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The Business Cycle with Nominal Contracts and Search Frictions

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Abstract

This paper examines a dynamic stochastic general equilibrium (DSGE) model containing flexible prices, search frictions and nominal wage contracts. It is assumed that the nominal hourly wage rate and the hours of work are jointly determined, so-called efficient bargaining, for each period. The frictional labor markets reasonably reflect the volatility of real variables and the fact that productivity is no longer countercyclical. As contract length increases, the volatilities of the unemployment rate and the vacancy rate increase sharply, but those of output and total hours worked do not appreciably change.
Keywords: Business Cycles, Labor Market Frictions, Nominal Wage Contracts

1 Introduction

This paper studies a dynamic stochastic general equilibrium (DSGE, henceforth) model containing flexible prices, search frictions and the wage contracts examined in Cho and Cooley (1995). As Janko (2008) argues, the Cho-Cooley framework does not capture the business cycle statistics of the U.S. economy, and this limits the theory considerably. Once Cho-Cooley's nominal wage contracts are incorporated into real business cycle models, nominal rigidities improve the monetary transmission and amplification mechanisms, but they lead to unrealistically high volatility among real variables as well as countercyclical productivity. In this paper, I attempt to incorporate Cho and Cooley (1995)'s monetary model to show that search frictions and wage bargaining play a crucial role in overcoming limitations of the model.

In this paper, I follow Cho and Cooley (1995)'s assumption that the future nominal wage rate is agreed to, but my approach differs with respect to contract regime: While the nominal wage rate in Cho and Cooley (1995) is derived from the decision rule of the model without contract, assuming that the contract wage rate is the expected market-clearing level of the wage rate, I derive the contract wage counterpart from the solution to a forward-looking Nash bargaining problem. This is a crucial component in explaining why firms and workers enter into the nominal contracts. Wage contracts in my model are based on bargaining between workers and firms due to the coordination failure raised from labor market frictions.

This paper also investigates the reason why the Cho-Cooley's nominal wage contracts incorporated into the equilibrium business cycle model hardly leads to results that better match the U.S. data, and thus it highlights the importance of labor market frictions and efficient bargaining, which reveals what has not yet been examined in the literature. Under the nominal contracting arrangements in Cho and Cooley (1995), employees and firms agree upon the nominal hourly wage rate in advance and firms are free to choose employment on the hours margin at the wage rate. This is referred to as the right-to-manage (RTM, henceforth) approach. Under Cho-Cooley's RTM framework, firms adjust to shocks during the contract period by choosing hours to equate the marginal product of labor to the realized real wage. Consequently, the volatility of hours worked and that of output are unrealistically high. This issue is raised not only for the flexible price model but for the New-Keynesian model. Christoffel et al. (2009) examine a New-Keynesian model with staggered wages and report that the model, combined with an RTM assumption, does not replicate the dynamics of hours worked, because hours per worker remain far too volatile relative to data.

On the other hand, the existing RTM framework does not allow employment to have an effort dimension, even though it could better capture actual labor contracts.[1] If labor input varies due to responses in effort as well as hours, the impact of wage rigidities on hours worked can be offset by variations in effort at work.[2] As a reasonable approximation for bargaining, therefore, I assume efficient bargaining under which the nominal wage rate and the hours of work are jointly determined. Little attention has been paid to the role of labor market frictions with efficient bargaining as a way to resolve Cho-Cooley's unrealistic degree of real variable volatility, and thus this paper is a contribution on this issue.

Once search frictions and the Cho-Cooley's nominal wage contracts are incorporated into the flexible price model, the model no longer generates unrealistically high volatility among real variables and countercyclical productivity. In all the models with different contract lengths, the volatilities of output and total hours worked are not greater than those found in the U.S. data, and productivity remains highly procyclical during the business cycle. In addition, the volatilities of the unemployment rate and the vacancy rate rise significantly as contract length rises.

There are several papers in the real business cycle literature that have studied the implications of nominal wage contracts in the transmission of monetary shocks. Cho (1993) first examines the quantitative implications of one-period nominal wage contract. Cho and Cooley (1995) study the properties of model economies with multi-period wage contracts. Cho et al. (1997) quantitatively estimate the welfare cost of nominal wage contracting. Recently, Janko (2008) adds empirically plausible labor adjustment costs to the Cho and Cooley (1995) model to overcome some of the shortcomings otherwise present with nominal wage rigidities. Janko (2008), however, does not discuss unemployment and vacancies.

When it comes to labor market frictions, Merz (1995) and Andolfatto (1996) first bring the concept into a real business cycle model. Later, Shimer (2005) discusses the lack of an amplification mechanism in the context of Mortensen and Pissarides (1994) search and matching model. Shimer (2005) finds that the wage bargaining process is a source of the inability that amplifies shocks. The bargaining wage is very volatile, because it absorbs most of the shocks, and the cyclical movements in firms' incentives to hire are dampened. As a result, Hall (2005) and others propose real wage rigidity which allows firms to have cyclical movement in their incentives to create jobs. Following

[1] Among others, see Trigari (2006) and Christoffel and Kuester (2008).

[2] Using a DSGE model with endogenous effort, Bils and Chang (2003) show that workers are willing to trade off exertion and hours in production.

Shimer (2005) and Hall (2005), many studies introduce real and nominal wage rigidities into DSGE models.[3] To my knowledge, this paper is the first work that revitalizes Cho and Cooley (1995)'s model, characterized by nominal wage contracts in a frictional labor market environment.

This paper is organized as follows. Section 2 presents the model and shows how the nominal contract wage rate is derived. Section 3 quantifies the model and presents the results of my analyses. Section 4 presents some concluding remarks.

2 Model

The model economy is a variant of Mortensen and Pissarides (1994) and Cho and Cooley (1995) which consists of households and firms.

