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Abstract

We construct an overlapping generations model in which parents vote on the tax rate that determines publicly provided education and offspring choose their effort in learning activities. The technology governing the accumulation of human capital allows these decisions to be strategic complements. In the presence of coordination failure, indeterminacy and, possibly, growth volatility emerge. This indeterminacy can be eliminated by an institutional mechanism that commits to a minimum level of public education provision. Given that, in the latter case, the economy moves along a uniquely determined balanced growth path, we argue that such structural differences can account for the negative correlation between volatility and growth.

Keywords: Human Capital, Economic Growth, Endogenous Taxation, Volatility

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1 Introduction

Investment in formal education is one of the most important intergenerational transfers. It is considered as a key factor of economic growth and income distribution. Several aspects of this investment have been analysed in the literature. For example, Glomm and Ravikumar (1992) examine the effects of public and private education on long-run growth and inequality. Bénabou (1996) considers how the growth performance of an economy is influenced by the degree of decentralization of government funding for education. Blankenau and Simpson (2004) show conditions under which the effect of public education spending on growth may be non-monotonic. Cremer and Pestieau (2006) study the design of optimal education policy. Finally, Kempf and Moizeau (2009) investigate the link between social segmentation, inequality and growth in an environment where education is a club good.

This paper complements the existing literature by highlighting the fact that, unlike the production of physical capital, the education process involves the decisions of two consecutive generations, parents and children. The first generation, parents/tax payers, provides the resources for education (e.g., teachers' salaries, buildings, equipment, etc.) and the second, children/students, the time and effort that are necessary to absorb knowledge. Moreover, insofar as the actions by one generation affect the outcome for the other, one of the important characteristics that may affect each generation's decisions and actions is its reflection of how the other will decide and act. Put differently, the education process entails the coordination of the decisions made by two generations, which may be strategic complements (see, Cooper and John 1988).[1]

The idea that there may be a coordination game inherent in the accumulation of human capital seems particularly relevant in the case of public education, where decisions are often made collectively by a large number of individuals through voting. More specifically, the voters' support towards public investment for education may depend on the extent to which the younger generation will provide the learning effort necessary for allowing them to 'reap' the benefits associated with more widely spread and qualitatively improved education services. Nevertheless, the return to the young generation's learning effort may be partially

[1] Strategic complementarity "implies that an increase in the action of all agents except agent *i* increases the marginal return to agent *i*'s action" (Cooper and John, 1988; p. 445). Hence agent *i* will respond by raising her activity level.

determined by the qualitative characteristics of the education sector – characteristics that depend on public investment. One expects that such cross-generational interactions will have important repercussions for human capital accumulation and, consequently, economic growth. Nevertheless, the aforementioned literature on public education and growth has largely neglected the strategic complementarities that are inherent to decisions of coexisting cohorts of agents with different objectives – decisions which jointly determine the formation of human capital.

Our analysis builds upon an overlapping generations model in which the engine of growth is the accumulation of human capital. The actions of both young (offspring) and adults (parents) affect the formation of human capital. In particular, the parents vote on the tax rate that determines the revenue available to the government for the provision of public education, while the offspring decide on the effort they devote towards learning activities. The technology governing the evolution of human capital allows these decisions to be strategic complements. Specifically, in equilibrium, the effort devoted by the young is an increasing function of the tax rate chosen by parents which, by itself, is an increasing function of the offspring's learning effort.

First, we show that a coordination failure may arise and multiple equilibria emerge. These equilibria are Pareto-ranked. They include an equilibrium in which both cohorts choose no provision (i.e., no effort by offspring and a zero tax rate chosen by parents), and equilibria entailing positive effort by the young and a positive tax rate by the adult voters. Thus, as in Redding (1996) and Palivos (2001) among others, our paper provides an explanation for the persistent disparities in the world distribution of incomes and growth rates, which differs from the "path dependence" hypothesis – a hypothesis that has been criticized on the basis that many industrialized countries did not happen to be rich at the initial stages of their development and yet the managed to cross the threshold level. [2] Why is it that currently poor countries cannot cross it?

Naturally, the idea that multiple growth paths may be attributed to failures of coordination offers support for some type of government intervention (be it in terms of economic policy or a more structural/institutional reform) that is designed to induce the

[2] Examples of path-dependent multiple equilibria are provided in the analyses of Azariadis and Drazen (1990), Galor and Zeira (1993), Ceroni (2001) and Chakraborty (2004) among others. For evidence regarding the existence of convergence clubs, see, for example, Quah (1997) and Canova (2004).

selection of the economically/socially "preferable" equilibrium. For this reason, we subsequently consider an institutional reform that could induce the selection of the "high growth/high welfare" equilibrium. In particular, we show that the commitment to a sufficiently high tax can achieve this objective and, therefore, eliminate growth indeterminacy.

In addition to the above, we are able to show that one of the most pervasive empirical regularities in macroeconomic data, the negative relation between output growth and its volatility, can be attributed to the intergenerational complementarities as well as to the public sector's institutional arrangements.[3] The explanation we offer works as follows. The existence of multiple growth equilibria possesses an additional explanatory power when it comes to the overall macroeconomic performance. In a dynamic setting, there is nothing to preclude the possibility that in some periods agents may choose actions associated with high growth while in other periods they may choose actions associated with low growth. Periods of strong economic activity may be followed by periods of weak economic activity and vice versa, depending on how some agents expect others to behave and act. Thus, growth indeterminacy is inherently linked with the idea of growth volatility. We show that the average growth rate in this case is lower compared to the uniquely determined growth rate that emerges in the presence of a minimum commitment to public education.

By providing an alternative suggestion, our analysis may be viewed as complementary to a series of theoretical papers that employ stochastic growth models in order to examine the impact of public policy on the growth-volatility nexus (e.g., Turnovsky, 2000; Blackburn and Pelloni, 2004; Chatterjee *et al.*, 2004; Varvarigos, 2007). In these models, *exogenous* variations in policy parameters cause changes in both the average growth rate and its volatility. In contrast, we attribute this relation to the structural characteristics pertaining to the *endogenous* determination of public policy.

