2I_\theta(X)$ is the coefficient of variation squared and we fall again on the decomposition rule given in Eq.13.

All the alternative methods proposed are exact decompositions. They are linked to the natural decomposition of their corresponding index. However, they reinforce the fact that the decomposition of an aggregate index may lead to different solutions of the contribution of the various components. The presence of the parameter $\alpha(X)$ corroborates with this situation. Thus, different decompositions of the same index will generate different results during empirical analyses. The question now posed is : which decomposition method is preferable and why?

In order to motivate the use of these alternative decompositions, it seems important to study their properties. Here, we focus our attention on the axiomatic properties of the natural decomposition defined by Shorrocks (1982).

1.2- Axiomatic properties

If K disjoint and exhaustive income sources are considered, the contribution of the income source k to the total inequality can be written $S^k\left(X^1, X^2, \ldots, X^K; X; K\right)$ and the proportional contribution is $s^k\left(X^1, X^2, \ldots, X^K; X; K\right)$.

The standard axioms for such a contribution function are:

Axiom1: (continuity, CONT) $S^k\left(X^1, X^2, \ldots, X^K; X; K\right)$ is a continuous function in X^k.

CONT insures that minor observational errors in incomes sources will generate minor changes in the contribution level.

Axiom2: (Symmetric treatment of income sources, SYM)

$S^k\left(X^1, X^2,\ldots,X^K; X; K\right)$ symmetrically treats the income sources if for any permutation π of 1,2,…,K, $S^{\pi(k)}\left(X^{\pi(1)}, X^{\pi(2)},\ldots,X^{\pi(K)}; X; K\right) = S^k\left(X^1, X^2,\ldots,X^K; X; K\right)$.

SYM means that, no significance is attached to how income sources are numbered.

Axiom3: (Independence of the level of disaggregation, ILD)

$S^k\left(X^1, X^2,\ldots,X^K; X; K\right) = S^k\left(X^k, X - X^k; X; 2\right) = S\left(X^k, X\right)$ for all k

ILD says that, the contribution of any income source does not depend on how many other types of income sources are distinguished.

Axiom4: (Population Symmetry, PSYM)

If M is any $n \times n$ permutation matrix,

$$S^k\left(X^1, X^2,\ldots,X^K; X; K\right) = S^k\left(X^1M, X^2M,\ldots,X^KM; XM; K\right)$$

PSYM indicates that the contribution of any income source does not depend on how individuals are numbered in the population. In other words, individuals are treated anonymously.

Axiom 5: (Two factors Symmetry, TFS)

For any $n \times n$ permutation matrix M , and for any income source X^1

$$S^1\left(X^1, X^1M; X^1 + X^1M; 2\right) = S^1\left(X^1M, X^1; X^1 + X^1M; 2\right)$$

TFS recommends that any income source and its permutation must be treated symmetrically and then assigned the same contributions value. This property was initiated by Shorrocks (1982) as an assumption on the income factor contribution function. Shorrocks used this condition to obtain the uniqueness of the decomposition given in Eq.6.

Axiom 6: (Exact decomposition, ED)

$$\sum_{k=1}^{K} S^k\left(X^1, X^2,...,X^K; X; K\right) = I(X) \text{ or } \sum_{k=1}^{K} s^k\left(X^1, X^2,...,X^K; X; K\right) = 1$$

Proposition 2:

1) *The alternative income source decomposition of the Variance and the Coefficient of Variations squared given in Eq.10 satisfies CONT, SYM, ILD, PSYM, TFS and ED.*

2) *The alternative income source decomposition of the Gini coefficient given in Eq.19 satisfies CONT, SYM, ILD, PSYM, ED but not TFS.*

3) *The natural decomposition of the General Entropy family of indices given in Eq.22 and the natural decomposition of Theil-T index satisfy CONT, SYM, ILD, PSYM, ED but not TFS.*

Proof: Since the proof is similar for the three cases, we will adopt only the case of I_{VAR} and I_{CV}.

