Crowdfunding: disintermediated investment banking

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Abstract

This paper introduces crowdfunding as a concept and model for the evolution of investment banking. Crowdfunding, an application of crowdsourcing, is defined as one party's attempt to finance a project by offering three types of investment opportunities to potential investors. The investment opportunities are donations, passive investments, and active investments. From this foundation I develop a model in which interdependent agents operate in a dynamic, discrete setting. Potential investors decide whether or not to invest in one of three opportunities each period while the entrepreneur sets the parameters of the game to maximize the probability of successful financing. I then simulate the model to analyze the effects changes in key parameters have on the results of the game.
1 Introduction

Crowdfunding is a financing model that answers three fundamental questions about how our economy operates:

1. Who decides which projects deserve financing?
2. How can we guarantee they represent the projects' target markets?
3. What can we do to systematically reduce entrepreneurs' exposure to the risk that they fail to cover their start-up costs?

Complex problems require simple solutions. What if customers provide financing? What if customers make the decisions and provide financing contingent on the delivery of product? That is the core of crowdfunding.

To develop the concept to a degree sufficient for modeling Section 1.1 discusses the context in which crowdsourcing and crowdfunding are relevant. Then it provides uncommonly general definitions of the two concepts. Section 1.2 is a review of what research has been done to date on both crowdfunding and its parent, crowdsourcing. With this understanding Section 1.3 analyzes several prominent implementations of crowdfunding, including EpicStep\(^1\), SellABand\(^2\), and Kickstarter\(^3\). What follows in Section 2 is the creation of a model largely based on how Kickstarter operates.

Section 3 uses the model to perform a series of simulations to improve our understanding of crowdfunding in practice and gain some insights into what entrepreneurs should do to use the platform successfully. Sections 4 and 5 conclude and offer suggestions for further research.

1.1 Context and Definitions

The internet's ubiquity in everyday life connects humanity like never before. Every aspect of life is being retooled by modern technology. However, three frictions restrict the digitization of our world: 1) certain social norms, 2) obsolete and/or inefficient laws and regulations, and 3) the dependence on something restricted by at least one of the other two frictions.

Patterns arise from industries embracing technology. Service industries generalize, standardize, and become disintermediated, low-touch read/write platforms. Common examples include travel, retail and publishing. Product industries specialize, automate, and outsource as many functions as possible. Greater efficiency, flexibility, and focus are now necessary to survive. The automotive and consumer electronics industries are excellent examples.

\(^1\)www.EpicStep.com
\(^2\)www.SellABand.com
\(^3\)www.KickStarter.com
With technology comes the efficient access to customers and often the use of crowdsourcing—a concept and process of value creation in existence and constant use for millennia—to reach greater levels of generalization of output. The advent and mass-adoption of the internet drastically increased crowdsourcing’s potential and scope of application; however, the concept pre-exists modern technology. A common example is academic research—society’s means of accumulating knowledge by rewarding\(^4\) many individuals for small contributions. Therefore, crowdsourcing should be defined as follows:

**Definition 1** “Crowdsourcing is the process of one party progressing towards a goal by requesting and receiving small contributions from many parties in exchange for a form of value to those parties.”

Crowdfunding is a subset of crowdsourcing, so it should be defined in the same form:

**Definition 2** “Crowdfunding is the process of one party financing a project by requesting and receiving small contributions from many parties in exchange for a form of value to those parties.”

1.2 Literature To Date

1.2.1 Crowdsourcing

Jeff Howe’s 2006 Wired article, *The Rise of Crowdsourcing* [7] is widely acknowledged as the first published use of the term crowdsourcing. In said article, Howe defines crowdsourcing as a democratic twist on outsourcing. To show its wide scope of application he explains its affect on three economic perspectives: the professional, the packager, and the tinkerer. He also touches on one of the functional components of crowdsourcing, the network.

“The most efficient networks are those that link to the broadest range of information, knowledge, and experience.”

This point brings us to Yochai Benkler’s *The Wealth of Networks* [4], the seminal analysis of networks and their role in our rapidly changing world. Benkler perceives a transition towards a decentralized industrial information economy where human capacity is the primary source of scarcity.

“I place at the core of the shift the technical and economic characteristics of computer networks and information. These provide the pivot for the shift toward radical decentralization of production.”