2.1 Households

There is a representative household consisting of a continuum of expected-utility-maximizing, infinitely lived individuals with measure 1. Each member has time-separable preferences over her consumption $c_t(i)$ and her labor supply $(h_t(i), n_t(i))$. He/She may be either employed by a firm, $n_t(i) = 1$, with the hours of work $h_t(i)$ or unemployed, $n_t(i) = 0$. Each member's period utility is given by

$$\begin{cases} \ln c_t(i) - B\frac{1}{1+\phi} h_t(i)^{1+\phi} & \text{if } n_t(i) = 1, \\ \ln c_t(i) & \text{if } n_t(i) = 0, \end{cases}$$

where $1/\phi$ denotes the elasticity of intertemporal substitution of leisure. Following Merz (1995), I assume that the household serves as a full insurance mechanism by pooling the resources of all its members. The household allocates total consumption to maximize the sum of household utility which can be obtained by equalizing the marginal utility of consumption of each household member. The household which makes all members obtain an identical consumption bundle acts as if it has a utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t - B\frac{h_t^{1+\phi}}{1+\phi} n_t \right\}, \tag{1}$$

[3]Major studies focus on real wage rigidities based on the New-Keynesian DSGE model, including Krause and Lubik (2007), Gertler and Trigari (2009), and Blanchard and Galí (2010). All studies employ Hall (2005)'s notion of a wage norm, but Gertler and Trigari (2009) assume that in each period a subset of firms and workers renegotiate wage contracts, and modify the conventional Mortensen and Pissarides (1994) model to allow for Calvo-type staggered wage contracts. On the other hand, Gertler et al. (2008) and Galí (2010) incorporate nominal wage rigidities into the New-Keynesian DSGE model. All those studies introduce nominal wage rigidities in the form of staggered nominal wage setting à la Calvo.

where $0 < \beta < 1$ denotes a discount factor, c_t consumption, h_t hours worked by each employed household member, and n_t the fraction of employed household members.

The households in this economy are required to hold money to purchase consumption goods. They face a cash-in-advance constraint of the form:

$$c_t \leq \frac{\widetilde{m}_{t-1} + (g_t - 1) M_{t-1}}{\widetilde{P}_t}, \tag{2}$$

where $\widetilde{m}_{t-1}$ is money carried over from the previous period, $(g_t - 1) M_{t-1}$ the lump-sum money transfer, and $\widetilde{P}_t$ the price level in period t. The gross growth rate of money, g_t, is known at the beginning of each period.

The budget constraint of the representative household can be expressed as

$$\begin{aligned} c_t + i_t + \frac{\widetilde{m}_t}{\widetilde{P}_t} &= \frac{\widetilde{W}_t^{t-j}}{\widetilde{P}_t} n_t h_t^{t-j} + (1 - n_t) b + r_t k_t \\ &\quad + \pi_t + \tau_t + \frac{\widetilde{m}_{t-1} + (g_t - 1) M_{t-1}}{\widetilde{P}_t}, \end{aligned} \tag{3}$$

where i_t denotes investment in capital, k_t; n_t the fraction of employed household members; $\widetilde{W}_t^{t-j}$ and h_t^{t-j} the nominal hourly wage rate and the hours of work, respectively, determined in period $t-j$ through bargaining; b unemployment insurance benefits; r_t the real rental rate of capital; π_t the profits that the household receives from firms; τ_t a lump-sum tax. The issue of how bargaining over nominal wages and hours worked takes place will be further analyzed in the next section.

Employment, n_t, evolves according to the following law of motion:

$$n_{t+1} = (1 - s) n_t + f_t (1 - n_t), \tag{4}$$

where s is an exogenous separation rate with which employees lose their job each period. $(1-s)n_t$ denotes the existing workforce at the beginning of period $t+1$ and $f_t (1 - n_t)$ new hires entering into employment agreement in period $t+1$, where f_t is the worker's job-finding probability.

Money is injected into the economy through lump-sum transfers. If I let g_t denote the growth rate of money in period t, money follows the process

$$M_t = g_t M_{t-1}. \tag{5}$$

The growth rate, g_t, is determined in the process

$$\ln g_t = \rho_g \ln g_{t-1} + \varepsilon_t^g, \tag{6}$$

where ε_t^g is a normal random variable with mean 0 and variance σ_g^2.

All consumption mechanisms for household members are equal, through full insurance arrangements. In equilibrium, I focus on the representative household's problem. I first divide all nominal variables, $\widetilde{m}_t$, $\widetilde{P}_t$, and $\widetilde{W}_t^{t-j}$, by the aggregate money stock, M_t. The representative household's maximization problem is then

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln \left(\frac{m_{t-1} + g_t - 1}{g_t P_t} \right) - B \frac{(h_t^{t-j})^{1+\phi}}{1+\phi} n_t \right\} \tag{7}$$

subject to

$$\begin{aligned} \frac{m_t}{P_t} + k_{t+1} &= \frac{W_t^{t-j}}{P_t \Pi_{i=1}^{j} g_{t+1-i}} n_t h_t^{t-j} + (1-n_t)\, b + (r_t + 1 - \delta)\, k_t + \pi_t + \tau_t \\ n_{t+1} &= (1-s)\, n_t + f_t (1-n_t), \end{aligned}$$

where $P_t \equiv \widetilde{P}_t / M_t$, $W_t^{t-j} \equiv \widetilde{W}_t^{t-j} / M_{t-j}$, and $m_{t-1} \equiv \widetilde{m}_{t-1} / M_{t-1}$. Note that the expression $\Pi_{i=1}^{j} g_{t+1-i}$ links the real hourly wage rate to the monetary shocks realized between period $t-j$ and period t.[4]