$$s^k_{VAR}\left(X^1, X^2,...,X^K; X; K\right) = $$

$$s^k_{CV}\left(X^1, X^2,...,X^K; X; K\right) = s^k_{Var} = s^k_{CV} = \frac{\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \mu\right)\left(1 + \frac{\alpha(X)}{x_i}\right)x_i^k}{Var(X)},$$

Hence it is straightforward that CONT, SYM, ILD, PSYM and ED are satisfied.
For TFS, we have seen that:

$$s^k_{VAR}\left(X^1, X^2,...,X^K; X; K\right) = s^k_{CV}\left(X^1, X^2,...,X^K; X; K\right) = \frac{Cov\left(X,\ X^k + \alpha(X)\frac{X^k}{X}\right)}{Var(X)}$$

Therefore, for any $n \times n$ permutation matrix M, and for any income source X^1,

$$s^1_{VAR}\left(X^1, X^1M; X^1 + X^1M; 2\right) = \frac{Cov\left(X^1 + X^1M,\ X^1 + \alpha(X^1 + X^1M)\frac{X^1}{X^1 + X^1M}\right)}{Var(X^1 + X^1M)}$$

Noting that, for any income source Y, Z, $Cov(Y,Z) = Cov(YM, ZM)$; $YMM = Y$;
$(Y + Z)M = YM + ZM$ and $\frac{Y}{Z}M = \frac{YM}{ZM}$,

We have

$$s^k_{VAR}\left(X^1, X^1M; X^1 + X^1M; 2\right) = \frac{Cov\left(X^1 + X^1M,\ X^1 + \alpha(X^1 + X^1M)\frac{X^1}{X^1 + X^1M}\right)}{Var(X^1 + X^1M)} =$$

$$\frac{Cov\left(X^1M + X^1MM,\ X^1M + \alpha(X^1 + X^1M)\frac{X^1}{X^1 + X^1M}M\right)}{Var(X^1 + X^1M)} =$$

$$\frac{Cov\left(X^1M + X^1,\ X^1M + \alpha(X^1 + X^1M)\dfrac{X^1M}{X^1M + X^1} \right)}{Var(X^1 + X^1M)} = s^1_{VAR}\left(X^1M, X^1; X^1 + X^1M; 2\right) \blacksquare$$

Another desirable property of income source decomposition is the additions stability of contributions. This property can be defined as follow:

Definition3: If axiom 3 holds**,** that is the contribution $S^k\left(X^1, X^2,\ldots,X^K; X; K\right) = S(X^k, X)$ of the income source k is independent of the degree of disaggregation of the total income X , The decomposition method satisfies the additions stability property if ,
for any couple X^1, X^2 of income sources, $S(X^1 + X^2; X) = S(X^1; X) + S(X^2; X)$

The usefulness of this property is clear. In many applications, certain groups of income sources are naturally clustered together. For example, the investment income might be split into interest, dividends, capital gains and rent. Satisfaction of the property guarantees that the contributions assigned to these income components sum to the contribution of investment income treated as a single unit.

Proposition 3:

The alternative income source decompositions defined:
- *In Eq.10 for the variance and the coefficient of variation squared*
- *In Eq.19 for the Gini coefficient*
- *In Eq.22 for the General Entropy family of indices*

all satisfy the additions stability property.

Proof: It is obvious since, for the three decompositions method, $s(X^k, X)$ or $S(X^k, X)$ is linear function in X^k. ■

Finally, the alternative decomposition methods for I_{VAR}, I_{CV}, I_G and I_θ $\left(for\, 1 \le \theta \le 2\right)$ satisfy the uniform additions and the additions stability properties. This in particular implies that their associated contribution functions verify: for any positive constant c and $e = (1,1,\ldots,1)$

$$s(X^k + ce, X) < s(X^k, X)$$

3- Regression-based to inequality decomposition under uniform additions property

Assuming that the income generating process is in the linear form:

$$x_i = \sum_{m=1}^{M} \beta_m x_i^m + \varepsilon_i \tag{22}$$

Where $x_i^1 = 1$ for all i, ε_i is the residual term, x_i^m $(m = 2,3,\ldots,M)$ are the independent variables usually taken to represent the characteristic of the household i such as age, education, household size, health etc.

