Second-degree price discrimination and universal access under (weighted average) price cap regulation

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Abstract

This paper analyzes the efficiency of the Price Cap regulatory scheme and its impact on universal access, when the monopolist is allowed to set a menu of alternative plans as part of a self-selection strategy (second-degree price discrimination) and the cap is calculated as the weighted average of the tariff plans he offers.

In this context, we characterized the solution of the monopolist; who, besides offering a menu of plans more distorted than the second-best outcome –even distorting the plan for high-valuation consumers– because the trade-off between efficiency and rents extraction is exhibited in different way, can exclude consumers who are willing to pay for the service –shutdown policy– despite of universal access obligation have been imposed by the regulator. This happens regardless of the weighted average price cap (WAPC) set by the regulator.

Consequently, if the Price Cap regulatory scheme is going to be used by the regulator, this mechanism must be applied to a single regulated plan, leaving the monopolist some flexibility to offer alternative plans that will be incentive compatible with this single regulated plan (tariff flexibility).

KEY WORDS: *price cap; non-linear tariffs; price discrimination; tariff flexibility; universal access.*

CLASSIFICATION JEL: L51; D52; L96

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1 Introduction

In several regulated industries –like telecommunications, electricity and gas–, regulators have opted to use the Price Cap regulatory scheme[1] at the expense of other schemes (e.g. rate of return) because of the incentives that this mechanism generates, mainly in terms of encouraging productive efficiency without discouraging investment[2]. However, industries regulated by Price Caps also have other characteristics: most regulated firms are multi-product, price-discriminating and, are also subjected to an universal access policy imposed by the regulator[3].

The regulation by Price Caps of multi-product firms, since the regulator has limited information about costs and/or demand for each one of the regulated services, is applied to a "basket" of services rather than to each of them; i.e. the regulator regulates a price index of the basket of services or a weighted average price. Note that the services of a basket can belong to perfectly separated markets (the monopolist could price discriminate in third-degree) or, to different market segments because the existence of different types of consumers (he could price discriminate in second-degree)[4]. On the other hand, the universal access policy is intrinsically linked to the tariff policy that the regulator implements. That is, an inadequate pricing by the regulator or the firm, with for example high fixed tariffs, may lead to non-participation of low-income consumers who had accessed the service if the tariffs had been properly set (i.e. they have willingness to pay for the service).

For purposes of this paper, we incorporate some of these characteristics (that have not been included in previous studies) in the efficiency analysis of the Price Cap regulatory scheme. In particular, we will assume a monopolist that can price discriminate in second-degree (this is the reason of defining a basket of services), but also faces an universal access obligation.[5]

[1] Also known as RPI - X regulation.

[2] Models studying this topic are conducted under the typical problem of asymmetric information about costs and/or demand between the regulator and the regulated firm (see Baron and Myerson, 1982). However, in this paper we will assume that no such asymmetric information exists.

[3] According to Li Ning and Willington (2009), universal access has 2 dimensions: (i) one related to willingness to pay of low-income consumers in service's coverage areas, and (ii) the other is the lack of coverage service in rural areas because of low demand and high costs.

In this paper, when referring to "universal access" we refer to (i). Inclusion of consumers related to (ii) will depend on how feasible is the implementation of a subsidy (permanently or temporarily) by the Government. Most countries have established funds, with contributions from the regulated firms themselves, to finance projects for low-income consumers.

[4] In this issue, Bertoletti and Poletti (1997) show that third-degree price discrimination generates Pareto improves when two-part tariffs are regulated, in comparison with the prohibition of any tariff differentiation. On the other hand, Vogelsang (1990) and, Li Ning and Willington (2009) show the same, although when the firm can price discriminate in second-degree.

[5] Without loss of generality, in this paper we refer to the telecommunications industry.

It should be noted that, when a firm is allowed to price discriminate in second-degree, the regulator sets the cap as the weighted average of all plans –each one corresponding to each consumer type– offered by the monopolist, which will be adjusted over time according to the productivity factor (X-factor) and variations in the index of input prices (RPI). The monopolist has some flexibility to offer a menu of plans that fulfill with the following characteristics: (i) allows different types of consumers self-select, and (ii) satisfies the cap set by the regulator. This cap setting and its application is defined as the Weighted Average Price Cap (WAPC) regulatory scheme.

In analyzing the efficiency of the WAPC regulatory scheme, we do not focus on incentives to productive efficiency, but in the existence of incentives to achieve allocative-efficient solutions. According to the economic literature, achieving allocative efficiency –when a basket of services is regulated by Price Cap– is possible under certain conditions. In relation to this issue, Bradley and Price (1988) and Vogelsang (1988) demonstrate that, when there are no changes in demand and costs, regulation by the WAPC leads to an iterative process that converges to a Ramsey pricing structure[6], whose speed of adjustment will depend not only of the X-factor but also the weights used at the beginning of the regulatory process. On the other hand, Vogelsang (1990)[7] shows that if the regulated firm offers two-part tariffs as an option, in addition to the initial tariff which should always offer (the constraint faced by the regulated firm), Pareto optimal outcomes can be achieved; however to this happen, the initial price vector (which always has to offer the regulated firm) must be properly regulated[8]. In short, these authors consider that the initial tariff regulation is important for the regulator, not only to induce productive efficiency[9] but also allocative efficiency or Pareto solutions[10].

Although all models mentioned above reach efficient outcomes when the regulator grants the regulated firm the possibility to set his own prices (also called price delegation), none of them shows the inefficiencies that could arise when the monopolist can price discriminate in second-degree and, simultaneously, is regulated by the WAPC regulatory scheme (a menu

[6]Note that Ramsey prices can be exclusionary, which contradicts our assumption that the regulator is interested in the universal access problem.

[7]Following the Incremental Surplus Scheme (ISS) due to Sappington and Sibley (1988), and the Sibley's ISS-R scheme (1989).

[8]For the initial tariff setting, Vogelsang (1990) proposed the rate of return regulation. However, firm-efficient regulation can also be used.

[9]Bernstein et al. (2006) mentioned that if the initial regulated pricing makes profits equal to zero –i.e. if the regulator ensures that the initial regulation of prices are oriented at cost–, with the application of the RPI-X factor, the monopolist will continue make zero profits.

[10]Note that for the case of Vogelsang (1990), the regulator should set tariffs for each type of consumer on the market, which proved to be an impractical solution just by the information asymmetries that held the regulator about the types of customers (only knows the distribution of them).

of plans from which the regulator validates the fulfillment of the WAPC)[11]. Therefore, in this context and assuming that there is no information asymmetry between the regulator and the regulated firm about costs and/or demand for the service, as shown below, the WAPC regulatory scheme would encourage the monopolist to offer socially inefficient plans, even more distorted than the second-best solution (allocative inefficiency). This happens because the monopolist, considering the WAPC constraint on its problem, can not raise the tariff of the high-valuation consumer's plan. The latter encourages the monopolist to extract informational rents from high-valuation consumers in another way: by reducing the number of minutes included in the high-valuation consumer's plan and shutting-down the low-valuation consumers. It must be emphasized that these distortions occur despite the initial setting of the weighted average price is cost oriented and the imposition of an universal access obligation.

