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Abstract
This article extends the seminal work of Granger and Yoo (2002) on hidden cointegration to panel data analysis. It shows how cumulative negative and positive changes can be constructed for each panel variable. It also shows how tests similar to the augmented Dickey-Fuller tests can be implemented to find out whether the cointegration is hidden in the panel or not. An application is provided to investigate the impact of permanent positive and negative shocks in the government expenditure on the national output in a panel of three countries.

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1. Introduction

Since the pioneer work of Granger (1981) cointegration analysis has become an integral part of applied econometrics when the underlying variables are measured across time. Based on Granger's definition, cointegration occurs in a situation in which a linear combination between integrated variables has one unit root less than the integration order of the variables in the model. The variables cointegrate if and only if they have common stochastic trends that cancel each other out. There is a massive literature on cointegration indicating its due importance. Cointegration analysis is important in empirical research in order to avoid spurious results based on a regression model. It is also important for analyzing the long-run relationships between the underlying variables combined with the short-run adjustment mechanism.

Testing for cointegration was initiated by Engle and Granger (1987) via implementing residual based tests for cointegration. More robust tests where suggested by Phillips (1987) as well as Phillips and Ouliaris (1990). Multivariate tests for cointegration were developed by Johansen (1988, 1991), Johansen and Juselius (1990) as well as Stock and Watson (1988). The idea to conduct tests for unit roots within a panel system originates from Quah (1994). Tests for panel cointegration were suggested by Pedroni (1999, 2004), Kao (1999) and Westerlund (2007), among others.

In all previous literature on cointegration testing, there was no separation between the impact of negative and positive shocks until Granger and Yoo (2002) introduced the concept of hidden cointegration for time series data. It is hidden in the sense that there might not be cointegration between the variables in the original format but when the impact of positive shocks is separated from the impact of negative shocks then cointegration might exist between the components of the variables. The aim of this article is to extend the concept of hidden cointegration to panel data analysis. The suggested tests in this paper are applied to investigate the impact of contractionary as well as expansionary fiscal policy on the economic performance in a panel consisting of Denmark, Norway and Sweden.

The article continues as the following. Section 2 introduces hidden panel cointegration analysis. Section 3 provides an application. The last section concludes the article. Finally, an appendix at the end of the article provides a simple mathematical example that shows the necessary condition for cointegration in a panel system.

2. **Hidden Panel Cointegration**

Consider the following variables that are integrated of the first degree, with the resultant solution for each that is found by the recursive approach:[1]

$$y_{i,t} = y_{i,t-1} + e_{i1,t} = y_{i,0} + \sum_{j=1}^{t} e_{i1,j}$$

[1] For the simplicity of expression we concentrate on the case where the panel model consists of two variables. The results can however be generalized in the sense that more independent variables can be included in the model.

$$x_{i,t} = x_{i,t-1} + e_{i2,t} = x_{i,0} + \sum_{j=1}^{t} e_{i2,j}$$

For i=1, …, m. Where m signifies the cross-sectional dimension (which is two in this particular case) and e is the disturbance term that is assumed to be a white noise process. The positive and negative shocks for each panel variable are defined as $e_{i1,t}^{+} \coloneqq Max\left(e_{i1,t}, 0\right)$, $e_{i2,t}^{+} := Max\left(e_{i2,t}, 0\right)$, $e_{i1,t}^{-} := Min\left(e_{i1,t}, 0\right)$ and $e_{i2,t}^{-} := Min\left(e_{i2,t}, 0\right)$. Using these results, the following can be obtained:

$$y_{i,t}^{+} = y_{i,0}^{+} + e_{i1,t}^{+} = y_{i,0} + \sum_{j=1}^{t} e_{i1,j}^{+}$$

$$x_{i,t}^{+} = x_{i,0}^{+} + e_{i2,t}^{+} = x_{i,0} + \sum_{j=1}^{t} e_{i2,j}^{+}$$

$$y_{i,t}^{-} = y_{i,0}^{-} + e_{i1,t}^{-} = y_{i,0} + \sum_{j=1}^{t} e_{i1,j}^{-}$$

$$x_{i,t}^{-} = x_{i,0}^{-} + e_{i2,t}^{-} = x_{i,0} + \sum_{j=1}^{t} e_{i2,j}^{-}$$

Assume that our dependent variable is y, and then the two potential panel cointegration equations for the components can be defined as

$$y_{i,t}^{+} = \alpha_i^{+} + \beta_i^{+} x_{i,t}^{+} + e_{i,t}^{+} \tag{1}$$

$$y_{i,t}^{-} = \alpha_i^{-} + \beta_i^{-} x_{i,t}^{-} + e_{i,t}^{-} \tag{2}$$

The positive cumulative shocks are cointegrated in the panel if $e_{i,t}^{+}$ is stationary. Likewise, the negative cumulative shocks are cointegrated in the panel if $e_{i,t}^{-}$ is stationary.[2]

[2] It should be mentioned that other combinations are also possible. Such as, cumulative positive changes of y as a function of cumulative negative changes of x, as well as cumulative negative changes of y as a function of cumulative positive changes of x.