A sample of observations $\left(x_i, x_i^m : i = 1,2,\ldots,n;\ m = 1,2,\ldots,M\right)$ can be used to estimate the model. The parameter β_m is interpreted as the effect of the independent variable X^m on the total income[1] (or per capital total income or logarithm of total income etc.) X.

Using OLS estimation leads to:

$$x_i = \sum_{m=1}^{M} \hat{\beta}_m x_i^m + \hat{\varepsilon}_i \tag{23}$$

$\hat{\beta}_m x_i^m$ can be viewed as the part of the household i's income (or expenditure) which is due to its endowment of the attribute x^m.

Thus Eq.23 can be used to decompose total income inequality as in section 1. By analogy with Eq.3, the proportional contribution to the total inequality of the attribute m is:

$$s^m = \frac{\hat{\beta}_m \sum_{i=1}^{n} a_i(X) x_i^m}{I(X)} \tag{24}$$

And the proportional contribution to the total inequality of the residual term is:

[1] In many empirical works, total expenditure or consumption is used rather than total income because of data availability

$$s^{\varepsilon} = \frac{\sum_{i=1}^{n} a_i(X)\hat{\varepsilon}_i}{I(X)} \qquad (25)$$

The standard errors[2] of these proportional contributions are :

$$\sigma(s^m) = \sigma(\hat{\beta}_m)\left|\frac{\sum_{i=1}^{n} a_i(X)x_i^m}{I(X)}\right| = \sigma(\hat{\beta}_m)\left|\frac{s^m}{\hat{\beta}_m}\right| \quad \text{if} \quad \hat{\beta}_m \neq 0 \qquad (26)$$

And under the homoscedastic residuals: $Var(\varepsilon_i) = \sigma_{\varepsilon}^2$

$$\sigma(s^{\varepsilon}) = \sigma_{\varepsilon}\left[\frac{\sum_{i=1}^{n}\left(a_i(X)\right)^2}{\left(I(X)\right)^2}\right]^{\frac{1}{2}} \qquad (27)$$

Therefore this principle can be applied to the alternative decompositions methods exposed in section1.

Variance and coefficient of variation squared

$$s_{Var}^m = s_{CV}^m = \hat{\beta}_m \frac{Cov\left(X,\ X^m + \alpha(X)\frac{X^m}{X}\right)}{Var(X)} \qquad (28)$$

and

$$\sigma(s_{Var}^m) = \sigma(s_{CV}^m) = \sigma(\hat{\beta}_m)\left|\frac{Cov\left(X,\ X^m + \alpha(X)\frac{X^m}{X}\right)}{Var(X)}\right| = \sigma(\hat{\beta}_m)\left|\frac{s_{Var}^m}{\hat{\beta}_m}\right| \qquad (29)$$

$$s_{Var}^{\varepsilon} = s_{CV}^{\varepsilon} = \frac{Cov\left(X,\ \hat{\varepsilon} + \alpha(X)\frac{\hat{\varepsilon}}{X}\right)}{Var(X)}\ ;\ \sigma(s_{Var}^{\varepsilon}) = \sigma(s_{CV}^{\varepsilon}) = \sigma_{\varepsilon}\left\{\frac{\frac{1}{n^2}\sum_{i=1}^{n}\left[\left(x_i - \mu\right)\left(1 + \frac{\alpha(X)}{x_i}\right)\right]^2}{\left(Var(X)\right)^2}\right\}^{\frac{1}{2}} \qquad (30)$$

[2] In fact, this is just an approximation of the standard errors computation. The correct standard errors are very quite complicated to compute as they require the use of bootstrap or the non trivial asymptotic distribution (Cowell F,A. and Fiolio Carlo,V; 2005)

