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May 2011

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May 14, 2011

Abstract We present a simple stochastic model in which heterogeneous agents accumulate wealth belonging to the capitalist or the working class, with profits generated by a stochastic multiplicative process and wages by an additive one. Class selection is based on a random process depending on wealth distribution and the profit rate. In general, playing the role of capitalist rises the probability of accumulating more and more wealth that, in turn, increases the probability to play again the role of the capitalist in following periods. This may give rise to an amplification mechanism leading to a persistent division in social classes. A scenario analysis is performed to explore the sensitivity of results to alternative assumptions on the propensity to consume/save and the fraction of wealth invested by capitalists in the risky process.

Keywords: wealth distribution; social classes; capitalist accumulation.

JEL Classification Numbers: P10, D31, C63.

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1 Introduction

The aim of this paper is to present a simple stochastic model with heterogeneous agents and class division in which wealth accumulation depends on the role agents play in the society: capitalists or workers. In the first case, they earn a profit generated by a stochastic multiplicative process; in the second one, they earn a wage generated by an additive stochastic process. This is similar to what assumed by Nirei and Souma (2007) in order to fit the empirical distribution of wealth in a two factor model, even though in that case there is no class division.[1] In this setting we introduce a random process of class selection according to which richer agents are more likely to become capitalists, and viceversa. Moreover, the rise of the profit rate leads to an increase of the average probability to become a capitalist (a sort of competition devise).

We study the model by means of computer simulation. In general, playing the role of capitalist rises the probability to accumulate more and more wealth that, in turn, increases the probability to play again the role of the capitalist in the following period. This gives rise to an amplification mechanism leading to a persistent division in social classes according to which agents which spend a long time as a capitalist are more likely to become richer, while a large majority of the population remains in the working class, with a small probability to access the capitalist role. We also perform a scenario analysis to check the robustness of simulation results to alternative parameter settings regarding the propensity to consume/save and the fraction of wealth invested by capitalists in the risky process.

2 The baseline model

Consider an artificial society composed of N agents, each of which has an initial endowement of wealth equal to ω_0 at time $t = 0$. For $t = 1, 2, ..., T$, agents will make decisions in order to produce income, consume and accumulate wealth. In each period t, an agent i can belong to one of the two *social classes*: capitalists and workers. We indicate with N_t^k and N_t^j the number of capitalists and workers in period t, respectively. Whether an agent will be a capitalist or a worker in period t depends on a random process that considers the distribution of individual wealth and the profit rate, π_{t-1}^r: indicating with ω_t^{max} the maximun value of agents' wealth in period $t-1$, we can construct an individual variable $p_t^i = \omega_{t-1}^i/\omega_{t-1}^{max} + \rho\pi_{t-1}^r$, where ρ is a positive parameter.[2] For each agent i, the value of

[1] In the Nirei-Souma model, all agents benefit from asset accumulation as well as from labour income in the following way: $a(t+1) = \gamma(t)a(t) + w(t) - c(t)$, where a is the asset, w is the wage, c is consumption, and t is time.

[2] We set a zero lower bound and a unitary upper bound for p_t^i.

another variable, $\tilde{p}_t^i$, is picked at random from a uniform distribution (0,1): if $p_t^i > \tilde{p}_t^i$, then agent i will be a capitalist in period t, otherwise a worker. So, it is more likely that a richer agent will be a capitalist, while a poorer agent a worker. Moreover, the higher the profit rate the higher the probability, for all agents, to become a capitalist; it follows an increase of competition among agents as capitalists, and viceversa.[3]

Capitalists start the *risky* economic activity with the aim of producing a profit and accumulate wealth. Let's assume that a capitalist – say the agent k – invests a fraction γ of its accumulated wealth: $\bar{\omega}_t^k = \gamma\omega_{t-1}^k$, where $0 < \gamma < 1$. Then, total wealth invested in period t is $\bar{\Omega}_t = \sum_k \gamma\omega_t^k$, $k = 1, 2, ..., N_t^k$. Individual profit is the result of a *multiplicative stochastic process*: $\pi_t^k = \phi\bar{\omega}_t^k$, where ϕ is a normally distributed random shock with positive mean μ_ϕ and finite standard deviation σ_ϕ.[4] Aggregate profits are: $\Pi_t = \sum_k \pi_t^k$, $k = 1, 2, ..., N_t^k$. Then, we define the profit rate as: $\pi_t^r = \Pi_t/\bar{\Omega}_t$.

Workers are paid a stochastic wage given by a normally distributed random shock ϵ with a positive mean μ_ϵ and a finite standard deviation σ_ϵ. The wage paids to the j^{th} worker at time t is the result of an *additive stochastic process*: $w_t^j = w_{t-1}^j + \epsilon$.[5] So, the the wage bill for the whole economy is: $W_t = \sum_j w_t^j$, $j = 1, 2, ..., N_t^j$.

