Determining the optimal moment for replacing equipment

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9 June 2011
DETERMINING THE OPTIMAL MOMENT FOR REPLACING EQUIPMENT  
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Summary  
In the present work I tried to reach two main objectives. The first consists in determining whether the theoretical methods regarding the ascertainment of the optimal moment of replacement can be used as tools in adopting a decision of replacement and upgrade of fixed mechanisms in a company. The second objective is to support those investors or managers who, having a technological professional educational background, do not have the possibility to finish studies in the economic field, with emphasis on the information and working techniques necessary for adopting and implementing a development investment decision. Many times the most efficient technical or high-performing solutions, proposed by engineers, do not provide the best solutions or financial effects, determined by economists. That is why there are often contradictions or even conflicts between the approach method and that of putting into practice the company’s activities, especially investments.

KEY WORDS  
Investments, development investments, fixed mechanisms, operating period, efficiency, optimal replacement moment, modernized plant, update coefficient.

WORK CONTENTS  
Investments represent capital investments achieved in order to obtain durable goods by which, in the future, revenues that will cover the initial investment and a profit will be obtained. From an economic perspective, the investments represent giving up the immediate use of some available resources in order to obtain future revenues (higher than the investment). From an accounting perspective, the investments represent freezing funds as mobile or immobile values destined or not to the current activity. From a financial perspective, investments represent actual expenses achieved in order to generate revenue flows that would ensure the recovery of initial expenses and profit gain. According to the purpose of launching the investment or the pursued objective, these are classified in:
- Investments of replacing the fixed capital
- Investments of upgrading and increasing productivity
- Investments of expansion, development or building new objectives
- Investments of scientific research, innovation
- Strategic investments

The economic efficiency of upgrading and developing fixed capital in production units  
The techno-economic restructuring of production units implies upgrading and reengineering them, so that they would assure an efficiency and productivity increase of the developed activities. Thus, an improvement of techno-economic parameters of the fixed capital, the increase of their work speed, the reduction of specific consumptions, the increase of the reliability, maintenance, etc. are achieved. Usually, through upgrade and reengineering there is ensured the replacement of physically outworn tools, that don’t correspond to the level achieved by the technological process. This upgrade form is known as there-equipment of sections. Along with re-equipment it is possible to choose development activities of existent units, by adding them to others’ existent fixed capital, so that the final resulted capacity would not represent a simple amount of the capacities pertaining to the old and new fixed capital, but, more than that, as a result of the fact that new tools better emphasize the capacities
previous to the investment process (the “bottlenecks” in the production process are eliminated, the quality level of the goods increases, etc.). Promoting the technological progress by developing, upgrading and reequipping the existent capacities represents a complex method which leads to obtaining one of the best results for the activity deployed in the production system.

Ensuring the normal development of economic activities of the economic units is made by means of their production mechanisms, formed by the circulating and fixed methods. The fixed methods represent that part of production mechanisms that participate in the work process with their entire application value, which are consumed and transmit their value to the product gradually created over the production cycles.

The fixed methods which represent the object of the exploiting, maintenance and repair activity are greatly diversified, the following branches being distinguished, due to their substantial weight, in the industrial domain: the car industry, the iron and steel industry, the chemical industry and the extraction industry.

According to their destination, the fixed industrial methods are classified in: buildings, special constructions (mines, drills, railways, etc.), force machines and energetic plants, work and tool machines, measurement devices and installations, control and tuning, means of transport, animals, plantations, tools and accessories.

The improvement of using existent fixed methods, a primary factor of economic increase, is achieved by organizing and ensuring their maintenance activity in optimal conditions.

The disintegration and degradation of fixed methods implies their replacement with new ones on the strength of fixed methods recovered through amortization.

