In a democracy, Bayrou would have won. Application of the Borda Fixed Point method to the 2007 French presidential elections

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■ Summary

Democratic nations are advised to have parliaments select the chief executive by the Borda Fixed Point method. The current practice of having direct popular elections using systems that have originated in history is inoptimal and actually quite undemocratic since winners are selected who don't reflect the national sentiment. The paper gives the example of the 2007 French presidential elections. Under the currently and historically grown system of run-off plurality Sarkozy got elected while the more democratic method of Borda Fixed Point would have generated Bayrou. The example uses reasonable assumptions on underlying micro preferences.

Introduction

The selection of a winner in a national election depends not only upon the person's qualities but also upon the voting method. In the 2007 French presidential elections Sarkozy was selected by the current voting method but it should not come as a surprise that under another system Bayrou would have won. What might come as a surprise however is that the current system is not really democratic and that there is a democratic system that would have favoured Bayrou.

The current French system is called run-off plurality. The two candidates with the most votes of the first round go on to the second round. The run-off plurality scheme has the property of destroying voter preferences, and a ruthless cut-off is used to achieve the simplicity of a final binary choice. An alternative is the Borda Fixed Point method. That method (i) uses all preferences, (ii) allows for some intensity of preference indicated by rank order, (iii) includes the condition that the winner should also win from its main contender, (iv) finds this main contender under the counterfactual that the winner would not partake. By the current system, Sarkozy got elected. By the Borda Fixed Point method Bayrou would have been elected.

What is relevant for the present discussion are not the candidates and their qualities but the properties of these methods. Run-off plurality is less democratic than Borda Fixed Point. In fact, the system used in France is quite undemocratic since the voters get a president that they would not prefer under optimal voting conditions.

It helps to see the wider picture. What would be a proper democratic system ? Given the widely differing opinions of voters and the long lists of candidates, it is generally advisable to use indirect representative democracy. In parliamen-

tary elections, parties advocate their preferences, the popular vote determines party sizes, whereupon the elected party professionals can arrive at the final choice of the executive by using both bargaining and more complex voting schemes. In this manner the information overload is reduced, both for voters who don't have to think about long lists of candidates and for voting mechanisms that don't have to calculate with millions of different preferences. Representative democracy also allows for bargaining that allows for optimal compromises.

This paper develops a voting example using the 2007 presidential elections in France. It is an example only, and the discussion below will emphasize the limitations of the example. For a general discussion, see Colignatus (2007). The following will discuss the data, use the voting routines of "The Economics Pack", run the two voting systems mentioned above, and will close with a conclusion that should not come as a surprise.

This paper does not discuss historical issues. They may be briefly indicated for perspective. For historical reasons, some countries still use direct presidential elections. Their methods of selection create all kinds of voting paradoxes and frequently lead to choices that do not properly reflect national sentiment. The main voting paradox is that the "popular vote" is said to serve the interests of the voters while instead that direct method goes against the voters' interest. Historically, the current suboptimal situation in national elections can best be explained by a serious deficiency by voting theorists. The explanation is stated in Colignatus (2007) and can be briefly summarized as follows. Starting with Kenneth Arrow in 1951 voting theorists have emphasized impossibilities and paradoxes in voting instead of designing systems that would work. This has resulted into a literature that is pervasive in its cynicism and anti-democractic inclination. Some authors even advise dictatorship. In a massive "betrayal by the clerks", voting theorists at the academia have entrenched points of view and are not open to the idea of optimal voting methods. As a result, democracy suffers greatly, not only in national elections but also in, say, union elections or company board elections. It helps to have examples that show avenues for improvement.

The data

For voting we need items (candidates), voter preferences, and voter weights.

The following data on France have been retrieved from Wikipedia (2007). This internet encyclopedia can be unstable, both over long periods of time (a shift of interest) and even at freak moments (a hacker), but the retrieved data fit some other reports in the media. The following table summarizes the data in a useful format.

The voting items are in the rows and the voting rounds are in the columns. The main contenders are Sarkozy and Royal, who move on to the second round. The third column gives an estimate how the Bayrou vote of the first round was split over Sarkozy and Royal in the second round, with a remainder of either blank or spoilt vote ("Zero2"). This table expresses an assumption that all votes for Le Pen in the first round went to Sarkozy in the second round, so that some 3.7 million from Bayrou's first round helped Sarkozy in the second round. The table also expresses the assumption that all votes of the smaller leftist groups in the first round went to Royal in the second round, so that some 2.6 million of Bayrou's first round helped Royal in the second round. The number of voters in the 2nd round was also a bit larger, with the difference indicated in the row "New".

