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## **The Formation of Wage Expectations in the Effort and Quit Decisions of Workers**

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### **Abstract**

While much work in macroeconomics considers the formation of price expectations, there has been relatively little work analyzing wage expectations. This study develops models in which workers form expectations of average wages in choosing levels of effort and on-the-job search, under the assumption that information on lagged average wages is free but other information is costly. Under reasonable conditions, workers' expectations are likely to be at least partly adaptive. It is argued that wage expectations may be more important than price expectations in explaining unemployment fluctuations.

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## **The Formation of Wage Expectations in the Effort and Quit Decisions of Workers**

### **I. Introduction**

In recent years economists have done a good deal of research on the issue of price expectations. One line of inquiry involves testing whether survey measures of expected inflation satisfy the criteria for rational expectations. In a second line of research, Mankiw and Reis (2002) assume that some firms operate with out-of-date information about optimal prices, and they demonstrate that this “sticky information” model can explain output and inflation dynamics better than a model with sticky prices. However, while price expectations have received much attention, a related issue that has been overlooked is workers’ expectations about average wages.

This study models the formation of workers’ wage expectations and argues that expectations of average wages may be at least as important as expectations of the price level in explaining unemployment fluctuations. Workers’ wage expectations are analyzed in the context of their effort and on-the-job search decisions, since these decisions depend on the relationship between a worker’s current wage and his or her expectations of the mean of the aggregate wage distribution. Under reasonable conditions, it is demonstrated that workers may make systematic errors in their wage expectations and that their expectations may be partly adaptive.

The motivation for this work is that systematic errors in wage expectations may have significant macroeconomic consequences. If workers’ expectations about average wages are not completely rational, they may appear to exhibit money illusion in their effort and quit decisions. In Section II it is argued that the wage expectations of workers may be more important than the price expectations of either firms or workers in explaining business cycle fluctuations.

Section III develops a model of information acquisition for the effort decision of workers. It is assumed that workers can form expectations of average wages both by using a free adaptive

expectations calculation and by acquiring costly information that yields a more accurate estimate. A worker seeks to minimize the sum of information acquisition costs and the utility loss that results from making decisions with imperfect information (since the effort exerted by a worker who forms incorrect wage expectations will be suboptimal). Two methods for acquiring information about average wages are considered: gathering and analyzing macroeconomic data and sampling wages at a subset of firms. In both cases it is demonstrated that workers' expectations do not necessarily satisfy the criteria for rational expectations and that their expectations are likely to be at least partly adaptive. The model is modified in Section IV to analyze the job search decision, which affects workers' quit behavior. Section V concludes and discusses implications of the model.

The main contributions of this study lie in identifying the benefits of information to workers and demonstrating why utility-maximizing workers may make systematic errors in their wage expectations.<sup>1</sup> In doing so, it provides theoretical justification for efficiency wage modeling in which effort and quits depend partly on lagged average wages. In addition, this study shows how microeconomic parameters affect the degree to which expectations are rational vs. adaptive and the weights placed on various lags of past wages.

## **II. Why Wage Expectations Matter**

There is an extensive literature that involves testing the rationality of price expectations, and much of this work uses the survey of economists conducted by Joseph Livingston and/or the household survey conducted by the Michigan Institute of Social Research. Taken together, these studies suggest that expectations are neither completely rational nor completely adaptive. On one hand, Evans and Gulamani (1984), Batchelor and Dua (1989), Roberts (1997), Thomas (1999), and Mankiw, Reis, and Wolfers (2003) find that expectations do not satisfy the criteria for

rational expectations, as they show that forecast errors can be predicted by information available at the time of the forecast (e.g., money supply growth, unemployment, the budget deficit, interest rates, the output gap, and lagged inflation). On the other hand, the findings of Mullineaux (1980), Gramlich (1983), and Baghestani and Noori (1988) indicate that expectations are not purely adaptive.<sup>2</sup> In addition, Fuhrer (1997) and Roberts (1998) demonstrate that expectations can be described as a mixture of rational and adaptive expectations.<sup>3</sup> Pfajfar and Santoro (2010) examine a cross section of individuals' inflation forecasts, and they find that some individuals have rational expectations, some form their expectations adaptively, while the expectations of others are based on adaptive learning and sticky information.

The assumption that expectations are not purely rational is made by Mankiw and Reis (2002) in their sticky information model. They assume that some firms (chosen randomly) operate with current information about optimal prices, while the remaining firms operate with out-of-date information, and they demonstrate that their model outperforms the sticky price model in explaining output and inflation dynamics.

In the studies that test the rationality of inflationary expectations and in the sticky information model, the variable of interest is price expectations. In contrast, the present study assumes that individuals have imperfect information about average wages. While this study differs from the rest of the literature by considering wage expectations rather than price expectations, the macroeconomic consequences of imperfect information may be more important for wage expectations than for price expectations.

Labor market outcomes are determined from the interaction between firms and workers. From the perspective of firms, it is not clear why imperfect information about the price level would significantly affect unemployment. Information on the price level is readily available from

the Bureau of Labor Statistics on a monthly basis, both for the aggregate economy and for many specific goods and services. Given the ease of accessing these statistics through the internet, it seems unlikely that firms would operate with information that is out of date. Even if firms lack perfect information about the price level, there is no obvious mechanism through which imperfect information would translate into large changes in unemployment. Mankiw and Reis consider firms' pricing and output decisions but do not consider their wage and employment decisions, and there is no reason why firms in their model would not continually set wages at their market-clearing level.

The behavior of workers is likely to be influenced more by their expectations about average wages than by their expectations about the price level. Workers make several decisions that may affect the wages set by firms: how much labor to supply, at what value to set their reservation wage, how hard to work, and how hard to look for another job. Theoretical considerations suggest that workers' labor market decisions are likely to depend more on relative wages than on real wages. The decisions concerning reservation wages (controlling for labor supply) and quits should depend on relative wages rather than on real wages, since these decisions are made by comparing wages at a given firm with opportunities elsewhere. There are two models that emphasize the effect of wages on workers' effort: the shirking model and the gift-exchange model. The relative wage is the relevant variable in the shirking model. In the gift-exchange model, effort could depend on either relative or real wages, depending on whether workers' loyalty is affected by the relationship between their pay and the market wage or by the relationship between their pay and the prices of goods and services. The one decision that unambiguously depends on real wages is labor supply. However, most studies find that the elasticity of labor supply with respect to the real wage is small.<sup>4</sup> In addition to these theoretical

arguments, empirical evidence also suggests that quits and effort depend more on workers' relative wages than on their real wages, as evidenced by the fact that real wages have increased dramatically over the past 60 years, yet we have not observed a significant decline in the quit rate or a significant increase in effort. Thus, it is reasonable to believe that workers' behavior is affected more by their expectations of average wages than by their expectations of the price level.

In addition, errors in wage expectations are probably larger than errors in price expectations. Since the Consumer Price Index is published monthly and is readily available, errors in price expectation are likely to be small. On the other hand, what matters for workers' effort and quits are their wages relative to average wages for workers in the same occupation with similar qualifications (e.g., experience and education), and this information is not easily obtainable. In fact, employers in Bewley's (1999) survey believed that their workers did not have a very precise idea about the wages at other firms.