**Determining the optimal moment of tool replacement**

The decision to invest represents a series of successive techno-economic options, regarding different aspects of the production system activity, such as: its optimal dimension, its placement, the technologies used, the types of tools acquired and their origin, the optimal service period of the tools, the efficient period of operating of the new objective created in the main, etc. To measure the economic efficiency of using a tool or a set of tools, it’s not enough to stop at appreciation criteria such as: recovery period of investments, the economic coefficient of efficiency in investment, etc., indicators that take into account only the annual effects occur as a result of the act of investing. Taking into account the economic life of the objective, the annual impact will multiply, it will propagate throughout the operating effectively. Static, this period coincides with the natural life time investment.

The growth process requires the calculation of economic efficiency in a dynamic, ever-advancing technical and economic. It should also be pointed out that the investment process is characterized by a coordinate in addition to the manufacturing process – time.

So in choosing the project an important role is to determine the optimal length of the objective operation, bringing together the effort that makes the production system, present and future expected impacts of its activities. Looking back, the production system with machines already in operation, the problem of determining the optimal timing of production now and expected effects of its future work.

Looking back, the production systems, with equipment already in use, the problem of determining the optimal timing of replacement of equipment and how it can be treated in the context of enlargement and development or modernization of the system.

The decisions of replacing should consider an important number of factors: the physical and moral wearing determined by the technologic progress in equipments and processes, the changing of demand on the market, the pressure of a stronger and stronger competition. All these lead to the necessity of some dynamic and flexible policies of replacing the old equipments with new ones after economic time periods. In establishing the models of
determining the optimal moment of replacing the equipments, one of the difficult problems is the lack of final data at the moment of analyze for the following parameters: the remained value, the rate of technologic changes, the exploitation costs, the maintenance, sustaining and repairing costs, the level of performance, etc.

That is why the evaluation is done based on presumptions or estimations. The problems of replacing the tools are divided into two large categories that consider:

- The elements whose performances damage in time, but which can be partially or totally rehabilitated by efficient programs of maintenance. In this category are included: the machine-tools, the tools and other industrial equipments. The costs of the damaged performances, the cost of maintenance, of keeping in use, are expected to raise in time. This category of problems is usually formulated in a determinist system.

- The elements whose performances are not damaged in time have a sudden fall, break, that’s why they must be immediately replaced. It’s the case of the components with a high degree of breaking.

The replacement programs take into consideration if the equipments will be individually replaced as they are damaging, or if it is necessary that they should be periodically replaced in group, which leads to the diminishing of the damaging probability. These considerations led to the concept of preventive maintenance, often formulated under the circumstances of risk and uncertainty. This approach to the problems of replacement is made in the context of building on the base of logic analysis. One of the frequently analyzed models is the Kaufman method which supposes replacing the tool when its maintenance and operating expenses overcome the total expenses (for the purchase of a new tool and for the upgraded repairing).

The calculating relation in the context of the method is:

\[
C_t = \frac{I_0 + \sum_{i=1}^{n} R_i \cdot \alpha^{b-1}}{\sum_{i=1}^{n} \alpha^{b-1}}
\]

in which: 
- \(C_t\) – are the total upgraded expenses;
- \(I_0\) – the purchase value of the tool;
- \(R\) – the expenses of maintenance, operating and repairing the tool;
- \(\alpha\) - the upgrading factor;
- \(\alpha\) - is the upgrading coefficient;
- \(n\) – the number of years that the calculation covers;

The tool will be replaced in the year \(h\), for which the following condition is achieved:

\[
R_{b+1} \geq \frac{I_0 + \sum_{i=1}^{n} R_i \cdot \alpha^{b-1}}{\sum_{i=1}^{n} \alpha^{b-1}} \iff R_{b+1} = \frac{I_0 + R_1 + R_2 \cdot \alpha^2 + \ldots + R_n \cdot \alpha^{n-1}}{\alpha^0 + \alpha + \alpha^2 + \ldots + \alpha^{n-1}}
\]

This method has the advantage that the influence of promoting the technological progress on the tools already existent in the unit is also considered.