The implications from our model share some similarities with those in the interesting and important, but largely neglected, paper of Glomm and Ravikumar (1995). They also show that the presence of endogenously determined public spending may generate, rather than eliminate, equilibrium indeterminacy, sending thus a cautionary message regarding the role of public policy. Nevertheless, there are also significant differences between their analysis and

[3] Evidence on the negative relation between output growth and its volatility is provided by Turnovsky and Chattopadhyay (2003), Hnatkovska and Loayza (2005) and Badinger (2010) among others.

ours. Firstly, they do not examine the relation between growth and volatility as we do in this paper. Secondly, the mechanism leading to their result is different as it rests on the ideas that, (i) the young generation's education effort depends on the expectation of the future tax rate that will be chosen by the same generation when it becomes old, and (ii) the chosen tax rate depends on aggregate human capital due to the fact that the 'warm glow' argument in the utility function is introduced with CRRA coefficient which is different in comparison to the one attached to the remaining utility arguments; therefore, their result is not due to strategic complementarities in the decision making process of two distinct cohorts of agents. Put differently, we find multiple equilibria even with simple functional forms that imply uniqueness in their model. Moreover, equilibria cannot be Pareto ranked in their model, whereas, in our case, the high-growth equilibrium yields higher welfare. Finally, when their parameter values allow multiple equilibria, they find an inverse relation between the tax rate and income. In contrast, our model shows that the high-growth equilibrium actually corresponds to the relatively high tax rate.

Although this last result appears to be in contrast to conventional wisdom, it is not completely at odds with existing empirical evidence, especially when considering the productive use of tax receipts in our model. While many analyses are unable to derive a decisive conclusion on the growth effects of taxation and public spending – see Myles (2000), Agell *et al.* (2006) and Bania *et al.* (2007) for example – when econometric methods account for the productive use of public spending (infrastructure investment, education etc.) then there is supportive evidence of positive effects from taxation/public spending to economic growth (e.g., Mofidi and Stone, 1990; Pereira, 1998; Kneller *et al.*, 1998; Cohen and Paul, 2004). In fact, in his survey of the relevant literature, Poot (2000) claims that "the most conclusive results in the literature relate to the positive impact of education expenditures on growth" (Poot, 2000; p. 516) – thus, providing further support on this element of our results.

The rest of the paper is structured as follows. Section 2 presents the general set-up of the model. Section 3 establishes the existence of multiple equilibria and analyzes its implications. Section 4 shows that growth indeterminacy can be eliminated with partial commitment on behalf of the government. Section 5 examines the same issue under alternative arrangements regarding the agents' timing of choices. Section 6 concludes.

2 The Basic Structure

We consider an overlapping generations economy in which time is discrete and indexed by $t = 0,1,2...$. Each period, a cohort of unit mass is born. Agents within the cohort are identical and live for two periods. They are 'young' (or 'offspring') in the first period of their lifetime and 'old adults' (or 'parents') in the second. All agents are endowed with one unit of time in each period. The young allocate it between activities that augment their human capital (e.g., formal schooling) and leisure. The old, on other hand, supply their time, combined with their human capital (determining knowledge, efficiency and expertise), inelastically to firms in exchange for the prevailing market wage. Adults are also 'voters' in the sense that they cast a vote on their preferred tax rate that the government imposes on their labour income. Their disposable income (i.e., the residual after taxation) finances their consumption. The revenues collected by the government are utilised so as to finance activities that support the qualitative characteristics of education (e.g., the quality of schools/colleges/universities, scholarships, research and teaching support etc.) and, therefore, promote the formation of human capital. The government abides by a balanced-budget rule each period.

An agent born in period t enjoys utility over her whole lifetime according to[4]

$$u^t = \ln(1-e_t) + \ln(c_{t+1}) + \ln(w_{t+2}h_{t+2}), \tag{1}$$

where e_t denotes schooling effort when young and c_{t+1} denotes consumption when old.[5] We implicitly assume that children's consumption is incorporated into the consumption of parents. The last term of the utility function indicates that parents are imperfectly altruistic towards their offspring. Specifically, a parent gets satisfaction from her offspring's realised income. This is meant to capture the idea that parents care about their offspring's future prospects and social status (both being enhanced through more advanced knowledge and/or increased income).

We assume that, when young, a person can pick up a fraction $v \in (0,1)$ of the existing (average) level of human capital H_t without effort. This may happen, for example, through

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[4] We choose equal weights in the utility function purely for simplicity. The more general case is analysed in Palivos and Varvarigos (2009).

[5] The superscript t on the left-hand side indicates the time of birth of the generation enjoying utility through this function. A similar notation applies to other functions below.

some type of home tutoring or by simple observation. The government provides goods and services that increase the potential human capital that a young person can acquire even further. Nevertheless, the young person must provide resources that take the form of effort (or foregone leisure), denoted by e_t, in order to benefit from the government's offer of education. Specifically, the formation of human capital takes place according to the learning technology[6]

$$h_{t+1} = vH_t + \varphi g_t e_t , \tag{2}$$

where g_t denotes public expenditure per student and the parameter $\varphi > 0$ captures the efficiency of the public education system.[7]

All adults are liable to income taxation. Therefore, they will meet their consumption needs out of their disposable income. Thus, the budget constraint during adulthood is

$$c_{t+1} = (1 - \tau_{t+1}) w_{t+1} h_{t+1} . \tag{3}$$

The government finances the provision of goods and services towards education by utilising its total revenue from labour income taxation $\tau_t w_t H_t$. Given that there is a unit mass of young agents and the population size remains constant, spending per student corresponds to

$$g_t = \tau_t w_t H_t . \tag{4}$$

The single and perishable consumption good that exists in this economy is produced and supplied by perfectly competitive firms, who employ efficient labour so as to produce Y_t units of output according to

$$Y_t = AH_t , \quad A > 0 . \tag{5}$$

Notice that besides the level of human capital, H_t also corresponds to the economy's available units of efficient labour, because adult agents (whose large population is normalised

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[6] This technology for the accumulation of human capital shares common features with de Gregorio and Kim (2000) and Ceroni (2001) among others. However, none of these have combined both effort (by offspring) and endogenously determined resources (by voters and the public sector) as complementary inputs within the same type of technology. Glomm and Ravikumar (1992, 1995) include both types of inputs in the formation of human capital, but they also assume that each input is essential for a positive stock of human capital.

[7] We assume a linear effect for g_t to guarantee an equilibrium with ongoing output growth. Recall that when an agent chooses e_t, she takes the effect of g_t as given, because this term is determined by old agents. Therefore, we can consider $\varphi g_t e_t$ as a composite input, in the same manner as we do for efficient labour in models with endogenously determined accumulation of human capital, without worrying about implications of maximisation under increasing returns.

to one) supply one unit of 'raw' time each. Profit maximisation implies that the equilibrium market wage per unit of (efficient) labour is $w_t = A \;\; \forall t$.