DataTable

	Round1	Round2	Bayrou2nd
Sarkozy	11448663	18983408	–
LePen	3834530	–	–
Sarkozy2	–	–	3700215
Royal	9500112	16790611	–
Leftist	4695327	–	–
Royal2	–	–	2595172
Bayrou	6820119	–	–
Other	420645	–	–
Valid	36719396	35774019	–
Zero	534846	1569450	–
Zero2	–	–	524732
New	–	–	89227
Total	37254242	37343469	–

The lower rows give blank or spoilt votes and people abstaining in the first round and joining up in the second round. The available data only give a net number and it may be that even more people joined while compensating for others dropping out. Overall turnout was about 84% so potentially there can be quite some flows here. The sizable increase in blank and spoilt votes in the second round is a bit curious since the whole system is intended such that there are only two options available in that second round and thus it makes little sense to try to insert a third option - though people apparently did.

Assumptions on preferences

Above assumptions already show that we lack proper data on the preferences. The available data are at the aggregate level, give us only a first and second choice for the total. At the micro level anything might be possible. This means that the discussion here is only indicative. But the example remains sufficiently clear for our stated purpose, to show that the current French voting system is inoptimal (even though it is better than the US system). Any example suffices, also a fabricated one. For the 2007 French presidential elections we can make some reasonable assumptions so that the example becomes a bit more realistic.

The following assumptions namely seem reasonable approximations. In this section we express the preferences by lists of names of decreasing preference. (Eventually the voting routines use lists with increasing preferences.)

(1) For voters on Sarkozy in the first round, perhaps a large share would vote for LePen as second choice. How many, is unknown. It is not so meaningful to use Le Pen's score against Chirac some years ago, since that was another candidate, and some years ago. We can insert an arbitrary split, say with 2 million voters. The majority of Sarkozy's voters would put Le Pen at the bottom of the list.

```
x = 2000000;
sA = {11448663 - x,
    {Sarkozy, Bayrou, Other, Zero, Abstain, Royal, LePen, Leftist}};
sB = {x, {Sarkozy, LePen, Bayrou, Other, Zero, Abstain, Royal, Leftist}};
```

(2) The voters for Le Pen would prefer Sarkozy in the second round and Bayrou above others.

```
l = {3834530, {LePen, Sarkozy, Bayrou, Other, Zero, Abstain, Royal, Leftist}};
```

(3) The Royal backing might have some subgroups who put Sarkozy or even LePen in second place. We neglect the possibility of such subgroups however, selecting only one single preference, that puts Bayrou in third place.

```
r = {9500112, {Royal, Leftist, Bayrou, Other, Zero, Abstain, Sarkozy, LePen}};
```

(4) The extreme left might rather abstain than vote for Royal or Bayrou. But overall it seems reasonable that we assign Royal to the second position and Bayrou to the third.

```
left =
   {4695327, {Leftist, Royal, Bayrou, Other, Zero, Abstain, Sarkozy, LePen}};
```

(5) The Bayrou vote is the one that we need to split.

```
bA = {3700215, {Bayrou, Sarkozy, Other, Zero, Abstain, Royal, LePen,
Leftist}};
bB = {2595172, {Bayrou, Royal, Other, Zero, Abstain, Sarkozy, LePen,
Leftist}};
bC = {524732, {Bayrou, Zero, Abstain, Sarkozy, Royal, Other, LePen,
Leftist}};
```

(6) For the other vote on the right we can assume the most interesting case that their second vote is for Sarkozy.

```
o = {420645, {Other, Sarkozy, Bayrou, Zero, Abstain, LePen, Royal, Leftist}};
```

(7) The zero (blank or spoilt) vote of the first round is marginal and gets an arbitray allocation. But they are not motivated to vote for the main contenders, so those drop to the lowest positions.

```
z = {534846, {Zero, Abstain, Other, Leftist, Bayrou, LePen, Royal, Sarkozy}};
```

(8) The people who abstained in the first round but joined in the second round are a curious lot. There can all kinds of psychological considerations here, but the simplest one is that their second vote is a protest, where they move from abstinention to zero. Note though that this is a net number, so the true process might be more complex.

```
a = {89227, {Abstain, Zero, Other, Leftist, Bayrou, LePen, Royal, Sarkozy}};
```