If firms pay efficiency wages, workers' imperfect information about average wages may have significant macroeconomic consequences, since firms take into account the reaction of workers in setting wages. In particular, an efficiency wage model with imperfect information about average wages can explain nominal wage rigidity.<sup>5</sup> For example, a firm that knows its workers' expectations are partly adaptive has an incentive to adjust wages slowly in response to contractionary shocks, out of concern that adjusting wages too quickly would adversely affect its workers' effort and quit behavior. This sluggish adjustment of nominal wages would likely cause unemployment to rise. Another desirable feature of models with efficiency wages and imperfect information is that a short-run Phillips curve can be derived from the profit-maximizing behavior of firms, as demonstrated in Campbell (2010).

### III. Expectation Formation in Choosing Optimal Effort

One explanation for a positive relationship between wages and effort is the shirking model of Shapiro and Stiglitz (1984), in which a higher wage raises the cost of job loss and induces workers to exert more effort. The cost of job loss depends negatively on the wages offered by other firms, which means that workers' effort depends on the relationship between their current wages and average wages in the rest of the economy.

In the shirking model, it is generally assumed that all firms pay the same wage, which means that workers know with certainty the average wage offered by other firms. In reality, however, this is not a reasonable assumption. Wages vary across employers, even for jobs that are similar, and workers generally do not have perfect information concerning the wages offered by other firms. Lacking perfect information, workers need to form expectations about wages elsewhere in order to provide the optimal amount of effort.

A worker who forms incorrect expectations of the average wage will exert a suboptimal level of effort and suffer a utility loss. A worker overestimating average wages will exert less than optimal effort, so that on average, the loss of future earnings resulting from the increased probability of dismissal will exceed the utility gain from lower effort. A worker who underestimates average wages will suffer the opposite type of utility loss.

This section models the information acquisition activities of workers, who incur costs from acquiring information and from making decisions with imperfect information. It is assumed that wages vary across firms and that the average wage is unobserved. While the mean of the wage distribution is unknown, workers have two tools to estimate this mean. First, they can observe past average wages at no cost and can predict the mean of the wage distribution from this old information. Second, they can obtain information about current average wages through



costly activities. Workers face the tradeoff that acquiring more information is costly but that it enables them to more accurately estimate the mean of the wage distribution, resulting in effort that is closer to its optimal level.

There has been little previous work on the methods used by workers to estimate average wages, so it is not clear how they form their estimates. In addition to looking at past wages, possible ways to estimate average wages include gathering and analyzing macroeconomic data (e.g., the growth rate of money, fiscal policy, and unemployment) and obtaining information on wages at a subsample of firms (e.g., by contacting firms, talking to friends, and reading help-wanted ads).<sup>6</sup>

It is assumed that the amount of information acquired by workers is determined by the following tradeoff. Suppose a worker seeks to maximize

$$(1) \quad E[U] = \sum_{t=1}^{\infty} \frac{1}{(1+\delta)^{t-1}} [\ln(c_t) + u(e_t)]$$

$$\text{s.t.} \quad \sum_{t=1}^{\infty} \frac{1}{(1+r)^{t-1}} c_t = \sum_{t=1}^{\infty} \frac{1}{(1+r)^{t-1}} [LI_t \Pr[Emp_t] + B(1 - \Pr[Emp_t])],$$

where  $\delta$  is the discount rate,  $c_t$  is consumption in period  $t$ ,  $u(e_t)$  is the utility or disutility of effort ( $e$ ) in period  $t$ ,<sup>7</sup>  $r$  is the interest rate,  $LI_t$  is labor income in period  $t$ ,  $\Pr[Emp_t]$  is the probability that a worker is employed in period  $t$ , and  $B$  represents the income of an unemployed worker. If it is assumed that the discount rate equals the interest rate (i.e.,  $\delta=r$ ), Campbell (2006) demonstrates that lifetime utility can be approximated by the equation

$$E[U] \approx \sum_{t=1}^{\infty} \frac{1}{(1+r)^{t-1}} [\ln(Y_t) + u(e_t)],$$

where  $Y_t$  represents income in period  $t$  (either labor income or unemployment benefits).

The expected utility of a worker employed at his or her current firm can be calculated by first considering the expected utility of a worker employed at the average outside firm and the expected utility of an unemployed worker. The expected utility of a worker employed elsewhere ( $V^{EO}$ ) can be expressed as

$$(2) \quad V_t^{EO} = \bar{w}_t + u(\bar{e}) + \frac{1}{1+r}[1-q-PD(\bar{e})]E_t V_{t+1}^{EO} + \frac{1}{1+r}[q+PD(\bar{e})]E_t V_{t+1}^{UN},$$

where  $\bar{w}_t$  is the log of the average wage,  $\bar{e}$  represents the effort the worker would provide at the average firm,  $q$  is the probability of a separation not related to the worker's effort,  $PD(\bar{e})$  is the probability of a worker being dismissed because of poor effort, and  $V^{UN}$  is the expected utility of an unemployed worker.

Assuming that separations occur at the end of a period and that hires occur at the beginning of a period, the expected present value of an unemployed individual's utility is

$$(3) \quad V_t^{UN} = (1-h) \left( \bar{w}_t + b - 1 + \frac{1}{1+r} E_t V_{t+1}^{UN} \right) + h V_t^{EO},$$

where  $h$  is the probability of an unemployed worker being hired,  $b$  is the ratio between unemployment benefits and the average wage, and  $\bar{w}_t + b - 1$  approximates the income accruing to an unemployed individual.<sup>8</sup>

Suppose that wages are expected to grow, on average, at the rate of  $g$  in each future period. Then  $E_t V_{t+1}^{EO} = V_t^{EO} + g$  and  $E_t V_{t+1}^{UN} = V_t^{UN} + g$ . From these relationships and from (2) and (3), the steady-state solution for  $V^{UN}$  is

$$V^{UN} = \frac{1}{r} \left[ (1+r)[\bar{w}_t + b - 1] + g + \frac{(1+r)^2 h[u(\bar{e}) + 1 - b]}{r + h + (1-h)[q + PD(\bar{e})]} \right].$$

The expected present value of utility for a worker employed at his or her current firm is

$$(4) \quad V_t = w_t + u(e_t) + \frac{1}{1+r} [1 - q - PD(e_t)] E_t V_{t+1} + \frac{1}{1+r} [q + PD(e_t)] E_t V_{t+1}^{UN},$$

where  $w_t$  is the log of the wage at a worker's current firm and  $PD(e_t)$  is the probability of dismissal for a worker who provides effort equal to  $e_t$ . If it is assumed that workers expect wages at their current firm to grow on average by  $g$  (the expected rate of aggregate wage growth), then  $E_t V_{t+1} = V_t + g$ . In a steady state,  $V$  is equal to

$$(5) \quad V = \frac{(1+r)[w_t + u(e_t)] + g}{r + q + PD(e_t)} + \frac{q + PD(e_t)}{r + q + PD(e_t)} V^{UN}$$

$$= \frac{(1+r)[w_t + u(e_t)]}{r + q + PD(e_t)} + \frac{g}{r}$$

$$+ \frac{q + PD(e_t)}{r + q + PD(e_t)} \frac{1}{r} \left[ (1+r)[\bar{w}_t + b - 1] + \frac{(1+r)^2 h[u(\bar{e}) + 1 - b]}{r + h + (1-h)[q + PD(\bar{e})]} \right].$$

Thus, the present value of a worker's utility can be expressed as  $V(w_t, \bar{w}_t, e_t)$ . This expression for  $V$  takes into account the disutility of effort and the utility of consumption, which depends on income in the current and future periods. Effort has two opposing effects on lifetime utility. An increase in effort reduces current utility, but it also reduces the probability of dismissal, which increases expected future income and thus raises expected lifetime consumption. Optimal effort is determined from the condition  $dV/de_t = 0$ .