The peruvian experience

In Peru, since september 2001, the tariffs for end users of fixed telephone service are regulated by Price Cap, specifically by the Tariff Regime of Weighted Average Price Cap. In particular, the regulator (the Supervisory Agency for Private Investment in Telecommunications - OSIPTEL) is the only responsible for calculating the WAPC, which is reduced according to the RPI-X factor; while the regulated firm sets the differentiated tariff plans so that end users self-select and, simultaneously, fulfills the WAPC set by the regulator.

The WAPC, according to the Concession Contract of Telefonica del Peru (TdP), is calculated for each basket of services offered by TdP. Calls from fixed-line subscribers are within the so-called "Basket D", whose tariff components are: the fixed monthly tariff (may include free minutes or not) and the per-minute tariff (for additional minute)[12]. Thus, each tariff component is considered as a service provided by the TdP, which weighted by the share of each service's revenue of the previous period on the basket's revenue, we obtain the WAPC[13]. It should be noted that each different tariff plan offered by TdP are also part of the basket

[11] There are only papers that studies the welfare effects of the thrid-degree price discrimination of a monopoly regulated by price cap (see Armstrong and Vickers, 1991; and Bertoletti and Poletti, 1997).

[12] The Concession Contract of TdP establishes two additional baskets of fixed-line services:

- Basket C: includes only the installation charge.
- Basket E: includes domestic and international long distance calls.

[13] In the TdP's contract, the WAPC is calculated from the following formula:

$$TT_{jn} = \sum T_{ijn-1} \left(\alpha_{ijn-1} \times \frac{T_{ijn}}{T_{ijn-1}} \right)$$

subject to:

D[14].

As you can see, those differentiated plans are three-part tariffs: the consumer pays a fixed fee for the right to access the service plus a certain amount of minutes (free minutes), and additionally pay a tariff for additional minute or excess beyond the free minutes. In this regard, Bagh and Bhargava (2006) and Lambrecht et al. (2007) show that firms will have incentives to offer plans with three-part tariffs if there is uncertainty about the consumers' future demand, which ultimately leads to a decrease in consumer surplus and an increase in firm's revenue. However, in this paper we will assume that no exist such information asymmetry and hence the monopolist only offers two-part tariffs or completely non-linear tariffs[15].

Furthermore, the application of the Price Cap regulatory scheme as mentioned in the Concession Contract of TdP, besides generates efficiency problems, also encourages the regulated firm to behave in some opportunistic way in the application of the RPI-X factor. By regulating a weighted average price the regulated firm has some freedom to "re-adjust" any of the tariff components (or services) and thus fulfill with the WAPC reductions resulting from the application of the RPI-X factor. This means that, reductions in the WAPC does not necessarily imply tariff reductions in all plans. For example, the regulated firm could decide to fulfill with the WAPC reductions, only reducing the per-minute tariff of some high-consumption plans[16] and, no reductions in the monthly fixed tariff of high-consumption plans and/or any of the tariffs for low-consumption plans. This avoidance behavior by the

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$$RT_{jn} = \sum \left(\alpha_{ijn-1} \times \frac{T_{ijn}}{T_{ijn-1}} \right) \leq F_n$$

where,
TT_{jn} = Price Cap of basket "j" during the quarter "n".
RT_{jn} = Ratio Ceiling of basket "j" during the quarter "n".
α_{ijn-1} = Weighting factor of service "i" that belongs to basket "j" during the previous quarter, given the share the service's revenue "i" on the basket's revenue "j".
T_{ijn} = Tariff of service "i" that belongs to basket "j" during the current quarter.
T_{ijn-1} = Tariff of service "i" that belongs to basket "j" during the previous quarter.
F_n = Control Factor for quarter "n"

$$F_n = (1 - X) \times \frac{IPC_{n-1}}{IPC_{n-2}}$$

IPC_n = Consumer Price Index of Metropolitan Lima for quarter "n".
X = Quarterly Productivity Factor.

[14] For example, for a firm offering 2 tariff plans, the weighted average price is calculated from 4 tariff components or services (2 fixed monthly fees and 2 per-minute charges).

[15] A natural extension of this model is the incorporation of this information asymmetry.

[16] The weights of the per-minute tariff of high-consumption plans might be high enough to fulfill the overall reduction of the WAPC.

regulated firm could increase the universal access problem.

Therefore, this paper attempt to demonstrate that in a context of WAPC regulation and second-degree price discrimination, allocative inefficiencies and some kind of monopolist's avoidance behavior arise. In the presence of these problems, it is recommended to design a regulatory scheme involving the regulation by Price Cap of a single plan (e.g. the "Classic Line" plan[17]), excepting that this regulated plan must always be offered by the monopolist as part of his menu of plans[18]. This will provide some flexibility to the monopolist to offer alternative plans but with the caveat that be incentive compatible with the regulated plan[19], thereby avoiding losses in consumer welfare.

The rest of the paper is structured as follows. In the second part, the main assumptions are made and two models are developed. The first one characterizes the solution to be reached if there were a social planner, who, not having asymmetric information about costs and/or demand and only knows the distribution of consumers types, may set socially optimal prices with zero profits for the monopolist. This model will be used as benchmark to compare efficiency gains. The second model presents the problem of the monopolist, who besides being able to price discriminate in second-degree, has to face the WAPC constraint. The inclusion of this restriction in the monopolist's optimization problem generates plans more distorted than the second-best outcome, even distorting the high-valuation consumer's plan. Finally, the third part presents some conclusions. The formal proofs are presented in the appendix.

2 The Models

Since we are interested in analyzing the efficiency level of the WAPC regulatory scheme, as mentioned above, it is assumed that the regulator has complete information about the monopoly's operating costs and/or total demand, but imperfect with respect to the types of consumers (both the regulator and the regulated firm only know the distribution of consumers types). In this regard, we will use the Li Ning and Willington (2009) basic model.

[17] In the case of TdP, the regulated plan would involve the regulation of one monthly fixed fee and one additional per-minute tariff, which also have to meet a minimum of quality requirements, among others.

[18] The restriction that the monopolist always has to offer the regulated plan jointly with other alternative plans, is very similar to the restriction imposed by Vogelsang (1990), in the sense that the regulated firm must always offer the initial tariff.

[19] The regulation of a single plan and the flexibility the monopolist has to offer alternative plans is defined as "tariff flexibility". See Li Ning and Willington (2009).

Consumers

It is assumed that there are two types of consumers: one of high valuation (h) and another of low valuation (l). The utility of being connected is u_i and consuming m minutes is $v_i(m)$, for $i = h, l$. The proportion of high-valuation consumers is α and low-valuation is β, where $\alpha + \beta = 1$.