The representative household aims at choosing contingent plans for $\{k_{t+1}, m_t\}$, taking the nominal hourly wage rate and the hours of work as given. The first-order conditions for the maximization imply

$$\mu_t = \beta E_t \left[\frac{1}{c_{t+1}} \frac{P_t}{g_{t+1} P_{t+1}} \right], \tag{8}$$

$$\mu_t = \beta E_t \left[\mu_{t+1} (r_{t+1} + 1 - \delta) \right], \tag{9}$$

where μ_t is the marginal utility of income in period t, i.e., the multiplier attached to the budget constraint. In equilibrium, it follows that $m_t = M_t = 1$. The equilibrium cash-in-advance constraint

[4] Let me define $\Pi_{i=1}^{j} g_{t+1-i} = 1$ if $j = 0$.

implies that consumption is the reciprocal of the price level,

$$c_t = \frac{1}{P_t}. \tag{10}$$

I denote by V_t the worker's surplus, if another household member is employed:[5]

$$V_t = \frac{W_t^{t-j}}{P_t \Pi_{i=1}^{j} g_{t-i+1}} h_t^{t-j} - b - B \frac{(h_t^{t-j})^{1+\phi}}{1+\phi} \frac{1}{\mu_t} + \beta (1 - s - f_t) E_t \left[\frac{\mu_{t+1}}{\mu_t} V_{t+1} \right]. \tag{11}$$

2.2 Firms

Firms produce output, y_t, using capital, k_t, and labor, $n_t h_t^{t-j}$, under a constant returns to scale technology:

$$y_t = z_t k_t^{\alpha} (n_t h_t^{t-j})^{1-\alpha}, \tag{12}$$

where z_t is an aggregate productivity shock, and h_t^{t-j} is the hours of work, determined through bargaining in period $t - j$. The productivity shock follows an AR(1) process in logs:

$$\ln z_t = \rho_z \ln z_{t-1} + \varepsilon_t^z, \tag{13}$$

where ε_t^z is a normal random variable with mean 0 and variance σ_z^2. It is assumed that ε_t^z is independent of ε_t^g and that the technology shock is known at the beginning of each period before decisions are made.

The hiring rate, x_t, is defined as the ratio of new hires, $q_t v_t$, to the existing employment, n_t:

$$x_t = \frac{q_t v_t}{n_t}, \tag{14}$$

where q_t is the probability that each vacancy, v_t, will be filled. The total employment is the sum of the number of workers not being laid off, $(1 - s)\, n_t$, and new hires, $q_t v_t$, equivalent to $x_t n_t$:

$$n_{t+1} = (1 - s)\, n_t + x_t n_t. \tag{15}$$

It is assumed that firms incur labor adjustment costs, denoted by $\frac{\kappa}{2} x_t^2 n_t$. Given the adjustment

[5] One can derive the worker's surplus by taking the derivative of the household's indirect utility function with respect to n_t, subject to the budget constraint and the law of motion for employment. It is expressed in terms of current consumption of final goods.

costs, the expected discounted sum of real profits for the firm is given by

$$F_t = z_t k_t^{\alpha} (n_t h_t^{t-j})^{1-\alpha} - \frac{\widetilde{W}_t^{t-j}}{\widetilde{P}_t} n_t h_t^{t-j} - r_t k_t - \frac{\kappa}{2} x_t^2 n_t + \beta E_t \left[\psi_{t+1} F_{t+1} \right], \tag{16}$$

where $\psi_{t+1} = \mu_{t+1}/\mu_t$. In what follows, I assume that matched firms and workers bargain about the nominal hourly wage rate and the hours of work and that the firms choose the amount of capital given the hours of work. Notice that the period real profits for the firm in Eq. (16) is linear in the level of employment, n_t, so that the value of the marginal worker for the firm, $J_t \equiv \partial F_t / \partial n_t$, is given by

$$J_t \;=\; z_t \overline{k}_t^{\alpha} (h_t^{t-j})^{1-\alpha} - \frac{\widetilde{W}_t^{t-j}}{\widetilde{P}_t} h_t^{t-j} - r_t \overline{k}_t - \frac{\kappa}{2} x_t^2 + \beta \left(1 - s + x_t\right) E_t \left[\psi_{t+1} J_{t+1} \right], \tag{17}$$

where $\overline{k}_t$ is k_t / n_t. The firm's hiring decision is found as the derivative of J_t with respect to x_t:

$$\kappa x_t = \beta E_t \left[\psi_{t+1} J_{t+1} \right]. \tag{18}$$

Rewriting Eq. (17) as the firm's value of the marginal worker gives:

$$J_t = z_t \overline{k}_t^{\alpha} (h_t^{t-j})^{1-\alpha} - \frac{W_t^{t-j}}{P_t \Pi_{i=1}^{j} g_{t-i+1}} h_t^{t-j} - r_t \overline{k}_t + \frac{\kappa}{2} x_t^2 + \left(1 - s\right) \kappa x_t. \tag{19}$$

Given the hours of work, the first-order condition with respect to $\overline{k}_t$ equalizes the marginal product of capital to its rental rate:

$$r_t = \alpha z_t \overline{k}_t^{\alpha - 1} (h_t^{t-j})^{1-\alpha}. \tag{20}$$

In this economy, there is another technology which describes how matches take place. The so-called matching technology or a matching function can be expressed as follows:

$$\mathcal{M}_t = \eta u_t^{\xi} v_t^{1-\xi}, \tag{21}$$

where $\mathcal{M}_t$ is the total number of matches or hires; u_t the number of unemployed workers; v_t the aggregate number of vacancies. Assuming that the size of labor force is fixed, the number of unemployed workers is $u_t = 1 - n_t$. The probability a firm fills its vacancy, q_t, is given by

$$q_t = \frac{\mathcal{M}_t}{v_t}, \tag{22}$$

and the probability an unemployed worker finds a job, f_t, is given by

$$f_t = \frac{\mathcal{M}_t}{u_t}. \tag{23}$$

Finally, combining the household's equilibrium budget constraint and the value of the firm, under the binding cash-in-advance constraint, yields the aggregate resource constraint:

$$c_t + k_{t+1} + \frac{\kappa}{2} x_t^2 n_t = y_t + (1-\delta) k_t. \tag{24}$$

2.3 Bargaining over Wages and Hours

The nominal wage contract set up in this section follows Cho and Cooley (1995), where at the beginning of each period agents agree to a contract arranged for j periods ahead at a time. In the case when $j = 2$, at time t workers and firms agree to a nominal wage rate for period $t+2$, and the firms pay to employees the nominal wage rate agreed upon in period $t-2$; at $t+1$, they agree to a wage rate for period $t+3$, and the firms pay to employees the nominal wage rate agreed upon in period $t-1$. This process repeats over time.