As indicated earlier, the electorate is comprised by the adults who cast a vote on their preferred tax rate. Therefore, the problem of an agent born it t is to choose e_t, c_{t+1} and τ_{t+1} so as to maximise (1) subject to (2), (3), (4), $0 \leq e_t \leq 1$, $0 \leq \tau_{t+1} \leq 1$, and $c_{t+1} \geq 0$, taking H_t, H_{t+1}, w_t, w_{t+1} and w_{t+2} as given. Of course, given that individuals are identical, the tax rate chosen by the representative parent is the one that will prevail in a democratic regime. Equivalently, we can substitute (2)-(4) in (1) and write lifetime utility as

$$u^t = \ln(1-e_t) + \ln[(1-\tau_{t+1})w_{t+1}(vH_t + \varphi e_t \tau_t w_t H_t)] + \ln[w_{t+2}(vH_{t+1} + \varphi e_{t+1}\tau_{t+1}w_{t+1}H_{t+1})]. \quad (6)$$

It is straightforward to check that the FOC associated with the problem of an individual who was born in period *t* can be eventually written as

$$\frac{1}{1-e_t} \geq \frac{\varphi \tau_t w_t H_t}{vH_t + \varphi \tau_t w_t H_t e_t}, \quad e_t \geq 0, \quad (7)$$

and

$$\frac{1}{1-\tau_{t+1}} \geq \frac{\varphi w_{t+1} H_{t+1} e_{t+1}}{vH_{t+1} + \varphi \tau_{t+1} w_{t+1} H_{t+1} e_{t+1}}, \quad \tau_{t+1} \geq 0, \quad (8)$$

with complementary slackness in both (7) and (8). Notice that we can use (8) to infer the tax rate that will be chosen by adults who were born in period $t-1$ (that is, the parents of agents born in period t).

3 Equilibrium Analysis

In this section we establish the existence of multiple equilibria and analyse their implications for macroeconomic outcomes as well as for institutional arrangements pertaining to public policy.

3.1 Multiplicity of Equilibria

We can view the situation described here as a game played between each pair of consecutive generations, that is, between parents and children. Each agent plays this game twice, once as a child and once as a parent. In any period t, a parent, who was born in period $t-1$, chooses the amount of resources that will be allocated to public education by casting a vote on the preferred tax rate τ_t. At the same time, the child chooses the amount of her time, or equivalently her effort, e_t, that she will devote on schooling. Nevertheless, when deciding her choice variable, each player takes the action of the other player as given. If we substitute the equilibrium conditions $h_t = H_t$ and $w_t = A$ in equations (7) and (8), both dated in period t, and take into account the constraints $0 \le e_t \le 1$ and $0 \le \tau_t \le 1$, we can derive the best response function of each generation, that is,

$$e_t = \max\left\{0, \frac{1}{2}\left(1 - \frac{\gamma}{\tau_t}\right)\right\}, \tag{9}$$

and

$$\tau_t = \max\left\{0, \frac{1}{2}\left(1 - \frac{\gamma}{e_t}\right)\right\}, \tag{10}$$

where $\gamma = v / \varphi A$. The equilibrium values of τ_t and e_t are given by the intersection of (9) and (10). Given that $\tau_t, e_t \in [0,1]$, we can summarise these solutions as

$$e_t \begin{cases} > 0 & \text{if} \quad \tau_t > \gamma \\ = 0 & \text{otherwise} \end{cases}, \quad \text{and} \quad \tau_t \begin{cases} > 0 & \text{if} \quad e_t > \gamma \\ = 0 & \text{otherwise} \end{cases}. \tag{11}$$

These results merit some discussion. First of all, we see that corner solutions are possible and, as a result, $e_t = \tau_t = 0$ is an equilibrium. Evidently, this is due to the effect of the composite term γ which stems, mainly, from the presence of the parameter v. The intuition is that, as long as $v > 0$, the marginal utility has a finite upper bound for zero values of e_t or τ_t. Therefore, such choices are possible due to the fact that utility may become monotonically decreasing in these arguments. Secondly, as indicated in (11), children will devote a positive level of effort or time in schooling only if they expect a minimum level of resources allocated to public education. Moreover, if positive, the effort on schooling depends negatively on *v,* the fraction of human capital transferred to the next generation without any effort, and positively on the efficiency of the schooling system (φ) and the wage rate (A) (see equation 9). Thirdly, parents behave analogously (see equations 10 and 11). In particular, they require a minimum level of effort from the students, before they decide to allocate a positive fraction of the existing resources on public education. This fraction, if positive, depends also negatively on v and positively on φ and A.

As indicated above, additional interior solutions with both e_t and τ_t being positive are possible as well, meaning that the model may actually admit multiple equilibria. The underlying cause of multiple equilibria in this framework is the strategic complementarity between the decisions of the two groups, which mutually reinforce one another (see Cooper and John, 1988). More specifically, as it can be seen form equations (9) and (10), a higher activity by one cohort of agents induces the other cohort to increase its activity as well. Once more, the presence of the parameter v (which implies a positive γ) is responsible for these effects. We can clearly see that when $v = 0$ ($\gamma = 0$), both solutions become invariant to each other. The intuition is that, for $v = 0$, the marginal utilities of both e_t and τ_t depend only on the relative weights of the utility arguments that they ultimately affect (in this case, the arguments are equally weighted). When $v > 0$, however, the marginal utility of e_t (τ_t) is increasing in τ_t (e_t). Following increases in these variables, individuals will restore the equilibrium by taking the appropriate action so as to reduce their marginal utility – something they can do with an increase in e_t (τ_t). In terms of intuition, a higher tax rate implies an increase in publicly provided education, therefore an increase in the benefits from devoting effort towards human capital accumulation. Similarly, a greater education effort by the young increases their parents' marginal utility benefit of foregoing consumption and choosing a higher tax rate, a benefit that is due to the presence of the 'warm-glow' element in their preferences.