Worker j's consumption function is: $c_t^j = c' w_t^j$, where $c' > 0$ is the propensity to consume labour income. Similarly, for capitalist k: $c_t^k = c''\pi_t^k$, where $c'' > 0$ is the propensity to consume the income earned as a capitalist.[6] So, aggregate consumption is: $C_t = \sum_i c_t^i$, $i = 1, 2..., N$. As time elapses, agents' wealth evolves as the result of the accumulation of non-consumed income, that is savings: the j^{th} worker's wealth in period t depends on the wage income minus consumption: $\omega_t^j = \omega_{t-1}^j + w_t^j - c_t^j$; instead, the k^{th} capitalist's wealth depends on the realised profit net of actual consumption: $\omega_t^k = \omega_{t-1}^k + \pi_t^k - c_t^k$. Society's aggregate wealth is $\Omega_t = \sum_i \omega_t^i$, $i = 1, 2, ..., N$.

3 Simulating the baseline model

We study the dynamic properties of this artificial society by means of computer simulation. Let's set the initial endowement $\omega_0 = 100$, the same for all agents. We investigate the properties of a society composed of $N = 10000$ agents for a time spam of $T = 1000$ periods.

[3]If the process results in an empty set of capitalists, one agent is picked at random to become a capitalist.

[4]In this version of the model we do not distinguish between real and financial investments; accordingly, profits derive from both real and/or financial activities we do not model explicitly.

[5]In each period, we compute the wage for all agents, then also for non-worker ones (which is just a potential income), so that if the worker j was a capitalist in period $t-1$, then w_{t-1}^j represents the wage the agent would have earned as a worker; this is used as the base to compute the current wage. Moreover, a zero lower bound holds for wages, given that they cannot be negative.

[6]If the profit realised by the capitalist k is negative in period t, then its consumption is set equal to zero.

The parameters are set as follows: $\gamma = 1/2$, $\rho = 0.01$, $c' = 0.8$, $c'' = 0.2$; hence, following Kaldor (1955), $c' > c''$. The mean of the additive stochastic process for wages as well as of the multiplicative stochastic process for profits is $\sigma_\epsilon = \sigma_\phi = 1/4$. The standard error of the two stochastic process is $\mu_\epsilon = \mu_\phi = 0.01$.

First of all, we can see that starting from perfect equality – that is, with a zero Gini index (henceforth, *gini*) – wealth distribution evolves towards an unequal outcome with few rich and many poor(see Figure 1, upper-left panel), with $gini = 0.2123$. It is worth to note that agents spend a different time in a role or in another: it emerges that a large fraction of the population has been part of the working class for the majority of simulation periods: the average time spent as a capitalist is around 10%; few agents exhibit a time spent as a capitalist larger than 20 or 30% of the time as shown in Figure 1, upper-right panel; accordingly, the lower-right panel shows that the size of the capitalist class is around 10% of the entire population of agents.

Clearly, there is a connection between class division and wealth distribution: the longer the period of time spent as capitalist the higher the amount of individual wealth (see lower-right panel of Figure 1). Then, for an agent in this artificial society the aim of accumulating wealth is better reached being a capitalist rather than a worker; at the same time, the accumulation of wealth leads to a higher probability to be a capitalist. This amplification mechanisms results in an unequal distribution of wealth which is connected to class division.

The profit rate oscillates around an average value of 0.0105 (obviously, very close to μ_ϕ), attracting more agents in the capitalist role when above the average, and viceversa. This positive average profitability leads to (an exponential) accumulation of wealth at an average growth rate of 0.0044 (henceforth, $g\bar{\Omega}$), with a standard error of 0.0025 (henceforth, $std\bar{\Omega}$). Then, the coefficient of variation is about 55% (henceforth, $CV\bar{\Omega}$).

4 Scenario analysis

In this section we explore the dynamics of the model in order to check the robustness of simulation results to alternative parameter settings. In particular, we are interested in investigating the role of the propensity to consume/save and of the parameter γ, that is the fraction of wealth invested by capitalists in the risky process.

4.1 Propensity to consume/save

In this subsection we analise the role of the parameters c' and c''. Let's consider a setting in which all agents have the same propensity to consume, for instance $c' = c'' = 1/2$, independently of class division. Simulation results are summarised in Figure 2: also in this case it emerges a right-skew distribution of agents' wealth (upper-left panel); for an agent, the time spent as a capitalist (upper-right panel) is now almost equal to 30%, as the average size of the capitalist class (lower-left panel); there is a positive relation between the accumulated wealth and the time spent as a capitalist (lower-right panel), even if weaker in this case than in the baseline model. As a consequence, a less unequal society emerges with $gini = 0.1646$. Finally, in this case $g\bar{\Omega} = 0.0040$, $std\bar{\Omega} = 0.0037$, and $CV\bar{\Omega}$ is higher than than 90%.