Suppose that a worker does not know with certainty the true mean of the wage distribution ( $\bar{w}_t$ ) and forms estimates of this mean, denoted by  $\bar{w}_t^e$ . Let  $e(\bar{w}_t)$  represent effort when a worker knows that the mean of the wage distribution is  $\bar{w}_t$  and  $e(\bar{w}_t^e)$  represent effort

when the worker estimates that the mean is  $\bar{w}_t^e$ . The utility loss resulting from incorrect expectations of average wages can be expressed as

$$(6) \quad VL(\bar{w}_t^e - \bar{w}_t) = V(w_t, \bar{w}_t, e(\bar{w}_t)) - V(w_t, \bar{w}_t, e(\bar{w}_t^e)),$$

with  $VL(0) = 0$  and  $VL'' > 0$ .

The assumption that  $VL'' > 0$  means that the utility loss rises at an increasing rate as the difference between  $\bar{w}_t^e$  and  $\bar{w}_t$  increases.<sup>9</sup>

The total expected utility loss (*TEUL*) equals the cost of acquiring information plus the expected utility loss from imperfect information, so that

$$(7) \quad TEUL = C(I) + E[VL(\bar{w}_t^e - \bar{w}_t)],$$

where  $C(I)$  is the cost of acquiring information. Approximating equation (7) with a second-order Taylor expansion around the point where  $\bar{w}_t^e = \bar{w}_t$  yields

$$TEUL \approx C(I) + E[VL(0)] + E[VL'(0)(\bar{w}_t^e - \bar{w}_t)] + \frac{1}{2} E[VL''(0)(\bar{w}_t^e - \bar{w}_t)^2].$$

This expression can be simplified by making the substitutions  $VL(0)=0$  and  $VL'(0) = 0$  and by using the fact that  $VL''(0)$  is a constant.<sup>10</sup> Accordingly,

$$(8) \quad TEUL \approx C(I) + \frac{1}{2} VL''(0) E[(\bar{w}_t^e - \bar{w}_t)^2],$$

where  $E[(\bar{w}_t^e - \bar{w}_t)^2]$  depends negatively on the quality of the worker's information. A worker acquires the amount of information that minimizes this expected utility loss.

To obtain an expression for  $E[(\bar{w}_t^e - \bar{w}_t)^2]$  it is necessary to make assumptions about demand and wage setting. It is assumed that demand follows an autoregressive process,

$$(9) \quad m_t = \rho m_{t-1} + \varepsilon_t,$$

where  $m_t$  is the log of demand, which is assumed to be unobserved, and  $\varepsilon_t$  is a white noise error with variance  $\sigma_\varepsilon^2$ . It is also assumed that the wage set by the  $i$ th firm is described by the relationship,

$$(10a) \quad w_{it} = \lambda m_t + (1 - \lambda) \bar{w}_t^e + \eta_t + \xi_{it},^{11}$$

where  $\bar{w}_t^e$  is the average worker's expectation of the average wage,  $\eta_t$  is a common white noise error with variance  $\sigma_w^2$ , and  $\xi_{it}$  is an idiosyncratic white noise error that sums to 0 across firms.

The presence of this idiosyncratic error term means that wages vary between firms. Aggregating across firms gives the following expression for the average wage ( $\bar{w}_t$ ):

$$(10b) \quad \bar{w}_t = \lambda m_t + (1 - \lambda) \bar{w}_t^e + \eta_t.$$

Taking expectations of both sides of (10b) yields

$$(11) \quad \bar{w}_t^e = m_t^e,$$

where  $m_t^e$  is the average worker's expectation of demand.

Three ways that workers may estimate average wages are observing past wages, collecting and analyzing macroeconomic data, and sampling wages at other firms. The next subsection develops a model of expectations formed by observing lagged wages. Then the

following subsections combine the information contained in lagged wages with costly information, either from macroeconomic variables or sampling wages at other firms.

### *Observing lagged wages*

This subsection models the wage expectations of a worker who uses information on lagged average wages. To estimate current average wages from past average wages, a worker is assumed to use information on average wages in period  $t-1$  ( $\bar{w}_{t-1}$ ) and his or her own prior expectations of average wages in period  $t-1$ . Workers are likely to differ in their wage expectations, and it will be assumed that individual wage expectations are normally distributed around the average wage expectation ( $\bar{w}_{t-1}^e$ ). Since the wage expectation of the average worker is what determines the firm's optimal wage, this subsection considers the average wage expectation. (The results are the same if we consider the wage expectation of an individual worker, except that  $\sigma_w^2$  would be replaced by  $\sigma_w^2 + (1-\lambda)\text{Var}(\zeta)$  in the equations below, where  $\zeta$  is white noise error term representing the difference between the average wage expectation and the individual's wage expectation.) The average worker obtains information about  $m_{t-1}$  from the relationship in (10b), which can be rewritten as

$$(12) \quad \bar{w}_{t-1} - (1-\lambda)\bar{w}_{t-1}^e = \lambda m_{t-1} + \eta_{t-1}.$$

A worker's expectation of demand (and thus his or her expectations of average wages) can be viewed as being determined from a Kalman filtering process, with (9) and (12) being the relevant equations. Assuming that the economy is in a steady-state equilibrium, a worker's estimate of  $m_t$  can be expressed as

$$(13) \quad m_t^e = \left( \rho - \lambda \frac{\rho v^2 \lambda}{\lambda^2 v^2 + \sigma_w^2} \right) m_{t-1}^e + \frac{\rho v^2 \lambda}{\lambda^2 v^2 + \sigma_w^2} [\bar{w}_{t-1} - (1-\lambda) \bar{w}_{t-1}^e]$$

$$= \frac{\rho \sigma_w^2 m_{t-1}^e + \rho v^2 \lambda [\bar{w}_{t-1} - (1-\lambda) \bar{w}_{t-1}^e]}{\lambda^2 v^2 + \sigma_w^2},$$

where

$$(14) \quad v^2 = \frac{1}{2} \sigma_\varepsilon^2 + \frac{-(1-\rho^2) \sigma_w^2 + \sqrt{(1-\rho^2)^2 (\sigma_w^2)^2 + 2\lambda^2 (1+\rho^2) \sigma_w^2 \sigma_\varepsilon^2 + \lambda^4 (\sigma_\varepsilon^2)^2}}{2\lambda^2}.$$

The variable  $v^2$  represents the variance of the difference between  $m_t^e$  and the true value of  $m_t$ .