To this end Benkler posits the possible reversal of two trends that defined the last century of economic development: centralization and commercialization.

\(^4\)The value received by researchers may be respect, praise, grants or teaching positions.

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“‘Centralization’ is a particular response to the problem of how to make the behavior of many individual agents cohere into an effective pattern or achieve an effective result. Its primary attribute is the separation of the locus of opportunities for action from the authority to choose the action that the agent will undertake.”

Commercialization is a logical reaction to centralization as agents migrate activities to the formal economy so they can take advantage of efficiencies derived from said centralization. For instance, consider the effects of urbanization on local informal activities. Excluding slums near large cities in developing countries, nearly all valuable activities are performed commercially. As opposed to a time when more was done informally as part of an active community.

Oppositely, Benkler persuades the reader of looming economic decentralization.

“‘Decentralization’ describes conditions under which the actions of many agents cohere and are effective despite the fact that they do not rely on reducing the number of people whose will counts to direct effective actions.”

Disintermediation is the complete decentralization of decision making; the middleman’s extinction and reincarnation as entirely passive infrastructure. For a number of industries the economics of crowdsourcing make disintermediation seemingly inevitable5.

Technology is driving economic and social change...

“...by lowering the capital costs required for effective individual action, these technologies have allowed various provisioning problems to be structured in forms amenable to decentralized production based on social relations, rather than through markets or hierarchies.”

Many such provisioning problems are being restructured this way; enough for researchers to pick up on the trend and analyze how corporations’ relationships with customers are evolving. Specifically, there is a thread of research being done in Germany. Piller, Moeslein, Stotko (2004) [14] investigate the value of integrating customers into the value chain. They define the ‘economies of integration’ as coming from three sources:

“(1) from postponing some activities until an order is placed, (2) from more precise information about market demands and (3) from the ability to increase loyalty by directly interacting with each customer.”

Such ‘economies of integration’ have proven to be immensely powerful in content industries, where consumers can now vote for outcomes in tv shows, join a global conversation about any live event and even vote on or contribute reporting of current events. Another famous example is Ikea, who requires users to construct furniture for themselves after delivery.

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5Publishing, Accessory Design, Reporting, Content Curation, Retail Banking, Research, Media Creation, Product Reviews, Travel, and many many more.
Kleemann, Voß, Rieder (2008) [10] interpret this shift “as the emergence of a new type of consumer: the working customer.” They go on to explain that:

“Customers are no longer the classic kings to be waited upon, but are more like co-workers, who assume specific elements of a production process that remains ultimately under the control of a commercial enterprise.”

Their Working Customer Thesis [15] is largely parallel to the rest of the crowdsourcing literature while offering a slant towards its affect on customers.

1.2.2 Crowdfunding

Thusfar the reader may have noticed the sources’ dates–most are from the last five years. The term crowdsourcing was coined only five years ago. Conceptually, crowdfunding is a subset of crowdsourcing. However, practically, crowdfunding is far more difficult to implement because of the various legal, technical and social complexities one can imagine arising from an online project financing platform. One crucial piece, trust, and the web of reputation, is in its infancy [8]. As explained earlier, an industry’s evolution is often restricted by dependencies on other industries, and the web of reputation is restricting crowdfunding’s growth. Nevertheless, its potential has been recognized and a discussion of what literature exists follows.

We must first differentiate between ex post facto and ex ante crowdfunding, as first described by Kappel (2009) [9]. Ex post facto crowdfunding is where investors or customers finance a completed product. Relevant examples are ‘pay what you want’ business models⁶. Ex ante crowdfunding is the more interesting model. In this case investors finance a project that has not been completed. They directly make the project’s realization possible by participating in the capital distribution process. The analysis in this paper focuses on ex ante crowdfunding.

Kevin Lawton and Dan Marom provide a valuable discussion of crowdfunding’s development and potential in The Crowdfunding Revolution [12]. They offer both positive and normative discussions of crowdfunding. For instance, they touch on its unique ability to scale.

“The power of crowds is not just gaining access to ideas, it’s also very much about using the collective wisdom as a sorting and leading indicator mechanism, which allows for scalability.”