In summary, we have these assumptions on the preferences, reading a high value on the left and a low value on the right.

```
PrefData = {sA, sB, l, r, left, bA, bB, bC, o, z, a}
```

9448663	{Sarkozy, Bayrou, Other, Zero, Abstain, Royal, LePen, Leftist}
2000000	{Sarkozy, LePen, Bayrou, Other, Zero, Abstain, Royal, Leftist}
3834530	{LePen, Sarkozy, Bayrou, Other, Zero, Abstain, Royal, Leftist}
9500112	{Royal, Leftist, Bayrou, Other, Zero, Abstain, Sarkozy, LePen}
4695327	{Leftist, Royal, Bayrou, Other, Zero, Abstain, Sarkozy, LePen}
3700215	{Bayrou, Sarkozy, Other, Zero, Abstain, Royal, LePen, Leftist}
2595172	{Bayrou, Royal, Other, Zero, Abstain, Sarkozy, LePen, Leftist}
524732	{Bayrou, Zero, Abstain, Sarkozy, Royal, Other, LePen, Leftist}
420645	{Other, Sarkozy, Bayrou, Zero, Abstain, LePen, Royal, Leftist}
534846	{Zero, Abstain, Other, Leftist, Bayrou, LePen, Royal, Sarkozy}
89227	{Abstain, Zero, Other, Leftist, Bayrou, LePen, Royal, Sarkozy}

Setting up the voting problem

We already mentioned the items:

    Items = {Sarkozy, LePen, Royal, Leftist, Bayrou, Other, Zero, Abstain};

We select the votes from the PrefData and check that their total is indeed the total of the second round.

    Votes = First /@ PrefData

    {9448663, 2000000, 3834530, 9500112, 4695327, 3700215, 2595172, 524732, 420645, 534846, 89227}

    % // Add

    37343469

For the voting routines we assign scores 1 to 8 to the candidates, giving the highest value to the candidate of the highest preference. The order of the Items determines where a score is put. For example, the voters for Sarkozy will assign value 8 to the first position, Sarkozy, 7 to their second choice Bayrou in the 5th position, and so on.

    prefs = PrefToList /@ (Pref @@ # &) /@ Reverse /@ Last /@ PrefData

$$
\begin{pmatrix}
8 & 2 & 3 & 1 & 7 & 6 & 5 & 4 \\
8 & 7 & 2 & 1 & 6 & 5 & 4 & 3 \\
7 & 8 & 2 & 1 & 6 & 5 & 4 & 3 \\
2 & 1 & 8 & 7 & 6 & 5 & 4 & 3 \\
2 & 1 & 7 & 8 & 6 & 5 & 4 & 3 \\
7 & 2 & 3 & 1 & 8 & 6 & 5 & 4 \\
3 & 2 & 7 & 1 & 8 & 6 & 5 & 4 \\
5 & 2 & 4 & 1 & 8 & 3 & 7 & 6 \\
7 & 3 & 2 & 1 & 6 & 8 & 5 & 4 \\
1 & 3 & 2 & 5 & 4 & 6 & 8 & 7 \\
1 & 3 & 2 & 5 & 4 & 6 & 7 & 8
\end{pmatrix}
$$

This gives all elements to set up the voting problem. The votes will be expressed as ratio's summing to 1.

    SetVotingProblem[Votes, Items, prefs]

$\{$Number of Voters $\to$ 11, Number of items $\to$ 8, Votes are nonnegative and add up to 1 $\to$ True,

    Preferences fit the numbers of Voters and Items $\to$ True, Type of scale $\to$ Ordinal,

    Preferences give a proper ordering $\to$ True, Preferences add up to $\to$ {36},

    Items $\to$ {Sarkozy, LePen, Royal, Leftist, Bayrou, Other, Zero, Abstain}, Votes $\to \left\{ \dfrac{9448663}{37343469}, \dfrac{2000000}{37343469}, \dfrac{3834530}{37343469}, \right.$

    $\left. \dfrac{3166704}{12447823}, \dfrac{1565109}{12447823}, \dfrac{1233405}{12447823}, \dfrac{2595172}{37343469}, \dfrac{524732}{37343469}, \dfrac{140215}{12447823}, \dfrac{178282}{12447823}, \dfrac{89227}{37343469} \right\} \}$