Once the labor market is characterized by search frictions, the firm and the worker enter into bargaining over the nominal hourly wage rate and hours worked under the generalized Nash bargaining framework. The nominal hourly wage rate, W_{t+j}^t, and hours worked, h_{t+j}^t, in time $t+j$ are agreed upon in period t by both parties, where the hourly wage rate and hours worked jointly maximize the Nash product after the aggregate shocks are realized:

$$\left(W_{t+j}^t, h_{t+j}^t\right) = \arg\max \left(\beta^j E_t \frac{\mu_{t+j}}{\mu_t} V_{t+j}\right)^{\gamma} \left(\beta^j E_t \frac{\mu_{t+j}}{\mu_t} J_{t+j}\right)^{1-\gamma}, \tag{25}$$

where γ denotes the worker's bargaining power in wage negotiations, and the surpluses for a matched worker and a matched firm are given by Eq. (11) and (19), respectively. At the time of the contract, the nominal wage is paid to all employees, and they supply the work as specified in the contract. Under this nominal wage contract rule, new hires are paid the same nominal wage pre-determined through bargaining between firms and workers who have agreed on a contract.

The first-order conditions with respect to the nominal hourly wage rate, W^t_{t+j}, and the hours of work, h^t_{t+j}, at time $t+j$, are given by

$$(1-\gamma)\, E_t \left[\frac{\mu_{t+j}}{\mu_t} V_{t+j} \right] = \gamma E_t \left[\frac{\mu_{t+j}}{\mu_t} J_{t+j} \right] \tag{26}$$

$$B \left(h^t_{t+j} \right)^{\phi} \frac{1}{\mu_t} = E_t \left[\frac{\mu_{t+j}}{\mu_t} (1-\alpha) z_{t+j} \overline{k}^{\alpha}_{t+j} (h^t_{t+j})^{-\alpha} \right]. \tag{27}$$

The nominal hourly wage rate chosen by a firm-worker match is derived from the expected discounted surplus of firms and workers. The hours of work is chosen by the match such that the marginal rate of substitution between consumption and leisure is equated to the expected discounted value of marginal product of labor. In this paper, the economies are approximated by log-linearization around the steady state, because those models with nominal contracts cannot be solved analytically.

3 Quantitative Analysis

3.1 Calibration

I set the discount factor β to .99 implying an interest rate of one percent per quarter. Capital's share of total income, α, is calibrated to be .33, and δ is set equal to .025. The worker's bargaining power in wage negotiations, γ, is set to .5 and the unemployment insurance benefits, b, to about 40 percent of the steady state (real) bargaining wage. The elasticity of the matching function, ξ, is set to .5, consistent with the literature. I set the steady-state value of worker's job-finding probability, f, to .6, which implies an average duration of unemployment of 1.67, as reported by Cole and Rogerson (1999). The steady-state unemployment rate, u, is set to 6 percent per quarter, and the labor force size is normalized to unity. Given the job-finding rate and the employment rate, the exogenous separation, s, is pinned down from the steady state version of the law of motion for employment, $n = f/(s+f)$, so that $s = .0383$. The steady state level of hours worked, h, is normalized to 1/3, and utility parameter B is adjusted accordingly. Following Chang and Kim (2006), I set the intertemporal substitution elasticity of leisure to .4, which implies $\phi = 2.5$. The adjustment cost parameter κ and the real bargaining wage W/P are jointly determined from the steady state of the model, so that $\kappa = 24.895$ and $W/P = 1.992$.

The parameters governing the money growth rate, ρ_g and σ_g, are set to .49 and .00623, respectively, as used by Cooley and Quadrini (1999). Finally, the parameters, ρ_z and σ_z, control the process for technology shocks. The parameter ρ_z is assigned the value .95, a value commonly used in the related literature. The parameter σ_z is set to .005, which allows the model economy with period-by-period wage bargaining to match the actual volatility of aggregate output, around 2.1%. Table 1 summarizes the set of parameters used in the simulation.

<<< Table 1 >>>

3.2 Findings

I investigate to what extent the model economy with wage contracts following Cho and Cooley (1995), as well as frictional labor markets, is able to amplify the monetary and real shocks. Table 2 presents the standard deviations of output and other key variables of interest. To evaluate predictive accuracy, I first present the relevant statistics obtained from the U.S. quarterly data between 1956 and 2005. The output measure y is production in the non-farm business sector. Consumption c is the sum of personal consumption expenditures of nondurables and services, deflated by the associated price indices plus real government consumption expenditures. Investment i is the sum of real private domestic investment and real personal consumption expenditures of durables. Employment n is measured, using the quarterly average number of non-farm employees. Hours h are average weekly hours for the non-farm business sector. Unemployment u is quarterly averages of monthly data from the Current Population Survey. Vacancy v is the quarterly average of monthly help-wanted indices, constructed by the Conference Board. The real wage w is real hourly compensation in the non-farm business sector. All data are seasonally adjusted and HP-filtered with smoothing parameter 1,600.