For more insight into how the crowd acts as a sorting mechanism we return to Benkler [4]:

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⁶Customers can set their own price according to their perceived valuation of the product or service. Recently bands such as Radiohead and Nine Inch Nails, as well as some authors, have used this business model to cover their costs. ‘Pay what you want’ is now frequently combined with luxury offerings for those customers with above average financial means and valuations.
“Filtering, accreditation, synthesis, and salience are created through a system of peer review by information affinity groups, topical or interest based.”

Imagine a vast network of peer groups, in a number of different forms, working together to curate and apply meaning to the seemingly insurmountable amount of content created on the internet every day. That is roughly how content flows through the internet and that is how something can move from obscurity to the nightly news in an afternoon.

The intuition behind such a network’s value goes beyond filtering, it extends to the wisdom of the crowd. Hayek explained the concept in a 1945 essay [6].

“Practically every individual has some advantage over all others because he possesses unique information of which beneficial use might be made, but of which use can be made only if the decisions depending on it are left to him or are made with his active cooperation.”

This is what brings value to such a network’s recommendations, and this is why Lawton and Marom believe affinity groups make crowdfunding a viable ‘long tail’ proposition7.

“Crowdfunding is in many ways, such a ‘long tail’ proposition: there are so many early-stage projects which don’t receive the attention of bigger investors, but yet the aggregate amount of potential projects is hard to fathom.”

But how does crowdfunding actually function? For this we turn to Schwienbacher and Larralde (2010) [16]. In a chapter they contributed to the Handbook for Entrepreneurial Finance they deconstruct entrepreneurs’ value propositions as belonging to three potential categories: donations, passive investments, and active investments. Donations do not yield the donor any tangible reward, as opposed to investments which do entitle investors to a reward of some form. Active investments differ from passive investments by granting investors the ability to directly affect the results of the entrepreneur’s project.

Many model the variety of value propositions as a form of price discrimination [3]. Interestingly Lambert and Schwienbacher (2010) [11] performed an empirical analysis and found that there is a significant negative correlation between the numbers of passive investments and active investments for a given project.

“This lends to expect that rewards and control are substitutes; investors may require more rewards if they cannot be involved in the happening of the initiative.”

Researchers have identified two other factors contributing to the success or failure of crowdfunding instances. Ward and Ramachandran (2010) [17] model the importance of peer effects on the contribution patterns of crowdfunding.

7As described in Chris Andersen’s 2004 Wired article [2].
“We find that investors are more influenced by information aggregating devices, such as top-5 popularity lists and by the information provided by projects in blog updates than by more granular information sources, possibly due to information overload.”

In their model information aggregating devices are the source of peer effects and these effectively provide potential investors with information that compares investment opportunities. Whereas their definition of network effects and the wisdom of the crowd is limited to the information signals created by investing trends in a single project. Thus, mechanisms such as Benkler’s affinity groups [4] are more important as a tool for determining the value of an item in a network than the value signals created by agents interacting with said item. When looking at the big picture of the network this makes sense; however, the information aggregation devices do not function without the presence of such information to aggregate – without the presence of these value signals. Their research thus reaffirms the power of large networks in applying meaning and verifying the value of content by showing that items on the surface of a network are more meaningful than items hidden deeper within, because, those items on the surface reach the surface by way of the network’s ability to filter and verify; thus creating value to the individual.


“The average distance between artists and investors is about 3,000 miles, suggesting a reduced role for spatial proximity.”

From experience we know this to be the case with activities that migrate to the internet. However, they note that geography does still play some role. In fact, local investments tend to occur earlier in the financing period and are less affected by other investors’ activities. This suggests that social relations, such as friends and family, are still an important part of any crowdfunding initiative.

Entrepreneurs now have a new alternative to traditional financing methods. Although crowdfunding is far from being commonly utilized by the general populace we have some understanding of the value it provides. Thus we can begin to compare its efficacy to traditional methods, and that is exactly what Belleflamme, Lambert, Schwienbacher (2010) [3] do in their paper Crowdfunding: Tapping the Right Crowd. They consider crowdfunding from an industrial organization perspective and develop two models. In the first, they associate crowdfunding with pre-ordering and price discrimination and find that:

“In situations where a firm can use crowdfunding and pre-sales to induce self-selection of high paying consumers, crowdfunding is preferred over external funding if the discount factor ($\delta$) is large enough and the fixed cost of production ($K$) is not too large.”