There is potentially a battery of the tests available in the literature that can be used for testing whether $e_{i,t}^{+}$ as well as $e_{i,t}^{-}$ is stationary or not. However, the well-known augmented Dickey-Fuller (ADF) test is the simplest one that can be used for this purpose. Assume that we wish to test for cointegration in the panel model (1). Then, the panel ADF test equation is the following:

$$e_{i,t}^{+} = \rho^{+} e_{i,t-1}^{+} + \sum_{l=1}^{k} \gamma_i^{+} \Delta e_{i1,t-l}^{+} + w_{i,t}^{+} \tag{3}$$

The optimal lag order l can be determined by minimizing an information criterion. The null hypothesis of no cointegration between the positive components is $\rho^{+} = 1$. It is also possible to allow for deterministic components such as individual drifts and trends in equation (3) if necessary. Based on the results provided by Kao (1999), the following test statistics can be used to test the null hypothesis of no panel cointegration:

$$ADF = \frac{t_{\rho^{+} + \sqrt{6m} \times \frac{\sigma_v}{2\sigma_{0v}}}}{\sqrt{\frac{\sigma_{0v}^2}{2\sigma_v^2} + \frac{3\sigma_v^2}{10\sigma_{0v}^2}}} \tag{4}$$

Where $t_{\rho^{+}}$ is the *t*-statistic for parameter ρ^{+} in equation (3). The variance is estimated as $\sigma_v^2 = \sigma_{e_1^{+}}^2 - \frac{\sigma_{e_1^{+},e_2^{+}}^2}{\sigma_{e_2^{+}}^2}$, and the long-run variance is estimated as $\sigma_{0v}^2 = \sigma_{0e_1^{+}}^2 - \frac{\sigma_{0e_1^{+},e_2^{+}}^2}{\sigma_{0e_2^{+}}^2}$.

Let $u_{it} = \begin{bmatrix} e_{i1,t}^{+} \\ e_{i2,t}^{+} \end{bmatrix}$. The variance-covariance for u_{it} is estimated as

$$\Sigma = \begin{bmatrix} \sigma_{e_1^{+}}^2 & \sigma_{e_1^{+},e_2^{+}} \\ \sigma_{e_1^{+},e_2^{+}} & \sigma_{e_2^{+}}^2 \end{bmatrix} = \frac{1}{mT} \sum_{i=1}^{m} \sum_{t=1}^{T} u_{it} u_{it}'$$

The long-run variance-covariance matrix is estimated via the kernel estimation approach as

$$\Omega = \begin{bmatrix} \sigma^2_{0e_1^+} & \sigma_{0e_1^+,e_2^+} \\ \sigma_{0e_1^+,e_2^+} & \sigma^2_{0e_2^+} \end{bmatrix}$$

$$= \frac{1}{m}\sum_{i=1}^{m}\left[\frac{1}{T}\sum_{t=1}^{T} u_{it}u'_{it} + \frac{1}{T}\sum_{t=1}^{T}\kappa(\tau/b)\sum_{t=\tau+1}^{T}(u_{it}u'_{it-\tau} + u_{it-\tau}u'_{it})\right]$$

Where κ is representing the kernel function and b is the bandwidth.

The ADF test as presented in equation (4) has a standard normal distribution asymptotically. For a proof see Kao (1999). To test for stationarity in the linear combination between negative components as presented in equation (2), a similar ADF test can be conducted. Other combinations are also possible.[3] A Gauss code for constructing cumulative sums of positive and negative changes of each variable for each cross sectional unit in the panel is available on request from the author.[4]

3. An Application

The suggested test for hidden panel cointegration is applied to investigating the long-run relationship between government spending and economic performance in Denmark, Norway and Sweden. Quarterly data is used during the period 1993-2010. The source of the data is the statistical bureau of each country. In order to capture real effects, the variables are expressed at constant prices. The cumulative sums of positive and negative shocks were constructed based on the procedure presented in the previous section. Prior to testing for panel cointegration, panel unit root tests were implemented by using the Im, Pesaran and Shin (2003) test. The results are presented in Table 1, which show that each panel variable has one unit root.

[3] It should be mentioned that it is also possible to test for hidden panel cointegration using other tests such as seven residual based testes suggested by Pedroni (1999, 2004) as well as panel version of the multivariate Johansen (1991) test as developed by Maddala and Wu (1999) based on the Fisher (1932) principle of deriving a combined test using the individual test results.

[4] Hatemi-J (2010) suggests tests for asymmetric causality using time series data. Tests for asymmetric panel causality are available in Hatemi-J (2011a). For estimating asymmetric impulse response functions and variance decompositions see Hatemi-J (2011b).

Table 1: The Results of Panel Unit Root Tests.

VARIABLE	H_0: I(1), H_1: I(0)	H_0: I(2), H_1: I(1)
S	0.1485	<0.0001
Y	0.1314	<0.0001
S^{+}	0.9999	<0.0001
Y^{+}	0.9494	<0.0001
S^{-}	0.9998	<0.0001
Y^{-}	0.9991	<0.0001

Notes

The denotation *S* stands for the log of government spending and *Y* is representing the log of GNP. The Im, Pesaran and Shin (2003) test is used to test for panel unit root. P-values are presented.