Appendix A derives this expression for  $v^2$  and demonstrates that

$$\frac{\partial v^2}{\partial \sigma_\varepsilon^2} > 0.$$

Substituting the relationship  $m_t^e = \bar{w}_t^e$  (from (11)) into (13) yields

$$(15a) \quad m_t^e = \rho \frac{\lambda(\lambda-1)v^2 + \sigma_w^2}{\lambda^2 v^2 + \sigma_w^2} m_{t-1}^e + \frac{\rho v^2 \lambda}{\lambda^2 v^2 + \sigma_w^2} \bar{w}_{t-1}, \quad \text{and}$$

$$(15b) \quad \bar{w}_t^{e,K} = \rho \frac{\lambda(\lambda-1)v^2 + \sigma_w^2}{\lambda^2 v^2 + \sigma_w^2} \bar{w}_{t-1}^{e,K} + \frac{\rho v^2 \lambda}{\lambda^2 v^2 + \sigma_w^2} \bar{w}_{t-1},$$

where  $\bar{w}_t^{e,K}$  represents the expectations formed with the Kalman filtering process. Solving (15b)

recursively results in the following solution for wage expectations:

$$(16) \quad \bar{w}_t^{e,K} = \frac{\rho v^2 \lambda}{\lambda^2 v^2 + \sigma_w^2} \sum_{j=1}^{\infty} \left( \rho \frac{\lambda(\lambda-1)v^2 + \sigma_w^2}{\lambda^2 v^2 + \sigma_w^2} \right)^{j-1} \bar{w}_{t-j}.$$

The sum of coefficients on lagged average wages is

$$\sum \text{coeff's on } \bar{w}_{t-j} = \frac{\rho v^2 \lambda}{(1-\rho)(\lambda^2 v^2 + \sigma_w^2) + \rho v^2 \lambda},$$

which equals 1 if  $\rho=1$  (i.e., demand follows a random walk).

The coefficients on the various lags of past average wages depend on  $\rho$ ,  $\lambda$ ,  $\sigma_w^2$ , and  $v^2$ ; in addition,  $v^2$  depends on  $\rho$ ,  $\lambda$ ,  $\sigma_w^2$ , and  $\sigma_\varepsilon^2$ . Thus, the coefficient on each lagged value of average wages depends on  $\rho$ ,  $\lambda$ ,  $\sigma_w^2$ , and  $\sigma_\varepsilon^2$ , which means that these coefficients are determined by the model's microeconomic parameters.

For a worker who uses only information from lagged average wages to predict the current average wage, the forecast error is

$$\begin{aligned} (17) \quad \bar{w}_t - \bar{w}_t^{e,K} &= \lambda m_t + (1-\lambda)\bar{w}_t^{e,K} + \eta_t - \bar{w}_t^{e,K} = \lambda(m_t - \bar{w}_t^{e,K}) + \eta_t \\ &= \lambda(m_t - m_t^e) + \eta_t, \end{aligned}$$

and the variance of the forecast error is

$$(18) \quad E[\bar{w}_t - \bar{w}_t^{e,K}]^2 = \lambda^2 v^2 + \sigma_w^2.$$

Workers can supplement the free information provided by lagged average wages by acquiring costly information that yields additional information about the mean of the wage distribution. Two types of costly information are considered: published macroeconomic data and a sample of wages at other firms.

#### *Supplementing information on lagged wages by acquiring macroeconomic information*

First, workers can collect and analyze macroeconomic variables such as unemployment, fiscal policy, and money growth in order to form expectations of nominal demand. Let  $I$



represent the amount of information acquired and  $c$  represent the cost of each unit of information, so that the total cost equals  $cI$ . This information is assumed to provide a noisy, but unbiased and serially uncorrelated estimate of demand ( $m_t^{e,I}$ ), so that

$$(19) \quad m_t^{e,I} = m_t + v_{I,t},$$

where  $v_{I,t}$  is a serially uncorrelated error term with variance  $\sigma_I^2$ . It is assumed that  $\sigma_I^2$  depends negatively on the amount of information acquired, that more information reduces this variance at a decreasing rate, and that this variance is positive if  $I < \infty$ . These assumptions imply that  $\sigma_I^2$  can be expressed as  $\sigma_I^2(I)$  with  $d\sigma_I^2/dI < 0$ ,  $d^2\sigma_I^2/dI^2 > 0$ , and  $\sigma_I^2(I) > 0$  for  $I < \infty$ .

There are two ways to model the behavior of a worker who uses information from lagged average wages and macroeconomic variables to form expectations of average wages. One is to incorporate the macroeconomic information into the Kalman filter described in the previous subsection. In this case, a worker's expectation of the average wage depends on lagged average wages and lagged macroeconomic information. The other approach is to model the information obtained from lagged wages (using the Kalman filter) and the information from the macroeconomic variables as separate processes and to use signal extraction to obtain the worker's wage expectation. The advantage of the first approach is that a consistent framework is used to model the information obtained from both sources. However, a drawback of this approach is that expectational errors from the macroeconomic variables may exhibit serial correlation, since expectations depend on lagged macroeconomic information in this framework, so past forecast errors may be correlated with current forecast errors. Also, contemporaneous information is not taken into account in the integrated framework. The second approach lacks the internal consistency of the first, but it has the advantages that expectation errors can be

modeled as serially uncorrelated and that contemporaneous information is taken into account.<sup>13</sup> Because there are advantages and disadvantages with both modeling strategies, both are presented. In the main body of the text, the information from lagged average wages and the macroeconomic variables are treated as separate processes with optimal signal extraction, while the integrated approach is discussed in Appendix B. The results are qualitatively the same, including the effects of demand variability and information costs on the amount of information acquired and on the degree to which expectations are rational vs. adaptive.<sup>14</sup>

Suppose the information gathered from lagged wages and the macroeconomic variables are treated as separate processes. Let  $\bar{w}_t^{e,I}$  represent the wage expectation of a worker who uses only information from macroeconomic variables to predict the current average wage. For this worker, the forecast error is

$$\begin{aligned}\bar{w}_t - \bar{w}_t^{e,I} &= \lambda m_t + (1 - \lambda)\bar{w}_t^{e,I} + \eta_t - \bar{w}_t^{e,I} = \lambda(m_t - \bar{w}_t^{e,I}) + \eta_t \\ &= \lambda(m_t - m_t^{e,I}) + \eta_t,\end{aligned}$$

and the variance of the forecast error is

$$(20) \quad E[\bar{w}_t - \bar{w}_t^{e,I}]^2 = \lambda^2 \sigma_I^2(I) + \sigma_w^2.$$

If a worker uses both lagged average wages and macroeconomic information to form his or her estimate of the average wage, signal extraction implies that the worker's expectation of the average wage can be expressed as

$$(21) \quad \bar{w}_t^e = \frac{[\lambda^2 \nu^2 + \sigma_w^2] \bar{w}_t^{e,I} + [\lambda^2 \sigma_I^2(I) + \sigma_w^2] \bar{w}_t^{e,K}}{\lambda^2 \sigma_I^2(I) + 2\sigma_w^2 + \lambda^2 \nu^2},$$

and that the variance of the difference between  $\bar{w}_t$  and  $\bar{w}_t^e$  is

$$(22) \quad E[\bar{w}_i - \bar{w}_i^e]^2 = \frac{[\lambda^2 \sigma_i^2(I) + \sigma_w^2][\lambda^2 \nu^2 + \sigma_w^2]}{\lambda^2 \sigma_i^2(I) + 2\sigma_w^2 + \lambda^2 \nu^2}.$$

Substituting (22) into (8) yields the following expression for the total expected utility loss:

$$TEUL \approx cI + \frac{1}{2}VL''(0) \frac{[\lambda^2 \sigma_i^2(I) + \sigma_w^2][\lambda^2 \nu^2 + \sigma_w^2]}{\lambda^2 \sigma_i^2(I) + 2\sigma_w^2 + \lambda^2 \nu^2}.$$