In their second model they identify crowdfunding as an entrepreneur’s attempt to inform consumers of their product’s value. Said model leads the authors to conclude that
crowdfunding can provide entrepreneurs with the incentive to increase quality and operate as a non-profit; i.e. it is often preferable for entrepreneurs more interested in quality and sales quantity rather than profit to use crowdfunding as opposed to traditional financing.

1.3 Current Implementations

Where is crowdfunding implemented today and who uses it? I identify three crowdfunding platforms and analyze their differences.

EpicStep.com is a donation platform for financing billboards. Users can propose billboards with locations, potential designs, and expected costs. Then other users make small contributions to make the billboards of their dreams realities. One notable success is a WikiLeaks billboard in Los Angeles.8

SellABand.com is a full-fledged implementation with donations, passive investments, and active investments. The scope is limited to independent musicians seeking funding to finish albums. A unique feature is the option to share revenue from album sales with investors. Passive and active investments are more general forms of debt and equity investments. Thus features such as revenue sharing may one day bridge the two in practice.

KickStarter.com has an unlimited scope and offers all three investment types. It is currently targeted towards creative projects as those generally require smaller initial investments; however, full-fledged entrepreneurial ventures have been launched from Kickstarter. Some noteworthy examples are TikTok9, an accessory that turns the latest iPod Nano into a wristwatch, Diaspora10, a decentralized online communication infrastructure project, and Minecraft: The Story of Mojang11, a feature-length documentary on a very successful indie computer game. These three projects raised $941,718.00, $200,641.00, and $210,297.00 respectively. We strongly recommend browsing Kickstarter as it is an important source of inspiration for the model that follows.

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8http://epicstep.com/campaign/95/support-wikileaks-los-angeles/
9http://kck.st/al8N3p
10http://kck.st/9QC2zk
11http://kck.st/hfds6N
2 The Model

2.1 Game Structure

The model is structured as a dynamic game in a discrete setting with interdependent agents. Time is denoted $t \in [0, T], T_d \text{ s.t. } T_d \geq T$. There are two agent types: the entrepreneur and the investors. At $t = 0$ the entrepreneur realizes all exogenous parameters and sets all endogenous parameters. Furthermore, the investors realize their independent and normally distributed valuations of the investment opportunities. At $t = T$ the game ends. If financing is successful (i.e. if the total amount raised $S_t \geq K$, the financing threshold) then products are delivered to all applicable investors at $t = T_d$.

2.2 Entrepreneur’s Problem

This model focuses on the interaction between a single entrepreneur and a limited number of potential investors\(^{12}\). The entrepreneur has an exogenously determined financial need $F$ required to realize his project. Therefore, he sets prices, costs, and quantity controls for investment opportunities to maximize the probability that $S_t > K \text{ s.t. } K - \text{costs} \geq F$.

2.3 Investment Opportunities

Entrepreneurs offer investors three types of investment opportunities: donations, passive investments, and active investments. Donations and passive investments grant investors the ability to passively participate in the project. Active investments grant investors the ability to actively participate in the project\(^{13}\). Passive and active participation are fixed costs to the entrepreneur denoted $c^{PI}$ and $c^{AI}$ respectively.

Passive and active investments also include a pre-order of the project’s output, a margin cost to the entrepreneur of $c^P$. Because of the lower margins earned on passive and active investments entrepreneurs frequently institute quantity controls denoted $Q^{PI}$ and $Q^{AI}$. There is also evidence that quantity controls induce consumers to purchase items at an earlier date from a fear of the product selling out\(^{13}\).

Donations, passive and active investments are offered at fixed prices $p^D$, $p^{PI}$, and $p^{AI}$.

\(^{12}\)A broader model could consider the existence of multiple entrepreneurs with differentiated projects competing for the attention of a limited pool of investors.

\(^{13}\)Some research has been done into the possibility that active investors create value for the entrepreneur by actively participating in important decisions \([15][5]\). Although my analysis does not specifically account for the possibility, it does not ignore it either. The $c^{PI}$ above could be thought of as $c^{PI}_{\text{net}}$, the net cost of active participation to the entrepreneur. I then assume this net cost is greater than zero. Thus, $c^{PI} = c^{PI}_{\text{net}} = c^{PI}_{\text{gross}} - v^{PI} > 0$. 

