Applying conditional DEA to measure football clubs’ performance: Evidence from the top 25 European clubs

Halkos, George and Tzeremes, Nickolaos

University of Thessaly, Department of Economics

June 2011

Online at https://mpra.ub.uni-muenchen.de/31278/
MPRA Paper No. 31278, posted 05 Jun 2011 15:03 UTC
Applying conditional DEA to measure football clubs’
performance: Evidence from the top 25 European
clubs

By

George E. Halkos* and Nickolaos G. Tzeremes

University of Thessaly, Department of Economics,
Korai 43, 38333, Volos, Greece

Abstract

This paper applies a probabilistic approach to investigate how the top European football clubs’ current value and debt levels influence their performance. Specifically, a bootstrapped conditional data envelopment analysis (DEA) is used in order to measure the effect of football clubs’ current value and debt levels on their obtained efficiency performances. The results indicate that football clubs’ current value levels have a positive influence up to a certain point. But as the current value increases the effect is neutral to football clubs’ performance. At the same time, the empirical evidence suggests that there is no influence on football clubs’ efficiencies associated with lower and medium football clubs’ debt levels while higher debt levels appear to have a direct negative effect.

Keywords: European football clubs; Data Envelopment Analysis; Nonparametric regression; Bootstrapping; Probabilistic approach.

JEL classification: C14, C69, L83

* Address for Correspondence: Professor George Halkos, Department of Economics, University of Thessaly, Korai 43, 38333, Volos, Greece. Email: halkos@econ.uth.gr, http://www.halkos.gr/ Tel.: 0030 24210 74920 FAX: 0030 24210 74701
I. Introduction


Our study, similarly to the ones already mentioned, by applying for the first time conditional DEA efficiency estimates (Daraio and Simar, 2005, 2006, 2007a, 2007b) investigates how clubs’ value and debt levels influence their performances. In contrast to the main research stream, instead of using data of a specific national football league, our study uses a sample of the top 25 European football clubs and proposes a composite index as output based on several long term football clubs’ success factors.
II. Data and Methodology

All the data are extracted from Forbes database (2010) and concern data recorded for the year 2009. In our DEA formulation we use one input and one composite output. The input used is football clubs’ revenues (measured in millions $) and one composite output which measures football clubs European and domestic trophies. The composite output contains the sum of the number of European champions’ cups (weighted by 5), UEFA cups/ Euroleague cups (weighted by 4), European cup winners’ cups (weighted by 3), Intercontinental cups (weighted by 3) and FIFA Club World cups (weighted by 3).

In addition the composite output contains also the sum of the number of domestic championships (weighted by 2) and domestic cups (weighted by 1). Both the number of the weighted domestic champions and domestic cups (includes all domestic cups, i.e. super cups, league cups, national cups, etc) are again weighted by FIFA world ranking score (FIFA, 2010). This extra weight has been added in order to reflect the different difficulty levels of obtaining a domestic cup and/ or championship among the different European leagues\(^1\). We also assume that club revenues are used from the clubs in order to buy the best (in term of football quality) possible managers and players which can lead to team success (based on world, European and domestic championships and cups).

Recent studies for the English Premier League suggest that revenues are related to clubs’ success (Hickman et al., 2008; Carmichael et al., 2010). Moving a step ahead and by applying a conditional efficiency measurement approach we examine in what way European football clubs’ current value (CurrVal-Z\(_1\)) and debt levels (Debt-Z\(_2\)) (measured in millions of $) affect their obtained efficiency levels.

\(^1\) We assume that it is not of the same difficulty to obtain a domestic championship or cup between the English, the Scottish, the Spanish, the German and the Italian football league. All the weights used in order for the composite output to be constructed are subjective and can be subject to criticism.
Table 1 presents the descriptive statistics of the variables used in our study. As can be realized table 1 reports several variations of the variables used indicated by the high standard deviation values. Finally, in our DEA setting we assume an output orientation suggesting by how much football clubs can increase their output while keeping the level of inputs fixed.