Gini coefficient

$$s_G^m = \hat{\beta}_m \frac{Cov\left(Rank,\ X^m + \alpha(X)\dfrac{X^m}{X}\right)}{Cov(rank, X)} \quad \text{and} \quad \sigma(s_G^m) = \sigma(\hat{\beta}_m)\left|\frac{Cov\left(Rank,\ X^m + \alpha(X)\dfrac{X^m}{X}\right)}{Cov(rank, X)}\right|$$

$$= \sigma(\hat{\beta}_m)\left|\frac{s_G^m}{\hat{\beta}_m}\right| \tag{31}$$

$$s_G^\varepsilon = \frac{Cov\left(Rank,\ \hat{\varepsilon} + \alpha(X)\dfrac{\hat{\varepsilon}}{X}\right)}{Cov(Rank, X)} \quad \text{and} \quad \sigma(s_G^\varepsilon) = \sigma_\varepsilon \frac{\dfrac{1}{n}\left\{\displaystyle\sum_{i=1}^{n}\left[\left(i - \frac{n+1}{2}\right)\left(1 + \frac{\alpha(X)}{x_i}\right)\right]^2\right\}^{\frac{1}{2}}}{Cov(Rank, X)} \tag{32}$$

Generalized Entropy family of indices

- For $1 < \theta \leq 2$,

$$s_\theta^m = \frac{\hat{\beta}_m}{\left(\theta^2 - \theta\right) I_\theta(X)}\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{\left(\dfrac{x_i}{\mu}\right)^\theta - 1}{x_i}\right) x_i^m\right] \tag{33}$$

and

$$\sigma(s_\theta^m) = \frac{\sigma(\hat{\beta}_m)}{I_\theta(X)\left(\theta^2 - \theta\right)}\left[\left|\frac{1}{n}\sum_{i=1}^{n}\left(\frac{\left(\dfrac{x_i}{\mu}\right)^\theta - 1}{x_i}\right) x_i^m\right|\right] \tag{34}$$

$$s_\theta^\varepsilon = \frac{1}{\left(\theta^2 - \theta\right) I_\theta(X)}\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{\left(\dfrac{x_i}{\mu}\right)^\theta - 1}{x_i}\right)\hat{\varepsilon}_i\right] \tag{35}$$

And

$$\sigma(s_\theta^\varepsilon) = \frac{\sigma_\varepsilon}{\left(\theta^2 - \theta\right) I_\theta(X)}\left[\frac{1}{n^2}\sum_{i=1}^{n}\left(\frac{\left(\dfrac{x_i}{\mu}\right)^\theta - 1}{x_i}\right)^2\right]^{\frac{1}{2}} \tag{36}$$

- For the Theil-T index

$$s_1^m = \frac{\hat{\beta}_m}{I_1(X)n}\sum_{i=1}^{n}\frac{x_i^m}{\mu}\ln\left(\frac{x_i}{\mu}\right) \text{ and } \sigma(s_1^m) = \frac{\sigma(\hat{\beta}_m)}{I_1(X)n}\left|\sum_{i=1}^{n}\frac{x_i^m}{\mu}\ln\left(\frac{x_i}{\mu}\right)\right| \qquad (37)$$

$$s_1^\varepsilon = \frac{1}{I_1(X)n}\sum_{i=1}^{n}\frac{\hat{\varepsilon}_i}{\mu}\ln\left(\frac{x_i}{\mu}\right) \text{ and } \sigma(s_1^\varepsilon) = \frac{\sigma_\varepsilon}{I_1(X)n}\left\{\sum_{i=1}^{n}\left[\frac{1}{\mu}\ln\left(\frac{x_i}{\mu}\right)\right]^2\right\}^{\frac{1}{2}} \qquad (38)$$

4- Applications

Illustration From Cameroonian rural Data

Data from the country's household survey known as ECAM ('Enquête Camerounaise auprès des ménages') is used. It is conducted every 5 years by the National Statistical Office in Cameroon. For illustration we consider ECAM II which corresponds to the year 2001 and we restraint the study to the rural zone of the Centre province of Cameroon (Povince du centre). The survey provides 390 observations of households in this rural zone.