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2.4 Investor’s Problem

Investors try to maximize their utility by deciding between four possible actions for each they are active. An active investor’s potential actions are to donate, to passively invest, to actively invest, or to wait and do nothing until \( t + 1 \). They are denoted as \( A_{i,t} = D, PI, AI, \) or \( WAIT \). Investors can only contribute once per game, thus an active investor is one who has waited every period until now. An inactive investor is one who decided to invest in some form at a previous \( t \); his actions at every \( t \) afterwards are denoted \( A_{i,t} = DONE \).

To make their decisions, investors are aware of the current and past states of the game, all prices, costs, quantity controls, times, the financing threshold, and commonly held expectations. The state of the game at time \( t \) is the number of each investment made prior to the investor’s decision and the amount of financing raised; these measures are denoted \( q^D_t, q^{PI}_t, q^{AI}_t, \) and \( S_t = p^D q^D_t + p^{PI} q^{PI}_t + p^{AI} q^{AI}_t \).

With this information investors develop linear forecasts using a lookback period of 3. Such forecasts will become useful in the estimation of the probability that financing will be successful and of the probability that passive or active investments will ‘sell out’ within the next period. The probabilities are evaluated as follows:

\[
P_{PI}^{t,t+1} = \max \left( 0, \min \left( 1, \frac{1}{Q^{PI}} \left( \frac{q^{PI}_{t-1} - q^{PI}_{t-3}}{3} + q^{PI}_t \right) \right) \right)
\]

(1)

\[
P_{AI}^{t,t+1} = \max \left( 0, \min \left( 1, \frac{1}{Q^{AI}} \left( \frac{q^{AI}_{t-1} - q^{AI}_{t-3}}{3} + q^{AI}_t \right) \right) \right)
\]

(2)

\[
P_T^S = \max \left( 0, \min \left( 1, \frac{1}{K} \left( \frac{S_{t-1} - S_{t-3}}{3} (T - t) + S_t \right) \right) \right)
\]

(3)

Equations (1) and (2) derive the probabilities with which passive and active investments will ‘sell out’ between \( t \) and \( t + 1 \) by forecasting the quantity of investments sold at \( t + 1 \) and dividing it by the quantity limit. The result is then bounded between zero and one as any probability must be. Equation (3) derives the probability with which the total amount raised exceeds the financing threshold at \( t = T \). It does so in a manner similar to the previous two equations, though it looks directly at period \( T \) for that is the relevant period for such a probability.

The active investor must now make a decision. To do so he evaluates the three investment opportunities, subject to availability, as well as the value of waiting. The equations follow.

**Value of Donation**

\[
V_{i,t}^D = P_T^S (\beta_i c^{PI} - p^D \delta^{T-t})
\]

(4)

**Value of Passive Investment (if \( q^{PI}_t < Q^{PI} \))**

\[
V_{i,t}^{PI} = P_T^S (\beta_i c^{PI} + \alpha_i c^{PI} \delta^{T-t} - p^{PI} \delta^{T-t})
\]

(5)
Value of Active Investment (if $q_{it}^{AI} < Q^{AI}$)

$$V_{i,t}^{AI} = P_{t,T}^S \left( \gamma_i c^{AI} + \alpha_i c^P \delta_{T_d-t} - p^{AI} \delta_{T-t} \right)$$ \hspace{20pt} (6)

Value of Waiting (if $t<T$

$$V_{i,t}^W = \max \left\{ 0, \  \delta^{-1} P_{t,T}^S \left( \beta_i c^{PI} - p^P \delta^{T-t+1} \right), \  \delta^{-1} P_{t,T}^S \left( 1 - P_{t,t+1}^{PI} \right) \left( \beta_i c^{PI} + \alpha_i c^P \delta^{T_{d-t+1}} - p^{PI} \delta^{T-t+1} \right), \  \delta^{-1} P_{t,T}^S \left( 1 - P_{t,t+1}^{AI} \right) \left( \gamma_i c^{AI} + \alpha_i c^P \delta^{T_{d-t+1}} - p^{AI} \delta^{T-t+1} \right) \right\}$$ \hspace{20pt} (7)

Action (if $A_{i,t-1} =$ Wait i.e. player is active)

$$A_{i,t} = \max \left\{ V_{i,t}^D, \  V_{i,t}^{PI}, \  V_{i,t}^{AI}, \  V_{i,t}^W \right\}$$ \hspace{20pt} (8)

Upon acting all aggregate variables and expectations are immediately updated. In a sense this creates n sub-periods $\forall t \in [0, T]$ as each agent potentially faces a unique environment within a given $t$.