Summary statistics for the models subject to both monetary and technology shocks, as well as the results from the monetary shocks and technology shocks, are presented. Statistics for the model economies are computed by simulating for 200 periods and by repeating the simulation 100 times. This highlights the role of each shock and enables to determine the relative importance of each shock, as nominal wage contracts and search frictions are introduced.

<<< Table 2 >>>

The results from the basic model with period-by-period wage bargaining show low volatility for the labor market variables. Compared to the volatility of output, for example, the relative standard deviations of employment (n), unemployment (u), and vacancies (v) are .12, 1.83, and 2.13, respectively. The results demonstrate that the model in which the nominal wage rate is Nash-bargained in every period lacks amplification mechanisms.[6] This result is consistent with Galí (2010) that realistic labor market frictions seem to have limited effects on the economy's equilibrium dynamics. The basic model shows that monetary shocks are not propagated in this economy. Technology shocks generate most of the observed volatility in output (y). The basic model with both shocks returns essentially the same results as the model with technology shocks only.

Cho and Cooley (1995) find unrealistically high fluctuations in output and total hours worked, as multi-period nominal wage contracts are incorporated. First, in the case of two-period contracts, output is more than twice as volatile as it is in the model without contracts. Second, total hours worked in the one-period contract case fluctuate more than output. These dramatic increases in volatility are attributed to the strong monetary transmission mechanism induced by nominal wage contracts. Under the nominal contracting arrangements in Cho and Cooley (1995), households and firms enter into a wage contract and agree upon the nominal wage set in advance. The specified equilibrium nominal wage satisfies that expected marginal product of labor is equalized to the expected marginal rate of substitution between consumption and leisure. In addition, the workers are assumed to cede the firm the right to determine the aggregate hours, leaving firms to maximize profits. The firms adjust to shocks during the contract period by choosing hours worked to equate the marginal product of labor to the realized real wage. Consequently, the volatilities of total hours worked and output are unrealistically high, and labor productivity becomes countercyclical.[7]

When search frictions are taken into account in the model, nominal wage contracts increase the volatility of variables but do not produce unrealistically high volatilities in output as well as in total hours worked. In the case of a four-period contract, the volatility of output and the relative volatility of total hours worked are 2.13 and .31, respectively. With respect to the labor market variables, multi-period nominal wage contracts result in higher degree of volatilities. Specifically, the volatility

[6]In order to have highly volatile labor market variables, Cooley and Quadrini (1999) set the worker's bargaining power (or the sharing parameter in their paper) in the range of 0.01-0.1. Decreasing this parameter leads to the higher volatility of both employment and unemployment. Note that the weaker bargaining power workers hold, the more rigid real wages become, if the worker's period value from unemployment is not time-varying. However, a standard parameter value in the literature is around 0.5.

[7]Bils and Chang (2003) also show that a model with sticky wages but no effort response predicts a strong negative relationship between labor productivity and hours worked.

of employment increased from .12 to .30, unemployment from 1.83 to 4.75, and vacancies from 2.13 to 7.14, under the four-period contract scenario. The corresponding impact on the volatilities of unemployment and vacancies is notably large.

Table 2 also demonstrates that nominal wage contracts play a crucial role in amplifying monetary shocks. In the case of a four-period contract, monetary shocks have more weights in generating fluctuations of employment, unemployment and vacancies than technology shocks. Interestingly, the impact of monetary shocks on the volatilities of the labor market variables, including real wages, has more significant effects than that of technology shocks.

Introducing nominal wage contracts slightly decreases the volatility of output from 2.11 to 2.09 (to 2.08) with a one-period contract (with a two-period contract) because hours worked are predetermined through efficient bargaining. In the basic model, for example, the log-linearized model expresses the output as

$$\widehat{y}_t = .33\widehat{k}_t + .67(\widehat{n}_t + \widehat{h}_t) + \widehat{z}_t.$$

All else being equal, the volatility of hours worked plays an important role for capturing the volatility of output. The hours of work for the multi-period contract case are less dependent on the state variables, including shock components, than those for the period-by-period contract case. In the case of $j = 2$, for example, the hours of work in period $t + 2$ are determined in period t when workers and firms form their expectations about the shocks to be realized two periods later. Since all shocks have zero mean, under the rational expectations assumption, the state variables become less persistent, and the hours of work for multi-period contracts are less dependent on the state variables. This generates less volatile hours worked displayed in Table 2.

The results from the four-period contract model are consistent with the fact that most fluctuations in total hours worked are due to the change in the number of employed. However, employment and hours worked from the model are about half as volatile as those from the data. Despite the modeling along the intensive and extensive margin, I do not consider labor force participation. The task incorporating labor force participation is not trivial, because the model predicts a negative relation between unemployment and nonparticipation. Without barriers which deter workers from moving between unemployment and nonparticipation, all the workers choose one out of two states, unemployment and nonparticipation, along the business cycle, and it leads to the counterfactual result.[8]

[8] In the U.S. data, unemployment is weakly positively correlated with nonparticipation. For studies on the labor

On the other hand, I investigate efficient bargaining through which nominal wages and hours worked are jointly determined. Blanchard and Fischer (1989) argue that actual labor contracts appear only to set a wage and to leave the employment decision to the firm. This is referred to as the right-to-manage approach on which the firm and the union bargain over the wage, and then the firm chooses employment freely to maximize profit. The paper of Cho and Cooley (1995) therefore examines the nominal contracts with the right-to-manage approach. As mentioned, allowing for right-to-manage bargaining generates excessive volatilities of real variables.

Unfortunately, nominal wage contracts and search frictions lead to large changes in the relative volatility of the real wages. None of the models with different contract lengths are able to match the relative volatility found in the U.S. data. For four-period contracts, for example, the real wage rate depends on the nominal wage rate determined in period $t-4$, monetary shock components, and other state variables. As contract length increases, long-term monetary shocks, realized from $t-4$ through t, make the real wages become more volatile than before.

