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Abstract

How does patent policy affect economic growth through human capital accumulation and endogenous fertility? In this study, we develop a scale-invariant R&D-based growth model to analyze an unexplored interaction between intellectual property rights, endogenous fertility, human capital accumulation and economic growth. We find that strengthening patent protection has (a) a positive effect on technological progress, (b) a negative effect on human capital accumulation through a higher rate of fertility, and (c) an ambiguous overall effect on economic growth. Furthermore, a stronger cultural preference for fertility strengthens the negative effect of patent policy relative to its positive effect on economic growth. Finally, we calibrate the model to provide a quantitative analysis on the relative strength of these opposing effects of patent policy.

JEL classification: O31, O34, O40.
Keywords: economic growth, endogenous fertility, patent policy.

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1 Introduction

How does patent policy affect economic growth through human capital accumulation and endogenous fertility? To analyze this question, we develop a scale-invariant R&D-based growth model. In the literature on R&D-driven economic growth, there has been a very important debate about the presence of counterfactual scale effects in the first-generation models, such as Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992). In response to this critique, subsequent generations of R&D-based growth models have been developed to remove the strong scale effect (i.e., a positive relationship between population size and long-run growth).[1] In these scale-invariant models, the long-run growth rate is either solely or partly determined by the population growth rate that is *assumed* to be exogenous. However, in a more realistic framework, the fertility rate should be treated as an endogenous variable chosen by optimizing households. In this study, we first develop a scale-invariant quality-ladder endogenous-growth model with endogenous fertility and human capital accumulation and then apply this growth-theoretic framework to analyze the effects of intellectual property rights on fertility, human capital accumulation, technological progress, and economic growth. To our knowledge, this interaction between patent policy, endogenous fertility, human capital accumulation and economic growth has never been explored in the literature. Furthermore, in recent vintages of R&D-based growth models, the long-run growth rate is usually increasing in the population growth rate (i.e., a *weak* scale effect); however, even this weak scale effect is not supported empirically.[2] Therefore, we follow Strulik (2005) to model human capital accumulation that generates a negative relationship between fertility and economic growth.

In the model, optimizing households choose the fertility rate by trading off the marginal utility of higher fertility against its marginal costs arising from (a) foregone wages, (b) the dilution of financial assets per capita, and (c) the dilution of human capital per capita. We find that strengthening patent protection that increases the market power of firms weakens the *foregone-wage* effect and the *human-capital-diluting* effect but strengthens the *asset-diluting* effect of fertility. On the one hand, weakening the foregone-wage effect and the human-capital-diluting effect leads to a higher fertility rate.

[1] See Jones (1999) for an excellent review on these subsequent generations of R&D-based growth models.

[2] See for example Strulik (2005) for a discussion.

On the other hand, strengthening the asset-diluting effect leads to a lower fertility rate. We find that the effects of patent policy on the dilution of financial assets and the dilution of human capital cancel each other. As a result, strengthening patent protection unambiguously increases fertility through weakening the foregone-wage effect, and this higher rate of fertility reduces human capital accumulation, which in turn leads to a negative effect on economic growth. Together with the positive effect of patent protection on R&D and technological progress, the overall effect on economic growth is ambiguous. Furthermore, we find that a stronger cultural preference for fertility (i.e., a larger value for the fertility-preference parameter) tends to strengthen the negative growth effect of patent policy.

The intuition of the above results can be explained as follows. Strengthening patent protection that increases the market power of firms raises the share of income that goes to monopolistic profits giving rise to a conventional positive effect on R&D and technological progress. However, it also reduces the share of income that goes to other factor inputs including labor. As a result of lower wages, the opportunity cost of non-market activities decreases; consequently, households reallocate their time from labor supply to non-market activities including childrearing. This is the weakening foregone-wage effect discussed above. The higher rate of fertility in turn reduces the rate of human capital accumulation by crowding out parents' time and reducing the amount of resources per child. Because economic growth is driven by both technological progress and human capital accumulation, the overall growth effect of patent policy is ambiguous.

We also calibrate the model to aggregate data of the US economy to provide a quantitative analysis. We find that the magnitude of the negative growth effect of patent policy through human capital accumulation crucially depends on a preference parameter on fertility and is increasing in its parameter value. Given a reasonable parameter value for the US economy, the negative effect of patent policy on human capital accumulation and economic growth is likely to be dominated by the positive effect through technological progress. However, as the strength of cultural preference for fertility increases, the negative growth effect of patent policy becomes quantitatively significant relative to the positive effect. For example, Fernandez and Fogli (2009) provide empirical evidence to show that preference on fertility varies across culture and has a significant effect on fertility outcomes.[3]

[3] Fernandez and Fogli (2009) use the past total fertility rate in the country of ancestry

Our study relates to the literature on endogenous fertility and R&D-driven growth for which Growiec (2006) provides an excellent review.[4] Jones (2001) develops a semi-endogenous growth model with endogenous fertility to analyze the emergence of rapid growth and demographic transitions.[5] To simplify their analysis, Jones (2001, 2003) and Growiec (2006) consider a model in which the allocation of inputs to R&D is exogenously determined. The present study differs from Jones (2001, 2003) and Growiec (2006) by developing a quality-ladder model in which both fertility and the allocation of factor inputs are endogenously determined through the market equilibrium. Therefore, our model follows more closely the footsteps of Connolly and Peretto (2003), who develop an R&D-based growth model with vertical and horizontal innovations to analyze demographic shocks and industrial policies that affect the costs of R&D and/or entry. However, our model differs from Connolly and Peretto (2003) by featuring human capital accumulation as well as creative destruction that gives rise to the importance of patent breadth that protects an innovation against previous innovations. Therefore, the present study complements their interesting analysis by analyzing another important set of industrial policy: the effects of intellectual property rights on fertility, human capital accumulation and economic growth.

Our study also relates to the literature on patent policy and economic growth. The seminal study in the literature on optimal patent design is Nordhaus (1969).[6] While studies in this patent-design literature mostly analyze patent policy in partial-equilibrium models, the present study follows more closely a related macroeconomic literature by analyzing the effects of patent policy in a quantitative dynamic general-equilibrium model. The seminal dynamic general-equilibrium analysis on optimal patent length is Judd

as a proxy for cultural preference on fertility and find that second-generation Americans whose ancestry is from countries with higher fertility rates tend to have more children.

[4]See also Barro and Becker (1989) for a seminal study on endogenous fertility in an overlapping-generation model with exogenous growth.

[5]See also Jones (2003), who analyzes the effects of an exogenous increase in the R&D share of labor chosen by the government. He finds that this policy change increases growth in the short run but decreases growth in the long run through a *lower* rate of fertility due to a *crowding-out* effect on labor supply. In the previous version of this study, see Chu and Cozzi (2011), we also consider a semi-endogenous-growth version of our model with endogenous fertility and derive a negative effect of patent breadth on long-run growth similar to Jones (2003) but our result is based on a *higher* rate of fertility through an *opportunity-cost* effect of lower foregone wages.

[6]See Scotchmer (2004) for a comprehensive review of this patent-design literature.

(1985), who finds that the optimal patent length can be infinite. Subsequent studies by Iwaisako and Futagami (2003) and Futagami and Iwaisako (2007) show that the optimal patent length is usually finite in the Romer model due to an additional distortionary effect on intermediate goods that is absent in Judd (1985).[7] While this branch of studies focuses on characterizing the optimal patent length, another branch of studies in the literature analyzes the effects of other patent-policy levers on innovation and growth. See, for example, Li (2001) on patent breadth,[8] O'Donoghue and Zweimuller (2004) on forward patent protection and patentability requirement, Cozzi (2001) and Cozzi and Spinesi (2006) on intellectual appropriability, Furukawa (2007, 2010) and Horri and Iwaisako (2007) on patent protection against imitation, and Chu (2009) on blocking patents. Some of these studies find that strengthening patent protection may generate a negative effect on innovation, and this finding is consistent with the detailed case studies analyzed in Jaffe and Lerner (2004), Bessen and Meurer (2008) and Boldrin and Levine (2008). The present study contributes to this literature by analyzing a novel effect through endogenous fertility that patent policy reduces human capital accumulation causing a negative effect on economic growth.