Another "extreme" with respect to the consumption/saving behaviour is assuming that $c' = 1$ and $c'' = 0$, a scenario in which, following Kalecki (1942), capitalists earn what they spend and workers spend what they earn. In this case, $g\bar{\Omega}$ is almost zero and $std\bar{\Omega} = 0.0158$, that is a more volatile system with respect to the cases analised above (because of a quasi-zero growth rate, $CV\bar{\Omega}$ is exceptionally high). Hence, a very unequal society emerges, as described by Figure 3, with $gini = 0.4557$. The lower growth rate of society's wealth with respect to the previous case (and also to the baseline model) suggests that this scenario is a very good one for a very small number of capitalists, while the large majority of the working class does not benefit from wealth accumulation. To sum up, in this case wealth accumulation is weaker for the society as a whole while it is stronger for the restricted class of capitalists. In addition, the economy suffers from a larger volatity of the wealth accumulation process.

Let's now analise two other "extreme" scenarios. The first one is based on the following assumption: $c' = c'' = 0.8$, that is workers as well as capitalists have a "high"propensity to consume. In this case (see Figure 4) a quite equalitarian society emerges, being the distribution of wealth well approximated by a normal distribution, with *gini* slightly higher than 0.1. Agents spend about half of their time in a role of in the other and the fraction of the population belonging to the capitalist class in each period is around 50% in the average. As a consequence, there is not a strong correlation between being a capitalist and having a large wealth. Moreover, $g\bar{\Omega} = 0.0016$ with $std\bar{\Omega} = 0.0047$. So, the economy grows less than in the baseline scenario, with a comparable aggregate volatility. However, this volatility is related to a smaller average growth rate, with a very high value of $CV\bar{\Omega}$ (almost 300%).

Things change when we explore an opposite scenario: $c' = c'' = 0.2$, that is workers as well capitalists have a "low" propensity to consume. This simulation is characterised by a *gini* of almost 0.2, signalling a higher degree of inequality with respect to the previous case. The wealth distribution shows a fat right tail, indicating the presence of very rich agents which have accumulated more and more playing the role of capitalist for a large fraction of their time (see Figure 5). However, inequality is not so strong as in the case with $c' = 1$ and $c'' = 0$: this results is due to the fact that both capitalists and workers have a high propensity to save; this increases the correlation between wealth accumulation and the time spent as a capitalist; at the same time, workers' saving increases their probability to become capitalists, so rising *social mobility* in the economy. Then, also in this case we observe the amplification mechanism according to which the time spent as capitalist increases the accumulation of wealth which in turn increases the probability of being a capitalist and so on is; its strength is mitigated, however, by the presence of large savings coming from workers' income. In this setting, $g\bar{\Omega} = 0.0058$ – a value larger than that of the baseline scenario, while $std\bar{\Omega} = 0.0045$, a value slightly larger than in the baseline model. Then, $CV\bar{\Omega}$ is about 130%.

Obviously, simulation results are sensible to assumptions on the propensity to consume of agents. In general, a low propensity to consume – a high propensity to save – boosts wealth accumulation, and viceversa. The clearer case is the one for which $c' = 1$ and $c'' = 0$, a Kaldorian-like scenario, leading to the more unequal society we analised, because of a consumption behaviour stritly related to class division: capitalist save and workers consume. Interestingly enough, even when the propensity to save is high, independently of the role played in the society – the case for which $c' = c'' = 0.2$ – an evident class structure emerges, although characterised by a less unequal distribution of wealth (however, not Gaussian as when $c' = c'' = 0.8$). For sure, the analysis suffers from a lack of investigation about the role of consumption and, in general, of the demand side of the economy. As we will discuss in the conclusions, this is one of the steps we are going to make in the next future in order to improve the model.

4.2 Riskiness and wealth accumulation

In this subsection we investigate the role of γ in shaping simulation results. This parameter represents the fraction of wealth invested by a capitalist in the risky process described above. In a sense, then, it is a proxy for the riskiness of wealth accumulation (but we will also suggest a slightly different interpretation). The parameter was set equal to 1/2 in the baseline model. Let's now check the behaviour of the simulation model for two different values of this parameter: $\gamma = 1/4$ and $\gamma = 3/4$, say a "low" and a "high" level of γ.

When $\gamma = 1/4$, $g\bar{\Omega} = 0.0042$ – a value slightly smaller than that emerging from the simulation of the baseline model (0.0044). With a lower value of γ, $std\bar{\Omega} = 0.0021$, then it is smaller than that of the baseline scenario. In this case, $CV\bar{\Omega}$ is almost 50%, while the same value is around 55% in the baseline scenario. Figure 6 shows that the average size of the capitalist class is larger than in the baseline scenario, around 1/4 of the population (instead of 10%), mitigating the 'being capitalist-wealth accumulation' amplification mechanism, so that $gini = 0.1616$ – which is smaller than in the baseline scenario (0.2123).

When $\gamma = 3/4$, $g\bar{\Omega} = 0.0044$ and $std\bar{\Omega} = 0.0030$. Then, the economy grows as in the baseline scenario (then a bit more with a "high" than a "low" γ) but with a slightly larger volatility. $CV\bar{\Omega}$ is almost 70% with respect to about 50% in the previous case and 55% in the baseline scenario. Moreover, $gini = 0.2664$ – a value larger than in the baseline scenario. In fact, the average size of the capitalist class (as well as the average time spent by each agent in the capitalist role) is 6-7%, leading to a more unequal society (see Figure 7).