The optimal amount of information is determined from the first-order condition,

$$(23) \quad \frac{dTEUL}{dI} = 0 = c + \frac{1}{2}VL''(0) \frac{\lambda^2[\lambda^2 \nu^2 + \sigma_w^2]^2}{[\lambda^2 \sigma_i^2(I) + 2\sigma_w^2 + \lambda^2 \nu^2]^2} \frac{d\sigma_i^2}{dI},$$

and the condition for an interior minimum to exist is

$$\frac{d^2TEUL}{dI^2} = \frac{1}{2} \frac{VL''(0)\lambda^2[\lambda^2 \nu^2 + \sigma_w^2]^2}{[\lambda^2 \sigma_i^2(I) + 2\sigma_w^2 + \lambda^2 \nu^2]^2} A > 0,$$

where

$$A = \frac{d^2\sigma_i^2}{dI^2} - \frac{2\lambda^2}{\lambda^2 \sigma_i^2(I) + 2\sigma_w^2 + \lambda^2 \nu^2} \left( \frac{d\sigma_i^2}{dI} \right)^2.$$

An interior minimum exist if  $A > 0$ . If this condition for an interior solution is not satisfied, workers acquire no information and  $I=0$ . If an interior solution does exist, the following comparative static results are obtained:

$$\frac{dI}{dc} = -2 \frac{[\lambda^2 \sigma_i^2(I) + 2\sigma_w^2 + \lambda^2 \nu^2]^2}{VL''(0)\lambda^2[\lambda^2 \nu^2 + \sigma_w^2]^2 A} < 0,$$

and

$$\frac{dI}{d\sigma_\varepsilon^2} = -2 \frac{\lambda^2[\lambda^2 \sigma_i^2(I) + \sigma_w^2]}{[\lambda^2 \sigma_i^2(I) + 2\sigma_w^2 + \lambda^2 \nu^2][\lambda^2 \nu^2 + \sigma_w^2]A} \frac{\partial \nu^2}{\partial \sigma_\varepsilon^2} \frac{d\sigma_i^2}{dI} > 0.$$

Thus, workers acquire less information as information becomes more costly and acquire more information as the variance of demand increases.<sup>15</sup>

Equation (21) can be expressed as

$$(24) \quad \bar{w}_t^e = \omega \bar{w}_t^{e,I} + (1 - \omega) \bar{w}_t^{e,K}, \quad \text{where}$$

$$(25) \quad \omega = \frac{\lambda^2 v^2 + \sigma_w^2}{\lambda^2 \sigma_I^2 (I) + 2\sigma_w^2 + \lambda^2 v^2}.$$

Accordingly, workers' expectations are a weighted average of the expectations formed with costly information and the expectations obtained from the Kalman filtering process. The expectations formed with costly information about macroeconomic variables have the characteristics of rational expectations,<sup>16</sup> and the expectations formed with the Kalman filter have the characteristics of adaptive expectations. Thus, workers' expectations of average wages can be viewed as a weighted average of rational and adaptive expectations, with  $\omega$  representing the weight placed on rational expectations. The effects of  $c$  and  $\sigma_\varepsilon^2$  on the degree to which expectations are rational are

$$\frac{d\omega}{dc} = -\frac{(\lambda^2 v^2 + \sigma_w^2) \lambda^2}{[\lambda^2 \sigma_I^2 (I) + 2\sigma_w^2 + \lambda^2 v^2]^2} \frac{d\sigma_I^2}{dI} \frac{dI}{dc} < 0, \quad \text{and}$$

$$\frac{d\omega}{d\sigma_\varepsilon^2} = \lambda^2 \frac{[\lambda^2 \sigma_I^2 (I) + \sigma_w^2] \frac{dv^2}{d\sigma_\varepsilon^2} - [\lambda^2 v^2 + \sigma_w^2] \frac{d\sigma_I^2}{dI} \frac{dI}{d\sigma_\varepsilon^2}}{[\lambda^2 \sigma_I^2 (I) + 2\sigma_w^2 + \lambda^2 v^2]^2} > 0.$$

Thus, expectations become more adaptive as the cost of information increases and become more rational as the variance of demand increases.

The degree to which expectations are rational vs. adaptive may depend on the actions of policymakers. For example, suppose that policymakers credibly announce a disinflationary monetary policy. This public announcement would be a source of free information about nominal demand, and this free information would result in expectations that are more rational. In addition, if policymakers announce a reduction in the growth rate of demand that is a significant departure from its previous value, individuals would view this as an increase in the variance of demand, also resulting in expectations that are more rational and less adaptive.

*Supplementing information on lagged wages by sampling wages at other firms*

A second way for workers to acquire additional information about the mean of the wage distribution is by randomly sampling wages at other firms. It is assumed that the log of wages is normally distributed with a variance of  $\sigma_{wD}^2$ . The prior mean and variance are assumed to equal  $\bar{w}_t^{e,K}$  and  $\lambda^2 v^2 + \sigma_w^2$ , respectively, based on the Kalman filtering process described earlier. In this case, closed-form solutions can be obtained for the amount of information acquired and for the value of  $\omega$ . Let  $I$  represent the number of wages sampled,  $c$  represent the cost of each observation, and  $\bar{w}_t^{e,I}$  represent the average of the wages sampled. The difference between  $\bar{w}_t^{e,I}$  and  $\bar{w}_t$  should be unbiased, serially uncorrelated, and unpredictable to other agents. Under the assumption that  $\bar{w}_t^{e,K}$  and  $\lambda^2 v^2 + \sigma_w^2$  are the workers' prior mean and variance, the mean and variance of the posterior distribution are

$$(26) \quad \bar{w}_t^e = \frac{(\lambda^2 v^2 + \sigma_w^2) I \bar{w}_t^{e,I} + \sigma_{wD}^2 \bar{w}_t^{e,K}}{\sigma_{wD}^2 + (\lambda^2 v^2 + \sigma_w^2) I}, \quad \text{and}$$

$$(27) \quad E[\bar{w}_t - \bar{w}_t^e]^2 = \frac{\sigma_{WD}^2 (\lambda^2 \nu^2 + \sigma_w^2)}{\sigma_{WD}^2 + (\lambda^2 \nu^2 + \sigma_w^2) I}.$$

In this case, the total expected utility loss is

$$TEUL \approx cI + \frac{1}{2} VL''(0) \frac{\sigma_{WD}^2 (\lambda^2 \nu^2 + \sigma_w^2)}{\sigma_{WD}^2 + (\lambda^2 \nu^2 + \sigma_w^2) I},$$

and the optimal amount of information is determined from the first-order condition,

$$(28) \quad \frac{dTEUL}{dI} = 0 = c - \frac{1}{2} VL''(0) \frac{\sigma_{WD}^2 (\lambda^2 \nu^2 + \sigma_w^2)^2}{[\sigma_{WD}^2 + (\lambda^2 \nu^2 + \sigma_w^2) I]^2}.$$

Solving this equation for  $I$  yields

$$(29) \quad I = \sqrt{\frac{\sigma_{WD}^2 VL''(0)}{2c}} - \frac{\sigma_{WD}^2}{\lambda^2 \nu^2 + \sigma_w^2}, \quad \text{with the boundary condition that } I \geq 0.$$

As in the previous case, workers acquire less information as the cost of acquiring information increases and acquire more information as aggregate demand becomes more variable.