Finally, this study relates to a growing literature on culture and economic growth. A recent empirical study by Tabellini (2010) provides evidence that cultural traits, such as trust, respect for others, confidence in individual self-determination, and emphasis on children's obedience, have significant causal effects on regional per capita income in Europe. Another interesting empirical study by Alesina and Giuliano (2010) analyzes the effects of family ties on economic outcomes, such as home production and labor force participation of women and youth. In terms of theoretical work, a seminal study by Galor and Moav (2002) shows that individual preferences on offspring quality affect the speed of transition to sustained economic growth. A subsequent study by Ashraf and Galor (2007) analyzes the relative advantage of two interesting cultural characteristics, namely, cultural assimilation and cultural diversity, at different stages of economic development. Another recent study by Chu (2007) argues that cultural variation in entrepreneurial overconfidence can play a role in causing different rates of economic growth across countries. The present study relates to this literature by showing that cultural preference

[7] See also Horowitz and Lai (1996).

[8] See also Chu (2011) for a quantitative analysis on uniform versus sector-specific optimal patent breadth in a two-sector quality-ladder growth model.

on fertility not only has a direct effect on economic growth but it may also have an indirect effect on growth through intellectual property rights.

The rest of this study is organized as follows. Section 2 describes the model. Section 3 derives the equilibrium allocation and analyzes the dynamics of the balanced growth path. Section 4 considers the effects of patent policy on economic growth and social welfare. The final section concludes.

2 A quality-ladder model with endogenous fertility and human capital accumulation

In this section, we develop a scale-invariant version of the Grossman-Helpman (1991) quality-ladder model. The key changes in our model are as follows. First, we consider endogenous fertility instead of exogenous fertility following the setup in Razin and Ben-Zion (1975) and Yip and Zhang (1997). Second, we allow for variable patent breadth as in Li (2001) in order to analyze the effects of patent policy. Third, we remove the strong scale effect through diluting R&D inputs by the scale of the economy following Laincz and Peretto (2006). In the literature, there are two seminal approaches to remove the strong scale effect. The first approach is the semi-endogenous growth model in which long-run economic growth is solely determined by the population growth rate.[9] The second approach is the second-generation model in which long-run economic growth is determined by both the population growth rate and the R&D share of labor.[10] In our model, economic growth depends on both the population growth rate and the share of human capital allocated to R&D resembling a second-generation model.[11] Finally, we introduce human capital accumulation as in Strulik (2005) to generate a negative effect of fertility on economic growth. Given that the quality-ladder model has been well-studied, we will describe the familiar features briefly to conserve space and discuss the new features in details.

[9] Early studies on the R&D-based semi-endogenous growth model include Jones (1995), Kortum (1997) and Segerstrom (1998).

[10] Early studies on the second-generation R&D-based *endogenous* growth model include Young (1998), Dinopoulos and Thompson (1998) and Peretto (1998).

[11] See Laincz and Peretto (2006) and Ha and Howitt (2007) for empirical evidence that supports the second-generation R&D-based growth model.

2.1 Households

There is a unit continuum of identical households. As is standard in the literature on endogenous fertility, households derive utility from fertility. Here we consider a continuous-time setup similar to Yip and Zhang (1997). However, considering a discrete-time setup with overlapping generations of households as in Razin and Ben-Zion (1975) would not change our results. The intergenerational utility of households is the discounted sum of per capita utility across time.[12] Specifically, the utility function of a household is given by

$$U = \int_0^\infty e^{-\rho t} u(c_t, n_t) dt, \tag{1}$$

where $u(c_t, n_t) = \ln c_t + \alpha \ln n_t$. c_t is the per capita consumption of final goods (numeraire), and n_t is the number of births per person at time t. Given N_t as the size of population, the total number of births is $\dot{N}_t = n_t N_t$. In this simple model with zero mortality, n_t is also the population growth rate. $\alpha > 0$ is a fertility-preference parameter, and $\rho > 0$ is the discount rate.

Each household maximizes (1) subject to the following asset-accumulation equation.

$$\dot{a}_t = (r_t - n_t) a_t + w_t l_t - c_t. \tag{2}$$

a_t is the amount of financial assets per capita, and r_t is the rate of return on assets. An increase in n_t reduces the amount of assets per capita, and we refer to this effect as the *asset-diluting* effect of fertility. w_t is the wage rate, and l_t is human-capital embodied labor supply. Each person has one unit of time to allocate between fertility, work and education. The time spent on fertility is given by $n_t/\theta < 1$, where $\theta > 0$ is a parameter that is negatively related to the time cost of fertility.[13] At time t, the stock of human capital per capita is h_t. Each person combines her remaining time endowment $1 - n_t/\theta$ with

[12] See Growiec (2006) for an interesting discussion on alternative ways of modelling endogenous fertility in the growth literature.

[13] We follow a common approach in the literature to assume that θ is independent of capital accumulation or technological progress; see also Yip and Zhang (1997) and Connolly and Peretto (2003). Otherwise, as technology or human capital accumulates, θ increases causing a lower time cost of fertility, which in turn leads to a rising fertility rate instead of a constant fertility rate (i.e., ruling out a balanced growth path). However, we think it is reasonable that parental human capital contributes to the health and education of children, and this positive effect is captured by the law of motion for human capital per capita in (4).

her human capital h_t for work l_t and education e_t subject to

$$h_t(1 - n_t/\theta) = l_t + e_t. \tag{3}$$

Increasing n_t reduces the amount of time available for work and education, and this setup captures the *foregone-wage* effect of fertility. The law of motion for human capital per capita is

$$\dot{h}_t = \xi e_t - (n_t + \delta)h_t, \tag{4}$$

where $\xi > \rho$ is a productivity parameter for human capital accumulation. $n_t h_t$ captures the *human-capital-diluting* effect of fertility as in Strulik (2005). The parameter $\delta \geq 0$ is the depreciation rate of human capital.

From standard dynamic optimization, the Euler equation is

$$\frac{\dot{c}_t}{c_t} = r_t - n_t - \rho, \tag{5}$$

and the consumption-fertility optimality condition is

$$\frac{\alpha}{n_t} = \frac{1}{c_t}\left[a_t + \left(\frac{1}{\theta} + \frac{1}{\xi}\right) w_t h_t\right]. \tag{6}$$

This condition equates the marginal utility of fertility given by α/n_t to the marginal utility of consumption (in response to a change in fertility) given by $\left[a_t + w_t h_t \left(1/\theta + 1/\xi\right)\right]/c_t$. The first term a_t/c_t captures the asset-diluting effect of fertility, and this effect is positively related to the value of assets per capita. The second term $\theta^{-1} w_t h_t/c_t$ captures the foregone-wage effect of fertility, and the third term $\xi^{-1} w_t h_t/c_t$ captures the human-capital-diluting effect of fertility. Both of these effects are positively related to the wage rate. From dynamic optimization, we can also derive an equilibrium condition that equates the returns on assets and human capital.

$$r_t = \frac{\dot{w}_t}{w_t} - \delta + \xi(1 - n_t/\theta). \tag{7}$$

We will show that this condition determines the equilibrium growth rate of human capital.