All in all, a "low" γ results in a slightly smaller growth of wealth accumulation and also in a lower aggregate volatility. Most importantly, the Gini index is now clearly smaller than in the baseline scenario. Instead, a "high" γ results in an average growth of wealth comparable to that of the baseline scenario, but more volatile and associated with a larger Gini index.

Again, the lack of an appropriate analysis of the demand side of the economy – which for sure we expect to have a very significant impact on macroeconomic dynamics – restrict the scope of model findings. This is a limitation that we want to overcome by extending the analysis beyond this basic version. For now, the experiments on the parameter γ suggest that the larger the fraction of wealth invested in the stochastic multiplicative process the more unequal is wealth distribution – although the average growth of wealth is quite the same with respect to the baseline model – and stronger are the effects of class division.

In a sense, the parameter γ could be interpreted as the amount of wealth required to perform the role of capitalist in the society. Accordingly, the larger γ the more likely is to observe an unequal society. Trying to make a parallel with Galor (2006), a possible interpretation is that when economic growth is led by huge investments in physical capital (assuming that this leads to a large fraction of wealth invested in the risky process), then a significant degree of inequality associated with a persistent class division emerges; on the contrary, if the access to the capitalist role could be quite likely for many agents, say when economic growth is led by investments in human capital (that could require a smaller amount of wealth to generate profits), then a less unequal society emerges. Anyway, in both cases credit and finance would play a fundamental role, suggesting another way to improve our modelling framework. For now, we can just say that the proposed random process of class selection implicitly means that agents' wealth is a sort of "collateral" in order to have the access to the capitalist role.

5 Concluding remarks and future research

We have presented a simple stochastic model with heterogeneous agents and class division. A major finding is that an unequal society may emerge starting from a perfectly equal one. Class selection is based on a random process depending on wealth distribution and the profit rate. In the baseline model, we assumed that capitalists have a "high" propensity to save, while workers have a “high” propensity to consume. In general, playing the role of capitalist rises the probability of accumulate more and more wealth that, in turn, increases the probability to play again the role of the capitalist in the following period. This gives rise to an amplification mechanism leading to a persistent division in social classes according to which agents which spend a long time as a capitalist are more likely to become richer, while a large majority of the population remains in the working class, being characterised by a small probability to access the capitalist role.

Exploring the model with different parameter settings has highlighted some relevant properties. A scenario in which capitalists save and workers consume leads to a very unequal society with a weak accumulation of aggregate wealth (and high volatility), while few agents become much richer than the rest of the population. A very different scenario is the one in which agents have the same propensity to consume, independently of class division. This mitigates wealth inequality, especially when all agents have a “high” propensity to consume: in this case the growth of aggregate wealth is smaller than in the baseline model; by contrast, wealth accumulation is stronger (but also more volatile) when all agents have a “high” propensity to save (with wealth inequality similar to that of the baseline model). Another result is that the larger the fraction of wealth invested in the risky process the more unequal the society. This suggests that when the amount of wealth required to access the capitalist role is “large” – maybe when economic growth is boosted by physical capital accumulation – an unequal society with a persistent class division emerges; by contrast, when this amount is “small” – maybe when economic growth is boosted by human capital accumulation – a less unequal society emerges. Hence, agents’ wealth can be interpreted as a “collateral” in order to access the capitalist role.

The proposed framework suffers from some limitations which restrict the scope of results. We have not modelled the demand side of the economy that, for sure, plays a fundamental role in the reality (or, at least, we think so). We will move along this direction in the next future to further improve the model by linking the stochastic multiplicative process of profit generation to aggregate demand and wages to capital accumulation. Another relevant aspect is related to credit and finance that we will analyse by introducing credit as an external source to finance the wealth accumulation process. Furthermore, we could introduce financial investments which allow agents to directly make financial profits (that is, outside of the “real sector”). All in all, we think that the modelling of the demand side and of credit/finance will allow us to develop a stochastic *macroeconomic* framework aimed at investigating some relevant aspects that have been also involved, according to our vision, in the evolution of the recent international crisis. It is very clear that the current model is just a first step in this direction.