<<< Table 3 >>>

Table 3 presents the correlations with output. Cho and Cooley (1995) report that productivity is negatively correlated with total hours worked, and that output is negatively correlated with real wages and is uncorrelated or positively correlated with prices. From the negative correlation between productivity and total hours worked, one can thus infer that productivity may be countercyclical.[9] By contrast, my model with different lengths of contracts correctly predicts a positive correlation between output and productivity (a): .99 with one-period contracts and .95 with four-period contracts. Only monetary shocks, however, make productivity countercyclical, which becomes more significant as the contract period gets longer. If monetary shocks dominate, I can then have countercyclical productivity as in Cho and Cooley (1995).

I also look at the correlation between productivity and total hours worked displayed in Table 3. The correlation coefficient of .91 supports a strong positive correlation with total hours worked in the basic model. As contract length increases, however, the correlation decreases. The correlation simulated from the model with four-period contracts, .43, is close to the observed level from the data, .27. It is also worth noting that the real wage rate is highly procyclical in the basic model

market transitions between employment and unemployment, and that in and out of the labor force, see Veracierto (2008).

[9]Cho and Cooley (1995) do not report the correlation between output and productivity.

with correlation .99, but its procyclicality weakens as the length of the contract increases. Thus, the correlation between output and real wages is .46 with four-period contracts, which fits closely to the observed positions.[10]

Although the models with nominal wage contracts and search frictions overcome limitations of Cho and Cooley (1995) successfully, the setup predicts weak correlations of unemployment and vacancies with output when contract length increases. The correlations of unemployment and vacancies are -.89 and .99 for one-period contracts, but -.61 and .46 for four-period contracts. Nevertheless, the model with four-period contracts exhibits highly negative correlations of unemployment for both shocks: -.98 for monetary shocks and -.96 for technology shocks, respectively. There is a similar pattern to correlations of vacancies with output.

<<< Table 4 >>>

Finally, Table 4 presents the correlations with unemployment. For different contract lengths, the model is able to produce a negative relationship between unemployment and vacancies, even though the correlation result of -.60 indicates a weaker relationship in the four-period contract case. Interestingly, the relative importance of monetary shocks in the negative relation between unemployment and vacancies increases at longer periods. When only monetary shocks are considered, the correlation is -.18 in the model with two-period contracts, while it is -.53 with four-period contracts.

The model is also able to account for a negative relationship between unemployment and total hours worked. The correlation between unemployment and total hours worked of the model with four-period contracts is -.96, close to the level seen in the U.S. data of -.94. As contract length increases, the volatilities of employment and unemployment are much more driven by monetary shocks than technology shocks, while hours worked remain less volatile for either shock. The cyclical behaviors of total hours worked then are explained mainly by those of employment, where employment moves in the exact opposite direction of unemployment over the business cycle.

On the other hand, the model shows that there is no systematic relationship between unemployment and real wages. The correlation is -.13 for the U.S. economy over the last fifty years, while the correlation is -.14 for three-period contracts and .14 for four-period contracts. The results do not seem to be quite different from data. For the relationship between unemployment and hours worked, however, none of the models have found a negative relationship at all in the U.S. data.

[10] When it comes to the price level, it is the reciprocal of consumption and moves in the opposite direction of consumption over the business cycle. The price level is negatively correlated with output.

4 Conclusion

This paper examines a dynamic stochastic general equilibrium (DSGE) model containing flexible prices, search frictions and nominal wage contracts. Due to search frictions, a firm-worker pair negotiates its future nominal wage rate based upon the expected value for the future surplus through the match. This study assumes efficient bargaining under which the trade takes place between the firm and the worker in both the wage rate and the hours of work.

Once the flexible price model accounts for search frictions as well as Cho-Cooley's nominal wage contracts, it hardly produces unrealistically high volatility of real variables or countercyclical productivity. In my model, as the contract period gets longer, the volatilities of the unemployment rate and the vacancy rate increase significantly. Moreover, the volatility of output does not increase for a longer contract period.

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Table 1: Parameter Values

Parameters	Descriptions	Values
β	discount factor	.99
δ	depreciation rate	.025
ξ	elasticity of the matching function	.5
α	capital share	.33
γ	worker's bargaining power	.5
b	unemployment insurance benefits	.7970
f	job-finding probability	.6
n	employment rate	.94
ϕ	inverse of the elasticity of intertemporal substitution of leisure	2.5
B	utility parameter	2.8661
κ	adjustment cost parameter	24.8954
s	exogenous separation rate	.0383
w	real bargaining wage	1.9924
ρ_z	aggregate shock persistence parameter	.95
ρ_g	persistence parameter of shock to the money growth rate	.49
σ_z	standard deviation of the aggregate shock	.005
σ_g	standard deviation of the shock to the money growth rate	.00623

Table 2: Standard Deviations

		Relative to the Volatility of Output								
	y	c	i	n	h	nh	u	v	w	a
U.S. Economy	2.09	.36	2.86	.61	.24	.74	5.31	6.36	.43	.50
Basic Model										
Both shocks	2.11	.74	2.39	.12	.12	.18	1.83	2.13	.81	.83
Monetary shock only	.04	8.61	26.77	.13	.45	.38	2.08	2.42	1.25	1.21
Technology shock only	2.11	.72	2.33	.12	.12	.18	1.82	2.12	.81	.83
1-Period Contract Model										
Both shocks	2.09	.74	2.38	.12	.11	.18	1.84	2.14	.92	.84
Monetary shock only	.04	8.69	27.61	.14	.39	.29	2.13	2.48	24.60	1.24
Technology shock only	2.09	0.73	2.32	.12	.11	.18	1.84	2.14	.80	.84
2-Period Contract Model										
Both shocks	2.08	.74	2.38	.13	.10	.19	2.09	2.88	1.04	.84
Monetary shock only	.10	3.30	11.06	1.09	.27	.92	17.11	36.49	14.15	.55
Technology shock only	2.08	.73	2.31	.12	.10	.18	1.90	2.26	.79	.84
3-Period Contract Model										
Both shocks	2.09	.74	2.39	.20	.09	.23	3.08	4.71	1.16	.84
Monetary shock only	.23	1.49	5.72	1.31	.22	1.18	20.55	35.99	7.84	.44
Technology shock only	2.08	.73	2.31	.13	.09	.18	2.04	2.51	.79	.84
4-Period Contract Model										
Both shocks	2.13	.74	2.41	.30	.09	.31	4.75	7.14	1.25	.82
Monetary shock only	.41	.93	4.21	1.36	.19	1.26	21.36	33.73	5.14	.43
Technology shock only	2.09	.73	2.31	.14	.08	.19	2.26	2.84	.77	.84