2.2 Final goods

Final goods are produced by competitive firms that aggregate intermediate goods using a standard Cobb-Douglas aggregator given by

$$Y_t = \exp\left(\int_0^1 \ln X_t(i)di\right). \tag{8}$$

$X_t(i)$ denotes intermediate goods $i \in [0,1]$. From profit maximization, the conditional demand function for $X_t(i)$ is

$$X_t(i) = Y_t/p_t(i), \tag{9}$$

where $p_t(i)$ is the price of $X_t(i)$.

2.3 Intermediate goods

There is a unit continuum of industries producing differentiated intermediate goods. Each industry is temporarily dominated by an industry leader until the arrival of the next innovation, and the owner of the new innovation becomes the next industry leader.[14] The production function for the leader in industry i is

$$X_t(i) = z^{q_t(i)} L_{x,t}(i). \tag{10}$$

The parameter $z > 1$ is the step size of productivity improvement, and $q_t(i)$ is the number of productivity improvements that have occurred in industry i as of time t. $L_{x,t}(i)$ is production labor in industry i. Given $z^{q_t(i)}$, the marginal cost of production for the industry leader in industry i is $mc_t(i) = w_t/z^{q_t(i)}$. It is useful to note that we here adopt a cost-reducing view of vertical innovation as in Peretto (1998, 1999).

Standard Bertrand price competition leads to a profit-maximizing price given by

$$p_t(i) = \mu(z,b)mc_t(i), \tag{11}$$

where $\mu = z^b > 1$ and $b \in (0,1)$ denotes patent breadth. In the original Grossman-Helpman (1991) model, the patentholder is assumed to have complete protection against imitation such that $b = 1$. Li (2001) considers a

[14] This is known as the Arrow replacement effect in the literature. See Cozzi (2007) for a discussion on the Arrow effect.

more general policy environment with incomplete patent protection against imitation such that $b \in (0, 1)$. Here we follow the formulation in Li (2001). From (9), the amount of monopolistic profit is

$$\pi_t(i) = \left(\frac{\mu - 1}{\mu}\right) p_t(i) X_t(i) = \left(\frac{\mu - 1}{\mu}\right) Y_t. \tag{12}$$

Therefore, a larger patent breadth b increases the markup μ and the amount of monopolistic profit improving the incentives for R&D. For the rest of this study, we use μ to measure the strength of patent protection. Finally, production-labor income is

$$w_t L_{x,t}(i) = \left(\frac{1}{\mu}\right) p_t(i) X_t(i) = \left(\frac{1}{\mu}\right) Y_t. \tag{13}$$

Equations (12) and (13) show that strengthening patent protection increases the share of profit income (i.e., π_t/Y_t) and decreases the share of wage income (i.e., $w_t L_{x,t}/Y_t$). Through these effects, patent policy affects the equilibrium rate of fertility.

2.4 R&D

Denote $v_t(i)$ as the share value of the monopolistic firm in industry i. Because $\pi_t(i) = \pi_t$ for $i \in [0, 1]$ from (12), $v_t(i) = v_t$ in a symmetric equilibrium that features an equal arrival rate of innovation across industries.[15] In this case, the familiar no-arbitrage condition for v_t is

$$r_t v_t = \pi_t + \dot{v}_t - \lambda_t v_t. \tag{14}$$

This condition equates the interest rate to the asset return per unit of asset. The asset return is the sum of (a) monopolistic profit π_t, (b) potential capital gain $\dot{v}_t$ and (c) expected capital loss $\lambda_t v_t$ from creative destruction for which λ_t is the arrival rate of the next innovation.

There is a unit continuum of R&D firms indexed by $j \in [0, 1]$. They hire R&D labor $L_{r,t}(j)$ for innovation. The zero-expected-profit condition of firm j is

$$v_t \lambda_t(j) = w_t L_{r,t}(j), \tag{15}$$

[15] We follow the standard approach in the literature to focus on the symmetric equilibrium. See Cozzi *et al.* (2007) for a theoretical justification for the symmetric equilibrium to be the unique rational-expectation equilibrium in the quality-ladder growth model.

where the *firm-level* arrival rate of innovation is

$$\lambda_t(j) = \overline{\varphi}_t L_{r,t}(j). \tag{16}$$

To remove the strong scale effect, we follow Laincz and Peretto (2006) to specify that $\overline{\varphi}_t$ is decreasing in the scale of the economy. Specifically, we assume that $\overline{\varphi}_t = \varphi L_{r,t}^{\phi-1}/(h_t N_t)^{\phi}$,[16] where $h_t N_t$ measures the scale of the economy and the parameter $\phi \in (0,1)$ captures the negative duplication externality commonly discussed in the literature; see for example Jones (1995) and Jones and Williams (2000). Given $L_{r,t} = \int_0^1 L_{r,t}(j)dj$, the *aggregate* arrival rate λ_t of innovation features decreasing returns to scale in $L_{r,t}$.[17]

3 Decentralized equilibrium

The equilibrium is a time path of allocations $\{c_t, n_t, h_t, l_t, N_t, Y_t, X_t(i), L_{x,t}(i), L_{r,t}(j)\}$ and a time path of prices $\{p_t(i), w_t, r_t, v_t\}$. Also, at each instance of time,

- households maximize utility taking $\{r_t, w_t\}$ as given;
- competitive final-goods firms produce $\{Y_t\}$ to maximize profit taking $\{p_t(i)\}$ as given;
- monopolistic intermediate-goods firms produce $\{X_t(i)\}$ and choose $\{L_{x,t}(i), p_t(i)\}$ to maximize profit taking $\{w_t\}$ as given;
- R&D firms choose $\{L_{r,t}(j)\}$ to maximize expected profit taking $\{w_t, v_t\}$ as given;

[16] In the previous version of this study, see Chu and Cozzi (2011), we consider a semi-endogenous-growth version of our model by specifying $\overline{\varphi}_t$ to be decreasing in aggregate technology. In that model, we find that patent breadth has the same effects on fertility as in the current framework. However, the current framework is more general because long-run growth depends also on the R&D share of human capital whereas this R&D share only plays a role on short-run growth but not on long-run growth in the semi-endogenous growth model. We would like to thank a referee for suggesting us to pursue the current formulation.

[17] We assume constant returns to scale at the firm level in order to be consistent with free entry and zero expected profit.

- the market-clearing condition for human-capital embodied labor supply holds such that $l_t N_t = L_{x,t} + L_{r,t}$;
- the market-clearing condition for final goods holds such that $Y_t = c_t N_t$; and
- the share value of monopolistic firms adds up to the total value of household assets such that $v_t = a_t N_t$.

The aggregate production function is given by

$$Y_t = Z_t L_{x,t}, \tag{17}$$

where aggregate technology Z_t is defined as

$$Z_t = \exp\left(\int_0^1 q_t(i)di \ln z\right) = \exp\left(\int_0^t \lambda_\tau d\tau \ln z\right). \tag{18}$$

The second equality of (18) applies the law of large numbers. Differentiating the log of (18) with respect to t yields the growth rate of aggregate technology given by

$$g_{z,t} \equiv \frac{\dot{Z}_t}{Z_t} = \lambda_t \ln z = (\varphi \ln z)\left(\frac{L_{r,t}}{h_t N_t}\right)^\phi. \tag{19}$$

As for the dynamics of the model, Proposition 1 shows that the economy is always on a unique and saddle-point stable balanced growth path.

Proposition 1 *Given a constant level of patent breadth μ, the economy immediately jumps to a unique and saddle-point stable balanced growth path along which each variable grows at a constant (possibly zero) rate.*

Proof. See Appendix A. ■

3.1 Balanced growth path

Given Proposition 1, we analyze the equilibrium allocation on the balanced growth path in this section. On the balanced growth path, the arrival rate of innovation is constant so that $L_{r,t}$ and $h_t N_t$ must grow at the same rate. The steady-state growth rate of technology is

$$g_z = (\varphi \ln z) s_r^{\phi}, \tag{20}$$

where we define $s_r \equiv L_{r,t}/(h_t N_t)$ and $s_x \equiv L_{x,t}/(h_t N_t)$ as the shares of human capital devoted to R&D and production, respectively.