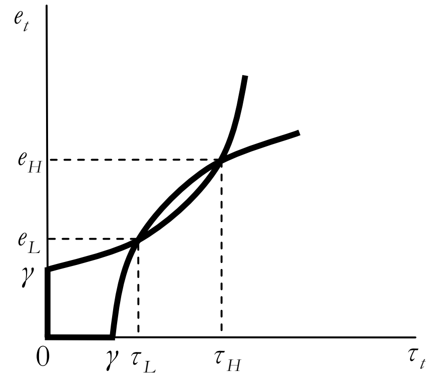

Figure 1. Multiple equilibria

The situation described above is depicted in Figure 1. We can describe it, more formally, in

Proposition 1. *There exist at most three pure strategy equilibria. These are* $\{0,0\}$, $\{e_L, \tau_L\}$ *and* $\{e_H, \tau_H\}$ *where,*

$$e_t = \begin{cases} e_L = \dfrac{1-\sqrt{1-8\gamma}}{4} \\ \\ e_H = \dfrac{1+\sqrt{1-8\gamma}}{4} \end{cases} . \tag{12}$$

and

$$\tau_t = \begin{cases} \tau_L = e_L \\ \\ \tau_H = e_H \end{cases} . \tag{13}$$

Proof: All proofs are relegated to the Appendix.

As long as $\gamma < 1/8$ (henceforth, a condition that we assume to hold) the results in (12) and (13) show that it is possible to get interior equilibria in addition to the corner solution.[8] The next step of our analysis is to examine whether the multiplicity of equilibria rests upon the presence of a coordination failure in the decision making process by the young and the old. In contrast to Glomm and Ravikumar (1995) and Palivos (2001), whose frameworks involve trade-offs that do not allow, in general, the Pareto ranking of equilibria, we can establish such ranking through

Lemma 1. *The three equilibria are ranked in the Pareto sense.*

To complete the characterisation of the different equilibria, we need to address the issue of their stability. In other words, we need to consider whether small perturbations in the

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[8] The equality between the equilibrium values of e_t and τ_t follows from the equal weights in the three arguments in the utility function (equation 1). For further details, see Palivos and Varvarigos (2009).

neighbourhood of each set of equilibrium choices will leave these choices unaffected or not. As it is known from the analysis of Cooper and John (1988), not all possible equilibria of a coordination game are unresponsive to such perturbations, as one of them may be locally unstable. In our model, such an equilibrium is represented by the point $\{e_L, \tau_L\}$. This becomes evident from the fact that $\partial e_L / \partial \gamma > 0$ and $\partial \tau_L / \partial \gamma > 0$ – results that are completely at odds with the nature of the best-response functions in (9) and (10). If anything, we would expect that both cohorts choose lower values when the composite parameter term γ is higher, as they actually do at $\{e_H, \tau_H\}$. Thus, the point $\{e_L, \tau_L\}$ represents nothing else but a threshold which, in conjunction with agents' expectations of how others will act, determines which of the two stable equilibria – i.e., $\{0,0\}$ or $\{e_H, \tau_H\}$ – will prevail. For example, consider that each cohort makes a choice x, where $x = e, \tau$. If one cohort expects the other to choose $x < x_L$ ($x > x_L$), then it will choose 0 (x_H). Anticipating this, the other cohort will choose $0 < x_L$ ($x_H > x_L$), thus verifying the initial expectation.[9]

3.2 Further Implications

As we have seen, when decisions by the young and the old within a given period are strategic complements, and in the presence of a coordination failure, the model can generate multiple equilibria. Moreover, what is particularly interesting with our analysis is the idea of output growth indeterminacy that arises because any of the two equilibria can prevail: for a given H_t, next period's human capital (which, in equilibrium, satisfies $h_{t+1} = H_{t+1}$) can take more than one possible values. The indeterminacy of equlibria, which, as argued intuitively above and shown formally in Sections 4 and 5 below, emerges because of *the endogenous determination of public policy*, has the following two additional implications.

Firstly, it results in indeterminacy of the growth rate of output and human capital, since $Y_{t+1} / Y_t = H_{t+1} / H_t = v + \omega e_t \tau_t$. Thus, our paper belongs to the strand of literature that

[9] Notice that this notion of instability differs from the one applied in variables that display an explicit dynamic pattern. More formally, let $e_t = f(\tau_t)$ and $\tau_t = \Phi(e_t)$ denote the best-response function of the children and the parents, respectively, when $e_t, \tau_t > 0$. A (Nash) equilibrium is said to be stable if, starting from any point in its neighbourhood, the adjustment process in which players take turns myopically playing a best response to each other's current strategies converges to the equilibrium. This requires that $f' < (\Phi^{-1})'$, which, using (9) and (10), is equivalent to $\tau > 1/4$. Hence, $\{e_L, \tau_L\}$ is unstable and $\{e_H, \tau_H\}$ is stable.

illustrates the stylised fact of 'club' convergence, without resorting to the problematic scenario in which growth/development paths depend on initial conditions or endowments – problematic in the sense that the suggestion that some countries are currently rich simply because they happened to be rich before does not appear to be historically accurate. Other analyses that arrive to similar conclusions, but under different settings, are those of Redding (1996) and Palivos (2001). In the former, strategic complementarities between workers and entrepreneurs imply that, over some range of parameter values, multiple growth equilibria may emerge. In the latter, the complementarities generated by the existence of family-size norms imply indeterminate fertility choices and, given the trade-off between child-rearing and educational attainment, multiple growth equilibria.

The second significant implication of indeterminacy is that the growth rate of output may not settle down to a balanced growth path, instead its behaviour may display a periodic pattern. In fact, any of the two equilibria $\{0,0\}$ and $\{e_H,\tau_H\}$ may prevail during each distinct period $t\in[0,\infty)$.[10] We can formalise this argument with

Proposition 2. *The growth rate of output may not be balanced, instead it may be volatile as it is given by* $\hat{\eta}_t \equiv \eta(\hat{e}_t,\hat{\tau}_t) = Y_{t+1}/Y_t = H_{t+1}/H_t = v + \omega\hat{e}_t\hat{\tau}_t$, *where* $\{\hat{e}_t,\hat{\tau}_t\}=\{0,0\}$ *or* $\{\hat{e}_t,\hat{\tau}_t\}=\{e_H,\tau_H\}$, *for any* $t\geq 0$.