For simplicity, we assume that both types of consumers have no uncertainty about their future demand and, thus pay a total transfer T_i, for $i = h, l$, that include the right to access the network and an amount of minutes[20]. Accordingly, the net utility of consuming m minutes is $u_i + v_i(m_i) - T_i(m_i)$, for $i = h, l$.

Additionally, it is assumed that v_i is differentiable, increasing and strictly concave. Also, it is assumed that $v_h'(m) > v_l'(m)$, $\forall m > 0$ and $\lim_{m_i \to \infty} v_i'(m_i) = 0$, for $i = h, l$.

The Firm

The firm's revenues come from the consumers' payments (T_h, T_l). The cost function is:

$$C(m) = A + g(m)$$

where A is the network sunk fixed cost and $g(m)$ is the variable cost that depends on the amount of minutes produced. It is assumed that $g'(m) > 0$ and $g''(m) = 0$.

In this context, the efficient or first-best quantities are defined as: $m_h^* \equiv \{m_h : v_h'(m_h) = g'\}$ and $m_l^* \equiv \{m_l : v_l'(m_l) = g'\}$; while the second-best quantities when the monopolist can price discriminate in second-degree is: $m_l^{SB} \equiv \{m_l : v_l'(m_l) = g'(.) + \alpha\, [v_h'(m_l) - g'(.)]\}$.

The Regulator

The regulator's objective is to maximize the consumers' surplus. To do it, he is responsible to: (i) determining the weighted average price cap ($T^R \geq \theta T_h + (1-\theta) T_l$) that the regulated firm must fulfill when offers alternative plans, where $\theta \in (0, 1)$ is the weighting of the high-

[20] With the non-existence of such uncertainty, a plan with a three-part tariff (or an additional per-minute tariff) does not hold (see Lambrecht, Seim and skier, 2007). In terms of the plans currently offered by TdP, it would imply that each plan would have a single tariff component.

valuation consumer's plan[21,22]; and (ii) calculating and applying the RPI-X factor which reduce the cap over time. Recall that in this paper we only analyze the (allocative) efficiency of using a WAPC scheme to regulate the monopolist's market power.

Since it is assumed that the regulator has complete information about the costs and/or demand, he can determine the socially optimal tariffs for the different types of consumers; i.e. the regulator can set a menu $\left\{\left(T_l^R, m_l^R\right), \left(T_h^R, m_h^R\right)\right\}$ such that consumers self-select (only knows the distribution of consumer types) and the monopoly does not make losses. From the socially optimal menu, the regulator could determine the socially optimal WAPC:

$$T^R = \theta T_h^R + (1-\theta) T_l^R \tag{PC}$$

such that the regulated firm always makes zero profits in each application of the RPI-X factor[23].

Moreover, because the regulator promotes a universal access policy, with the regulated menu $\left\{\left(T_l^R, m_l^R\right), \left(T_h^R, m_h^R\right)\right\}$ ensures the low-valuation consumer participation; i.e. we assume that holds:

$$u_l + v_l(m_l) - T_l \geq U_l \tag{UA}$$

where U_l is the reservation utility of such consumer[24].

Timing

1. Regulator sets the weighted average price cap (T^R).

2. Monopolist offers alternative plans: $\{(T_h,\ m_h),\ (T_l,\ m_l)\}$ such that fulfills with the weighted average price cap (T^R) set by the regulator.

[21] It is noteworthy that in regulated industries, θ is defined as the share of the revenue that represents the high-valuation consumer's plan on the monopolist's total revenue. However, for its calculation the regulator uses information from immediately preceding periods. This is why θ is considered as a parameter in this paper.

[22] $\theta \in (0,1)$ means we are assuming that, in previous periods, there were low-valuation consumers willing to pay for the service. According to Rodríguez (2004), in periods prior to the privatization of TdP (and therefore before the application of the RPI-X factor), 22.3% of consumers who had no fixed-telephone at home in 2003, once had access to it; being the main cause of service cancellation the inability to pay a higher fixed monthly tariff.

[23] This is consistent with the economic literature regarding the initial tariff setting, before the first application of the RPI-X factor, must be regulated to costs (monopolist break-even) to ensure that the Price Cap regulation be allocative and productively efficient.

[24] For the plan $\left\{\left(T_l^R, m_l^R\right), \left(T_h^R, m_h^R\right)\right\}$ ensures the low-valuation consumer participation, we assume that the fixed costs –A– are not as high and the regulator finds socially profitable offer a plan to such consumer. Otherwise, the solution would be trivial.

3. Consumers choose a plan and consume.

2.1 Central Planner's Model: The socially optimal solution

As mentioned above, on the assumption of complete information about the monopoly's costs and/or demand for the service, the regulator can set a menu of socially optimal plans $\left\{\left(T_l^R, m_l^R\right), \left(T_h^R, m_h^R\right)\right\}$ such that consumers self-select and the monopolist makes zero profits. This menu of regulated plans will serve as a benchmark, in terms of allocative efficiency achieved, when we will discuss the WAPC regulatory scheme.

The regulator's problem is:

$$\underset{\{T_h^R, T_l^R, m_h^R, m_l^R\}}{Max} \quad \alpha[u_h + v_h(m_h^R) - T_h^R] + \beta[u_l + v_l(m_l^R) - T_l^R] \tag{PR}$$

subject to:

$$\begin{aligned}
\alpha T_h^R + \beta T_l^R - A - g(\alpha m_h^R + \beta m_l^R) &\geq 0 && \text{(R1)}\\
u_h + v_h(m_h^R) - T_h^R &\geq U_h && \text{(R2)}\\
u_l + v_l(m_l^R) - T_l^R &\geq U_l && \text{(R3)}\\
u_l + v_l(m_l^R) - T_l^R &\geq u_l + v_l(m_h^R) - T_h^R && \text{(R4)}\\
u_h + v_h(m_h^R) - T_h^R &\geq u_h + v_h(m_l^R) - T_l^R && \text{(R5)}\\
T_h^R, T_l^R, m_h^R, m_l^R &\geq 0
\end{aligned}$$

Note that (R2) is not binding and is always satisfied (by assumption). The solution to the regulator's problem is summarized in the following proposition.