Workers' expectations can be expressed as

$$(30) \quad \bar{w}_t^e = \omega \bar{w}_t^{e,I} + (1 - \omega) \bar{w}_t^{e,K},$$

where

$$(31) \quad \omega = \frac{(\lambda^2 \nu^2 + \sigma_w^2) I}{\sigma_{WD}^2 + (\lambda^2 \nu^2 + \sigma_w^2) I}.$$

Substituting (29) into (31) yields

$$(32) \quad \omega = 1 - \frac{1}{\lambda^2 \nu^2 + \sigma_w^2} \sqrt{\frac{2\sigma_{WD}^2 c}{VL''(0)}}, \quad \text{with the boundary condition that } \omega \geq 0.$$

Since the expectations formed by sampling wages have the characteristics of rational expectations, (30) implies that expectations are a weighted average of rational and adaptive expectations, with  $\omega$  again measuring the degree to which expectations are rational. From (32),  $\omega$  depends negatively on the cost of sampling wages and depends positively on  $\nu^2$ , which depends positively on  $\sigma_\varepsilon^2$ . As before, the positive dependence of  $\omega$  on  $\sigma_\varepsilon^2$  means that expectations become more rational as the variance of demand increases.

The dependence of  $\omega$  on the variance of demand may provide an explanation for Lucas's (1973) finding that countries with greater variability in inflation experience smaller increases in output in response to nominal demand shocks. As demonstrated in this section, expectations become relatively more rational and relatively less adaptive as the variance of demand increases. As expectations become more rational and less adaptive, Campbell (2010) shows that the Phillips curve becomes steeper, and Campbell (2009) shows that the aggregate supply curve becomes steeper, which means that a given nominal demand shock has a smaller effect on real output.

#### **IV. Expectations Formation in Choosing Optimal Job Search Intensity**

Section III develops a model in which workers acquire information about average wages in choosing their level of effort. Another decision workers make is how much time to spend looking for a different job, and this decision also depends on the ratio of their own wages to average wages elsewhere. Workers need to estimate the mean of the wage distribution to determine how much time to devote to job search, and their process of information acquisition can be modeled in the same way as in Section III. In fact, the job search decision is probably

more important for macroeconomic outcomes than the effort decision described in Section III (based on the shirking model), since survey evidence suggests that theories involving turnover are much more relevant than theories involving shirking in explaining the rigidity of wages.<sup>17</sup>

In deciding how much job search to undertake, workers face the tradeoff that job search is costly, but that it may enable them to find a job that will yield higher income in the future. The present value of a worker's expected lifetime utility can be denoted  $V(w_t, \bar{w}_t, s_t)$ , where  $w_t$  and  $\bar{w}_t$  are defined as in Section III, and  $s_t$  measures the worker's job search intensity. If  $s(\bar{w}_t)$  represents the optimal amount of search when a worker knows that the mean of the wage distribution is  $\bar{w}_t$  and  $s(\bar{w}_t^e)$  represents the optimal amount of search when the worker estimates that the mean is  $\bar{w}_t^e$ , the utility loss resulting from imperfect expectations of average wages is

$$VL(\bar{w}_t^e - \bar{w}_t) = V(w_t, \bar{w}_t, s(\bar{w}_t)) - V(w_t, \bar{w}_t, s(\bar{w}_t^e)).$$

Using a model similar to the one developed in Section III, it can be demonstrated that, in choosing how much time to devote to job search, workers' expectations of average wages will likely be at least partly adaptive.

Workers will quit their present jobs if they find a more attractive job at a different firm. The probability that a worker quits depends on two factors: 1) the difference between the worker's current wage and the actual mean of the wage distribution, and 2) the worker's search intensity. The worker's search intensity matters because, controlling for the first factor, the probability of finding one of the jobs that offers a higher wage depends on how hard he or she searches. Accordingly, the probability of a worker quitting can be expressed as

$$q = f(w_t - \bar{w}_t, s(\bar{w}_t^e)).$$



Since quits depends on job search intensity, and since job search depends on  $\bar{w}_t^e$ , workers' quit propensities depend on their expectations of average wages, as well as on the actual average wage. If workers' wage expectations are partly adaptive, the fact that quits are a function of average wage expectations means that quits depend on lagged average wages.

Campbell (1995) finds support for the hypothesis that quits depend partly on lagged average wages. In this study, quit rates in 2-digit SIC manufacturing industries are regressed on current and lagged industry wages and on current and lagged values of average manufacturing wages. Industry wages have a negative effect on quits, while average manufacturing wages have a positive effect. The hypothesis that industry wages and average manufacturing wages have an equal (but opposite) long-run effect on industry quits cannot be rejected. However, it is found that industry quit rates respond almost immediately to industry wages, but respond with a relatively long lag to average manufacturing wages, suggesting that expectations of average wages are partly adaptive.

## V. Conclusion

This study develops a model in which workers can obtain costly information that enables them to more accurately predict the mean of the aggregate wage distribution; they then use this information to choose their levels of effort and job search. It is demonstrated that workers' expectations do not necessarily satisfy the criteria for rational expectations and that expectations are likely to be at least partly adaptive. The degree to which expectations are rational vs. adaptive depends on the cost of acquiring information and on the variability of demand.

The main contributions of this work lie in highlighting the importance of workers' wage expectations and in identifying the benefits of information about average wages. This information is valuable to workers because it enables them to make better decisions about their

effort and job search. In addition, this study shows how microeconomic parameters affect the degree to which expectations are rational vs. adaptive and how these parameters affect the coefficients on the various values of lagged wages. The model developed here provides a rationale for efficiency wage modeling in which expectations about average wages are partly adaptive.

This study considers workers' effort and job search decisions, since these choices are related to efficiency wage theory. Another labor market decision that could be modeled in a similar framework is the reservation wage of unemployed workers. If unemployed individuals have imperfect information about average wages, they may use a process similar to that described in Sections III and IV to estimate the mean of the wage distribution. In this case, their reservation wages will depend partly on lagged average wages, which means they may exhibit money illusion in deciding whether to accept a given job offer.

This study also provides an alternative explanation for Lucas's finding that nominal demand shocks have a smaller effect on real output in countries with greater inflation variability. It is demonstrated that workers' expectations become relatively more rational and relatively less adaptive as the variability of demand increases. As expectations become relatively more rational, the Phillips curve and the aggregate supply curve become steeper, so that nominal shocks have a smaller effect on real output.

One implication of this study is that more work should be done on the issue of how workers estimate wages elsewhere. While there has been a great deal of research on the formation of price expectations, there has been relatively little (if any) research on the formation of wage expectations. This study discusses several methods that workers may use in estimating average wages, such as sampling wages at other firms and analyzing macroeconomic variables.

In addition, workers may employ other methods to estimate average wages. Investigating the ways that workers predict average wages would yield insights into the response of effort and quits to macroeconomic shocks. Since firms take the response of workers into account in setting wages, such research may help economists understand the reasons for sluggish nominal wage adjustment.