In the activity of any production system, determining the optimal moment of replacing the fixed capital generates two facilities:

✓ knowing the efficient period of operating, the production system has the possibility to ensure funds of necessary investments, so that the cassation of the fixed capital would correspond to the moment of reequipping certain sections and workshops. The elaboration of a rigorous program of investments depends largely on establishing the optimal moment of replacement;
necessary indications for the management activity can be obtained if the period of efficient operating of the fixed capital ($D_e$) is compared to the normal period of its writing-off ($D$). Practically, three situations may appear:

- $D_e = D$, neutral situation;
- $D_e > D$, the inequality reflects the situation of an accelerated amortization process, which represents a positive fact, as the respective production system can restructure the investments funds much earlier, which creates the perspective of developing and upgrading its own activity.
- $D_e < D$, the most unfavorable situation, is to be avoided by any means. It reflects the situation in which the fixed capital must be disabled before being amortized.

Starting from the duality of problems regarding the economic efficiency (imposed by the alternative minimum-maximum), we will present, in what follows, models based on one hand on the criteria of minimizing the upgraded total costs, and on the other hand on maximizing the upgraded total revenues. The variant of calculating the optimal period of a tool operating and the variant for a tool lot/group/line (is calculated only with discrete values) are both considered. For the remaining value of the tool in year $T$ one of the following methods of amortization can be used: in constant quotas, regressive and progressive.

**Methods to maximize the effects**

In the case of a single plant discounted net income $V_h$ in year $h$ ($h' = 1, ..., T$) is given by:

$$V_h = \sum_{i=1}^{T} \frac{P_i - C_i - I_0 + \frac{S(T)}{(1+a)^t}}{(1+a)^t}$$

where:

- $P_i$ - is the annual production value obtained on the plant in the year $h$
- $C_i$ - annual cost of maintenance and operation of equipment, in the year $h$
- $I_0$ - purchase value of the equipment
- $S(T)$ - the remaining value of the unit in the year of replacement $T$
- $T$ - optimal year of replacement or optimal duration of economic life of the equipment
- $a$ - discount rate

We notice that the relation summed up the discounted net income throughout the economic life of the plant ($T$), plus the remaining value of equipment. The update is done at the time of purchase of equipment.

This model presented above assumes that the system uses production equipment until the end of its life ($T$) for a certain type of production, then it will be sold for the value $S(T)$. Usually, however, production continues for an indefinite period of time and requires the purchase of a group of plants. Since the length of the string is infinite, the total discounted net value and thus economic life for each plant group will be the same.

For a group of plants discounted net income $V_h$ in the year $h$ ($h = 1, ..., T$) is:

$$V_h = \left[ \sum_{i=1}^{T} \frac{P_i - C_i - I_0 + \frac{S(T)}{(1+a)^t}}{(1+a)^t} \right] \left[ \frac{1}{1 + \frac{1}{(1+a)^T} + \frac{1}{(1+a)^{2T}} + ...} \right]$$

where the sizes that appear were mentioned. Note: $x = 1/(1 + a)^T$ and using the approximation formula of a Taylor series, in the case of a operation with one variable, we obtain:

$$1 + x + x^2 + ... = \frac{1}{1 - \frac{x}{(1-a)^T}} = \frac{(1+a)^T}{(1+a)^T - 1}$$
Therefore,

\[ V_h = \left[ \sum_{k=1}^{T} \frac{P_k - C_h}{(1 + \alpha)^k} - I_0 + \frac{S(T)}{(1 + \alpha)^T} \right] \left[ \frac{(1 + \alpha)^T}{(1 + \alpha)^T - 1} \right] \]

Therefore,

**Methods to minimize costs**

In the case of a single plant, discounted total costs \( C_h \), in year \( h = 1, \ldots, T \) is calculated using the ratio:

\[ C_h = \sum_{k=1}^{T} \frac{C_h}{(1 + \alpha)^k} + I_0 - \frac{S(T)}{(1 + \alpha)^T} \]

where the quantities that occur have already been defined.