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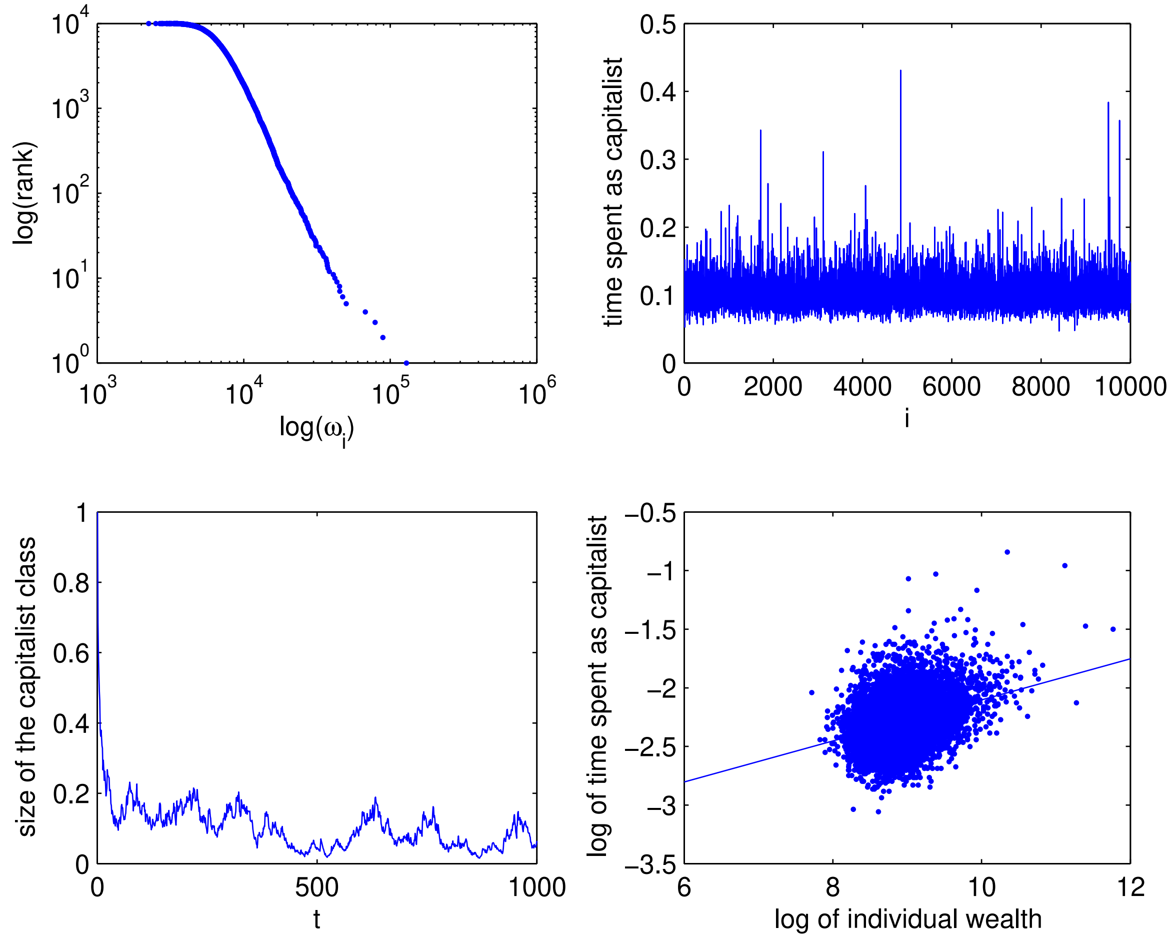

Figure 1: Wealth distribution and class division: *upper-left panel*, rank-size plot of individual wealth; *upper-right panel*, fraction of time each agent has spent as a capitalist; *lower-left panel*: fraction of agents belonging to the capitalist class as time elapses; *lower-right panel*: logarithm of individual wealth vs. logarithm of time spent as capitalist.

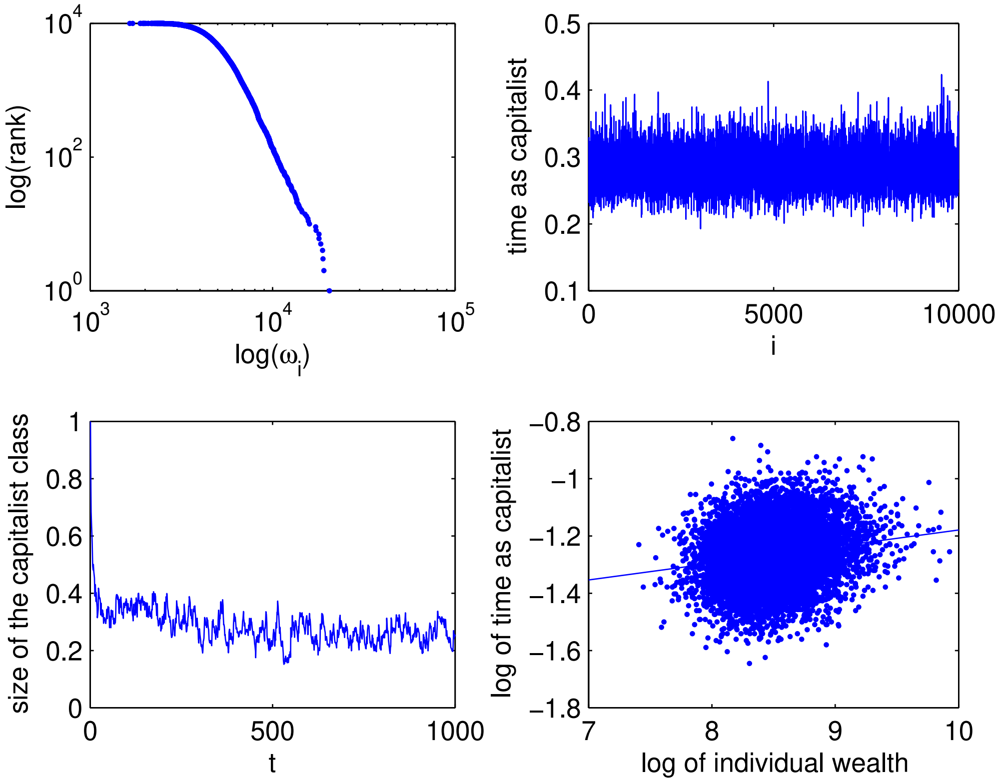

Figure 2: Wealth distribution and class division with $c' = c'' = 1/2$: *upper-left panel*, rank-size plot of individual wealth; *upper-right panel*, fraction of time each agent has spent as a capitalist; *lower-left panel*: fraction of agents belonging to the capitalist class as time elapses; *lower-right panel*: logarithm of individual wealth vs. logarithm of time spent as capitalist.