## Appendix A

Derivation of  $v^2$  :

$$v^2 = \rho^2 v^2 + \sigma_\varepsilon^2 - \frac{\rho^2 v^4 \lambda^2}{\lambda^2 v^2 + \sigma_w^2} \quad 18$$

$$\lambda^2 v^4 + v^2 \sigma_w^2 = \rho^2 v^2 \sigma_w^2 + \lambda^2 v^2 \sigma_\varepsilon^2 + \sigma_\varepsilon^2 \sigma_w^2$$

$$\lambda^2 v^4 + (\sigma_w^2 - \rho^2 \sigma_w^2 - \lambda^2 \sigma_\varepsilon^2) v^2 - \sigma_\varepsilon^2 \sigma_w^2 = 0$$

$$v^2 = \frac{\lambda^2 \sigma_\varepsilon^2 - (1 - \rho^2) \sigma_w^2 + \sqrt{(1 - \rho^2)^2 \sigma_w^4 - 2(1 - \rho^2) \sigma_w^2 \lambda^2 \sigma_\varepsilon^2 + \lambda^4 \sigma_\varepsilon^4 + 4\lambda^2 \sigma_w^2 \sigma_\varepsilon^2}}{2\lambda^2}$$

$$v^2 = \frac{1}{2} \sigma_\varepsilon^2 + \frac{-(1 - \rho^2) \sigma_w^2 + \sqrt{(1 - \rho^2)^2 (\sigma_w^2)^2 + 2\lambda^2 (1 + \rho^2) \sigma_w^2 \sigma_\varepsilon^2 + \lambda^4 (\sigma_\varepsilon^2)^2}}{2\lambda^2}.$$

Derivation of  $\partial v^2 / \partial \sigma_\varepsilon^2$  :

$$\frac{\partial v^2}{\partial \sigma_\varepsilon^2} = \frac{1}{2} + \frac{\left[ (1 - \rho^2)^2 (\sigma_w^2)^2 + 2\lambda^2 (1 + \rho^2) \sigma_w^2 \sigma_\varepsilon^2 + \lambda^4 (\sigma_\varepsilon^2)^2 \right]^{\frac{1}{2}} [\lambda^2 (1 + \rho^2) \sigma_w^2 + \lambda^4 \sigma_\varepsilon^2]}{2\lambda^2} > 0.$$

## Appendix B

In modeling the information obtained from lagged average wages and from the macroeconomic variables with a Kalman filter, the relevant equations are (9), (12), and (19):

$$m_t = \rho m_{t-1} + \varepsilon_t,$$

$$\bar{w}_{t-1} - (1 - \lambda)\bar{w}_{t-1}^e = \lambda m_{t-1} + \eta_{t-1}, \quad \text{and}$$

$$m_t^{e,I} = m_t + \nu_{I,t}.$$

In a Kalman filtering framework, these equations can be expressed as

$$(B1a) \quad m_t = \rho m_{t-1} + \varepsilon_t, \quad \text{and}$$

$$(B1b) \quad \begin{bmatrix} \bar{w}_{t-1} - (1 - \lambda)\bar{w}_{t-1}^e \\ m_{t-1}^I \end{bmatrix} = \begin{bmatrix} \lambda \\ 1 \end{bmatrix} m_{t-1} + \begin{bmatrix} \nu_{w,t-1} \\ \nu_{I,t-1} \end{bmatrix}.$$

From Ljungqvist and Sargent (2004, p. 123, eq. 5.6.2 and 5.6.3a) a worker's expectation of  $m_t$  is

$$(B2) \quad m_t^e = \left( \rho - \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} \lambda \\ 1 \end{bmatrix} \right) m_{t-1}^e + \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} \bar{w}_{t-1} - (1 - \lambda)\bar{w}_{t-1}^e \\ m_{t-1}^I \end{bmatrix} \\ = (\rho - \lambda K_1 - K_2) m_{t-1}^e + K_1 \bar{w}_{t-1} - K_1 (1 - \lambda) \bar{w}_{t-1}^e + K_2 m_{t-1}^I,$$

where

$$\begin{aligned} \begin{bmatrix} K_1 & K_2 \end{bmatrix} &= \rho \nu^2 \begin{bmatrix} \lambda & 1 \end{bmatrix} \left( \begin{bmatrix} \lambda \\ 1 \end{bmatrix} \nu^2 \begin{bmatrix} \lambda & 1 \end{bmatrix} + \begin{bmatrix} \sigma_w^2 & 0 \\ 0 & \sigma_I^2 \end{bmatrix} \right)^{-1} \\ &= \rho \nu^2 \begin{bmatrix} \lambda & 1 \end{bmatrix} \begin{bmatrix} \lambda^2 \nu^2 + \sigma_w & \lambda \nu^2 \\ \lambda \nu^2 & \nu^2 + \sigma_I^2 \end{bmatrix}^{-1} \end{aligned}$$

$$\begin{aligned}
&= \rho v^2 [\lambda \quad 1] \frac{1}{\lambda^2 v^2 \sigma_l^2 + \sigma_w^2 v^2 + \sigma_w^2 \sigma_l^2} \begin{bmatrix} v^2 + \sigma_l^2 & -\lambda v^2 \\ -\lambda v^2 & \lambda^2 v^2 + \sigma_w^2 \end{bmatrix} \\
&= \frac{[\rho v^2 \lambda \sigma_l^2 \quad \rho v^2 \sigma_w^2]}{\lambda^2 v^2 \sigma_l^2 + \sigma_w^2 v^2 + \sigma_w^2 \sigma_l^2}.
\end{aligned}$$

Combining (11) and (B2) yields the relationships,

$$(B3) \quad m_t^e = (\rho - K_1 - K_2)m_{t-1}^e + K_1 \bar{w}_{t-1} + K_2 m_{t-1}^l, \quad \text{and}$$

$$(B4) \quad \bar{w}_t^e = (\rho - K_1 - K_2)\bar{w}_{t-1}^e + K_1 \bar{w}_{t-1} + K_2 m_{t-1}^l.$$

By solving (B4) recursively, the following solution for wage expectations is obtained:

$$(B5) \quad \bar{w}_t^e = K_1 \sum_{j=1}^{\infty} (\rho - K_1 - K_2)^{j-1} \bar{w}_{t-j} + K_2 \sum_{j=1}^{\infty} (\rho - K_1 - K_2)^{j-1} m_{t-j}^l.$$

From Ljungqvist and Sargent (2004, p. 123, eq. 5.6.3b), the value of  $v^2$  is

$$v^2 = \rho^2 v^2 + \sigma_\varepsilon^2 - \rho^2 v^4 [\lambda \quad 1] \left( \begin{bmatrix} \lambda \\ 1 \end{bmatrix} v^2 [\lambda \quad 1] + \begin{bmatrix} \sigma_w^2 & 0 \\ 0 & \sigma_l^2 \end{bmatrix} \right)^{-1} \begin{bmatrix} \lambda \\ 1 \end{bmatrix}$$

$$v^2 = \rho^2 v^2 + \sigma_\varepsilon^2 - \rho^2 v^4 [\lambda \quad 1] \frac{\begin{bmatrix} v^2 + \sigma_l^2 & -\lambda v^2 \\ -\lambda v^2 & \lambda^2 v^2 + \sigma_w^2 \end{bmatrix}}{\lambda^2 v^2 \sigma_l^2 + \sigma_w^2 v^2 + \sigma_w^2 \sigma_l^2} \begin{bmatrix} \lambda \\ 1 \end{bmatrix}$$

$$v^2 = \rho^2 v^2 + \sigma_\varepsilon^2 - \frac{\rho^2 v^4 (\lambda^2 \sigma_l^2 + \sigma_w^2)}{\lambda^2 v^2 \sigma_l^2 + \sigma_w^2 v^2 + \sigma_w^2 \sigma_l^2}$$

$$v^2 \lambda^2 v^2 \sigma_I^2 + v^2 \sigma_w^2 v^2 + v^2 \sigma_w^2 \sigma_I^2 = \rho^2 v^2 \sigma_w^2 \sigma_I^2 + \sigma_\varepsilon^2 \lambda^2 v^2 \sigma_I^2 + \sigma_\varepsilon^2 \sigma_w^2 v^2 + \sigma_\varepsilon^2 \sigma_w^2 \sigma_I^2$$