Proposition 1 *: If the regulator is interested in universal access, (R3) is active and the solution to (PR) is:* $m_l^{SB} < m_l^R < m_l^*$ *and* $m_h^R = m_h^*$*. The payments* $(T_l^R,\ T_h^R)$ *are such that (R1) and (R5) are binding.*

Proof. See appendix. ∎

Proposition 1 shows that, if the regulator had the same information handled by the monopolist and set the menu of alternative plans instead the monopolist, he would set less distorted plans than those plans that the monopolist would set. In the typical screening problem, the monopolist faces the typical trade-off between efficiency and informational rents extraction, having incentives to distort the plan for low-valuation consumers with the purpose of extracting the informational rents from the high-valuation consumers. The monopolist obtains

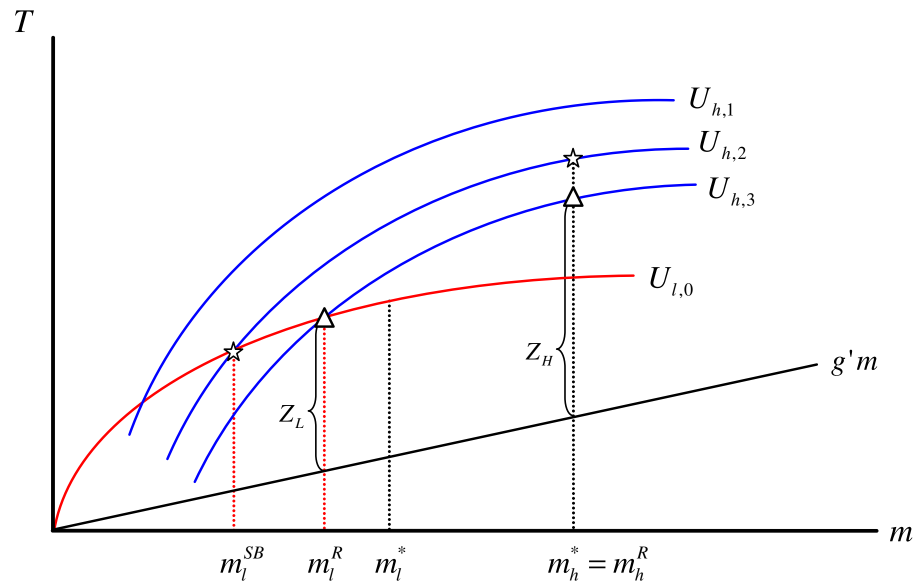

Figure 1: Socially optimal regulation

extranormal profits in detriment of the surplus of high-valuation consumers (recall that low-valuation consumer always participates and reaches his reservation utility, U_l). Therefore, if the regulator had the ability to set the monopolist's menu of plans (because he knows the costs and/or demand), then he would set plans that could avoid any informational rents extraction (which are passed on to consumers) and the corresponding loses in efficiency. The gains in social welfare when the regulator sets the menu of plans (instead of the monopolist) are derived from Corollary 2.

Corollary 2 *: The monopolist's profits in the second-best solution are greater than zero.*

Proof. See appendix. ■

By concavity of the monopolist's profit function, there is a unique plan to the right of m_l^{SB} such that the monopolist self-finance ($\pi = 0$), the low-valuation consumer keeps his reservation utility, U_l, and the high-valuation consumer reaches a lower indifference curve (higher utility level)[25], providing unambiguous gains in social welfare with the regulated plan. Figure 1 shows the results mentioned above, where $\alpha Z_H + \beta Z_L = A$.

Consequently, considering the results above, the regulator has the task of establishing a regulatory scheme that not only encourages the monopolist to be productively efficient, but

[25] The redistribution of rents to consumers could also benefit low-valuation consumers, reaching a lower indifference curve (higher utility level). However, for simplicity it has been assumed that the low-valuation consumer will always be on his reservation utility, U_l.

also to offer alternative plans that generates, at least, a similar level of allocative efficiency than the obtained in proposition 1. However, as will be shown in the next section, this will be impossible to achieve if the regulator uses the WAPC regulatory scheme.

2.2 Weighted Average Price Cap Model

The WAPC regulatory scheme is characterized because the regulator, rather than establish a menu of regulated plans, delegates the pricing to the monopolist (he has better information for it). However, the regulator is responsible for setting the cap as a weighted average price (T^R), which will be reduced according to the RPI-X factor (also calculated by the regulator). Additionally, the regulator maintains a universal access policy.

The monopolist will price discriminate in second-degree in order to maximize his profits. This imply the design of a menu of alternative plans $\{(T_l, m_l), (T_h, m_h)\}$ such that encourages consumers to reveal their type and, simultaneously, fulfill with the weighted average price cap set by the regulator; i.e. carry out $\theta T_h + (1-\theta)T_l \leq T^R$.

The monopolist's problem is defined as:

$$\underset{\{T_h, T_l, m_h, m_l\}}{Max} \; \alpha T_h + \beta T_l - A - g(\alpha m_h + \beta m_l) \tag{PM}$$

subject to:

$$\begin{aligned}
u_h + v_h(m_h) - T_h &\geq U_h && \text{(R2')}\\
u_l + v_l(m_l) - T_l &\geq U_l && \text{(R3')}\\
u_l + v_l(m_l) - T_l &\geq u_l + v_l(m_h) - T_h && \text{(R4')}\\
u_h + v_h(m_h) - T_h &\geq u_h + v_h(m_l) - T_l && \text{(R5')}\\
\theta T_h + (1-\theta)T_l &\leq T^R && \text{(R6)}\\
T_h^R, T_l^R, m_h^R, m_l^R &\geq 0 &&
\end{aligned}$$

Note that (R2') is not binding and is always satisfied (by assumption).

It is noteworthy that, if the WAPC constraint (R6) is not included, the monopolist's problem above is reduced to the simple screening problem, in which the monopolist does not know the types of consumers and offers a menu of plans such that consumers self-select. Obviously, the solution to this screening problem is the second best, which was introduced in the assumptions of the monopolist.

The solution to the monopolist's problem is summarized in the following proposition.

Proposition 3 *: Under the WAPC regulatory scheme, if the regulator imposes an universal access policy and the monopolist's fixed costs (A) are not as high, then (R3'), (R5') and (R6) are actives and the solution of (PM) is:* $m_{l2} < m_l^{SB}$ *and* $m_{h2} < m_h^*$*. The payments* (T_{l2}, T_{h2}) *are such that (R3') and (R6) are binding.*

Proof. See appendix. ■

Proposition 3 tell us that, under the WAPC regulatory scheme, the monopolist still has incentives to extract informational rents from high-valuation consumers, inducing inefficient allocation results. By delegating the setting of the menu of plans to the monopolist, in the classical screening model he will maximize his profits by setting plans equal to the second-best outcome $\{(T_l^{SB}, m_l^{SB}), (T_h^{SB}, m_h^*)\}$; i.e. he will face the typical trade-off between efficiency (distorting m_l and T_l) and rent extraction (increasing T_h but with the same m_h^*).

Now, when the WAPC constraint is imposed, the monopolist may continue extracting informational rents, although not by increasing T_h over the second best –T_h^{SB}– but by reducing m_h; i.e. the trade-off between efficiency and rent extraction is presented in different way: the distortion of the low-valuation consumer's plan (on his reservation utility –U_l–) is accompanied by a distortion of m_h from the first best –m_h^*–. If this distortion does not occur, then the monopolist fulfills with the WAPC constraint but would not be maximizing profits; or otherwise.