The idea of volatile growth is absent from the analysis of Redding (1996) because he employs a framework in which the economy terminates at the end of the second period, implying that interactions among agents occur only once: consequently, multiple equilibria cannot be considered as a sign of periodic fluctuations in economic activity. In this respect, our framework shares more similarities with the analysis of Palivos (2001) in the sense that both employ full-fledged dynamic settings which allow interactions between agents to occur

[10] An important feature that leads to the emergence of multiple equilibria is the assumption that individuals decide optimally on how much time or effort they devote towards learning activities. Some may argue that the introduction of compulsory schooling may invalidate this idea, in which case multiple equilibria (and growth volatility) disappear. However, there are strong arguments against this conjecture. First, even if attendance to some basic education (schooling) is mandatory, there are always elements in the education system that are not compulsory (e.g., higher education). Second, even if someone interprets e_t in the narrow sense of 'schooling' (which we do not), there are still qualitative characteristics that justify our approach: although individuals may have to spend a fixed amount of time at school, how much effort they are going to devote during their studies is an element of their own decision making process. The large differences in performance among pupils is certainly not an outcome related solely to innate abilities, therefore the element of personal effort is still crucial.

at every distinct period. Like we do in this paper, he recognises that multiplicity and indeterminacy are sources of growth volatility. In a framework which is closer to ours, Glomm and Ravikumar (1995) also discuss the possibility of cycles due to the presence of multiple equilibria. In their model, these arise (under some parameter specifications) because the future tax rate, which depends on future income, affects current education decisions which, partially, determine future income due to the accumulation of human capital.

Notwithstanding these common equilibrium implications, our particular framework offers new dimensions in two different respects: firstly, in the type of interactions that generate these effects and, secondly, in the implications for public policy. The latter issue is particularly pertinent, that is why we discuss it and analyse it formally in the subsequent part of our paper.

4 Equilibrium with Partial Commitment

Given that the equilibria are Pareto-ranked, there is a clear scope for government intervention that will induce the selection of the "high growth/high welfare" equilibrium, represented by the pair $\{e_H, \tau_H\}$. Nevertheless, the preceding analysis has shown that the underlying source of indeterminacy and, therefore, growth volatility is inherently linked to the endogenous determination of public policy itself. For this reason, it may be instructive to seek a more institutional-oriented arrangement that could act as the desired selection mechanism.

As it will transpire from the following analysis, such an institutional mechanism exists. Suppose that, irrespective of the choices made by voters (which may approximate the ideological stance of different political parties), the government commits a minimum fraction $s \in (0,1)$ of the economy's output for public education expenditures. Given the other assumptions of the model, s is also the minimum tax rate that adults will pay irrespective of their choices. However, they may choose to vote for a tax rate which is higher than s. Denoting this incremental tax (i.e., the increment over s) that adults may vote for by q_t, and given that the total amount of tax revenues supports public expenditures towards the formation of human capital, the lifetime utility function is now given by

$$\begin{aligned} u^t &= \ln(1-e_t) + \ln\{(1-s-q_{t+1})w_{t+1}[vH_t + \varphi e_t(s+q_t)w_t H_t]\} \\ &\quad + \ln\{w_{t+2}[vH_{t+1} + \varphi e_{t+1}(s+q_{t+1})w_{t+1}H_{t+1}]\}. \end{aligned} \tag{14}$$

It should be noted here that a scenario in which governments commit to the provision of a certain fraction of GDP or of tax revenue towards education spending is not just a theoretical construction; instead there are instances where such mechanisms have been actually implemented. For example, Section 8 of Article XVI of the California state constitution, added by Proposition 98 of 1988, establishes a minimum funding level or guarantee for K–12 education and community colleges (Leyden, 2005).

Now, the lifetime utility in (6) is replaced by the one in (14). Following the same steps as before, it is straightforward to establish that the best response functions are given by

$$e_t = \max\left\{0, \frac{1}{2}\left(1 - \frac{\gamma}{s + q_t}\right)\right\}, \tag{15}$$

and

$$q_t = \max\left\{0, \frac{1}{2}\left(1 - 2s - \frac{\gamma}{e_t}\right)\right\}. \tag{16}$$

Solving (15) and (16) simultaneously, we find that e_t is the same as the one given by (12), while

$$q_t = \begin{cases} q_L = \tau_L - s \\ \\ q_H = \tau_H - s \end{cases}, \tag{17}$$

where, τ_L and τ_H are given in (13). Thus, the results in (12) and (17) allow us to derive

Proposition 3. *As long as* $s \in (\tau_L, \tau_H)$, *there exists a unique equilibrium* $\{e_H, q_H\}$. *Consequently, the growth rate of the economy is balanced and equal to* $\breve{\eta} \equiv \eta(\hat{e}_t, \hat{\tau}_t) = Y_{t+1} / Y = H_{t+1} / H_t$ $= v + \omega e_H (s + q_H)$. *Furthermore, it is* $s + q_H = \tau_H$.

The preceding analysis shows that an institutional arrangement that commits a sufficient fraction of output towards public education can act as an efficient mechanism that will induce the coordination towards a unique equilibrium. In particular, the equilibrium selected will replicate the "high growth/high welfare" equilibrium which we derived in a preceding part of our analysis. The equilibrium is illustrated in Figure 2.

This result is quite intuitive. A sufficiently high s will induce the young to choose a relatively high effort towards learning activities. The adult voters recognise this and respond optimally by choosing a positive increment over the minimum tax rate s. This choice induces even higher levels of learning effort by the young. Thus, it verifies the adults' expectations that induced them to support public education through a sufficiently high overall tax rate.

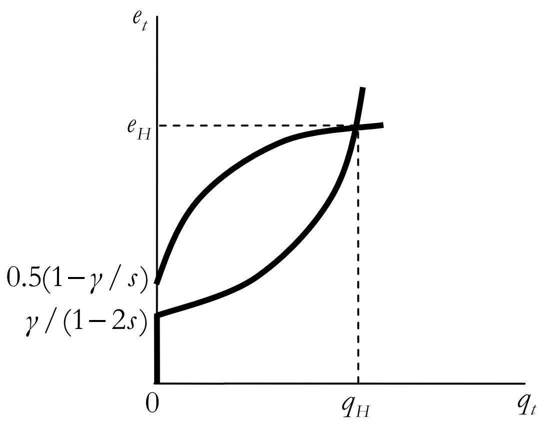

Figure 2. Unique equilibrium

4.1 Public Education Spending and the Relation between Growth and Volatility

Due to its ability to induce the selection of the "high growth/high welfare" equilibrium, the commitment to a sufficiently high share of public education can also eliminate growth volatility. Therefore, the structural characteristics pertaining to the endogenous determination of public spending on education may allow us to derive a novel explanation for one of the most pervasive macroeconomic facts, i.e., the relation between output growth and its volatility.