Given that each variable in the panel has one unit root, it is crucial to conduct tests for panel cointegration. The results of these tests based on equation (4) are presented in Table 2, which indicate that there is no cointegration between government spending and output in the panel sample for these three countries. Tests for cointegration between cumulative positive and negative components also indicate no panel cointegration.

Table 2: The Results of Panel Cointegration Tests.

VARIABLES IN THE MODEL	H_0: I(1), H_1: I(0)
$(Y\ \ ,S\ \ )$	0.5122 (8)
(Y^{+},S^{+})	1.7930 (8)
(Y^{-},S^{-})	-0.3572 (5)
(Y^{-},S^{+})	0.5724 (8)
(Y^{+},S^{-})	2.6156 (8)

Notes

The values in the parenthesis indicate the bandwidth in the kernel estimation. The null hypothesis of no panel cointegration is rejected at the 5% significance level if the estimated test value is lower than -1.64.

4. **Conclusions**

The aim of this article is to extend the hidden cointegration tests of time series data as developed by Granger and Yoo (2002) to hidden cointegration tests of panel data. It is shown how cumulative sums of positive and negative changes can be constructed. An augmented Dickey-Fuller test for a panel system can be implemented to test the null hypothesis of no panel cointegration between different components of the underlying variables. A user friendly Gauss algorithm is produced to transform the panel variables into their respective components.

The suggested test procedure is applied to investigating the impact of government spending on economic output in a panel of three countries—namely Denmark, Norway and Sweden. The results show that there is no panel cointegration between these variables or their components. This surprising finding of no panel cointegration between GNP and government spending in Denmark, Norway and Sweden might provide empirical support for the existence of the Ricardo equivalence theorem in these countries. According to this theorem, economic agents are rational in the sense that they will conclude any increase in the government expenditure now means an equivalent increase in the taxes in the future. Hence, they will adjust their behavior in such a way that there will be no real effects of increases in the government expenditure on the economy.

In this particular case, the same cointegrational inference was obtained in the panel regardless if the impact of positive shocks is separated from the impact of negative shocks or not. This might be indicative of the strength of the empirical findings. Nevertheless, future applications of the test might indicate cases in which the panel cointegration is indeed hidden. In addition, separating the impact of positive shocks from the negative ones might be informative *per se*, in order to capture the potential asymmetry that might prevail.

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Appendix

An Example regarding Panel Cointegration

Let us be more explicit about panel cointegration between two integrated variables by a simple example. Consider the following two non-stationary panel variables:

$$Z_{i,t} = Z_{i,t-1} + \varepsilon_{i1,t} \tag{A1}$$

$$W_{i,t} = Z_{i,t-1} + \varepsilon_{i2,t} \tag{A2}$$

Where $\varepsilon_{i1,t}$ and $\varepsilon_{i2,t}$ are two white noise error terms. Assuming that the initial values are zero, continuous substitutions give the following solutions:

$$Z_{i,t} = \sum_{j=1}^{t} \varepsilon_{i1,j} \tag{A3}$$

$$W_{i,t} = \sum_{j=1}^{t-1} \varepsilon_{i1,j} + \varepsilon_{i2,t} \tag{A4}$$

By taking the first difference of each variable, we obtain

$$\Delta Z_{i,t} = Z_{i,t} - Z_{i,t-1} = \sum_{j=1}^{t} \varepsilon_{i1,j} - \sum_{j=1}^{t-1} \varepsilon_{i1,j} = \varepsilon_{i1,t} \tag{A5}$$

$$\begin{aligned} \Delta W_{i,t} = W_{i,t} - W_{i,t-1} &= \sum_{j=1}^{t} \varepsilon_{i1,j} + \varepsilon_{i2,t} - \left(\sum_{j=1}^{t-1} \varepsilon_{i1,j} + \varepsilon_{i2,t-1} \right) \\ &= \varepsilon_{i1,t} + \varepsilon_{i2,t} - \varepsilon_{i2,t-1} \end{aligned} \tag{A6}$$

That is, each variable becomes stationary after taking the first difference. Hence, the variables are integrated of the first order, denoted I(1). Now the question is if these two panel variables are cointegrated. Denote Y_t as the difference between the two variables, i.e.

$$Y_{i,t} = W_{i,t} - Z_{i,t} = \sum_{j=1}^{t} \varepsilon_{i1,j} - \left(\sum_{j=1}^{t-1} \varepsilon_{i1,j} + \varepsilon_{i2,t} \right) = \varepsilon_{i1,t} - \varepsilon_{i2,t} \tag{A7}$$

This difference is clearly a stationary process. That is, a linear combination of the non-stationary variables is stationary in the panel, which in turn means that the variables cointegrate with (1.0, -1.0) as cointegrating vector. It should be noted that the variables cointegrate because their stochastic trend cancels each other out. In another word, the variables have a common stochastic trend. This is the case in the hidden panel cointegration analysis also. In order to have hidden panel cointegration, there must be at least one common stochastic trend between the cumulative components of the positive or negative shocks in the panel.