$$[\lambda^2 \sigma_I^2 + \sigma_w^2](v^2)^2 + [\sigma_w^2 \sigma_I^2 - \rho^2 \sigma_w^2 \sigma_I^2 - \lambda^2 \sigma_\varepsilon^2 \sigma_I^2 - \sigma_\varepsilon^2 \sigma_w^2]v^2 - \sigma_\varepsilon^2 \sigma_w^2 \sigma_I^2 = 0$$

$$v^2 = \frac{(\lambda^2 \sigma_I^2 + \sigma_w^2) \sigma_\varepsilon^2 - (1 - \rho^2) \sigma_w^2 \sigma_I^2 + \sqrt{(1 - \rho^2)^2 (\sigma_w^2)^2 (\sigma_I^2)^2 - 2(1 - \rho^2) \sigma_w^2 \sigma_I^2 (\lambda^2 \sigma_I^2 + \sigma_w^2) \sigma_\varepsilon^2 + (\lambda^2 \sigma_I^2 + \sigma_w^2)^2 (\sigma_\varepsilon^2)^2 + 4(\lambda^2 \sigma_I^2 + \sigma_w^2) \sigma_\varepsilon^2 \sigma_w^2 \sigma_I^2}}{2(\lambda^2 \sigma_I^2 + \sigma_w^2)}$$

$$v^2 = \frac{1}{2} \sigma_\varepsilon^2 + \frac{-(1 - \rho^2) \sigma_w^2 \sigma_I^2 + \sqrt{(1 - \rho^2)^2 (\sigma_w^2)^2 (\sigma_I^2)^2 + 2(1 + \rho^2) \sigma_w^2 \sigma_I^2 (\lambda^2 \sigma_I^2 + \sigma_w^2) \sigma_\varepsilon^2 + (\lambda^2 \sigma_I^2 + \sigma_w^2)^2 (\sigma_\varepsilon^2)^2}}{2(\lambda^2 \sigma_I^2 + \sigma_w^2)}$$

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## Footnotes

<sup>1</sup> Conlisk (1988) discusses theoretical reasons for why rational behavior may result in expectations that are not unbiased. Under the assumption that it is costly to form accurate expectations of next period's price, he demonstrates that optimal forecasts may be a weighted average of an unbiased estimate obtained from agents' costly optimization activities and a "free estimator," which may be determined from an adaptive expectations process. However, Conlisk does not identify the benefits of information.

<sup>2</sup> Mullineaux (1980) and Gramlich (1983) find that, controlling for lagged inflation, other macroeconomic variables have significant effects on expectations, which suggests that economists and households use more than just past inflation to predict future inflation. Additional evidence that expectations are not purely adaptive comes from Baghestani and Noori (1988), who find that survey respondents predict inflation more accurately than an ARIMA model, implying that their expectations depend on more than just lagged inflation.

<sup>3</sup> Fuhrer (1997) estimates Phillips curves in which expected inflation depends on a weighted average of lagged inflation and actual future inflation. He can reject the hypothesis that expectations are completely rational, but cannot reject the hypothesis that they are completely adaptive. However, he demonstrates that inflation dynamics are predicted more accurately by a model with mixed rational and adaptive expectations than by a model with completely adaptive expectations. Roberts (1998) shows that survey forecasts of inflation can be explained by a model in which part of the population has rational expectations and the rest has adaptive expectations.

<sup>4</sup> Blundell and MaCurdy (1999) report on previous estimates of labor supply elasticities in Tables 1 and 2 of their study. The average uncompensated labor supply elasticity reported in these tables is 0.086 for men and 0.689 for married women. In addition, Card (1991, p. 22) reviews several previous studies of labor supply and concludes, "Taken together, the literature suggests that the elasticity of intertemporal substitution is surely no higher than 0.5, and probably no higher than 0.20."

<sup>5</sup> In conventional efficiency wage models, efficiency depends on the actual real or relative wage of workers, so these models are unable to explain nominal wage rigidity.

<sup>6</sup> An additional way to form expectations of average wages is to use information on expected price inflation, since wage inflation and price inflation are highly correlated. As discussed in Carroll (2003), evidence suggests that some households use the forecasts of professional forecasters to form their expectations of price inflation.

<sup>7</sup> While the overall utility of effort may be positive, the marginal utility of effort will be negative for a utility-maximizing worker.

<sup>8</sup> The expression  $\bar{w}_t + b - 1$  is approximately equal to  $\ln(\bar{W}_t) + \ln(B) - \ln(\bar{W}_t) = \ln(B)$ , where  $\bar{W}_t$  is the level of the average wage, since  $\bar{w}_t = \ln(\bar{W}_t)$ ,  $b - 1 \approx \ln(b)$ , and  $b = B / \bar{W}_t$ .

<sup>9</sup> For example, if the utility loss is represented by the quadratic equation,  $VL = \theta(\bar{w}_t^e - \bar{w}_t)^2$ , then  $VL''(0) = 2\theta$ . In the analysis in Section III,  $VL(\bar{w}_t^e - \bar{w}_t)$  is treated as a general functional form. However, an equation representing this loss can be obtained if specific assumptions are made about the utility function of workers and the probability of dismissal. In an unpublished appendix, the effort model of Campbell (2006) is used to derive a specific expression for  $VL(\bar{w}_t^e - \bar{w}_t)$ . (This appendix is available from the author upon request.)

<sup>10</sup> The fact that  $VL'(0) = 0$  is obtained from the relationship  $VL' = dVL / d(\bar{w}_t^e - \bar{w}_t) = (\partial VL / \partial e)(\partial e / \partial(\bar{w}_t^e - \bar{w}_t))$ , where  $\partial VL / \partial e = 0$  at the point  $\bar{w}_t^e = \bar{w}_t$ .

<sup>11</sup> Equation (10a) can be derived from the model of Campbell (2010). While this equation is not explicitly derived in Campbell (2010), a derivation of this equation is provided in Campbell (2008). The present study adds error terms to the models of Campbell (2008, 2010).

<sup>12</sup> See Ljungqvist and Sargent (2004, p. 123, equation 5.6.2) for an exposition of this equation.

<sup>13</sup> Which approach best describes the behavior of individuals depends on whether they take into account information beyond the most recent in forming their expectations.

<sup>14</sup> An earlier version of this paper which takes the first approach throughout is available at [http://www.niu.edu/econ/research\\_series/ImpInfo0210.pdf](http://www.niu.edu/econ/research_series/ImpInfo0210.pdf).

<sup>15</sup> Consistent with these results, Sethi and Franke (1995) develop a model in which agents can choose whether to use a costless adaptive expectations procedure or pay to acquire information that allows them to predict the true outcome with certainty. They find that agents are more likely to acquire this information when optimization costs are low or when the economy is characterized by a "high degree of exogenous variability."

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<sup>16</sup> In this study, rational expectations are defined as expectations that are unbiased, serially uncorrelated, and unpredictable to other agents.

<sup>17</sup> In Campbell and Kamlani's (1997) and Bewley's (1999) surveys of employers, respondents indicated that they viewed labor turnover as being much more important than shirking in explaining wage rigidity.

<sup>18</sup> See Ljungqvist and Sargent (2004, p. 123, equation 5.6.3b) for an exposition of this equation. In this study,  $v^2$  is equivalent to  $\Sigma$  in Ljungqvist and Sargent. The present study assumes that the economy is in a steady-state equilibrium, so that  $\Sigma_{t+1}=\Sigma_t$ .