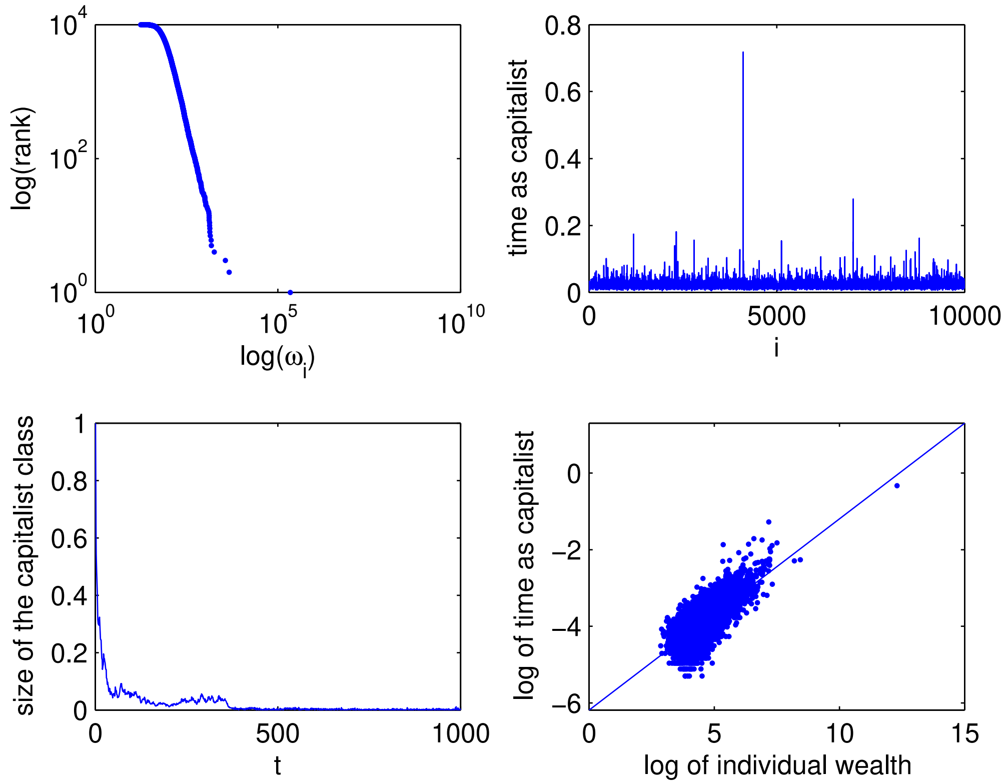

Figure 3: Wealth distribution and class division with $c' = 1$ and $c'' = 0$: *upper-left panel*, rank-size plot of individual wealth; *upper-right panel*, fraction of time each agent has spent as a capitalist; *lower-left panel*: fraction of agents belonging to the capitalist class as time elapses; *lower-right panel*: logarithm of individual wealth vs. logarithm of time spent as capitalist.