Despite the fact that some early economists conjectured that temporary and long-term movements in economic activity are inherently linked, it is only recently that a growing body of literature considered the analysis of the fundamentals behind this link as a research question worth pursuing. This strand of literature was further stimulated by an increasing number of empirical analyses (see footnote 3) showing that growth rates are significantly – and, mainly, inversely – correlated, on average, with proxies of their variability. Until recently, theoretical studies have explored this issue with the construction and solution of stochastic endogenous growth models – i.e., models in which (extrinsic) uncertainty is introduced through the incorporation of some RBC-type real and/or monetary shocks in frameworks that endogenise the process of productivity improvements. As a result, in models such as those of Turnovsky (2000), Blackburn and Pelloni (2004), Chatterjee *et al.*, (2004) and Varvarigos (2007), the decisions governing the formation of the reproducible factor of production (be it physical or human capital) respond optimally to the realisation of stochastic shocks, therefore the average growth rate is affected by the volatility of these shocks. Using such frameworks, the authors have examined how *exogenous variations in policy parameters* affect both the average growth rate and its volatility.

Our analysis can be viewed as providing an alternative suggestion – mainly, the idea that both differences in growth rates and the incidence of growth volatility are inherently linked to the structural characteristics of *endogenously determined economic policy*. On the one hand, growth volatility is an outcome related to the manner through which public policy is endogenously determined; on the other hand, such volatility may be eradicated by an institutional mechanism that commits some fraction of the economy's output towards public education spending. We outline the main implication from this idea in

Proposition 4. *There is a negative correlation between volatility and growth, in the sense that the average growth rate of the economy that may undergo fluctuations is lower than the average growth rate of the economy in which a minimum commitment eliminates these fluctuations. That is,* $E(\hat{\eta}_t) < E(\breve{\eta})$.

Contrary to the existing literature, our model does not rely on exogenously introduced random shocks so as to generate growth volatility. Rather, it is the intrinsic uncertainty that is inherent in strategically complement decisions, when these are subjected to coordination failure, which is responsible for growth volatility.[11] When the structural characteristics of the economy render such failures absent, variability disappears and the complementary actions by agents are conducive to the formation of human capital. Given that elements of public

[11] A recent contribution by Wang and Wen (2006) also examines the relation between endogenously driven cycles and growth. They do so in a completely different setting however. In their model, imperfectly competitive firms set prices one period in advance. Given that the decisions by firms within the industry are strategic complements, each one faces extrinsic uncertainty concerning its competitors' actions – uncertainty which can be self-fulfilling and, thus, lead to sunspot equilibria. They show that, under certain parameter restrictions, the mean growth rate of an economy perturbed by sunspot shocks is lower than the growth rate of an economy in which sunspot shocks do not emerge.

policy are embedded to these structural characteristics, our framework suggests that a macroeconomic phenomenon – that is, the correlation between output growth and its volatility – may be attributed to issues pertaining to the field of public economics.

What needs to be stressed at this point is that although we provide an alternative explanation to that of the aforementioned analyses of stochastic endogenous growth, we do not view these different suggestions as being mutually exclusive. Rather, we view our result as complementary to existing ones given that nothing precludes the possibility that the relation between growth and volatility nests factors that relate to both exogenous shocks and failures of coordination. So far, the existing empirical literature has not provided a definite answer on which type of framework is more appropriate in identifying the underlying forces that govern the relation between economic growth and macroeconomic volatility. However, existing empirical studies have actually shown that models including coordination failures can be successful in capturing pervasive stylised facts of actual business cycles – see for example, Cooper and Haltiwanger (1996). Such evidence provides definite support for our result's validity in suggesting a complementary explanation for the existence of a relation between volatility and output growth.

5 Outcomes with Full Commitment

So far, our formal analysis has been based on a scenario where choices by both the young and the old are formed through some type of coordination game. In this Section, we will reconsider these interactions under an alternative set-up. In particular, we will examine cases in which one of the two cohorts acts as a Stackelberg leader in the game that determines the optimal choices.

5.1 The Adult Voters as Leaders

We shall assume that the old decide on the tax rate first and, following this announcement, the young decide on their education effort. Effectively, this scenario is conceptually similar to the one described in Section 4. In particular, the government (through the voting behaviour of the old) commits to the level of public education spending reflected in the chosen tax rate τ_t.

More specifically, this approach entails that we solve the problem of a young person in period t so as to get her best-response function $e_t = f(\tau_t)$. The old adults of generation $t-1$ will take account of this when choosing their preferred tax, $\tilde{\tau}_t$; therefore, the effort level chosen by the young will be $\tilde{e}_t = f(\tilde{\tau}_t)$. Maximising (1) with respect to e_t yields

$$\frac{1}{1-e_t} = \frac{\varphi\tau_t w_t H_t}{vH_t + \varphi\tau_t w_t H_t e_t}. \tag{18}$$

Solving (18) for e_t gives

$$e_t = \frac{1}{2}\left(1 - \frac{v}{\varphi\tau_t w_t}\right). \tag{19}$$

Our next step is to substitute (19) into the $t-1$ variant of the utility function given in (1). Eventually, we get

$$\ln(1-e_{t-1}) + \ln[(1-\tau_t)w_t(vH_{t-1} + \varphi\tau_{t-1}w_{t-1}H_{t-1}e_{t-1})] + \ln\left[\frac{w_{t+1}}{2}(vH_t + \varphi\tau_t w_t H_t)\right]. \tag{20}$$

Maximising (20) with respect to τ_t yields

$$\frac{1}{1-\tau_t} = \frac{\varphi w_t H_t}{vH_t + \varphi\tau_t w_t H_t}. \tag{21}$$

Now, we can substitute $w_t = A$ and $\gamma = v/\varphi A$ in (21), and solve for τ_t to get

$$\tau_t = \frac{1-\gamma}{2} \equiv \tilde{\tau}. \tag{22}$$

Finally, substituting (22) in (19) gives us

$$e_t = \frac{1}{2}\left(\frac{1-3\gamma}{1-\gamma}\right) \equiv \tilde{e}. \tag{23}$$

As long as the previously imposed restriction $\gamma < 1/8$ still applies (which we assume it does), these solutions satisfy $\gamma < \tilde{\tau} < 1$ and $\gamma < \tilde{e} < 1$ as required. Thus, we can present our next result in the form of

Proposition 5. *The equilibrium growth rate is equal to* $\tilde{\eta} \equiv Y_{t+1}/Y_t = H_{t+1}/H_t = v + \omega\tilde{\tau}\tilde{e}$*, for every* $t \geq 0$.

It is evident that, in this scenario, the equilibrium is unique and the possibility of growth variability has disappeared. In terms of intuition, the intrinsic uncertainty that pertained choices when these were made through a coordination game has vanished. The old understand that an increase in the tax rate increases the willingness of the young to forego part of their leisure activities, simply because the benefits from doing so are higher. Consequently, they decide the tax rate that will induce children to provide the relatively high education effort that will satisfy their parents.