Notes: Data (1956:I-2005:IV) are seasonally adjusted and HP filtered with smoothing parameter 1,600. y = production in non-farm business; c = real personal consumption expenditures of non-durable goods and services + real government consumption expenditures; i = real gross private investment + real personal consumption expenditures of durables ; n = employed persons in the non-farm business sector; h = average weekly hours; nh = total hours worked; u = unemployed persons from the Current Population Survey; v = help-wanted indices; w = real hourly compensation; a = labor productivity (y/nh).

Table 3: Correlations with Output

	correlations with output								
	c	i	h	nh	u	v	w	a	(a, nh)
U.S. Economy	.65	.94	.72	.87	-.84	.90	.24	.70	.27
Basic Model									
Both shocks	.87	.89	.56	.94	-.89	.99	.99	.99	.91
Monetary shock only	.33	-.18	-.61	-.40	-.94	.95	.93	.95	-.65
Technology shock only	.90	.91	.57	.94	-.89	.99	.99	.99	.92
1-Period Contract Model									
Both shocks	.87	.89	.52	.91	-.90	.99	.84	.99	.87
Monetary shock only	.18	-.03	-.81	-.75	-.96	.99	.11	.99	-.85
Technology shock only	.90	.91	.52	.91	-.90	.99	.98	.99	.87
2-Period Contract Model									
Both shocks	.87	.89	.46	.87	-.86	.78	.70	.99	.81
Monetary shock only	-.03	.44	-.86	.85	-.93	.15	-.30	.37	-.17
Technology shock only	.90	.91	.47	.89	-.93	.99	.97	.99	.84
3-Period Contract Model									
Both shocks	.87	.89	.36	.77	-.71	.55	.58	.98	.65
Monetary shock only	.09	.70	-.83	.94	-.97	.38	-.47	-.27	-.59
Technology shock only	.90	.91	.41	.88	-.95	.97	.96	.99	.83
4-Period Contract Model									
Both shocks	.87	.90	.22	.67	-.61	.46	.46	.95	.43
Monetary shock only	.28	.84	-.81	.96	-.98	.51	-.54	-.49	-.72
Technology shock only	.90	.91	.34	.88	-.96	.94	.96	.99	.83

Notes: $Corr\,(y,n) = -Corr\,(y,u)$ and $Corr\,(y,P) = -Corr\,(y,c)$

Table 4: Correlations with Unemployment

	c	i	h	nh	v	w	a
U.S. Economy	-.57	-.78	-.56	-.94	-.93	-.13	-.29
Basic Model							
Both shocks	-.93	-.65	-.19	-.75	-.94	-.93	-.90
Monetary shock only	-.24	.10	.63	.41	-.95	-.90	-.91
Technology shock only	-.96	-.66	-.19	-.75	-.94	-.93	-.90
1-Period Contract Model							
Both shocks	-.93	-.66	-.15	-.80	-.94	-.82	-.90
Monetary shock only	-.25	.10	.72	.59	-.95	-.11	-.91
Technology shock only	-.96	-.68	-.15	-.80	-.94	-.95	-.90
2-Period Contract Model							
Both shocks	-.82	-.71	-.11	-.85	-.70	-.54	-.83
Monetary shock only	.20	-.58	.77	-.98	-.18	.41	-.01
Technology shock only	-.96	-.74	-.17	-.85	-.91	-.98	-.92
3-Period Contract Model							
Both shocks	-.59	-.66	.04	-.91	-.57	-.14	-.59
Monetary shock only	.03	-.77	.78	-.99	-.41	.52	.49
Technology shock only	-.94	-.80	-.21	-.89	-.86	-.98	-.94
4-Period Contract Model							
Both shocks	-.45	-.64	.22	-.96	-.60	.14	-.38
Monetary shock only	-.16	-.90	.77	-.99	-.53	.58	.65
Technology shock only	-.90	-.85	-.23	-.92	-.82	-.96	-.94

A Steady State and Linearized Economy

The appendix presents the steady state of the model economy with period-by-period nominal wage and hours worked bargaining, and the equilibrium conditions linearized around the steady state. Further discussion of the linearized economy with multi-period nominal wage bargaining is in a separate technical appendix available upon request.

A.1 Steady State

Given the steady state job-finding (f) and the employment rate (n), the exogenous separation is derived from the employment dynamics:

$$s = \frac{1-n}{n} f,$$

Note that the firm's hiring rate is equal to the separation rate, i.e., $x = s$. From the household's first-order condition, the real interest rate is given by $r = 1/\beta - 1 + \delta$. Given the steady state hours of work (h), the capital-to-employment rate ($\overline{k}$) is equal to

$$\overline{k} = \left(\frac{\alpha}{r}\right)^{1/(1-\alpha)} h.$$

The aggregate capital stock, output, and investment are given by $k = n\overline{k}$, $y = k^{\alpha}(nh)^{1-\alpha}$, and $i = \delta k$, respectively.