**Table 1: Descriptive statistics of the variables used**

<table>
<thead>
<tr>
<th>External Variables</th>
<th>Input</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Current Value($mil)</td>
<td>Debt($mil)</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>597.080</td>
<td>218.238</td>
</tr>
<tr>
<td><strong>Std</strong></td>
<td>443.374</td>
<td>338.197</td>
</tr>
<tr>
<td><strong>Min</strong></td>
<td>194.000</td>
<td>0.002</td>
</tr>
<tr>
<td><strong>Max</strong></td>
<td>1870.000</td>
<td>1284.000</td>
</tr>
</tbody>
</table>

**Output components**

<table>
<thead>
<tr>
<th></th>
<th>Intercontinental Cup</th>
<th>FIFA Club World Cup</th>
<th>Domestic Championships</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>0.56</td>
<td>0.08</td>
<td>13.80</td>
</tr>
<tr>
<td><strong>Std</strong></td>
<td>1.00</td>
<td>0.28</td>
<td>12.70</td>
</tr>
<tr>
<td><strong>Min</strong></td>
<td>0.00</td>
<td>0.00</td>
<td>2.00</td>
</tr>
<tr>
<td><strong>Max</strong></td>
<td>3.00</td>
<td>1.00</td>
<td>51.00</td>
</tr>
</tbody>
</table>

**Output components**

<table>
<thead>
<tr>
<th></th>
<th>European Champions Cups</th>
<th>Uefa Cups/Euroleague Cups</th>
<th>European Cup Winners Cup</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>1.600</td>
<td>0.840</td>
<td>0.800</td>
</tr>
<tr>
<td><strong>Std</strong></td>
<td>2.432</td>
<td>1.143</td>
<td>0.913</td>
</tr>
<tr>
<td><strong>Min</strong></td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td><strong>Max</strong></td>
<td>9.000</td>
<td>3.000</td>
<td>4.000</td>
</tr>
</tbody>
</table>

**Output components**

<table>
<thead>
<tr>
<th></th>
<th>Domestic Cups</th>
<th>FIFA country Ranking</th>
<th>Composite Output</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>13.48</td>
<td>7.04</td>
<td>27.29</td>
</tr>
<tr>
<td><strong>Std</strong></td>
<td>13.04</td>
<td>8.88</td>
<td>33.82</td>
</tr>
<tr>
<td><strong>Min</strong></td>
<td>2.00</td>
<td>1.00</td>
<td>1.46</td>
</tr>
<tr>
<td><strong>Max</strong></td>
<td>57.00</td>
<td>35.00</td>
<td>142.00</td>
</tr>
</tbody>
</table>

Based on the work by Koopmans (1951) and Debreu (1951) the production set $\Psi$ constrains the production process and is the set of physically attainable points $(x, y)$:

$$\Psi = \left\{ (x, y) \in \mathbb{R}_+^{N+M} \mid x \text{ can produce } y \right\},$$

(1)

where $x \in \mathbb{R}_+^N$ is the input vector and $y \in \mathbb{R}_+^M$ is the output vector.
Then the output oriented efficiency boundary \( \partial Y(x) \) is defined for a given \( x \in \mathcal{N}_+ \) as:
\[
\partial Y(x) = \{ y \mid y \in Y(x), \lambda y \notin Y(x), \forall \lambda > 1 \},
\]
and the Debreu-Farrell output measure of efficiency for a production unit can be defined as:
\[
\lambda(x,y) = \sup \{ \lambda \mid (x,\lambda y) \in \Psi \}
\]

In equation (3) by construction \( \lambda(x,y) \geq 1 \) and technical efficiency is achieved when \( \lambda(x,y) = 1 \). As suggested by several authors (Førsund and Sarafoglou, 2002; Førsund et al., 2009), Hoffman’s (1957) discussion regarding Farrell’s (1957) paper was the first to indicate that linear programming can be used in order to find the frontier and estimate efficiency scores, but only for the single output case. Later, Boles (1967, 1971) developed the formal linear programming problem with multiple outputs identical to the constant returns to scale (CRS) model in Charnes et al. (1978) who named the technique as data envelopment analysis (DEA).