5.2 The Young as Leaders

Although it represents a less reasonable scenario, we shall briefly discuss the case where the young are the ones who commit to a certain effort towards learning activities. We do this purely as a means of illustrating the robustness of our main results regarding the implications of endogenous public policy. In terms of a concrete real-life example, we may think of scholarships and/or tuition fee waivers that are provided on the basis of students' success on achieving some performance targets.

In this case, when the young choose e_t, they take account that $\tau_t = \Phi(e_t)$. Based on this, they choose their optimal learning effort $\overline{e_t}$ which, subsequently, determines the chosen tax rate by adults through $\overline{\tau_t} = \Phi(\overline{e_t})$.

Rewriting (1) in terms of u^{t-1} and maximising with respect to τ_t, we obtain the best-response function

$$\tau_t = \frac{1}{2}\left(1 - \frac{v}{\varphi e_t w_t}\right). \tag{24}$$

Next we substitute (24) in (1) and maximise with respect to e_t. We get

$$e_t = \frac{1-\gamma}{2} \equiv \overline{e}\ , \tag{25}$$

which, after substituting in (24), leads us to

$$\tau_t = \frac{1}{2}\left(\frac{1-3\gamma}{1-\gamma}\right) \equiv \overline{\tau}\ . \tag{26}$$

The results in (25) and (26) indicate that, once again, the equilibrium and, therefore, the growth rate of output are uniquely determined. In terms of intuition, the young understand that by foregoing some of their leisure will increase the adults' benefit from foregoing part of their consumption, in order to support a higher tax rate. As a result, they devote the amount of learning effort that will provide adults with the incentive to choose relatively high public spending on education. Finally, since $\overline{\tau}\,\overline{e} = \tilde{\tau}\tilde{e}$, the growth rate in this case is the same as the one derived in Proposition 5.

Finally, a straightforward comparison between the result in Proposition 5 and the corresponding result in Proposition 1 allows us to establish

Proposition 6. *There is a negative correlation between volatility and growth, in the sense that the growth rate of the economy that may undergo fluctuations is always lower than the growth rate of the economy in which such fluctuations are absent. That is,* $\hat{\eta}_t < \tilde{\eta} \quad \forall t$.

Once more, the intuition for this outcome is related to the fact that, in the absence of coordination failure, indeterminacy disappears and complementary actions by agents are conducive to the formation of human capital. In fact, in this case we get an even stronger result: volatile growth rates are not only lower on average, but also at any moment in time.

6 Conclusions

In the preceding analysis, we have sought to analyse the implications from the fact that the education process entails coordination of the decisions made by distinct generations of agents. Among other results, we offered a novel explanation for the, empirically observed, negative correlation between volatility and growth. In particular, we argued that this may be due to the structural characteristics of the endogenous determination of public policy when this affects the accumulation of a growth promoting factor – in our case, human capital. Furthermore, our framework lies in the class of models that are able to explain convergence in 'growth clubs' without resorting to the idea of differences in initial conditions. In terms of policy implications, our analysis suggests that a credible policy of commitment towards growth promoting factors (such as education in our particular framework) could lead to both an increase in output growth and, as an added benefit, a reduction in the incidence of aggregate variability.

As mentioned in the Introduction the intergenerational complementarities identified and analyzed in this paper seem particularly relevant in the case of public education. Nevertheless, in Palivos and Varvarigos (2010) we analyze the case of private education in a similar framework. There we show that in the case of private education there exists also an intergenerational externality, since when the young decide how much effort to devote on education, they realise that this decision will affect their future spending on their children's education. Therefore, the parents' learning effort depends on their children's effort, which also depends on their own children's effort and so on ad infinitum. In the end, because of the existence of indirect effects, it is not clear whether the decisions made by two consecutive generations are strategic complements. Moreover, the additional channel of interaction generates rich dynamics that may lead to periodic as well as aperiodic (i.e., chaotic) equilibria. Finally, in such a framework, the scope for Pareto-improving government intervention is limited.

Recently there has been a growing literature on the determination and the implications of public funding of education through voting (see Bearse *et al.* 2005, de la Croix and Doepke 2009 and the references therein). In this literature there is some form of heterogeneity, which makes voting non-trivial. In addition, the parent's utility typically depends on spending on her own consumption and on her children's education. Nevertheless, the alternative specification, which is often used in the literature and adopted here, where some variant of the human capital of the children (be it the income or the services generated from it) enters the parental utility is equally plausible. The implications of this specification, especially in the presence of the aforementioned intergenerational complementarities, though interesting and potentially significant, remain largely unexplored. We view this as a fruitful avenue for future work.

Appendix

Proof of Proposition 1

First, notice that the origin is an equilibrium, since it lies on both best response functions. Moreover, the best response function of the children, described by equation (9), is upward sloping and concave for $\tau_t > \gamma$, while that of the parents, equation (10), is upward sloping and convex when solved in terms of e_t. Hence, the two curves can intersect at most twice. Next, substitute (10) in (9) and manipulate algebraically to derive the quadratic equation

$$e_t^2 - \frac{1}{2}e_t + \frac{\gamma}{2} = 0,$$

whose solution is the one given by equation (12). Similarly, we can substitute (9) in (10) so as to get the quadratic equation

$$\tau_t^2 - \frac{1}{2}\tau_t + \frac{\gamma}{2} = 0,$$

whose solution is given in equation (13). ■

Proof of Lemma 1

Consider the utility of the old adult/parent during period t. Using (1), it can be written as

$$u^{t-1}(e_t, \tau_t) = \Psi^{t-1} + \ln(1-\tau_t) + \ln(v + \omega\tau_t e_t),$$

where $\omega = \varphi A$ and $\Psi^{t-1} = \ln(1-e_{t-1}) + 2\ln[AH_{t-1}(v + \omega\tau_{t-1}e_{t-1})]$. We can also write the utility of the young adult/offspring during period t as

$$u^{t}(e_t, \tau_t) = \Xi^{t} + \ln(1-e_t) + \ln(v + \omega\tau_t e_t),$$

where $\Xi^t = \ln(1-\tau_{t+1}) + 2\ln[AH_{t+1}(v + \omega\tau_{t+1}e_{t+1})]$. Using the results in (12) and (13) we get

$$1-e_L = 1-\tau_L = \frac{3+\sqrt{1-8\gamma}}{4}, \qquad 1-e_H = 1-\tau_H = \frac{3-\sqrt{1-8\gamma}}{4},$$

and

$$\tau_L e_L = \frac{\left(1-\sqrt{1-8\gamma}\right)^2}{16},\ \tau_H e_H = \frac{\left(1+\sqrt{1-8\gamma}\right)^2}{16}.$$