Our analysis has two parts. First we begin with the evaluation of the contribution of various consumption sources to the total consumption inequality. Second, we introduce the regression-based method to estimate the determinant of the total consumption (as proxy of the total income) inequality.

3.1- Consumption sources factors decomposition

We decompose the total consumption into nine types of consumption sources. For every household we have the corresponding consumption source per capita: (a) food (b) Clothing (c) Housing; (d) House servicing (e) Health (f) Transport and communication (g) Education (h) Personal Treatment (i) leisure.

Table 1 gives the statistics characteristic of these nine variables. The second column of the table contains the average share of the total consumption of each consumption source and the third column represents their coefficient of variation.

Table 1: <u>Statistic characteristics of consumption sources variables</u>

Consumption Sources	Consumption share	Coefficient of variation	Share flows by Mean				
			Min	First quartile	Second quartile	Third quartile	Max
Food	0,4899	0,9289	0,06	0,51	0,77	1,15	10,54
Clothing	0,0456	1,6386	0,00	0,20	0,52	1,08	14,85
Houses servicing	0,0450	2,0164	0,00	0,30	0,56	1,06	29,21
Health	0,0956	1,9058	0,00	0,17	0,43	0,89	19,21
Transport	0,0789	2,3077	0,00	0,18	0,42	0,93	26,68
Education	0,0401	2,4618	0,00	0,00	0,23	0,94	26,14
Personnel care	0,0161	3,2055	0,00	0,17	0,53	1,05	59,27
Leisure	0,0042	3,5923	0,00	0,00	0,00	0,34	32,07
Housing	0,1846	0,9551	0,13	0,46	0,73	1,11	8,41
Total	1,0000	0,7895	0,07	0,53	0,76	1,20	6,49

Source: Own calculation on ECAM II database

We observe that the most important source of consumption is Food which represents almost 49% of the total consumption. Food is also the consumption source with relative smaller coefficient of variation but this coefficient of variation is greater than the total consumption one. In the same spirit, but with lower acuity, we have Housing and Heath which represent respectively 18.46% and 9.56% share of the total consumption. Thus we presage important role in total consumption inequality increases contributions of these variables.

Table 2: <u>Consumption sources contributions to inequality</u>

Consumption Sources	Gini index			CV squared			Theil-T
	Natural or $\alpha = 0$	$\alpha = 1$	$\alpha = \mu$	Natural or $\alpha = 0$	$\alpha = 1$	$\alpha = \mu$	-
Food	0,461	0,461	0,439	0,454	0,454	0,435	0,428
Clothing	0,044	0,044	0,042	0,047	0,047	0,046	0,043
Houses servicing	0,047	0,047	0,045	0,055	0,055	0,056	0,055
Health	0,121	0,121	0,147	0,116	0,116	0,134	0,142
Transport	0,109	0,109	0,130	0,138	0,138	0,160	0,156
Education	0,035	0,035	0,027	0,042	0,042	0,039	0,036
Personnel care	0,018	0,018	0,020	0,016	0,016	0,017	0,019
Leisure	0,006	0,006	0,007	0,005	0,005	0,006	0,007
Housing	0,160	0,160	0,141	0,126	0,126	0,107	0,114
Total	1,000	1,000	1,000	1,000	1,000	1,000	1,000

Source: Own calculation on ECAM II database

In Table 2, we have computed the inequality contributions of the sources consumption. For illustration and comparison we have retained seven decomposition rules: Gini and CV squared with $\alpha = 0$, $\alpha = 1$, $\alpha = \mu$, and the natural decomposition of Theil-T index. Notably, the different decomposition methods are concordant. They assign almost the same amount of contributions to each consumption source. As noticed, our expectation is satisfied by all the seven decompositions rules. Food constitutes the most important consumption inequality-increasing source followed far-off by Housing and Health. In this particular analysis, the difference is not sensitive between decomposition rules satisfying the uniform additions property and those which do not satisfy this property. However, this is not a general rule. As we have already mentioned, this situation will arrive whenever among the consumption sources, there is none of them with a large average total consumption share, which is evenly distributed. Particularly, when one of the attributes (or income inequality determinant) is constant, the effect of the uniform additions may be perceptive. This will be exemplified with the regression-base model.