Following Zelenyuk and Zheka (2006, p.149) we apply the assumption of CRS due to the fact that it enables to obtain greater discriminative power, which in turn would result in larger variation of the regressand. In addition, since we examine the 25 European football clubs with the highest values, we are not expecting great differences among their sizes. This formulation can be expressed as:
\[
\dot{\Psi}_{\text{CRS}} = \{ (x,y) \in \mathbb{R}^{N+M} \mid y \leq \sum_{i=1}^{n} \gamma_i y_i; x \geq \sum_{i=1}^{n} \gamma_i x_i \text{ for } (\gamma_1,...,\gamma_n) \}
\]
such that \( \gamma_i \geq 0, i = 1,...,n \)

which then can be computed by solving the following linear program:
\begin{align*}
\hat{\lambda}_{crs} &= \sup \{ \lambda | \lambda y \leq \sum_{i=1}^{n} \gamma_i y_i, x \geq \sum_{i=1}^{n} \gamma_i x_i \text{ for } (\gamma_1, \ldots, \gamma_n) \} \\
&\quad \text{such that } \gamma_i \geq 0, i = 1, \ldots, n \}
\end{align*}

(5)

**II.1 A bootstrap approach for bias correction of the efficiency estimator**

Simar and Wilson (1998, 2000, 2008) suggest that DEA estimators were shown to be biased by construction. They introduced an approach based on bootstrap techniques (Efron, 1979) to correct and estimate the bias of the DEA efficiency indicators. The bootstrap bias estimate for the original DEA estimator \( \hat{\lambda}_{CRS}(x, y) \) can be calculated as:

\begin{align*}
BIAS_B \left( \hat{\lambda}_{CRS}(x, y) \right) &= B^{-1} \sum_{b=1}^{B} \hat{\lambda}_{CRS,b}(x, y) - \hat{\lambda}_{CRS}(x, y) 
\end{align*}

(6)

Furthermore, \( \hat{\lambda}_{CRS,b}(x, y) \) are the bootstrap values and \( B \) is the number of bootstrap replications (2000 replications has been used). Then a biased corrected estimator of \( \lambda(x, y) \) can be calculated as:

\begin{align*}
\hat{\lambda}_{CRS}(x, y) &= \hat{\lambda}_{CRS}(x, y) - BIAS_B \left( \hat{\lambda}_{CRS}(x, y) \right) \\
&= 2 \hat{\lambda}_{CRS}(x, y) - B^{-1} \sum_{b=1}^{B} \hat{\lambda}_{CRS,b}(x, y)
\end{align*}

(7)

Following the conditional efficiency estimators Daraio and Simar (2005, 2006, 2007a, 2007b) and the theoretical background of their statistical properties (Jeong et al. 2010) the conditional output efficiency measure can be defined as:

\begin{align*}
\lambda(x, y|z) &= \sup \{ \lambda | S_{y|x,z}(\lambda y|x, z) > 0 \}
\end{align*}

(8)

where \( Z \in \mathcal{R} \) denotes the external variables. Then the conditional survival function introduced in (8) can be estimated as:
In equation (9) K(.) is the Epanechnikov kernel and \( h \) is the bandwidth, which has been calculated following a fully automatic data-driven approach as has been indicated by Bădin et al. (2010). Since our external variables \( (Z_1 \text{ and } Z_2) \) are correlated (Pearson correlation = 0.65, P-value=0.000) and this will lead us to biased conditional efficiency estimates, we apply the Mahalanobis transformation (Mardia et al., 1979), following Cherchye et al. (2007) in order to de-correlate the variables and thus to apply a sequential kernel estimation as if all external variables were independently distributed. This will enable us to observe separately the effect of the external variables on the obtained efficiency estimates.