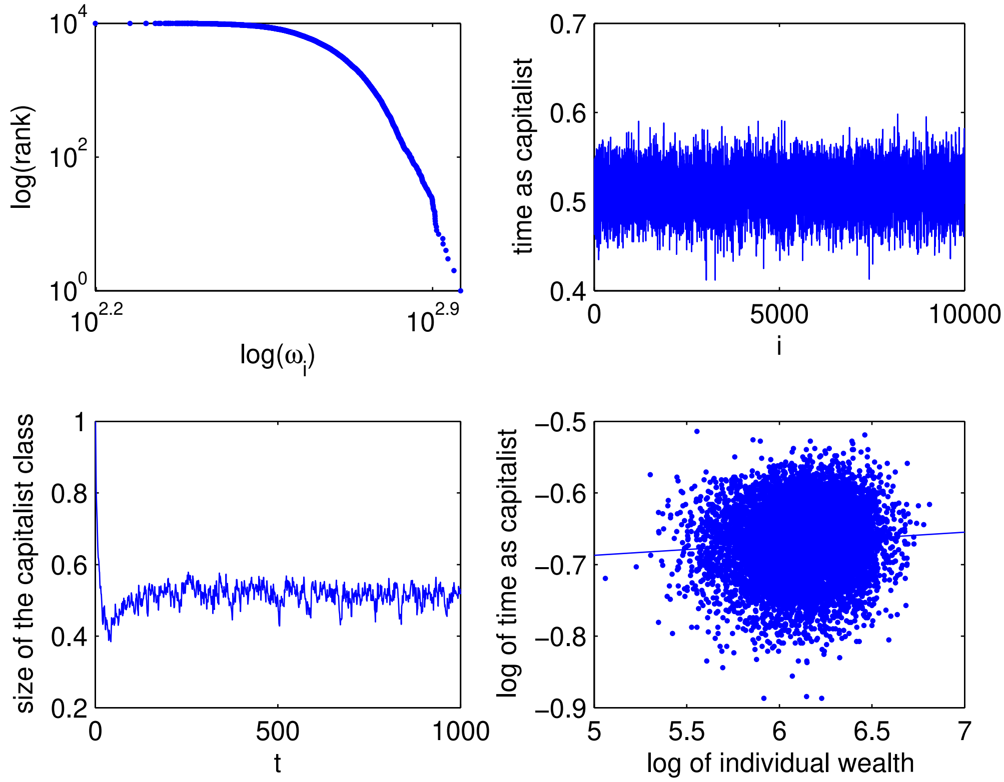

Figure 4: Wealth distribution and class division with $c' = c'' = 0.8$: *upper-left panel*, rank-size plot of individual wealth; *upper-right panel*, fraction of time each agent has spent as a capitalist; *lower-left panel*: fraction of agents belonging to the capitalist class as time elapses; *lower-right panel*: logarithm of individual wealth vs. logarithm of time spent as capitalist.

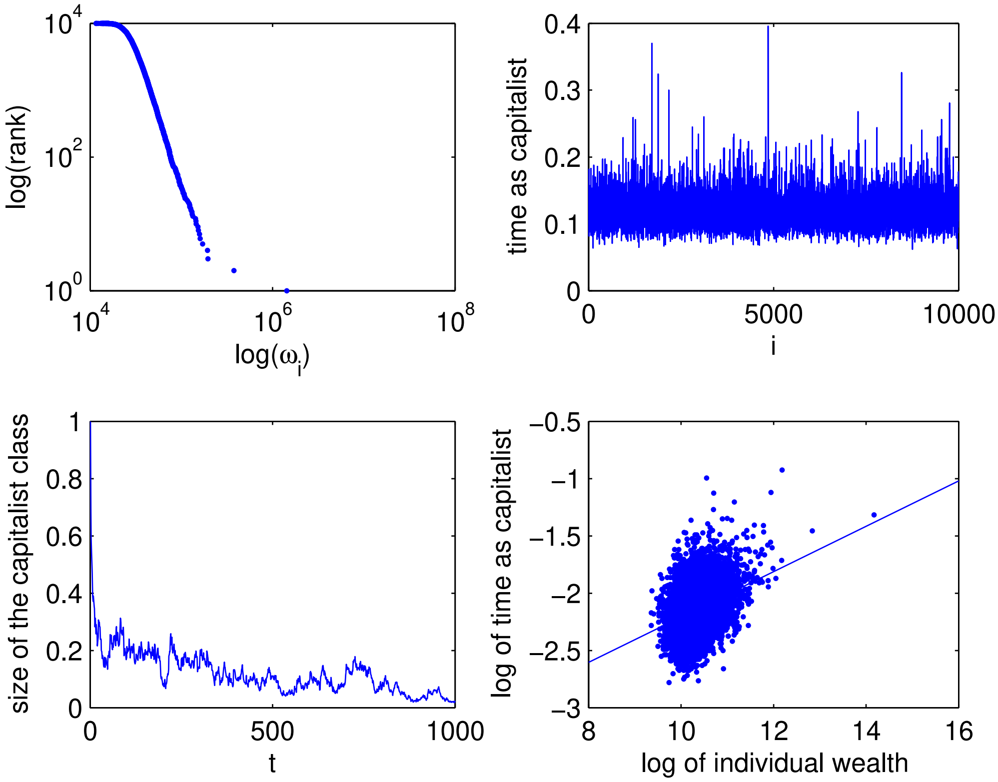

Figure 5: Wealth distribution and class division with $c' = c'' = 0.2$: *upper-left panel*, rank-size plot of individual wealth; *upper-right panel*, fraction of time each agent has spent as a capitalist; *lower-left panel*: fraction of agents belonging to the capitalist class as time elapses; *lower-right panel*: logarithm of individual wealth vs. logarithm of time spent as capitalist.