3.1 Determinants of the income inequality: The regression-based model

The variables we use to explain per capita income[3] (consumption) are listed in the first column of table 3. We include age of the head of household. We have also used its squared value to account for life-cycle effect but it was not statistically significant and was removed from the analysis. We also include a set of binary indicators of the educational level of the head of household. Household size and composition are represented by two variables: family size and the fraction of working age in the family. The working age in the family is taking to be from 20 to 60. The economic resource of the household is essentially represented by per capita land owned. We also include a dummy indicator for the household head being a male. Since the Catholic religion plays a central role in the region, we have included a dummy variable for the head of the household being catholic. The second column of the table 3 contains the means of explanatory variables and it reveals that 72.7% of household heads are male and more than 77% are catholic.

[3] We suppose that total consumption is a good proxy of total income.

Table 3 : **Explanatory Variables and Linear income Generating Equation**

	Explanatory Variables statistic				Linear income Equation		
	Mean	Coefficient of variation	Min	Max	Estimated coefficients	Standard errors	income share
Household size	5,300	0,688	1	21	-21205,5***	2536,548	-0,4268
Age of the head of household	49,748	0,312	20	98	2125,737**	686,900	0,4015
Primary level education	0,488	1,023	0	1	48297,886*	27116,514	0,0896
Secondary level education	0,272	1,634	0	1	139495,3***	32212,836	0,1443
High level education	0,025	6,156	0	1	322051,8***	59898,039	0,0314
Land per capita	1,813	2,941	0	60	8332,197***	1616,943	0,0574
Adult as % size of family	0,400	0,707	0	1	159269,5***	34465,487	0,2424
Catholic as head religion	0,778	0,533	0	1	35104,351*	20463,379	0,1038
Male head	0,727	0,612	0	1	-44627,8*	22881,055	-0,1233
Constant term	1	0,000	1	1	126341,9**	56150,232	0,4797

Source: Own calculation on ECAM II database using SPSS program.

***coefficient significant at 1%, **coefficient significant at 5%, *coefficient significant at 10%. R^2=0.364 and R^2_{adj} =0.348.

Dependent variable = Total household Income per capita. Nb observations = 389

Income Inequality Decomposition

The filth column of table 3 gives the coefficient of the linear equation estimated by the OLS method and the sixth column gives standard errors. Notably, almost all estimated coefficients are statistically significant (five of them : Household size , Secondary level of education, High level of education, Land per capita and Adult as % size of family are strongly significant at 99% level of confidence) and they have the expected sign. Household size have a negative effect while the effect of Adult as % size of family is positive, per capita income decreases with the size of the family but increases with the Fraction of working-age adults. The three variables accounting for education each have a positive effect and the impact of these effects increases with the level of schooling. It is worth noting that the High level education and the Secondary level education effects are highly significant while the significance of the Primary level education is lower. This confirms that per capita income increases with the level of education of household head. Age has a positive effect and male-headed households have lower income per capita than female-headed households, but the effect of gender issues is not highly significant. Per capita income increases with Land owned per capita as it does with the head of households being catholic. However, the effect of the catholic religion is relatively low since it is statistically significant only at 90% level of confidence.

The seventh column corresponds to a measure of the relative magnitude of the effect of the explanatory variables on the income. It gives average income share $\hat{\beta}_m \bar{x}_m / \bar{x}$ of each explanatory variable. Note that none of the educational variables induces a large share of average income. On the contrary, the demographic variables such as Household size (negatively), Age and Fraction of working-age adults present considerable income share. The constant term also plays a substantial role in average income sharing. Consequently, we expect relatively large influence of these variables on income inequality contributions.