The Figure 2 shows the mechanism behind the menu of plans designed by the monopolist, as well the inefficient results generated by the application of the WAPC regulatory scheme. If the WAPC constraint does not exist, the monopolist has the flexibility to set a menu of plans and will maximize his profits distorting the high-valuation consumer's plan with the intention of extracting informational rents from the high-valuation consumers; i.e. he will set the menu $\{(T_l^{SB}, m_l^{SB}), (T_h^{SB}, m_h^*)\}$. However, offering a second-best menu implies that the WAPC constraint will not be fulfilled –$\theta T_h^{SB} + (1-\theta)T_l^{SB} > T^R$– (see the stars for both types of consumers in Figure 2).

In this situation, the monopolist has two alternatives: (i) reduce T_h^{SB} (distorting m_h^*) so that, together with T_l^{SB}, the WAPC constraint is fulfilled[26]; or, (ii) reduce T_l^{SB} (distorting m_l^{SB}) so that, together with T_h^{SB}, the WAPC constraint is fulfilled[27]. Choosing the first alternative the monopolist's profits would be affected in $\alpha[v_h'(m_h) - g'(.)]$; while with the sec-

[26] In the Figure 2, this involves moving from the second best –stars for both types of consumers– to a menu of star –for low-valuation consumer– and cross –for high-valuation consumer–, $\{(m_l^{SB}, T_l^{SB}), (m_h, T_h)\}$.

[27] That is, moving from the second best to a menu of circles for both types of consumers, $\{(m_l, T_l), (m_h, T_h^{SB})\}$.

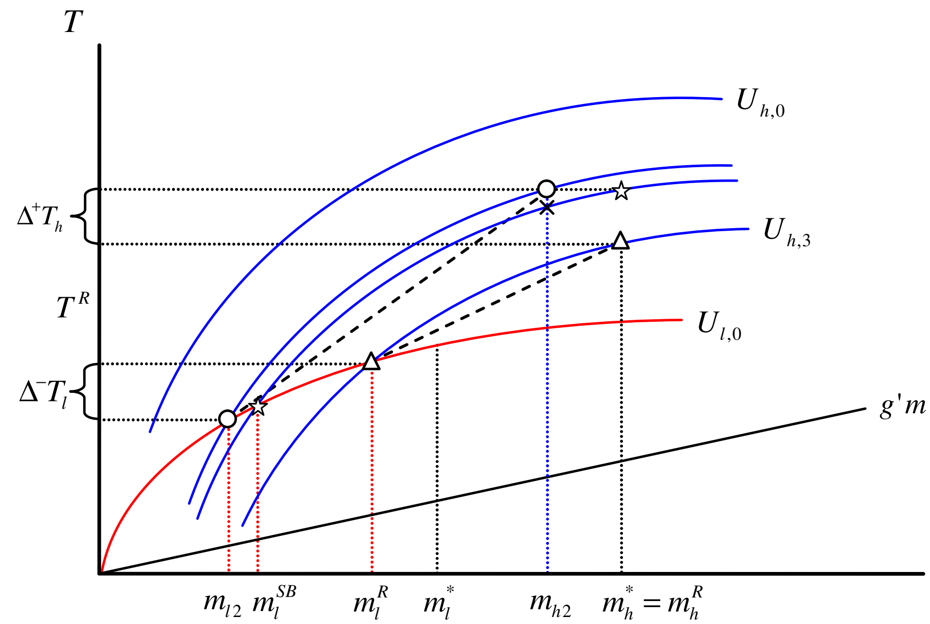

Figure 2: Distortions in the WAPC model

ond alternative would be affected in $\beta[v_l'(m_l) - g'(.)] - \alpha g'(.)$[28]. Since the monopolist's profit function is concave in m (see Corollary 2), $\beta[v_l'(m_l) - g'(.)] > \alpha v_h'(m_h)$ and the monopolist will choose the alternative (ii). In words, the monopolist will distort the low-valuation consumer's plan beyond the second best ($m_l < m_l^{SB}$) with the intention to continue extracting informational rents from high-valuation consumers by offering a plan with the same payment (T_h^{SB}) but a lower amount of minutes ($m_h < m_h^*$). In terms of Figure 2, the incorporation of the WAPC constraint leads to the monopolist offering a menu of plans equal to the circles for both types of consumers.

Thus, under the WAPC regulatory scheme, the monopolist offers a menu of alternative plans more distorted than the second-best outcome and the socially optimal (allocative inefficiency), generating losses in social welfare. The low-valuation consumer remains on his reservation utility –U_l– and the high-valuation consumer reaches a higher indifference curve (lower utility level) than the one reached in the second best; besides both types of consumers are consuming inefficient quantities.

Furthermore, it should be noted that this result still remains inefficient despite the weighted average price cap (T^R) is calculated from the solution of the central planner of

[28]Note that by reducing the amount of minutes consumed –m–, $g'(.)$ is the reduction in costs for producing one minute less and $v'(m)$ is the reduction in revenue to stop selling one minute.

section 2.1[29]. Note that the distortions in m do not explicitly depend on T^R; that is, regardless of the value of T^R, the monopolist will distort the menu of plans with the intention to continue extracting informational rents, which ultimately leads to the inefficient results shown in Proposition 3.

Despite the findings, the monopolist can still continue extracting rents under certain situations. Obviously, the monopolist's profits depend on the proportion of each type of consumer, being strictly increasing in α. This means that the monopolist can implement a shutdown policy of low-valuation consumers –if there is a sufficiently large α– to extract all informational rents from high-valuation consumers, such as classical theory of contracts refers to (see Laffont and Martimort, 2002).

According to Laffont and Martimort (2002), the monopolist will implement a shutdown policy of low-valuation consumers if:

$$\alpha[v_h(m_l^{SB}) - v_l(m_l^{SB})] \geq (1-\alpha)[T_l^{SB} - g' m_l^{SB}] \tag{SP}$$

that is, if the expected costs of leave informational rents to high-valuation consumers (for the low-valuation consumer participation) are at least as large as the expected benefits from transacting with the low-valuation consumer at the second-best outcome. From this, we define α^{SB} as:

$$\alpha^{SB} \equiv \{\alpha : \alpha[v_h(m_l^{SB}) - v_l(m_l^{SB})] = (1-\alpha)[T_l^{SB} - g' m_l^{SB}]\} \tag{D1}$$

In this regard, to implement a shutdown policy under the WAPC regulatory scheme, the monopolist will not take in consideration the extra benefits generated by the shutdown policy at the second-best solution (i.e. from α^{SB} which induces the shutdown policy at the second best), but those that will generate the shutdown policy from the solution presented in Proposition 3. The latter implies that, under the WAPC regulatory scheme, there also exist a α^{PC} –sufficiently large but lower to α^{SB}– that generates the shutdown of low-valuation consumer. Thus, considering that the monopolist would implement a optimal shutdown policy[30], α^{PC} is defined as:

$$\alpha^{PC} \equiv \{\alpha : \alpha[v_h(m_{l2}) - v_l(m_{l2})] = (1-\alpha)[T_{l2} - g' m_{l2}]\} \tag{D2}$$

[29] T^R set from T_h^R and T_l^R, which makes zero profits. Recall that Bernstein et al. (2006) and Vogelsang (1990), highlight the importance of the initial pricing before the application of the RPI-X mechanism.