Combining (13) and (17) yields $w_t = Z_t/\mu$, which implies

$$\frac{\dot{Z}_t}{Z_t} = \frac{\dot{w}_t}{w_t} = \frac{\dot{c}_t}{c_t} + n_t + \rho + \delta - \xi(1 - n_t/\theta), \tag{21}$$

where the second equality of (21) is derived by substituting (5) into (7). The steady-state growth rate of consumption per capita is

$$g_c = g_y - n = g_z + g_h, \tag{22}$$

where g_y is the steady-state growth rate of Y_t. In other words, our model features two engines of growth (i.e., technological progress g_z and human capital accumulation g_h). Substituting (22) into (21) yields

$$g_h = \xi(1 - n/\theta) - n - \rho - \delta. \tag{23}$$

Therefore, the growth rate of human capital per capita is decreasing in n. The first negative effect (i.e., $-\xi n/\theta$) arises from the crowding out of fertility on time endowment. The second negative effect (i.e., $-n$) is the human-capital-diluting effect of fertility. Finally, the growth rate of consumption c_t is

$$g_c = g_z + g_h = (\varphi \ln z) s_r^{\phi} - (1 + \xi/\theta)n + \xi - \rho - \delta. \tag{24}$$

Equation (24) shows that economic growth g_c is increasing in s_r and decreasing in n. In other words, by introducing human-capital accumulation into the semi-endogenous growth model, we are able to generate a negative relationship between fertility and economic growth as in Strulik (2005). Furthermore, in our model, endogenous fertility generates an additional negative effect on human capital accumulation through the crowding out of time endowment that is absent in the Strulik exogenous-fertility model.

Using (13) and (15), we derive the first equation for solving the steady-state equilibrium n^* as follows.

$$\frac{v_t \lambda_t}{L_{r,t}} = w_t = \frac{Y_t}{\mu L_{x,t}} \Leftrightarrow \frac{s_r}{s_x} = (\mu - 1)\frac{\lambda}{\rho + \lambda}, \tag{25}$$

where $\lambda = \varphi s_r^{\phi}$. The second equation for solving the model can be obtained by combining the time-endowment constraint and the labor-market clearing condition.

$$1 - \frac{n}{\theta} = \frac{l_t}{h_t} + \frac{e_t}{h_t} = s_r + s_x + \frac{e_t}{h_t}. \tag{26}$$

From (4), the steady-state growth rate of h_t is

$$g_h \equiv \frac{\dot{h}_t}{h_t} = \xi \frac{e_t}{h_t} - n - \delta. \tag{27}$$

Equating (27) and (23) yields

$$\frac{e_t}{h_t} = 1 - \frac{n}{\theta} - \frac{\rho}{\xi}, \tag{28}$$

which describes a negative relationship between n and e_t/h_t. Using (28), we can simplify (26) to

$$\frac{\rho}{\xi} = s_r + s_x. \tag{29}$$

Combining (25) and (29) yields the following polynomial function that solves the equilibrium s_r^* as an implicit function in μ.

$$\varphi \mu s_r^* + \rho (s_r^*)^{1-\phi} = \varphi(\mu - 1)\rho/\xi. \tag{30}$$

Taking the total differentials of (30), we obtain

$$\frac{ds_r^*}{d\mu} = \frac{\varphi s_x^*}{\varphi \mu + (1-\phi)\rho(s_r^*)^{-\phi}} > 0. \tag{31}$$

Therefore, the R&D share s_r^* of human capital is increasing in μ, and this is the standard positive effect of patent breadth on R&D through a larger share of monopolistic profits. Equations (29) and (31) together imply that the production share s_x^* of human capital is decreasing in μ.

4 Effects of strengthening patent protection

To solve for the equilibrium fertility rate n^*, we make use of the consumption-fertility optimality condition in (6).[18]

$$\begin{aligned}\frac{\alpha}{n^*} &= \frac{a_t}{c_t} + \frac{1}{\theta}\left(\frac{w_t h_t}{c_t}\right) + \frac{1}{\xi}\left(\frac{w_t h_t}{c_t}\right) \\ &= \left(\frac{\mu-1}{\mu}\right)\frac{1}{\rho+\varphi(s_r^*)^{\phi}} + \frac{1}{\theta}\left(\frac{1}{\mu s_x^*}\right) + \frac{1}{\xi}\left(\frac{1}{\mu s_x^*}\right), \qquad (32)\end{aligned}$$

where s_r^* and s_x^* are implicit functions in μ. Equation (32) determines the equilibrium n^* as a function in μ. As for the comparative statics of n^* with respect to μ, we need to consider all the general-equilibrium effects of μ on n^*. The first term on the right-hand side of (32) captures the asset-diluting effect of fertility. For a given s_r^*, a larger patent breadth strengthens this effect by increasing a_t/c_t (i.e., the ratio of asset value to consumption) and leads to a lower rate of fertility. The second term on the right-hand side of (32) captures the foregone-wage effect of fertility. For a given s_x^*, a larger patent breadth strengthens this effect by decreasing $w_t h_t/c_t$ (i.e., the ratio of wage income to consumption) and leads to a higher rate of fertility. The third term on the right-hand side of (32) captures the human-capital-diluting effect of fertility. For a given s_x^*, a larger patent breadth also strengthens this effect by decreasing $w_t h_t/c_t$ and leads to a higher rate of fertility.

Although there are two positive effects and one negative effect, we nonetheless derive an unambiguously positive effect because the human-capital-diluting effect and the asset-diluting effect of fertility cancel each other. To see this result, we first differentiate α/n^* with respect to μ and then substitute (25) and (31) into the resulting expression to obtain

$$\begin{aligned}\frac{\partial \alpha/n^*}{\partial \mu} &= \frac{1}{\mu}\left(\frac{1}{\rho+\varphi(s_r^*)^{\phi}}\right)\left(\frac{1}{\mu} - \phi\frac{\varphi}{\varphi\mu + (1-\phi)\rho(s_r^*)^{-\phi}}\right) \\ &\quad - \frac{1}{\mu s_x^*}\left(\frac{1}{\theta}+\frac{1}{\xi}\right)\left(\frac{1}{\mu} - \frac{\varphi}{\varphi\mu + (1-\phi)\rho(s_r^*)^{-\phi}}\right). \qquad (33)\end{aligned}$$

It can be shown that

$$\frac{\partial \alpha/n^*}{\partial \mu} < 0 \Leftrightarrow \frac{s_x^*}{\rho+\varphi(s_r^*)^{\phi}} < \frac{\rho}{\varphi\mu(s_r^*)^{\phi}+\rho}\left(\frac{1}{\theta}+\frac{1}{\xi}\right). \qquad (34)$$

[18] It is useful to recall that $a_t = v_t/N_t$.

Applying (25) and (30), this inequality further simplifies to $1/\theta > 0$. Therefore, unless the foregone-wage effect is absent (i.e., $\theta \to \infty$), n^* is increasing in μ for $\phi \in (0,1)$.[19] Differentiating (24) with respect to μ yields

$$\frac{\partial g_c^*}{\partial \mu} = \underbrace{\frac{\partial g_z^*}{\partial \mu}}_{>0} + \underbrace{\frac{\partial g_h^*}{\partial \mu}}_{<0} = (\varphi \ln z)\underbrace{\frac{\partial (s_r^*)^{\phi}}{\partial \mu}}_{>0} - (1+\xi/\theta)\underbrace{\frac{\partial n^*}{\partial \mu}}_{>0}. \tag{35}$$

Furthermore, (32) and (33) imply that the value of $\partial n^*/\partial \mu$ is strictly increasing in α, whereas (30) implies that $\partial (s_r^*)^{\phi}/\partial \mu$ is independent of α. We summarize our main results in Proposition 2.