The game is run until $t > T$.

3 Simulations

I programmed a simulator in Excel using VBA. Using assumed parameter values I perform monte carlo simulations in batches of 100 realizations, each realization containing 28 periods (4 weeks). Then I record the average and standard deviation of $q_T^{D}$, $q_T^{PI}$, $q_T^{AI}$ and $S_T$. With this data I calculate the probability\(^{14}\) of the aforementioned aggregate values exceeding or matching their relevant quantity limits or thresholds. Moreover, I see how changes in individual parameters affect the aggregates’ means and the probability of successful financing. A summary of my results can be found in the Appendix.

\(^{14}\)Assuming the aggregates’ sample means are normally distributed by way of a central limit theorem.
4 Conclusion

“Value is created and money is made by using networks—not by trying to own yet another commodities network.”

–Pipes - The Dumber the Better by Bob Frankston

Modern technology decentralizes, and eventually, it disintermediates—giving users independent access to previously private, and often analog, networks. Relationship Banking is such a network. We have discussed how and why crowdfunding is theoretically superior; it is more efficient, it is scalable, it offers price discrimination, it is ‘wiser’, it distributes risk and it democratizes access to capital markets. Moreover, we developed and simulated a model that gives us key insights into how certain variables affect the entire round of financing. With this understand we should consider the feasibility of crowdfunding on a larger scale.

Crowdfunding is a dilated model of what disintermediated investment banking could look like. As mentioned earlier, debt and equity are simply particular forms of passive and active investments. With standardized business reporting, a modernized system of laws and regulations, and a web of reputation capital market access could become commoditized infrastructure.

As equity markets evolved so should capital markets in all shapes and sizes. Commercial and Investment Banking are operating on obsolete processes, and, they are in the unique political and economic position where they could push other industries forward to make room for their own evolution.15

5 Suggestions for Further Research

Crowdfunding requires and deserves far more academic attention than it has received to date. There are numerous opportunities for research to teach us more about the value crowdfunding provides and how it should be harnessed. On the empirical side, we need open access to data. The information is public as it is created but there is no simple way to access data from completed projects or record data from current projects. We should open a dialogue with crowdfunding platforms to persuade them of the value in granting researchers access to their data.

We can also do more to find the theoretical optimal structure of a crowdfunding platform. One possible innovation is revenue-sharing promotions. Let investors promote projects they have invested in and then receive a small part of the financing they bring in if the financing threshold is reached. Such a feature may be restricted to active investors to provide them with a greater incentive to invest as well as to add some legitimacy to the promotions (the promoter has a stake in the project as well).

15The alternative is guerilla-style innovation and competition by parties such as those mentioned in Section 1.3. Parallels can be drawn to the publishing and media distribution industries.
Kevin Lawton and Dan Marom [12] suggest giving entrepreneurs greater control over who can actively invest in their projects. They suggest integrating social networks and the reputational web into crowdfunding to give all users a better idea of who they are working with. Doing so can create a more collaborative financing environment in which who you are and the value you provide could be substitutable for financial assets.

Furthermore, the model in this paper provides numerous opportunities for improvement. One such opportunity is the addition of a dynamic number of aware investors. This would more accurately model how agents become aware of projects and create investment curves that are more balanced and possibly changing in waves as opposed to curves that spike at the boundaries of the financing period. Another opportunity is to model the entrepreneurs as more complex product-creating firms that exist to serve customers beyond potential investors. Doing so allows one to model passive and active investments as debt and equity and see how those compare to the investment opportunities modeled in this paper.
A Appendix

A.1 Simulations Log

For access to the full workbook, including all code, please visit:

http://dl.dropbox.com/u/641880/fine547bjr.zip
A.2 Key Graphs

\textbf{prob ST > K | varying Td}

\textbf{pD vs ST}

- mean S
- plus 1 std dev
- minus 1 std dev
References