For a plant group discounted total costs \( C_h \), in the year \( h = 1, \ldots, T \) are:

\[ C_h = \left[ \sum_{k=1}^{T} \frac{C_h}{(1 + \alpha)^k} + I_0 - \frac{S(T)}{(1 + \alpha)^T} \right] \left[ \frac{(1 + \alpha)^T}{(1 + \alpha)^T - 1} \right] \]

where the remaining value calculation was made with the same algorithm as the previous criterion.

In all cases considered above it was taken into account the fact that the plant was replaced with a similar one, meaning with the same technical and economic performance. In reality, the new plant is more efficient, resulting in an increased productivity for the production system.

**Substantiating the optimal decision for purchasing plants**

A key role in order to achieve the objectives of upgrading and modernization of production systems is played by the development of production capacity, which provides economic efficiency increase to a greater extent, enabling the primary mobilization of intensive economic growth factors. Thus, the purchase of new and more efficient plants provides reduced material costs, further improving production, reducing energy consumption and quality of finished products. Therefore, there arises the need to substantiate the actual total effort by taking into account not only the value of investments, but also of the expenses with plant maintenance and repair, production cost structure and the influence of the time factor upon these efforts.

A method of determining the direction of resource allocation, of choosing the optimal option for the purchase of plants respectively, is the method to minimize the amount to be recovered annually. In this case, the annual cost with repairs is discounted when we buy the plant and then, adding up the value of purchase, it is weighted with the factor for the recovery of the funds used, in order to determine the amount to be recovered, having in view the fact of ensuring full recovery of the amount spent on purchase, maintenance and repairmen of the plant. The relation for calculation is:

\[ K = [I_0 + \sum_{k=1}^{D} R_k \cdot (1 + \alpha)^{-k}] \cdot \frac{\alpha \cdot (1 + \alpha)^D}{(1 + \alpha)^D - 1} \]

Where:

- \( K \) – the annual amount to be recovered
- \( I \) - the purchase value of the plant
- \( a \) – discount coefficient
- \( R \) – plant maintenance and repair costs, in the year \( h \)
- \( D \) – lifetime of the objective

The discounted coefficient level can be given by the bank interest granted for the amounts that the production system keeps in the account, or by the average coefficient of economic efficiency within the industry or activity for which the calculation is made for.
Example: Let there be substantiated the decision to purchase some threading machines, characterized by the indicators in the following table:

<table>
<thead>
<tr>
<th>Type of plant</th>
<th>Purchase value [um x 10]</th>
<th>Average annual expenses related to repairing [um x 10]</th>
<th>Lifetime[ut]</th>
<th>Discounted Coefficient [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>48500</td>
<td>380</td>
<td>15</td>
<td>6</td>
</tr>
<tr>
<td>B</td>
<td>54650</td>
<td>310</td>
<td>13</td>
<td>6</td>
</tr>
</tbody>
</table>

Applying the relation for calculating the amount to be recovered annually, we obtain:

\[
K_A = \left[ 48500 + \sum_{k=1}^{15} 380 \times (1+0.06)^{-k} \right] \times \frac{0.06 \times (1+0.06)^{15}}{(1+0.06)^{15} - 1} = 5370 \times 10^3 \text{ um},
\]

\[
K_B = \left[ 54650 + \sum_{k=1}^{13} 310 \times (1-0.06)^{-k} \right] \times \frac{0.06 \times (1+0.06)^{13}}{(1+0.06)^{13} - 1} = 6479 \times 10^3 \text{ um},
\]