Proposition 2 *An increase in the strength of patent protection μ increases the equilibrium fertility rate n^* and decreases the growth rate g_h^* of human capital. However, it also increases the R&D share s_r^* of human capital and the growth rate g_z^* of technology. Therefore, the overall effect of μ on the growth rate g_c^* of consumption is ambiguous. If the fertility-preference parameter α is sufficiently large, then the negative effect of μ on g_c^* through g_h^* dominates the positive effect through g_z^*.*

Proof. Proven in the text. ■

Furthermore, we find that as households value fertility more (i.e., a larger α), they choose a higher rate of fertility n^*. As a result of higher population growth, the economy exhibits a lower growth rate of human capital per capita. Therefore, a stronger cultural preference for fertility also has a direct negative effect on economic growth.

Proposition 3 *An increase in the fertility-preference parameter α increases the equilibrium fertility rate n^* and decreases the growth rates of human capital and consumption $\{g_h^*, g_c^*\}$.*

Proof. First, note that a larger α increases n^* from (32). Then, note that a larger n^* reduces g_h^* from (23) and g_c^* from (24). ■

[19] In the previous version of this study, see Chu and Cozzi (2011), we consider the special case of $\theta = 1$ and find that n^* is independent of μ in this knife-edge case.

4.1 Welfare analysis

In this section, we analyze the welfare effects of strengthening patent protection. First, we derive the welfare of households in the market equilibrium. Then, we also derive the first-best optimal allocation. On the balanced growth path, (1) simplifies to the following welfare expression that applies to both the market equilibrium and the first-best allocation.

$$U = \frac{1}{\rho}\left(\ln c_0 + \frac{g_c}{\rho} + \alpha \ln n\right), \tag{36}$$

where c_0 is initial consumption per capita. Using $c_t = Y_t/N_t$ and (17), initial consumption can be expressed as

$$c_0 = Z_0 h_0 s_x, \tag{37}$$

where Z_0 and h_0 are the initial *exogenous* levels of technology and per capita human capital, respectively. The steady-state growth rate of consumption is $g_c = g_z + g_h$, where g_z is given by (20) and g_h is given by (27). The resource constraint in (3) can be re-expressed as

$$1 = \frac{n}{\theta} + s_r + s_x + \frac{e_0}{h_0}, \tag{38}$$

where we will normalize $h_0 = 1$ and simply use $e \equiv e_0/h_0$ for convenience.

Substituting some of the above conditions into (36) and dropping the exogenous terms yield

$$U = \frac{1}{\rho}\left(\ln s_x + \frac{\varphi \ln z}{\rho} s_r^{\phi} + \frac{\xi e - n}{\rho} + \alpha \ln n\right). \tag{39}$$

Under the market equilibrium denoted by superscript *, differentiating (39) with respect to patent strength μ yields

$$\rho\frac{\partial U^*}{\partial \mu} = \underbrace{\frac{\partial \ln s_x^*}{\partial \mu}}_{-} + \frac{\varphi \ln z}{\rho}\underbrace{\frac{\partial (s_r^*)^{\phi}}{\partial \mu}}_{+} + \frac{\xi}{\rho}\underbrace{\frac{\partial e^*}{\partial \mu}}_{-} - \frac{1}{\rho}\underbrace{\frac{\partial n^*}{\partial \mu}}_{+} + \alpha\underbrace{\frac{\partial \ln n^*}{\partial \mu}}_{+}. \tag{40}$$

Strengthening patent protection has the following effects on welfare. First, it decreases the production share s_x^* of human capital, which has a negative effect on welfare by reducing the initial level of consumption. Second, it increases the R&D share s_r^* of human capital, which has a positive effect on welfare by increasing the growth rate of technology. Third, it decreases human-capital investment e^* as implied by (28) giving rise to a negative effect on welfare through a lower growth rate of human capital. Finally, it increases the fertility rate n^*, which has a negative effect on welfare through a lower growth rate of human capital as well as a direct positive welfare effect. Whether the overall effect of μ on U^* is positive or negative is an empirical question that we will explore in the quantitative analysis in the next section.

As for the first-best allocation denoted by superscript **, we maximize (39) subject to (38) and obtain

$$s_x^{**} = \frac{\rho}{\xi}, \tag{41}$$

$$(s_r^{**})^{1-\phi} = \frac{\phi\varphi \ln z}{\xi}, \tag{42}$$

$$n^{**} = \frac{\alpha\rho}{1+\xi/\theta}, \tag{43}$$

$$e^{**} = 1 - \left(s_x^{**} + s_r^{**} + \frac{n^{**}}{\theta} \right). \tag{44}$$

The comparative statics with respect to the parameters are quite intuitive. Comparing (41) and (29), we find that $s_x^{**} > s_x^*$ because $s_r^* > 0$; in other words, the decentralized market allocates an insufficient share of human capital to production. Comparing (42) and (30), we find that if $\phi \ln z \geq \mu - 1$, then $s_r^{**} > s_r^*$ because $(s_r^*)^{1-\phi} < \varphi(\mu-1)/\xi$ from (30). In other words, R&D underinvestment occurs if either ϕ or z is sufficiently large. Intuitively, a larger ϕ implies a smaller degree of the negative duplication externality and a larger z implies a larger degree of the positive externality from z to technological progress g_z^* as shown in (20). Given R&D underinvestment, patent policy μ may help to mitigate this market failure. As for the comparison between n^{**} and n^*, we first note that the equilibrium fertility rate n^* is increasing in μ from Proposition 2. Then, from (32),

$$\lim_{\mu \to 1} n^* = \frac{\alpha\rho}{1+\xi/\theta}, \tag{45}$$

because $\lim_{\mu\to 1} s_r^* = 0$ from (30) and $\lim_{\mu \to 1} s_x^* = \rho/\xi$ from (29). Therefore, as μ approaches one, n^* approaches n^{**}. Given that n^* is increasing in

μ, we have $n^* > n^{**}$ for $\mu > 1$; in other words, the fertility rate chosen by households under the decentralized equilibrium is suboptimally high. Finally, as μ approaches one, we have (a) $n^* = n^{**}$, (b) $s_x^* = s_x^{**}$, and (c) $s_r^{**} > s_r^* = 0$; therefore, $\lim_{\mu \to 1} e^* > e^{**}$. As μ increases above one, e^* decreases towards e^{**}, whereas s_r^* increases towards s_r^{**}; however, s_x^* and n^* deviate from their optimal values. As μ becomes sufficiently large, e^* may fall below e^{**}, and s_r^* may rise above s_r^{**}. In the quantitative analysis, we will compute the welfare changes from increasing patent strength μ.

4.2 Quantitative analysis

In this section, we calibrate the model to examine quantitatively the effects of patent breadth on technological progress, fertility, human capital accumulation, economic growth and social welfare. In the previous section, we show that strengthening patent protection has both positive and negative effects on economic growth. In this section, we calibrate the model to examine which effect is likely to dominate.

There are nine structural parameters $\{\rho, \delta, \phi, \alpha, \theta, \varphi, \mu, z, \xi\}$ that are relevant for this numerical exercise. First, we set the discount rate ρ to a standard value of 0.04. As for the depreciation rate of human capital, Stokey and Rebelo (1995) consider a range between 3% and 8% to be reasonable for the US economy, so we set δ to an intermediate value of 0.055. As for the returns to scale in the R&D process, Kortum (1992) estimates a parameter similar to ϕ and finds that its value is 0.2; therefore, we set ϕ to 0.2.[20] We consider a range of values for the fertility-preference parameter $\alpha \in \{1, 2, 4, 8\}$. Finally, we use the following five empirical moments to pin down the values of the remaining five parameters. We consider a long-run population growth rate of 1% for the US economy, and the equilibrium condition for n^* is given by (32). As for the arrival rate of innovation, we use the estimate in Laitner and Stolyarov (2011) to set $\lambda^* = \varphi(s_r^*)^{\phi}$ to 0.17, which also takes on an intermediate value within the range considered by Acemoglu and Akcigit (2011).