Taking account of these results, we can show that $u^{t-1}(e_H, \tau_H) > u^{t-1}(e_L, \tau_L)$ and $u^{t}(e_H, \tau_H) > u^{t}(e_L, \tau_L)$ as long as

$$\frac{3+\sqrt{1-8\gamma}}{3-\sqrt{1-8\gamma}} < \frac{16v + \omega\left(1+\sqrt{1-8\gamma}\right)^2}{16v + \omega\left(1-\sqrt{1-8\gamma}\right)^2}.$$

After some extensive algebra, the last expression reduces to

$$\gamma < 1/2,$$

which holds. Thus, $u^{t-1}(e_H,\tau_H) > u^{t-1}(e_L,\tau_L)$ and $u^t(e_H,\tau_H) > u^t(e_L,\tau_L)$ hold simultaneously. With this result in mind, it is sufficient to show that $u^{t-1}(e_L,\tau_L) > u^{t-1}(0,0)$ and $u^t(e_L,\tau_L) > u^t(0,0)$ so as to prove that the equilibria are Pareto ranked. Both of these inequalities are satisfied as long as

$$\frac{3+\sqrt{1-8\gamma}}{4} > \frac{16\upsilon}{16\upsilon + \omega\left(1-\sqrt{1-8\gamma}\right)^2},$$

holds. Some algebraic manipulation can reduce this expression to

$$(1-6\gamma)^2 > (1-8\gamma)(1-2\gamma)^2 \Rightarrow$$

$$0 > -32\gamma^3,$$

which, of course, holds with a positive γ. In conclusion, $u^j(0,0) < u^j(e_L,\tau_L) < u^j(e_H,\tau_H)$ for $j = t-1, t$ and for every $t \geq 0$. ■

Proof of Proposition 2

The proof regarding the volatility of the growth rate of output follows from the absence of any intertemporal element in each cohort's choices, as it is obvious from equations (9) and (10). The value of the growth rate of output and its equality with that of human capital can be seen immediately from equations (2), (3), (5) and $h_t = H_t$. ■

Proof of Proposition 3

Given the result in (17) and the non-negativity constraint in q_t, it is obvious that $q_L = 0$. Thus, to prove the result, it is sufficient to show that an equilibrium with $q_t = 0$ and $e_t = \frac{1}{2}\left(1-\frac{\gamma}{s}\right)$ cannot exist. This will be the case if

$$q_t = \frac{1}{2}\left[1-2s-\frac{\gamma}{\frac{1}{2}\left(1-\frac{\gamma}{s}\right)}\right] > 0$$

or, equivalently,

$$\frac{s-\gamma}{s} > \frac{2\gamma}{1-2s}.$$

The above inequality can be equivalently expressed as $k(s) < 0$, where

$$k(s) = s^2 - \frac{s}{2} + \frac{\gamma}{2}.$$

Obviously, it is $k(0) > 0$, $k' = 2s - \frac{1}{2}$ (which can be either positive or negative) and $k'' = 2 > 0$. Furthermore, there are two roots satisfying $k(s) = 0$ and these are given by

$$s_1 = \frac{1 - \sqrt{1 - 8\gamma}}{4} \quad \text{and} \quad s_2 = \frac{1 + \sqrt{1 - 8\gamma}}{4}.$$

Therefore, for $s \in (s_1, s_2)$, it is $k(s) < 0$ and $q_t > 0$. Consequently, as long as $s \in (s_1, s_2)$, we conclude that an equilibrium with $q_t = 0$ and $e_t = \frac{1}{2}\left(1 - \frac{\gamma}{s}\right)$ does not exist. ■

Proof of Proposition 4

Let us compute the average growth rates we derive in both cases, over a number of periods T. For the case where a unique equilibrium is selected, the growth rate is balanced therefore

$$E(\breve{\eta}) = \frac{\sum_{t=0}^{T} \breve{\eta}}{T} = \breve{\eta} = v + \omega e_H (s + q_H).$$

However, for the case with multiple equilibria, the growth rate during a particular period may be either $\hat{\eta}_t = \eta^0 = v$ or $\hat{\eta}_t = \eta_H = v + \varphi\omega e_H \tau_H$. Now, let us assume that the equilibrium pair $\{e_H, \tau_H\}$ prevails in only a fraction $\pi \in (0,1)$ of the total number of periods spanning from $t = 0$ to $t = T$. This implies that a number of $(1-\pi)T$ periods will see the selection of the equilibrium pair $\{0, 0\}$. The average growth rate is thus equal to

$$E(\hat{\eta}_t) = \frac{\sum_{t=0}^{T} \hat{\eta}_t}{T} = \frac{(1-\pi)T\eta^0 + \pi T \eta_H}{T} = (1-\pi)\eta^0 + \pi\eta_H = v + \pi\omega e_H \tau_H.$$

From our existing results, we know that $\eta_H = \breve{\eta}$ because $s + q_H = \tau_H$. Therefore, $\eta^0 < \breve{\eta}$. Hence, comparison of the two average growth rates reveals that $E(\hat{\eta}_t) < E(\breve{\eta})$. ■

Proof of Proposition 5

The proof follows immediately from equations (2), (4) and the equilibrium condition $w_t = A \; \forall t$. ■

Proof of Proposition 6

Given our analysis and results so far, it suffices to show that $e_H \tau_H < \tilde{e}\tilde{\tau}$. It is

$$e_H \tau_H = \left(\frac{1+\sqrt{1-8\gamma}}{4} \right)^2 = \frac{1+2\sqrt{1-8\gamma}+1-8\gamma}{16},$$

and

$$\tilde{e}\tilde{\tau} = \frac{1}{2}\left(\frac{1-3\gamma}{1-\gamma} \right)\frac{(1-\gamma)}{2} = \frac{1-3\gamma}{4}.$$

Then, for $e_H \tau_H < \tilde{e}\tilde{\tau}$ we want

$$\frac{1+2\sqrt{1-8\gamma}+1-8\gamma}{16} < \frac{1-3\gamma}{4} \quad \Rightarrow \quad 4\gamma(1+\gamma) > 0,$$

a condition that is indeed true. Therefore, we conclude that $\hat{\eta}_t < \tilde{\eta}$. ■

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