Table 4: Contribution to the income inequality

	Gini index			CV squared			Theil-T
	Natural or $\alpha = 0$	$\alpha = 1$	$\alpha = \mu$	Natural or $\alpha = 0$	$\alpha = 1$	$\alpha = \mu$	-
Household size	0,29258	0,29258	1,45689	0,1755	0,1755	0,8034	0,88700
	(0,0350)	(0,0350)	(0,1743)	(0,0210)	(0,0210)	(0,0961)	(0,1061)
Age of the head of family	0,00796	0,00796	-0,53542	-0,0001	-0,0001	-0,3241	-0,36728
	(0,0026)	(0,0026)	(0,1730)	(0,000)	(0,000)	(0,1047)	(0,1187)
Primary level education	-0,02663	-0,02663	-0,20118	-0,0186	-0,0186	-0,1173	-0,12953
	(0,0150)	(0,0150)	(0,1130)	(0,0104)	(0,0104)	(0,0659)	(0,0727)
Secondary level education	0,03606	0,03606	-0,05252	0,0338	0,0338	-0,0330	-0,05963
	(0,0083)	(0,0083)	(0,0121)	(0,0078)	(0,0078)	(0,0076)	(0,0138)
High level education	0,06032	0,06032	0,08616	0,0569	0,0569	0,0766	0,08617
	(0,0112)	(0,0112)	(0,0160)	(0,0106)	(0,0106)	(0,0142)	(0,0160)
Land per capita	0,04573	0,04573	0,01971	0,0592	0,0592	0,0489	0,04781
	(0,0089)	(0,0089)	(0,0038)	(0,0115)	(0,0115)	(0,0095)	(0,0093)
Adult as % size of family	0,07038	0,07038	-0,18035	0,0682	0,0682	-0,0802	-0,09552
	(0,0152)	(0,0152)	(0,039)	(0,0148)	(0,0148)	(0,0174)	(0,0207)
Catholic as head religion	0,00004	0,00004	-0,15156	0,0016	0,0016	-0,0868	-0,09734
	(0,000)	(0,000)	(0,0883)	(0,0009)	(0,0009)	(0,0506)	(0,0567)
Male head of family	0,00650	0,00650	0,16744	0,0046	0,0046	0,1036	0,12278
	(0,0033)	(0,0033)	(0,0859)	(0,0024)	(0,0024)	(0,0531)	(0,0629)
Constant term	0,00000	0,00000	-0,66461	0,0000	0,0000	-0,3934	-0,44865
	(0,0000)	(0,0000)	(0,2954)	(0,0000)	(0,0000)	(0,1748)	(0,1994)
Regression residual	0,50706	0,50706	1,05544	0,6189	0,6189	1,0024	1,05420
	(0,0507)	(0,0507)	(0,1660)	(0,0404)	(0,0404)	(0,0908)	(0,0926)
Total	1,00000	1,00000	1,00000	1,0000	1,0000	1,0000	1,00000