[20] Jones and Williams (2000) consider a lower bound for ϕ to be about 0.5 based on empirical estimates for the social rate of return to R&D. In this study, we intentionally choose a small value for ϕ in order for TFP growth g_z not to be overly responsive to the R&D share of GDP. In our calibration, the elasticity of TFP growth with respect to the R&D share of GDP is about 0.2. If we set ϕ to a higher value of 0.5, the elasticity increases to about 0.5. However, while R&D share of GDP in the US has been steadily rising, TFP growth shows no significant upward trend.

We set the equilibrium R&D share of GDP to 0.03 for the US economy, and this share is given by $S_r^* \equiv wL_r/Y$ in the model.

$$S_r^* = \left(\frac{\mu - 1}{\mu}\right)\frac{\lambda^*}{\rho + \lambda^*}. \tag{46}$$

We set the growth rate $g_z^* = \lambda^* \ln z$ of total factor productivity (TFP) to 1% and the growth rate $g_c^* = g_z^* + g_h^*$ of consumption per capita to 2%. In other words, we consider a useful benchmark in which technological progress and human capital accumulation contribute equally to economic growth. Given a chosen value for each of $\{\rho, \delta, \phi, \alpha\}$, these five empirical moments determine the values of $\{\theta, \varphi, \mu, z, \xi\}$ respectively. The calibrated parameter values are reported in Table 1.

Table 1: Calibration

α	θ	φ	μ	z	ξ
1.0	0.048	0.443	1.038	1.061	0.145
2.0	0.026	0.465	1.038	1.061	0.185
4.0	0.018	0.500	1.038	1.061	0.266
8.0	0.014	0.550	1.038	1.061	0.427

Given these calibrated parameter values, we consider a counterfactual policy experiment by increasing patent breadth such that μ increases from 1.038 to 1.061 (i.e., patent breadth $b = \ln \mu / \ln z$ increases from 0.64 to 1.00). The numerical results are reported in Table 2. We see that S_r^* (i.e., the R&D share of GDP) increases by over one half. On the one hand, strengthening patent protection has a positive effect on technological progress. For all values of α, the arrival rate of innovation increases from 0.170 to 0.186 whereas the growth rate of technology increases from 1% to 1.095%. On the other hand, strengthening patent protection raises the fertility rate from 1% to roughly 1.003% and decreases the growth rate of human capital. The magnitude of the decrease in g_h^* depends on α and is increasing in its parameter value. For a small value of α, the positive effect of μ through technological progress dominates the negative effect through human capital accumulation giving rise to a positive overall effect on economic growth g_c^*. For a sufficiently large value of α, the negative effect of μ through g_h^* becomes quantitatively significant and may completely offset or even dominate the positive effect through g_z^* giving rise to a slightly negative overall effect on g_c^*. As for social welfare, we find that it increases and the welfare gain ΔU (expressed in terms of equivalent variation in consumption flow) is slightly over 0.5% of consumption per year. If we decompose the welfare effects according to (36), the welfare gain mostly comes from (a) a higher consumption growth rate g_c^* when α is small and (b) a higher fertility rate n^* when α is large; in all cases, the welfare gain is partially offset by a reduction in initial consumption c_0.

Table 2: Policy experiment ($\mu = 1.061$)

α	S_r^*	λ^*	g_z^*	n^*	g_h^*	g_c^*	ΔU
1.0	0.047	0.186	1.095%	1.002%	0.991%	2.085%	0.569%
2.0	0.047	0.186	1.095%	1.003%	0.978%	2.073%	0.567%
4.0	0.047	0.186	1.095%	1.003%	0.954%	2.048%	0.561%
8.0	0.047	0.186	1.095%	1.003%	0.904%	1.999%	0.551%
$\mu = 1.038$	0.030	0.170	1.000%	1.000%	1.000%	2.000%	n/a

From this quantitative analysis, we conclude that whether the positive or negative effects of patent policy on economic growth dominates depends on the empirical value of the fertility-preference parameter α. Here we consider the calibrated values of n/θ (i.e., the fraction of time spent on fertility) to narrow down the empirical range of α. Using the calibrated values of θ in Table 1, one can show that $\alpha \in \{1, 2, 4, 8\}$ corresponds to the following calibrated values of $n/\theta \in \{0.21, 0.38, 0.57, 0.73\}$. According to the American Time Use Survey from 2005 to 2009, an average person in households with youngest child under six years old spends less than 3 hours per day for child caring as a primary activity.[21] Assuming an average of 16 hours of non-sleeping time per day, the fraction of time spent on child caring in the US data is close to the lower bound of the calibrated values of n/θ implying that the empirical value of α should be reasonably small in the US. Therefore, for the US economy, the positive effect of patent policy through technological progress is likely to dominate the negative effect through human capital accumulation. However, as the strength of cultural preference for fertility increases, the negative effect of patent policy through human capital accumulation becomes quantitatively significant and offsets the positive effect through technological progress.

[21] Persons in households with older children spend even less time for child caring.

5 Conclusion

In this study, we have developed a scale-invariant quality-ladder model with endogenous fertility and human capital accumulation to analyze the effects of patent policy on economic growth. We find that although strengthening patent protection has a positive effect on technological progress, it also has a negative effect on human capital accumulation. As a result, the overall effect on economic growth is ambiguous. In the quantitative analysis, we find that the relative magnitude of these two effects depends on the empirical value of a preference parameter on fertility. Calibrating this parameter to a reasonable value for the US economy, we find that the positive effect through technological progress is likely to dominate the negative effect through human capital accumulation rendering a positive overall effect on economic growth. However, for a culture that has a stronger preference for fertility, the negative growth effect of patent policy through endogenous fertility and human capital accumulation would be quantitatively more significant. This theoretical result implies that if developing countries have a stronger preference for fertility than developed countries, then the negative effect of strengthening patent protection on economic growth would be larger in developing countries, and this implication may partially explain why some developing countries are reluctant to strengthen patent protection.[22] Therefore, an interesting direction for future research would be to empirically examine these effects across countries.

References

[1] Acemoglu, D., and Akcigit, U., 2011. Intellectual property rights policy, competition and innovation. manuscript.

[2] Aghion, P., and Howitt, P., 1992. A model of growth through creative destruction. *Econometrica* 60, 323-351.

[3] Alesina, A., and Giuliano, P., 2010. The power of the family. *Journal of Economic Growth* 15, 93-125.

[22] We would like to thank a referee for providing this interesting suggestion.

[4] Ashraf, Q., and Galor, O., 2007. Cultural assimilation, cultural diffusion and the origin of the wealth of nations. CEPR Discussion Papers No. 6444.

[5] Barro, R., and Becker, G., 1989. Fertility choice in a model of economic growth. *Econometrica* 57, 481-501.

[6] Bessen, J., and Meurer, M., 2008. *Patent Failure: How Judges, Bureaucrats, and Lawyers Put Innovators at Risk*. Princeton University Press.

[7] Boldrin, M., and Levine, D., 2008. *Against Intellectual Monopoly*. Cambridge University Press.

[8] Chu, A., 2007. Confidence-enhanced economic growth. *The B.E. Journal of Macroeconomics* (Topics), Vol. 7, Article 13.

[9] Chu, A., 2009. Effects of blocking patents on R&D: A quantitative DGE analysis. *Journal of Economic Growth* 14, 55-78.