[30] A shutdown policy of low-valuation consumer is optimal if (SP) holds.

Now we present the solution to the monopolist's problem when he implements a shutdown policy of low-valuation consumer.

Proposition 4 *: For any $\alpha \geq \alpha^{PC}$, the imposition of an universal access obligation in a WAPC regulatory scheme is not restrictive, and the monopolist will only offer a plan to high-valuation consumers, such that: $m_{h3} < m_h^*$ and $T_{h3} = \frac{1}{\theta}T^R$.*[31]

Proof. See appendix. ■

The flexibility the monopolist has to set a menu of plans for both types of consumers (main characteristic of WAPC regulatory scheme), also implies that he has enough flexibility to decide not offer a plan for low-valuation consumer if it is not profitable, as long as fulfills with the WAPC constraint. That is, the shutdown of low-valuation consumer may occur regardless if the universal access obligation is imposed or not[32].

In terms of Proposition 4, although the regulator imposes an universal access obligation to the monopolist, the latter may find some situation –when $\alpha \geq \alpha^{PC}$– in which it is not profitable to offer a plan to low-valuation consumers. In such situations, the monopolist will prefer to shutdown low-valuation consumers to focus only in the extraction of all informational rents from the high-valuation consumers by offering a plan on his reservation utility –U_h–.

However, as shown in Proposition 3, the informational rents extraction are presented in a different way. According to the classical literature of contract theory, with the shutdown of the low-valuation consumer, the monopolist should be able to offer an efficient plan to high-valuation consumers (T_h, m_h^*). In contrast, this does not happen under the WAPC regulatory scheme because, the increase in T_h is limited by the presence of the WAPC constraint, which ultimately encourages the monopolist to extract rents by distorting m_h from the first best –m_h^*–. This distortion will be such that the high-valuation consumer reaches its reservation utility –U_h–.

Figure 3 shows the results mentioned above. In it we can see how the solution to the monopolist's problem, presented in Proposition 3, changes to an α increases[33]. Then we can say that there is an α associated to a second-best solution (black stars for both types of consumers) from which the monopolist still might not have incentives to implement a shutdown policy of low-valuation consumers –i.e. $\alpha[v_h(m_l^{SB}) - v_l(m_l^{SB})] < (1-\alpha)[T_l^{SB} -$

[31]We uses the subscript $h3$ to differentiate the solution of monopolist's problem in Proposition 4 and the solution presented in Proposition 3.

[32]The universal service obligation fulfillment is not verifiable by the regulator, because he does not possess complete information about the existence of low-valuation consumers who are willing to pay for the service and not access it by the lack of supply plans for these type of consumers.

[33]For each α there is an unique second-best solution, which is distorted to fulfill with the WAPC constraint.

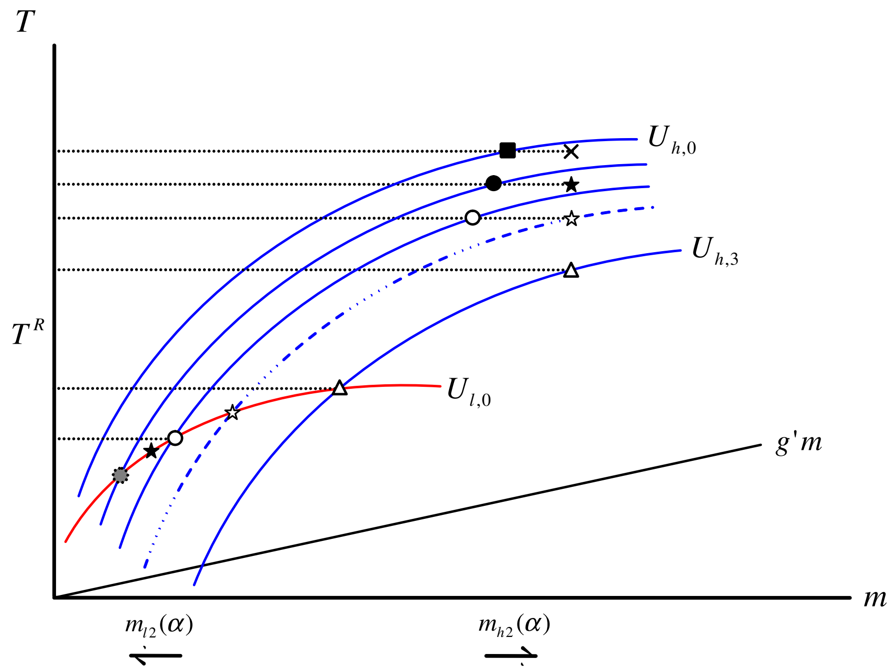

Figure 3: Distortions due to shutdown of low-valuation consumers

$g'm_l^{SB}]$–, but with the distortions presented in Proposition 3 may find it unprofitable to offer a plan for the low-valuation consumer –i.e. $\alpha[v_h(m_{l2}) - v_l(m_{l2})] > (1-\alpha)[T_{l2} - g'm_{l2}]$–. Thus, for any $\alpha \geq \alpha^{PC}$, the monopolist decides to shutdown low-valuation consumer to focus in the informational rents extraction from high-valuation consumers by increasing T_h, such that the WAPC constraint is fulfilled (the cross for high valuation consumer); however, the monopolist still has margin to continue extracting rents (note that cross is not on the reservation utility $U_{h,0}$), which are obtained by reducing m_h (the monopolist offers the black square and extracts all surplus from the high-valuation consumer).

It is worth mentioning that, as consequence from the monopolist's decision of doesn't offer a plan to low-valuation consumer, the monopolist would be restricting access to consumers who could access to the market but they don't do it just by lack of plans. In general, for any $\alpha \geq \alpha^{PC}$, the low-valuation consumers are shutdown from the market, although the reasons for this may be different[34]. However, if for α^{SB} the monopolist is able to cover his total costs and $\alpha[v_h(m_l^{SB}) - v_l(m_l^{SB})] < (1-\alpha)[T_l^{SB} - g'm_l^{SB}]$, then for any $\alpha \in (\alpha^{PC}, \alpha^{SB})$ there is clear that low-valuation consumers were shutdown from the market when they did have the

[34] Obviously, one cause for shutdown a certain types of consumers is the preference of the monopolist to extract informational rents instead of provide the service to these types of consumers. Another cause is the high fixed costs (mainly sunk) that restrict the access to certain types of consumers.

willingness to pay for the service and enter to the market; that is, if the universal access obligation would not been imposed, the monopolist would set a second-best menu of plans and both types of consumers would be participating in the market.