Source: Own calculation on ECAM II database
Standard errors are in parentheses

We now tackle the decomposition of the inequality by income determinants, results are in table 4. Our mainly concern here is to compare different results obtained by the three inequality indices (Gini, CV squared and Theil-T) and the two decomposition methods (natural decomposition and the proposed alternative method). Notably, the seven decomposition rules in table 4, in term of similarity of the sign and the contribution amount assigned to each income determinant, can be subdivided into two homogeneity groups. The first group is composed by the natural decomposition (or $\alpha = 0$) of the Gini and the CV squared indices and the alternative decomposition of these indices with $\alpha = 1$. The second group consists of the alternative decomposition method of the Gini, the squared CV with $\alpha = \mu$ and the Theil-T index natural decomposition. This clearly demonstrates the effect of the parameter α on the alternative method in one hand and the impact of the uniform addition property to the inequality decomposition in the other hand. Note that the alternative decomposition methods for Gini and the alternative decomposition for CV squared indices with $\alpha = 1$ satisfy the uniform additions property but the weight of parameter α is too light to capture the influence of the second term (see Eq.11 and Eq.19b) of the decomposition rule. Thus, these two decompositions rule practically assigned the same contribution to every variable as their respective natural decomposition; this is no surprise. According to the first group of decompositions methods, the constant term contributes nothing to the inequality. While Household Size constitutes the major contributor to inequality-increasing, followed far away by Fraction of working-age adults. All four decomposition rules of this group assign little amount to the education variables. We note that, here, Primary level education acts to reduce inequality, but only by about 2%. Except the null contribution of the constant term which represents a considerable share of average income, these results are not very far from our expectation as mentioned above. However, it is surprising that Age of head of household, which has an important role in explaining income and which average income share is relatively large, has a much smaller income inequality contribution.

All decomposition rules in the second group assign the same sign and the same level of amount to proportional contribution of each variable. This, in particular, shows that the alternative Gini decomposition rule with $\alpha = \mu$ neatly agree with the Theil-T natural decomposition as well known for the alternative decomposition of the CV squared[4] (Morduch and Sicular (2002)). Consider now the variables Household size, Age and Fraction of working-age adults. Each of them has a large income average share and they are relatively evenly distributed (see table 3). As expected, the three decomposition rules give a substantial inequality reduction for Age and Fraction of working-age adults as inequality increases for Household size. A similar observation holds for Catholic households headed and Male households headed but with a little less acuity. The Catholic religion reduces inequality while being a male household-head operates to increase inequality. Of course, the constant term contributes toward reducibly inequality and we note that this reduction is little more important with the Gini alternative decomposition than the others decomposition methods of the group. The two groups of decomposition methods give each a positive contribution to the Regression residual with more importance in the second group[5]. This gap can be, in part, explained by the compensation due to the negativity of the constant term contribution when the uniform addition property holds.

5- Concluding remarks

The paper has provided a new factor decomposition rules for classical inequality indices such as Gini index and the squared coefficient of variation. From the onset it was presumed that the decomposition rule must satisfy the uniform additions property. The approach adopted consists in adding a corrective term to the natural Shorrocks decomposition formula. We have obtained a parametric family of decomposition rules with several basic properties conserved and which make it an attractive family of decompositions procedures. The parameter of the decomposition family, which is defined as a real function of the total income can be seen as a bridge between the natural decomposition, which satisfy normalization axiom, and the others members of family which satisfy uniform additions property. It is important to point out that the impact of the uniform addition property in the decomposition rule is in proportion with the magnitude of the parameter value chosen. This brings out the natural question of

[4] It is no surprise that the alternative decomposition rule of the CV squared (with $\alpha = \mu$) being in concordance with the natural decomposition of the Theil-T index. As we have already mentioned, the former coincides with the natural decomposition of the entropy index with $\theta = 2$ while the latter is the natural decomposition of the entropy index with $\theta = 1$.

[5] It is well known that, in case of the natural decomposition of the CV squared, the Regression residual contributes $1 - R^2$ to income inequality.

the right value of the parameter, which is, of course, a very complicated problem. Nevertheless, empirical results are presented to demonstrate the use of the proposed procedure and to contrast our results with others decompositions rules. Noticeably, when the average of the total income is taken to be the parameter value, the results obtained with the two indices (Gini and CV squared) are in perfect concordance with those obtained from the natural decomposition rule of the Theil-T index which is considered by several authors (Morduch and Sicular 2002, Giammatteo 2007) as the most preferable (this is very surprising in respect with the large popularity and suitable properties of the Gini index) factor decomposition rule. Thus, the paper has, in particular, provided a solution to the problem of finding, for the Gini index, a correct factor decomposition rule which satisfies the uniform additions property. However, it is important to accomplish the axiomatization of this decomposition rule in future research.

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