Then in order to establish the influence of an external variable on the efficiency scores obtained a scatter of the ratios \( \hat{Q}_Z \left( \frac{\hat{x}}{\hat{y}} | z \right) \) against \( Z \) (in our case as mentioned there are two external factors) and its smoothed non-parametric regression line it would help us to analyse the effect of \( Z \) on the European football teams’ performance. Following Racine (2008) we use a known local polynomial method due to the fact that does not suffer from edge bias and can enable us to describe better the underlying data generating process (DGP) compared to the local constant estimator (Nadaraya, 1965; Watson, 1964). According to Daraio and Simar (2005) if the regression line is increasing it indicates that \( Z \) is unfavourable to the teams’ efficiency levels whereas if it is decreasing then it is favourable.
III. Empirical results and conclusions

Following the methodology presented previously, Table 2 presents the results obtained of the efficiency analysis under the CRS assumption. The European football clubs are presented in Table 2 in a descending order based on their current values. The second column represents the original efficiency estimates \( \hat{\lambda}_{DEA}(x, y) \), whereas the third represents their biased corrected efficiency estimates \( \hat{\lambda}_{DEA, BC}(x, y) \). Similarly, the conditional efficiency scores on football clubs’ current value levels \( \hat{\lambda}_{DEA}(x, y|z_1) \) are presented in the fourth column alongside side with their biased corrected conditional efficiency estimates \( \hat{\lambda}_{DEA, BC}(x, y|z_1) \). Finally, the last two columns present the conditional on debt efficiency estimates \( \hat{\lambda}_{DEA}(x, y|z_2) \) and their biased corrected conditional efficiency estimates \( \hat{\lambda}_{DEA, BC}(x, y|z_2) \).

As can be realized from the descriptive values, football clubs’ current value and debt levels appear to decrease their performance levels (in average terms) both for original and biased corrected estimates. In addition high standard deviation values have been reported for all the efficiency estimators indicating high performance variations among the European football clubs. Figure 1 illustrates the nonparametric estimates of the regression functions (using the conditional and unconditional biased corrected CRS efficiency estimates) and their variability bounds of pointwise error bars using asymptotic standard error (Hayfield and Racine, 2008).
### Table 2: Efficiency scores of conditional and unconditional measures