[10] Chu, A., 2011. The welfare cost of one-size-fits-all patent protection. *Journal of Economic Dynamics and Control* 35, 876-890.

[11] Chu, A., and Cozzi, G., 2011. Cultural preference on fertility and the long-run growth effects of intellectual property rights. MPRA Papers No. 29059.

[12] Connolly, M., and Peretto, P., 2003. Industry and the family: Two engines of growth. *Journal of Economic Growth* 8, 115-148.

[13] Cozzi, G., 2001. Inventing or spying? Implications for growth. *Journal of Economic Growth* 6, 55-77.

[14] Cozzi, G., 2007. The Arrow effect under competitive R&D. *The B.E. Journal of Macroeconomics* (Contributions), Vol. 7, Article 2.

[15] Cozzi, G., Giordani, P., and Zamparelli, L., 2007. The refoundation of the symmetric equilibrium in Schumpeterian growth models. *Journal of Economic Theory* 136, 788-797.

[16] Cozzi, G., and Spinesi, L., 2006. Intellectual appropriability, product differentiation, and growth. *Macroeconomic Dynamics* 10, 39-55.

[17] Dinopoulos, E., and Thompson, P., 1998. Schumpeterian growth without scale effects. *Journal of Economic Growth* 3, 313-335.

[18] Fernandez, R., and Fogli, A., 2009. Culture: An empirical investigation of beliefs, work, and fertility. *American Economic Journal: Macroeconomics* 1, 146-177.

[19] Furukawa, Y., 2007. The protection of intellectual property rights and endogenous growth: Is stronger always better? *Journal of Economic Dynamics and Control* 31, 3644-3670.

[20] Furukawa, Y., 2010. Intellectual property protection and innovation: An inverted-U relationship. *Economics Letters* 109, 99–101.

[21] Futagami, K., and Iwaisako, T., 2007. Dynamic analysis of patent policy in an endogenous growth model. *Journal of Economic Theory* 132, 306-334.

[22] Galor, O., and Moav, O., 2002. Natural selection and the origin of economic growth. *Quarterly Journal of Economics* 117, 1133-1192.

[23] Grossman, G., and Helpman, E., 1991. Quality ladders in the theory of growth. *Review of Economic Studies* 58, 43-61.

[24] Growiec, J., 2006. Fertility choice and semi-endogenous growth: Where Becker meets Jones. *The B.E. Journals of Macroeconomics* (Topics), Vol. 6, Article 10.

[25] Ha, J., and Howitt, P., 2007. Accounting for trends in productivity and R&D: A Schumpeterian critique of semi-endogenous growth theory. *Journal of Money, Credit and Banking* 39, 733–774.

[26] Horii, R., and Iwaisako, T., 2007. Economic growth with imperfect protection of intellectual property rights. *Journal of Economics* 90, 45-85.

[27] Horowitz, A., and Lai, E., 1996. Patent length and the rate of innovation. *International Economic Review* 37, 785-801.

[28] Iwaisako, T., and Futagami, K., 2003. Patent policy in an endogenous growth model. *Journal of Economics* 78, 239-258.

[29] Jaffe, A., and Lerner, J., 2004. *Innovation and Its Discontents: How Our Broken System is Endangering Innovation and Progress, and What to Do About it*. Princeton University Press.

[30] Jones, C., 1995. R&D-based models of economic growth. *Journal of Political Economy* 103, 759-784.

[31] Jones, C., 1999. Growth: With or without scale effects. *American Economic Review* 89, 139-144.

[32] Jones, C., 2001. Was an industrial revolution inevitable? Economic growth over the very long run. *The B.E. Journals of Macroeconomics* (Advances), Vol. 1, Article 1.

[33] Jones, C., 2003. Population and ideas: A theory of endogenous growth. In Aghion, P., Frydman, R., Stiglitz, J., and Woodford, M., (eds.) *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*. Princeton University Press.

[34] Jones, C., and Williams, J., 2000. Too much of a good thing? The economics of investment in R&D. *Journal of Economic Growth* 5, 65-85.

[35] Judd, K., 1985. On the performance of patents. *Econometrica* 53, 567-586.

[36] Kortum, S., 1992. Inventions, R&D and industry growth. Ph.D. Dissertation, Yale University.

[37] Kortum, S., 1997. Research, patenting, and technological change. *Econometrica* 65, 1389-1419.

[38] Laincz, C., and Peretto, P., 2006. Scale effects in endogenous growth theory: An error of aggregation not specification. *Journal of Economic Growth* 11, 263-288.

[39] Laitner, J., and Stolyarov, D., 2011. Derivative ideas and the value of intangible assets. manuscript.

[40] Li, C.-W., 2001. On the policy implications of endogenous technological progress. *Economic Journal* 111, C164-C179.

[41] Nordhaus, W., 1969. *Invention, Growth, and Welfare: A Theoretical Treatment of Technological Change.* The MIT Press.

[42] O'Donoghue, T., and Zweimuller, J., 2004. Patents in a model of endogenous growth. *Journal of Economic Growth* 9, 81-123.

[43] Peretto, P., 1998. Technological change and population growth. *Journal of Economic Growth* 3, 283-311.

[44] Peretto, P., 1999. Cost reduction, entry, and the interdependence of market structure and economic growth. *Journal of Monetary Economics* 43, 173-195.

[45] Razin, A., and Ben-Zion, U., 1975. An intergenerational model of population growth. *American Economic Review* 65, 923-933.

[46] Scotchmer, S., 2004. *Innovation and Incentives.* The MIT Press.

[47] Segerstrom, P., 1998. Endogenous growth without scale effects. *American Economic Review* 88, 1290-1310.

[48] Stokey, N., and Rebelo, S., 1995. Growth effects of flat-rate taxes. *Journal of Political Economy* 103, 619-550.

[49] Strulik, H., 2005. The role of human capital and population growth in R&D-based models of economic growth. *Review of International Economics* 13, 129-145.

[50] Tabellini, G., 2010. Culture and institutions: Economic development in the regions of Europe. *Journal of the European Economic Association* 8, 677-716.

[51] Yip, C., and Zhang, J., 1997. A simple endogenous growth model with endogenous fertility: Indeterminacy and uniqueness. *Journal of Population Economics* 10, 97-110.

[52] Young, A., 1998. Growth without scale effects. *Journal of Political Economy* 106, 41-63.

Appendix A: Proof of Proposition 1

In this proof, we first derive an autonomous system of two differential equations in $s_{r,t}$ and $s_{x,t}$. Then, we show that this dynamic system is characterized by global instability. Because $s_{r,t}$ and $s_{x,t}$ are jump variables, they must jump to the steady state. Finally, we show that stationary s_r^* and s_x^* imply a stationary fertility rate n^*, which in turn implies a stationary e_t^*/h_t^*.