The run-off plurality routine

To properly model run-off plurality we would have to account for shifts in preferences and participation between the rounds. The candidates would keep the same name but they might shift their political positions. The following routine however is simple, assumes that the numbers and preferences are exactly the same for the first and second round, and it does not account for abstination and zero votes other than including them in the items. By consequence we get slightly different numbers than the official figures. De denominators include not just the "valid" votes but also the zero and new ones. The 1.6 million "zero" votes of the second round are allocated to either Sarkozy or Royal, so that in this simulation run both get slightly more votes. (PM. To get the same result in the second round as in the official figures we might substract the 1.6 million zero votes from both rounds, and distribute this proportionally. It is needlessly complicated to do this just for this example.)

`RunOffPlurality[] // N`

CheckVote::adj : NumberOfItems adjusted to 2

$\Big\{$First $\rightarrow$

$\left\{ \text{Sum} \rightarrow \begin{pmatrix} \text{Abstain} & 0.00238936 \\ \text{Bayrou} & 0.182632 \\ \text{Leftist} & 0.125734 \\ \text{LePen} & 0.102683 \\ \text{Other} & 0.0112642 \\ \text{Royal} & 0.254398 \\ \text{Sarkozy} & 0.306577 \\ \text{Zero} & 0.0143223 \end{pmatrix}, \text{Ordering} \rightarrow \begin{pmatrix} 0.00238936 & \text{Abstain} \\ 0.0112642 & \text{Other} \\ 0.0143223 & \text{Zero} \\ 0.102683 & \text{LePen} \\ 0.125734 & \text{Leftist} \\ 0.182632 & \text{Bayrou} \\ 0.254398 & \text{Royal} \\ 0.306577 & \text{Sarkozy} \end{pmatrix}, \text{Max} \rightarrow \{\text{Sarkozy}, 0.306577\}, \text{Select} \rightarrow \{\} \right\},$

$\text{Sum} \rightarrow \begin{pmatrix} \text{Royal} & 0.466338 \\ \text{Sarkozy} & 0.533662 \end{pmatrix}, \text{Ordering} \rightarrow \begin{pmatrix} 0.466338 & \text{Royal} \\ 0.533662 & \text{Sarkozy} \end{pmatrix}, \text{Max} \rightarrow \{\text{Sarkozy}, 0.533662\}, \text{Select} \rightarrow \text{Sarkozy} \Big\}$

The Borda Fixed Point routine

Using the Borda Fixed Point routine, Bayrou is selected.

`BordaFP[]`

Bayrou

It appears that Bayrou would already have been selected by the Borda method itself but his position is also stable in the sense that adding or removing a candidate would not quickly change his winning status.

BordaAnalysis[] // N

$\Big\{$Select → Bayrou, BordaFPQ → {True},

   WeightTotal → {4.99957, 2.53173, 4.88274, 3.47337, 6.58486, 5.444, 4.53948, 3.54426},

$$\text{Position} \rightarrow (\,5.\,), \text{Ordering} \rightarrow \begin{pmatrix} 2.53173 & \text{LePen} \\ 3.47337 & \text{Leftist} \\ 3.54426 & \text{Abstain} \\ 4.53948 & \text{Zero} \\ 4.88274 & \text{Royal} \\ 4.99957 & \text{Sarkozy} \\ 5.444 & \text{Other} \\ 6.58486 & \text{Bayrou} \end{pmatrix}\Big\}$$

In this case we have lumped together all "other" candidates into one person, "who" comes as a surprising second. Perhaps our results have been overly influenced by this ? The following section shows that this is not the case.

Reducing the number of candidates

From the list of preferences we can also select just the three main contenders. In that case the preferences range from 1 to 3 instead of 1 to 8. The main conclusion does not change. The run-off plurality scheme selects Sarkozy while Borda FP selects Bayrou.