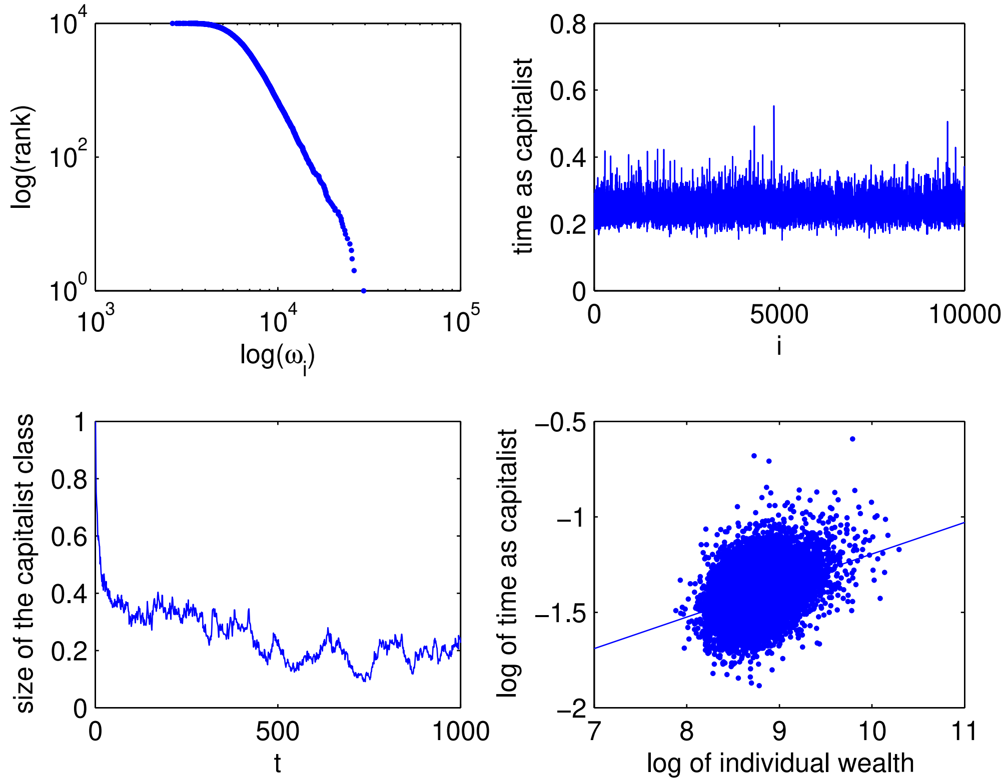

Figure 6: Wealth distribution and class division with $\gamma = 1/4$: *upper-left panel*, rank-size plot of individual wealth; *upper-right panel*, fraction of time each agent has spent as a capitalist; *lower-left panel*: fraction of agents belonging to the capitalist class as time elapses; *lower-right panel*: logarithm of individual wealth vs. logarithm of time spent as capitalist.

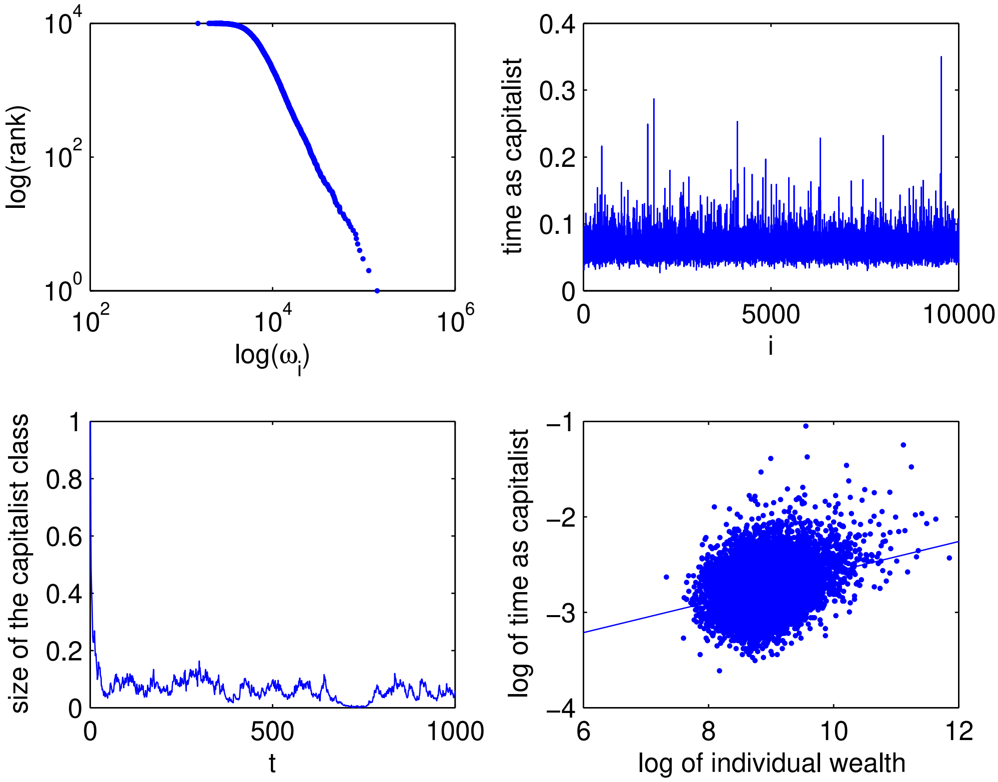

Figure 7: Wealth distribution and class division with $\gamma = 3/4$: *upper-left panel*, rank-size plot of individual wealth; *upper-right panel*, fraction of time each agent has spent as a capitalist; *lower-left panel*: fraction of agents belonging to the capitalist class as time elapses; *lower-right panel*: logarithm of individual wealth vs. logarithm of time spent as capitalist.