Taking the log of $Y_t = c_t N_t$ and $Y_t = Z_t L_{x,t}$ and then differentiating with respect to t yield

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{c}_t}{c_t} + n_t = \frac{\dot{Z}_t}{Z_t} + \frac{\dot{L}_{x,t}}{L_{x,t}}. \tag{A1}$$

Combining (A1) and (21) yields

$$\frac{\dot{L}_{x,t}}{L_{x,t}} = \xi\left(1 - \frac{n_t}{\theta}\right) - \rho - \delta. \tag{A2}$$

Taking the log of $s_{x,t} \equiv L_{x,t}/(h_t N_t)$ and differentiating with respect to t yield

$$\frac{\dot{s}_{x,t}}{s_{x,t}} = \frac{\dot{L}_{x,t}}{L_{x,t}} - \frac{\dot{h}_t}{h_t} - n_t. \tag{A3}$$

Substituting (A2) and (4) into (A3) yields

$$\frac{\dot{s}_{x,t}}{s_{x,t}} = \xi\left(1 - \frac{n_t}{\theta} - \frac{e_t}{h_t}\right) - \rho. \tag{A4}$$

Substituting (3) and $l_t = (s_{r,t} + s_{x,t})h_t$ into (A4) yields

$$\dot{s}_{x,t} = s_{x,t}\left[\xi(s_{r,t} + s_{x,t}) - \rho\right]. \tag{A5}$$

To derive our second differential equation, we first rewrite (25) as

$$\frac{s_{r,t}}{s_{x,t}} = \mu\frac{v_t \lambda_t}{Y_t}, \tag{A6}$$

where $\lambda_t = \varphi s_{r,t}^{\phi}$. Differentiating the log of (A6) with respect to t yields

$$(1-\phi)\frac{\dot{s}_{r,t}}{s_{r,t}} = \frac{\dot{s}_{x,t}}{s_{x,t}} + \frac{\dot{v}_t}{v_t} - \frac{\dot{Y}_t}{Y_t}. \tag{A7}$$

Substituting (14) into (A7) yields

$$(1-\phi)\frac{\dot{s}_{r,t}}{s_{r,t}} = \frac{\dot{s}_{x,t}}{s_{x,t}} + r_t + \lambda_t - \frac{\pi_t}{v_t} - \frac{\dot{Y}_t}{Y_t}. \tag{A8}$$

Substituting (5) and the first equality of (A1) into (A8) yields

$$(1-\phi)\frac{\dot{s}_{r,t}}{s_{r,t}} = \frac{\dot{s}_{x,t}}{s_{x,t}} + \rho + \lambda_t - \frac{\pi_t}{v_t}. \tag{A9}$$

Substituting (12) and (A6) into (A9) yields

$$(1-\phi)\frac{\dot{s}_{r,t}}{s_{r,t}} = \frac{\dot{s}_{x,t}}{s_{x,t}} + \rho + \lambda_t \left[1 - (\mu-1)\frac{s_{x,t}}{s_{r,t}}\right], \tag{A10}$$

Finally, we substitute (A5) and $\lambda_t = \varphi s_{r,t}^{\phi}$ into (A10) to obtain

$$\dot{s}_{r,t} = \frac{s_{r,t}}{1-\phi}\left[\xi(s_{r,t}+s_{x,t}) + \varphi s_{r,t}^{\phi} - \varphi(\mu-1)s_{x,t}/s_{r,t}^{1-\phi}\right]. \tag{A11}$$

Differential equations (A5) and (A11) describe the dynamics of $(s_{x,t}, s_{r,t})$.

Next we draw a phase diagram based on the system of differential equations in (A5) and (A11). From (A5), the $\dot{s}_{x,t} = 0$ locus in R^2_{++} is given by

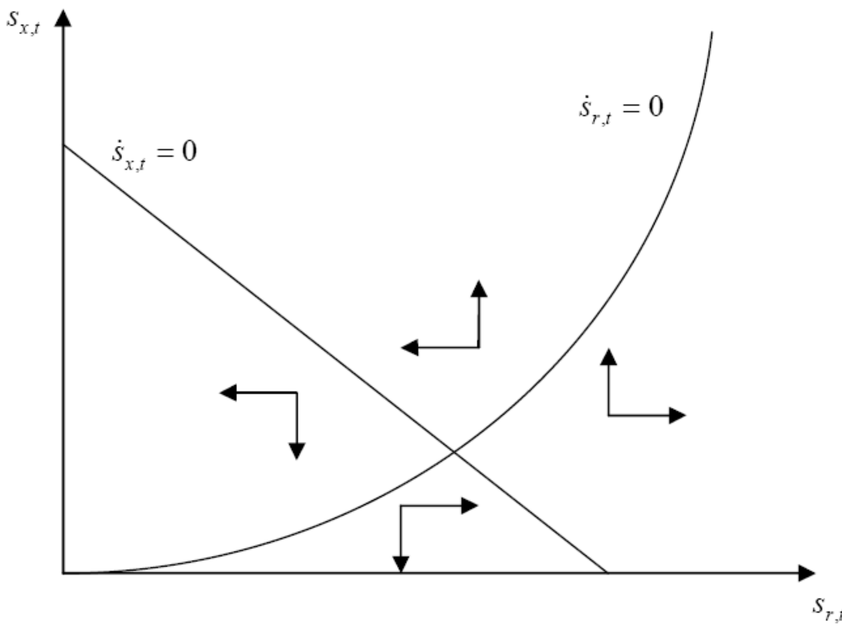
$$s_{x,t} = \rho/\xi - s_{r,t}, \tag{A12}$$

which describes a negative relationship between $s_{x,t}$ and $s_{r,t}$. From (A11), the $\dot{s}_{r,t} = 0$ locus in R^2_{++} is given by

$$s_{x,t} = \frac{\xi s_{r,t} + \varphi s_{r,t}^{\phi}}{\varphi(\mu-1)/s_{r,t}^{1-\phi} - \xi}, \tag{A13}$$

for $s_{r,t}^{1-\phi} < \varphi(\mu-1)/\xi$. Equation (A13) describes a positive relationship between $s_{x,t}$ and $s_{r,t}$. As $s_{r,t}^{1-\phi}$ approaches $\varphi(\mu-1)/\xi$, $s_{x,t}$ goes to infinity. Equation (30) implies that $(s_r^*)^{1-\phi} < \varphi(\mu-1)/\xi$. The phase diagram is plotted in Figure 1 along with the dynamics.

Figure 1: Phase diagram



The dynamical system is characterized by global instability in R^2_{++}, so that the jump variables $s_{x,t}$ and $s_{r,t}$ shall immediately start from their steady state values. More specifically, the Jacobian of the differential-equation system (A5) and (A11), computed at the interior steady state $(s_x^*, s_r^*) \in R^2_{++}$, is:

$$\begin{pmatrix} \xi s_x^*, & \xi s_x^* \\ \frac{s_r^*}{1-\phi}\left[\xi - \varphi(\mu-1)/(s_r^*)^{1-\phi}\right], & \frac{s_r^*}{1-\phi}\left[\xi + \varphi\phi(s_r^*)^{\phi-1} + (1-\phi)(\mu-1)s_x^*/(s_r^*)^{2-\phi}\right] \end{pmatrix},$$

with determinant equal to

$$\frac{\xi s_x^* s_r^*}{1-\phi}\left[\frac{\varphi\phi}{(s_r^*)^{1-\phi}} + \frac{(1-\phi)(\mu-1)s_x^*}{(s_r^*)^{2-\phi}} + \frac{\varphi(\mu-1)}{(s_r^*)^{1-\phi}}\right] > 0.$$

Since both determinant and trace are positive, the differential-equation system (A5) and (A11) is indeed locally unstable.

Next we use (6) to show that the stationarity of $s_{x,t} = s_x^*$ and $s_{r,t} = s_r^*$ implies n_t is also stationary. Equation (13) and $Y_t = c_t N_t$ imply that $w_t h_t / c_t = 1/(\mu s_x^*)$. We apply $Y_t = c_t N_t$ and $a_t = v_t / N_t$ to derive $a_t / c_t = v_t / Y_t = (s_r^*)^{1-\phi} / (\mu \varphi s_x^*)$, where the second equality follows from (A6). Therefore, we can express (6) as

$$\frac{\alpha}{n_t} = \frac{1}{\mu s_x^*} \left(\frac{(s_r^*)^{1-\phi}}{\varphi} + \frac{1}{\theta} + \frac{1}{\xi} \right), \tag{A14}$$

which implies $n_t = n^*$. Finally, from (3) and $l_t = (s_r^* + s_x^*) h_t$, the stationarity of $n_t = n^*$ implies that e_t / h_t must be stationary as well. Q.E.D.