Therefore, despite the regulator imposes an universal access obligation, the use of the WAPC regulatory scheme allows that: (i) the monopolist can implement a shutdown policy of low-valuation consumers –ceasing to offer the service to consumers willing to pay for such service–, (ii) distortions on the menu of plans are maintained and, (iii) once implemented the shutdown policy, there is nothing to prevent the extraction of all surplus from high-valuation consumers.

The effect of using the WAPC regulatory scheme on the monopolist's menu of plans is summarized in the following Corollary.

Corollary 5 *Under WAPC regulatory scheme, if* $(1-\alpha)v_l'(m_l) < g'(.)$ *then* $\frac{dm_h}{dm_l} > 0$, $\forall m_l$ *and* $\pi > \pi^{SB}$.

Proof. See appendix. ■

That is, while the regulator decides to use the WAPC regulatory scheme, the monopolist, even with the incorporation of such constraint on his problem, will still have flexibility to set the menu of plans and will prefer to distort it with the intention to continue extracting informational rents.

A profit-maximizing monopolist, by distorting m_l will also distort m_h to cover losses caused by the fulfillment of the WAPC constraint. The profits will depend on the profits size obtained due to changes in m_l and m_h, which could be even greater than those achieved in the second-best solution.

3 Conclusions

We analyze the Price Cap regulatory scheme from the point of view of allocative efficiency and universal access, in particular when the RPI-X mechanism is applied to a weighted average price, which is calculated from the menu of plans offered by the monopolist as part of a second-degree price discriminating strategy.

In this context, the regulation by Weighted Average Price Cap (WAPC), which imply services or goods grouped into baskets, generates additional incentives for the monopolist to extract informational rents from high-valuation consumers, distorting the plans of both types of consumers beyond the second-best outcome ($m_l < m_l^{SB}$ and $m_h < m_h^*$) and leading to social welfare losses. This happen despite that the initial prices were set to costs (tariffs that make zero profits), contradicting what until now the economic literature has proposed about it.

It has also been demonstrated that the WAPC constraint does not meet the primary objective of limiting the monopolist's market power. On the one hand, the monopolist can implement a shutdown policy of low-valuation consumers (for $\alpha \geq \alpha^{PC}$, despite that the regulator impose a universal access obligation), whose main consequence is the non-provision of service to consumers who are willing to pay for the service. On the other hand, this regulatory scheme does not prevent the monopolist extracts all high-valuation consumers' surplus, if the shutdown policy is implemented. The use of this regulatory scheme seems to be the cause of what happened in the Peruvian telecommunications market, since there were consumers that had been in the fixed-telephone market before the WAPC was implemented, and subsequently left it.

In that sense, the policy recommendation is that when the regulator allows the monopolist to price discriminates in second-degree, the latter should not be regulated by Price Cap, calculated as the weighted average of the plans offered. However, if the regulator wants to keep using the Price Cap scheme, the RPI-X factor should be applied to a single plan (e.g. the plan "Classic Line"), leaving some flexibility to the monopolist to offer alternative plans (see Li Ning and Willington, 2009).

It is noteworthy that the regulation of a single plan (tariff flexibility) will allow to transfer the gains in productivity, through the RPI-X factor application, to all consumers. This happen because the monopolist must offer alternative plans which are incentive compatible with the unique regulated plan. It also avoids any elusive behavior by the monopolist in the RPI-X application, benefiting all types of consumers.

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A Appendix

Proof. **[Proposition 1]** The Lagrangian of the regulator's problem, when he is interested in the universal access problem (R3 active), is as follows:

$$\begin{aligned} L &= \alpha[u_h + v_h(m_h^R) - T_h^R] + \beta[u_l + v_l(m_l^R) - T_l^R] + \lambda[\alpha T_h^R + \beta T_l^R - A - g(\alpha m_h^R + \beta m_l^R)] \\ & \chi[u_l + v_l(m_l^R) - T_l^R - U_l] + \varphi[v_h(m_h^R) - T_h^R - v_h(m_l^R) + T_l^R] \end{aligned}$$

The first order conditions:

$$T_h^R : -\alpha + \lambda\alpha - \varphi = 0 \qquad \Longrightarrow \quad \varphi = \alpha(\lambda - 1) \quad ; \quad \lambda > 1$$

$$T_l^R : -\beta + \lambda\beta - \chi + \varphi = 0 \qquad \Longrightarrow \quad \chi = \lambda - 1$$

$$\begin{gathered} m_h^R : \alpha v_h'(m_h^R) - \lambda\alpha g'(.) + \varphi v_h'(m_h^R) = 0 \\ v_h'(m_h^R) = g'(.) \end{gathered}$$

$$\begin{aligned} m_l^R : \beta v_l'(m_l^R) - \lambda\beta g'(.) + \chi v_l'(m_l^R) - \varphi v_h'(m_l^R) = 0 \\ v_l'(m_l^R) = \frac{\lambda(1-\alpha)}{\lambda - \alpha} g'(.) + \frac{\alpha(\lambda - 1)}{\lambda - \alpha} v_h'(m_l^R) \\ v_l'(m_l^R) = g'(.) + \alpha \left(\frac{\lambda - 1}{\lambda - \alpha} \right) \left[v_h'(m_l^R) - g' \right] \end{aligned}$$

By concavity of $v(m)$ and since $v_h'(m_h^R) = g'(.)$, we conclude that $v_l'(m_l^{SB}) > v_l'(m_l^R)$. Therefore, the plan for low-valuation consumer –set by the regulator– is less distorted than the second-best solution; that is, the low-valuation consumer will consume a greater amount of minutes than the second-best quantity (approaching to the efficient outcome).

Moreover, from the low-valuation consumer's incentive compatible constraint (R5) as an equality:

$$\begin{aligned} u_h + v_h(m_h^R) - T_h^R &= u_h + v_h(m_l^R) - T_l^R \\ T_h^R &= T_l^R + v_h(m_h^R) - v_h(m_l^R) \end{aligned}$$

Substituting above in the monopolist's participation constraint (R1) as an equality, the payments are:

$$\begin{aligned} T_l^R &= A + g(.) - \alpha v_h(m_h^R) + \alpha v_h(m_l^R) \\ T_h^R &= A + g(.) + (1-\alpha) v_h(m_h^R) - (1-\alpha) v_h(m_l^R) \end{aligned}$$

■

Proof. [Corollary 2] This demonstration is taken from Li Ning and Willington (2009).