The steady state bargained wage and hours of work satisfy

$$\begin{aligned} \frac{W}{P} h &= \gamma \left((1-\alpha)\overline{y} + \frac{\kappa}{2} x^2 + f\kappa x \right) + (1-\gamma) \left(b + B \frac{h^{1+\phi}}{1+\phi} \frac{1}{\mu} \right), \\ \lambda &= (1-\alpha)\overline{k}^{\alpha} h^{-\alpha}, \end{aligned}$$

where $\overline{y} = \overline{k}^{\alpha} h^{1-\alpha}$ and the marginal rate of substitution of leisure $\lambda = Bh^{\phi}/\mu$. In the steady state, the first-order condition for firm's hiring rate can be expressed as

$$\begin{aligned} \kappa x &= \beta \left\{ (1-\alpha)\overline{y} - \frac{W}{P} h + \frac{\kappa}{2} x^2 + (1-s)\kappa x \right\} \\ &= \beta \frac{(1-\alpha)\overline{y} - \frac{W}{P} h}{1 - \beta \left(1 - s + \frac{x}{2}\right)}. \end{aligned}$$

The steady state real wage, $\frac{W}{P}$, solves the following:

$$\left(1-(1-\gamma)\rho+\gamma\frac{\beta\left(f+\frac{x}{2}\right)}{1-\beta\left(1-s+\frac{x}{2}\right)}\right)\frac{W}{P}h=\gamma(1-\alpha)\overline{y}\left(1+\frac{\beta\left(f+\frac{x}{2}\right)}{1-\beta\left(1-s+\frac{x}{2}\right)}\right)+(1-\gamma)\frac{\lambda h}{1+\phi},$$

where ρ is the replacement ratio of unemployment insurance benefits such that $b=\rho(W/P)h$. Given the real bargained wage, the labor adjustment cost parameter κ is determined. From the market clearing condition, the aggregate consumption is given by

$$c=y-i-\frac{\kappa}{2}x^2n.$$

The steady state level of price is $P=1/c$, and the Lagrange multiplier is then $\mu=\beta/P$. The parameter of the utility function B is determined by

$$B=\lambda\mu h^{-\phi}.$$

A.2 Linearized Model Economy

In this subsection, the log-linearized model economy with period-by-period bargaining is presented. The log-linearized equations of the representative household's first-order conditions are given by

$$\widehat{\mu}_t = \widehat{P}_t-E_t\left[\widehat{g}_{t+1}\right], \tag{A-1}$$

$$\widehat{\mu}_t = E_t\widehat{\mu}_{t+1}+\left[1-\beta\left(1-\delta\right)\right]E_t\widehat{r}_{t+1}. \tag{A-2}$$

The log-linearized cash-in-advance constraint and budget constraint (resource constraint in equilibrium) are given by

$$\begin{aligned}\widehat{c}_t &= -\widehat{P}_t,\\ \widehat{y}_t &= \frac{c}{y}\widehat{c}_t+\frac{i}{y}\widehat{i}_t+\frac{\kappa}{2}x^2\frac{n}{y}\left(2\widehat{x}_t+\widehat{n}_t\right).\end{aligned}$$

Output, the hiring rate, and capital demand are given by

$$\begin{aligned}
\widehat{\overline{y}}_t &= \widehat{z}_t + \alpha \widehat{\overline{k}}_t + (1-\alpha)\,\widehat{h}_t \\
\widehat{x}_t &= E_t\widehat{\mu}_{t+1} - \widehat{\mu}_t + \beta E_t \widehat{x}_{t+1} \\
&\quad + \frac{\beta}{\kappa x} E_t \left[(1-\alpha)\overline{y}\widehat{\overline{y}}_{t+1} - \frac{W}{P} h \left(\widehat{W}_{t+1} - \widehat{P}_{t+1} + \widehat{h}_{t+1} \right) \right] \\
\widehat{r}_t &= \widehat{z}_t - (1-\alpha)\,\widehat{\overline{k}}_t + (1-\alpha)\widehat{h}_t,
\end{aligned}$$

where $\widehat{y}_t = \widehat{n}_t + \widehat{\overline{y}}_t$ and $\widehat{k}_t = \widehat{n}_t + \widehat{\overline{k}}_t$. The law of motion for the capital stock and the shock process are

$$\begin{aligned}
\widehat{k}_{t+1} &= (1-\delta)\,\widehat{k}_t + \delta \widehat{i}_t \\
\widehat{z}_t &= \rho_z \widehat{z}_{t-1} + \widehat{\varepsilon}^z_t.
\end{aligned}$$

The linearized nominal wage and hours worked are given by

$$\begin{aligned}
\frac{W}{P} h(\widehat{W}^t_t - \widehat{P}_t + \widehat{h}_t) &= \gamma \left((1-\alpha)\overline{y}\widehat{\overline{y}}_t + \kappa x(x+f)\widehat{x}_t + \kappa x f \widehat{f}_t \right) + (1-\gamma)\frac{\lambda h}{1+\phi}(\widehat{\lambda}_t + \widehat{h}_t) \\
\widehat{\lambda}_t &= \phi \widehat{h}_t - \widehat{\mu}_t \\
&= \widehat{z}_t + \alpha \widehat{\overline{k}}_t - \alpha \widehat{h}_t.
\end{aligned}$$

The linearized labor market variables are summarized as

$$\widehat{\mathcal{M}}_t = \xi \widehat{u}_t + (1-\xi)\,\widehat{v}_t$$

$$\widehat{n}_{t+1} = (1-s)\,\widehat{n}_t + s\widehat{\mathcal{M}}_t$$

$$\widehat{u}_t = -\frac{1-u}{u}\widehat{n}_t$$

$$\widehat{f}_t = \widehat{\mathcal{M}}_t - \widehat{u}_t$$

$$\widehat{q}_t = \widehat{\mathcal{M}}_t - \widehat{v}_t$$

$$\widehat{x}_t = \widehat{q}_t + \widehat{v}_t - \widehat{n}_t.$$