| Football Clubs            | $\hat{\lambda}_{DEA} (x, y)$ | $\hat{\lambda}_{DEA,BC} (x, y)$ | $\hat{\lambda}_{DEA} (x, y|z_1)$ | $\hat{\lambda}_{DEA,BC} (x, y|z_1)$ | $\hat{\lambda}_{DEA} (x, y|z_2)$ | $\hat{\lambda}_{DEA,BC} (x, y|z_2)$ |
|--------------------------|-------------------------------|---------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| Manchester United FC     | 1.681                         | 1.773                           | 1.766                             | 1.896                             | 3.051                             | 3.768                             |
| Real Madrid FC           | 1.000                         | 1.209                           | 1.000                             | 1.256                             | 1.000                             | 1.352                             |
| Arsenal FC               | 1.752                         | 1.814                           | 1.974                             | 2.073                             | 3.836                             | 4.367                             |
| Bayern Munich FC         | 1.350                         | 1.449                           | 1.400                             | 1.538                             | 1.198                             | 1.366                             |
| Arsenal FC               | 1.141                         | 1.198                           | 1.203                             | 1.286                             | 1.879                             | 2.231                             |
| AC Milan FC              | 1.105                         | 1.112                           | 1.060                             | 1.138                             | 1.493                             | 1.765                             |
| Barcelona FC             | 1.051                         | 1.186                           | 1.014                             | 1.177                             | 1.274                             | 1.672                             |
| Chelsea FC               | 2.380                         | 2.457                           | 2.759                             | 2.891                             | 1.949                             | 2.110                             |
| Juventus FC              | 2.053                         | 1.096                           | 1.135                             | 1.201                             | 2.588                             | 3.189                             |
| Schalke 04 FC            | 1.511                         | 1.560                           | 1.838                             | 1.925                             | 2.783                             | 3.044                             |
| Tottenham Hotspur FC     | 1.342                         | 1.384                           | 1.532                             | 1.605                             | 2.816                             | 3.126                             |
| Olympique Lyonnais FC    | 1.914                         | 2.040                           | 2.510                             | 2.693                             | 2.770                             | 2.986                             |
| AS Roma FC               | 1.902                         | 1.971                           | 2.366                             | 2.490                             | 2.422                             | 2.618                             |
| Internazionale Milan FC  | 1.097                         | 1.142                           | 1.174                             | 1.241                             | 2.469                             | 2.962                             |
| Hamburg SV FC            | 1.269                         | 1.309                           | 1.301                             | 1.366                             | 2.736                             | 3.016                             |
| Borussia Dortmund FC     | 1.130                         | 1.166                           | 1.215                             | 1.273                             | 2.417                             | 2.669                             |
| Manchester City FC       | 1.281                         | 1.374                           | 1.377                             | 1.485                             | 2.062                             | 2.225                             |
| Werder Bremen FC         | 1.279                         | 1.331                           | 1.486                             | 1.580                             | 2.350                             | 2.546                             |
| Newcastle United FC      | 1.429                         | 1.486                           | 1.561                             | 1.666                             | 2.545                             | 2.756                             |
| VfB Stuttgart FC         | 1.375                         | 1.471                           | 1.615                             | 1.741                             | 2.329                             | 2.513                             |
| Aston Villa FC           | 1.092                         | 1.137                           | 1.000                             | 1.098                             | 1.707                             | 1.845                             |
| Olympique Marseille FC   | 1.461                         | 1.523                           | 1.093                             | 1.180                             | 2.715                             | 2.940                             |
| Celtic FC                | 1.079                         | 1.133                           | 1.274                             | 1.367                             | 1.577                             | 1.703                             |
| Everton FC               | 1.115                         | 1.168                           | 1.325                             | 1.415                             | 1.704                             | 1.841                             |
| Glasgow Rangers FC       | 1.000                         | 1.075                           | 1.175                             | 1.281                             | 1.114                             | 1.201                             |
| Mean                     | 1.349                         | 1.423                           | 1.486                             | 1.594                             | 2.191                             | 2.472                             |
| Std                      | 0.346                         | 0.350                           | 0.480                             | 0.491                             | 0.693                             | 0.779                             |
| Min                      | 1.000                         | 1.075                           | 1.000                             | 1.098                             | 1.000                             | 1.201                             |
| Max                      | 2.380                         | 2.457                           | 2.759                             | 2.891                             | 3.836                             | 4.367                             |

The results indicate that football clubs’ current value levels (subfigure 1a) have a positive value up to a certain point. However as the current value increases the effect is neutral to football clubs’ performance. In addition it is reported that extremely high football clubs’ current values may have even a negative value on football clubs’ performance. In addition it appears that lower to medium levels of football clubs’ debt levels (subfigure 1b) has a neutral effect on football clubs’ performance. However there are strong evidences that higher debt values can affect negatively football clubs’ efficiency levels.
Finally, in terms of policy implications it appears that when comparing the top European football clubs, their determinants of higher efficiency (in terms of the number of domestic and European club trophies) are not based on their higher revenue and value levels. The conditional DEA approach proved to be a vital tool for showing that other factors like managerial efficiency (Fizel and D’Itri, 1996, 1997; Dawson et al., 2000) and team spirit (Scully, 1974) may be more important when comparing the top European football clubs.
References