It appears that variant B contains a higher annual amount to be recovered, although the annual costs of maintenance and repair of equipment are lower. This is because the type B plant has a higher purchase value while for plant A the expenditures that will be made in future period are higher (an amount spent today has an economic value much greater than the same amount spent in a future period). The relation can be applied with good results, if the level of production capacity and production value are equal, regardless of the plant type. However, a certain growth of the purchase value of the plant leads to its proper increase of the efficiency of the production value achieved on that plant. Also, a large amount of spending on maintenance and operation of the plant provides a higher level of use of its direct effects on economic performance. In this case it is necessary to calculate the annual to-be-recovered costs indicator, per one unit of effect (production, profit, etc.), noted with \( K \). The relation of calculation in this case is:

\[
K = \frac{K}{\sum_{k=1}^{D} P_k (1+\alpha)^{-k}}
\]

The two investment alternatives are not comparable in terms of operation time of the two plants. In this case, it is necessary to ensure compatibility by putting a number of plants under research, which should provide an operating time equal to the lowest common multiple of the operating times for the plants analyzed.

Regarding the cases presented, it was considered that the payment of the plant is fully provided by the production system capital. If payment is provided part by the production system capital and the difference is provided by means of loans, the annual cost to be recovered is calculated using the relation:

\[
K = \left[ I \times \sum_{k=1}^{D} C_k (1+\alpha)^{-k} + \sum_{k=1}^{D} R_k \times (1+\alpha)^{-k} - V (1+\alpha)^D \right] \times \frac{\alpha \times (1+\alpha)^D}{(1+\alpha)^D - 1}
\]

where:
- \( D \) - is the duration for the repayment of the credit
- \( C \) - credit repayment quotas in the year \( h \)
- \( V \) - the value of the plant to be paid
- \( I \) - advance payment from the fund’s own production system

Analyzing the previous relation it shows that it is more profitable (annual costs are lower) to make that purchase of plants on loan, if the loan interest is lower (in case A, if the plant is
purchased based on partial credit with interest of 6%, repaid in 10 ut, the annual costs to be recovered will be of 4376x10 um) and is unenforceable if the interest is higher (at the same event the interest rate is 12% and duration for the repayment of 5 years, costs are of 5486x10 um). Under these conditions it follows that there is some level of interest until the loan is convenient to make the purchase of debt and purchase equipment that becomes ineffective, as sources of financing or credit combined form.

This level of interest can be determined by successive tests, with the results obtained from the two relations to be equal. This is in case the repair costs have a discrete evolution. If the evolution is continuous, then this continuity has to be mentioned. In the calculation relation, the repairs are a operation of time or another indicator, which is directly influenced by it (such an indicator can be the annual profit).

\[
K = \left[ I_0 + \sum_{k=1}^{D} f(P_k) \times (1 + \alpha)^{-k} \right] \times \frac{\alpha(1+\alpha)^D}{(1+\alpha)^D - 1}
\]

If the plant purchase is made through double financing, that is one part of the equity and the rest on credit, the relation becomes:

\[
K = \left[ I_p + \sum_{k=1}^{D} C_k (1 + \alpha)^{-k} + \sum_{k=1}^{D} f(P_k) \times (1 + \alpha)^{-k} - V_j (1 + \alpha)^D \right] \times \frac{\alpha(1+\alpha)^D}{(1+\alpha)^D - 1}
\]

and in this case, if the economic effects level is different, we should calculate specific costs to be recovered annually.

CONCLUSIONS

Regarding the issue of replacement equipment study, one important issue is that of determining the optimal timing of replacement due to their physical and moral wear.

From an economic perspective, the depreciation of equipment determines the reduction of both the quantitative and qualitative performance. Replacement involves additional costs regarding acquiring and installing new equipment. Therefore, the optimal time for replacement must be chosen so as to achieve the best compromise between system performance and cost of replacement.

From the above results that the level of expenses on the allocation of resources to buying and maintaining plants depends on the purchase amount, the loan interest and the annual expenses for repairing and maintenance. Substantiating the decision to purchase plants using mathematical modeling, has a particular importance because in market economy the production of machinery, equipment, production lines are at a high level of diversification. Only by linking the efforts with the economic effects through the system of indicators and taking into account a certain priority economic criterion, can we properly underpin purchasing decisions.

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