ReduceVotingProblem[{Sarkozy, Royal, Bayrou}]

CheckVote::adj : NumberOfItems adjusted to 8

CheckVote::adj : NumberOfItems adjusted to 3

$\Big\{$Number of Voters → 11, Number of items → 3, Votes are nonnegative and add up to 1 → True,

   Preferences fit the numbers of Voters and Items → True, Type of scale → Ordinal,

   Preferences give a proper ordering → True, Preferences add up to → {6},

   Items → {Sarkozy, Royal, Bayrou}, Votes → $\Big\{\dfrac{9448663}{37343469}, \dfrac{2000000}{37343469}, \dfrac{3834530}{37343469}, \dfrac{3166704}{12447823},$

   $\dfrac{1565109}{12447823}, \dfrac{1233405}{12447823}, \dfrac{2595172}{37343469}, \dfrac{524732}{37343469}, \dfrac{140215}{12447823}, \dfrac{178282}{12447823}, \dfrac{89227}{37343469}\Big\}\Big\}$

Preferences

$$\begin{pmatrix} 3 & 1 & 2 \\ 3 & 1 & 2 \\ 3 & 1 & 2 \\ 1 & 3 & 2 \\ 1 & 3 & 2 \\ 2 & 1 & 3 \\ 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

```
RunOffPlurality[] // N
```

CheckVote::adj : NumberOfItems adjusted to 2

$$\left\{ \text{First} \rightarrow \left\{ \text{Sum} \rightarrow \begin{pmatrix} \text{Bayrou} & 0.199344 \\ \text{Royal} & 0.380132 \\ \text{Sarkozy} & 0.420524 \end{pmatrix}, \text{Ordering} \rightarrow \begin{pmatrix} 0.199344 & \text{Bayrou} \\ 0.380132 & \text{Royal} \\ 0.420524 & \text{Sarkozy} \end{pmatrix}, \text{Max} \rightarrow \{\text{Sarkozy}, 0.420524\}, \text{Select} \rightarrow \{\} \right\},$$

$$\text{Sum} \rightarrow \begin{pmatrix} \text{Royal} & 0.466338 \\ \text{Sarkozy} & 0.533662 \end{pmatrix}, \text{Ordering} \rightarrow \begin{pmatrix} 0.466338 & \text{Royal} \\ 0.533662 & \text{Sarkozy} \end{pmatrix}, \text{Max} \rightarrow \{\text{Sarkozy}, 0.533662\}, \text{Select} \rightarrow \text{Sarkozy} \right\}$$

```
BordaFP[]
```

Bayrou

```
BordaAnalysis[] // N
```

$$\left\{ \text{Select} \rightarrow \text{Bayrou}, \text{BordaFPQ} \rightarrow \{\text{True}\}, \right.$$

$$\left. \text{WeightTotal} \rightarrow \{1.95419, 1.84647, 2.19934\}, \text{Position} \rightarrow (3.), \text{Ordering} \rightarrow \begin{pmatrix} 1.84647 & \text{Royal} \\ 1.95419 & \text{Sarkozy} \\ 2.19934 & \text{Bayrou} \end{pmatrix} \right\}$$

Strategy

If Bayrou had made it to the second round then he would have beaten Sarkozy. One can imagine different voting strategies coming into play. Royal could have known this already and withdrawn from the race. Vanity gave her a president who may be opposite to her ideals. But perhaps if she had dropped from the race, voters would have started regarding Bayrou as a socialist candidate, so that he might have lost his overall appeal. So, Royal's supporters might have been wiser than she is and voted for Bayrou already in the first round. Perhaps many Sarkozy voters voted for Royal to make sure that she got more votes than Bayrou.

Whatever all that may be, given above preferences, Bayrou would beat Sarkozy, which is one reason why he is a Borda Fixed Point winner. The Borda Fixed Point method is not immune to strategic voting but less sensitive than current systems.

```
WinnerOfPair[Sarkozy, Bayrou]
```

Bayrou

Conclusion

Above discussion has taken aggregate data from the 2007 French presidential elections, added some arbitrary though not unreasonable assumptions on the underlying preferences, and showed that the winner depends, once the preferences are given, upon the voting method. The situation can be judged on the properties of these methods.

Democratic nations are advised to have parliaments select the chief executive by the Borda Fixed Point method. The current practice of having direct popular elections using systems that have originated in history is inoptimal and actually quite undemocratic.

The main conclusion is for students of voting theory, and in particular for those without vested interests in current theoretical errors. Voting theory both in the academic journals and in public statements needs to be realigned with Colignatus (2007) otherwise democratic nations get wrong advice.

Literature

Colignatus is the preferred name of Thomas Cool in science

Colignatus (2007), "Voting theory for democracy", 2nd edition, Thomas Cool Econometrics & Consultancy

Wikipedia (2007), Data on the French elections, http://en.wikipedia.org/wiki/Politics_of_France (Retrieved May 9 2007)