Since the regulator sets plans such that the monopolist makes zero profits, it is enough to show that the monopolist's profit function is monotonically decreasing in m_l as the socially optimal plan moves along the low-valuation indifference curve U_l (i.e. $T_l^R = u_l + v_l(m_l^R) - U_l$). The monopolist's profit function can therefore be rewritten as:

$$\pi = \alpha \left[u_l + v_l \left(m_l \right) - U_l + v_h(m_h) - v_h(m_l) - g' m_h \right] + \beta \left[u_l + v_l \left(m_l \right) - U_l - g' m_l \right] - A$$

Differentiating with respect to m_l:

$$\frac{\partial \pi}{\partial m_l} : \alpha v_l' \left(m_l \right) - \alpha v_h'(m_l) + \beta v_l' \left(m_l \right) - \beta g'$$

Note that from the latter equation, we get the second-best solution (the m_l that maximizes the profit function). Taking into account the Proposition 1 (i.e. evaluating at $m_l = m_l^R$), we have:

$$\begin{aligned} & v_l' \left(m_l^R \right) - \alpha v_h'(m_l^R) - \beta g' \\ & v_l' \left(m_l^R \right) - \left(\frac{\lambda - \alpha}{\lambda - 1} \right) v_l'(m_l^R) + \left(\frac{\lambda}{\lambda - 1} \right) \beta g' - \beta g' \end{aligned}$$

$$-v_l'(m_l^R) + g' < 0$$

By Proposition 1, it was shown that for m_l^R the profits are zero and $m_l^R > m_l^{SB}$. The above shows that for quantities greater than m_l^{SB}, the profit function is decreasing on m_l and, therefore, the monopolist's profits are greater than zero at m_l^{SB}. ■

Proof. [Proposition 3] The Lagrangian of the monopolist's problem is:

$$\begin{aligned} L &= \alpha T_h + \beta T_l - A - g(\alpha m_h + \beta m_l) + \phi[u_l + v_l(m_l) - T_l - U_l] \\ &\quad + \xi[v_h(m_h) - T_h - v_h(m_l) + T_l] + \gamma_2[T^R - \theta T_h - (1-\theta) T_l] \end{aligned}$$

The first order conditions are:

$$T_h : \alpha - \xi - \gamma_2\theta = 0 \qquad \Longrightarrow \xi = \alpha - \gamma_2\theta > 0$$

$$T_l : \beta + \xi - \phi - \gamma_2(1-\theta) = 0 \qquad \Longrightarrow \phi = 1 - \gamma_2 > 0 \ ; \quad \gamma_2 < 1 \quad ; \quad \gamma_2 > 0$$

$$m_h : -\alpha g'(.) + \xi v_h'(m_h) = 0$$
$$v_h'(m_{h2}) = \frac{\alpha}{\alpha - \gamma_2\theta} g'(.) \qquad ; \qquad \alpha > \gamma_2\theta$$

$$m_l : -\beta g'(.) + \phi v_l'(m_l) - \xi v_h'(m_l) = 0$$
$$v_l'(m_l) = \frac{1-\alpha}{1-\gamma_2} g'(.) + \frac{\alpha - \gamma_2\theta}{1-\gamma_2} v_h'(m_l)$$
$$v_l'(m_{l2}) = g'(.) + \alpha \left[\frac{\alpha - \gamma_2\theta}{\alpha(1-\gamma_2)} v_h'(m_l) - \frac{\alpha - \gamma_2}{\alpha(1-\gamma_2)} g'(.) \right] \qquad ; \qquad \gamma_2 < 1 \qquad ; \qquad \alpha > \gamma_2\theta$$

The payments are:

$$\begin{aligned} T_l &= u_l + v_l(m_l) - U_l \\ T_h &= \frac{1}{\theta} T^R - \frac{1-\theta}{\theta} [u_l + v_l(m_l) - U_l] \end{aligned}$$

Therefore, since $\alpha > \gamma_2\theta$ then $v_h'(m_{h2}) > v_h'(m_h^*)$ and $v_l'(m_{l2}) > v_l'(m_l^{SB})$. ■

Proof. [**Proposition 4**] By Corollary 2 is shown that the profit function is strictly decreasing on m_l when the constraint (R3') is binding (i.e. the low-valuation consumer gets his reservation utility $U_{l,0}$). Consequently, by the Intermediate Value Theorem, we conclude that for each α, there is a unique solution (T_l^{SB}, m_l^{SB}), and hence (T_{l2}, m_{l2}) when applying the WAPC regulatory scheme. The latter is true because the profit function is strictly increasing on α (see Li Ning and Willington, 2009).

Therefore, for any $\alpha \geq \alpha^{PC}$, the monopolist will not offers a plan for low valuation consumer (the universal access is not guaranteed), and the monopolist's problem is defined as follows:

$$\underset{\{T_h, m_h\}}{Max} \ T_h - A - g(m_h) \tag{PM'}$$

subject to:

$$\begin{aligned} u_h + v_h(m_h) - T_h &\geq U_h & \text{(R2')} \\ \theta T_h &\leq T^R & \text{(R6')} \\ T_h^R, m_h^R &\geq 0 \end{aligned}$$

The Lagrangian is as follows:

$$L = T_h - A - g(m_h) + \rho[u_h + v_h(m_h) - T_h - U_h] + \gamma_3[T^R - \theta T_h]$$

The first order conditions are:

$$T_h : 1 - \rho - \gamma_3\theta = 0 \qquad \Longrightarrow \rho = 1 - \gamma_3\theta > 0$$

$$\begin{aligned} m_h : -g'(.) + \rho v_h'(m_h) = 0 \\ v_h'(m_{h3}) = \frac{1}{1-\gamma_3\theta} g'(.) \end{aligned}$$

Considering that $1 - \gamma_3\theta > 0$, we have that $v_h'(m_{h3}) > v_h'(m_h^*)$ and thus $m_{h3} < m_h^*$. The payment T_h is:

$$\begin{aligned} T^R &= \theta T_h + (1-\theta)T_l \\ T_{h3} &= \frac{1}{\theta} T^R \end{aligned}$$

■

Proof. **[Corollary 5]** From Proposition 3 we have that the monopolist will distorts both m_l^{SB} (reducing T_l^{SB}) and m_h^* (holding T_h^{SB}) to fulfill with the WAPC constraint. In this context, the distortions in plans of low and high-valuation consumers, respectively, will impact on the monopolist's profits in the following way:

$$\pi = \alpha T_h + \beta v_l(m_l) - A - g(\alpha m_h + \beta m_l)$$

$$\begin{aligned} \frac{\partial \pi}{\partial m_l} &= \beta[v_l'(m_l) - g'(.)] & \text{(C1)} \\ \frac{\partial \pi}{\partial m_h} &= -\alpha g'(.) & \text{(C2)} \end{aligned}$$

From condition (C1) and concavity of $v_l(m)$, increases in m_l cut down the monopolist's profits in $\beta[v_l'(m_l) - g'(.)]$; while from condition (C2), increases in m_h raise his profits in $\alpha g'(.)$. Thus, for any distortion of m_l^{SB} as consequence of the fulfillment the WAPC constraint, the monopolist will always distorts m_h from the first-best outcome; typical reaction of a profit-maximizing monopolist, who in this case is minimizing losses caused by the fulfillment the WAPC constraint. This applies for any $g'(.) > 0$.

Therefore, if the gains of distorting m_h offset the losses of distorting m_l –i.e. $(1 - \alpha)v_l'(m_l) < g'(.)$–, the monopolist will make greater profits than those achieved in the second-best outcome. ■