Exploding offers and buy-now discounts

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Abstract

A common sales tactic is for a seller to encourage a potential customer to make her purchase decision *quickly*, before she can investigate rival deals in the market. We consider a market with sequential consumer search in which firms can achieve this either by making an exploding offer (which permits no return once the consumer leaves) or by offering a buy-now discount (which makes the price paid for immediate purchase lower than the regular price). We show that firms often have an incentive to use these sales techniques, regardless of their ability to commit to their selling policy. We examine the impact of these sales techniques on market performance. Inducing consumers to buy quickly not only reduces the quality of the match between consumers and products, but may also raise market prices.

Keywords: Consumer search, oligopoly, price discrimination, high-pressure selling, exploding offers, buy-now discounts, costly recall.

1 Introduction

Selling techniques are rarely a focus of economic research, although they are an important aspect of the consumer experience in many markets. One controversial sales method forces the consumer to decide quickly whether to buy. Methods of encouraging a quick decision include a seller refusing to sell to a customer unless she buys immediately (a sales tactic for which we use the term "exploding offer"), or the seller telling the potential customer that she will pay a higher price if she decides to purchase at a later date (we say the seller then offers a "buy-now discount"). In his account of sales practices, Cialdini (2001, page 208) reports:

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> Customers are often told that unless they make an immediate decision to buy, they will have to purchase the item at a higher price later or they will be unable to purchase it at all. A prospective health-club member or automobile buyer might learn that the deal offered by the salesperson is good for that one time only; should the customer leave the premises the deal is off. One large child-portrait photography company urges parents to buy as many poses and copies as they can afford because "stocking limitations force us to burn the unsold pictures of your children within 24 hours". A door-to-door magazine solicitor might say that salespeople are in the customer's area for just a day; after that, they, and the customer's chance to buy their magazine package, will be long gone. A home vacuum cleaner operation I infiltrated instructed its sales trainees to claim that, "I have so many other people to see that I have the time to visit a family only once. It's company policy that even if you decide later that you want this machine, I can't come back and sell it to you."

There are other examples of exploding offers: an academic journal may offer to publish a paper if the author submits it immediately before trying her luck with another outlet, or a seller of life insurance may give a quote to a consumer which is valid only for 10 days, knowing that it will take the consumer more than 10 days to generate another quote given the medical tests required.

A less extreme sales tactic than banning return is to offer a discount for immediate sale. Bone (2006, pp. 71-73) documents how a home improvement company offers its potential customers a regular price for the agreed service, together with a discounted price—which was termed a "first call discount"—if the customer agrees immediately. Robinson (1995) discusses other examples of buy-now discounts, such as a prospective tenant who is offered an apartment for \$900 per month but to whom the landlord offers \$850 if she agrees immediately, or a car dealer trying to close a deal who offers a further \$500 off the price if the buyer accepts now, so (as he claims) he can then make his sales quota for that month.

This paper examines a seller's incentive to discriminate against customers who wish to buy later, after investigating rival offers. It is natural to study this issue in the context of sequential search, where consumers search for a suitable product and/or for a low price.[1] There are three leading models of sequential consumer search, each of which is relevant for

[1] We use a model with rational consumers. There are many other methods to induce sales which rely on more psychological factors. These include attempts to make the prospective buyer "like" the seller (e.g., by claiming similar interests, family or social background) or attempts to make the buyer feel obligated to the seller (e.g., by means of a "free gift"). Cialdini (2001) describes these and other sales techniques in more detail, and Bone (2006) illustrates their use in the two companies he studies. Bone (page 90) describes the use of an extreme tactic: the seller "burst into tears" when the sale appeared to be in difficulty, claiming she would be in trouble with her boss if she didn't make the sale. Rotemberg (2010) presents a model in which sellers, by investing in sales effort, can directly affect a prospective buyer's utility from the seller's item or her disutility from not buying the seller's item.

the analysis in this paper. First, Diamond (1971) proposes a model in which sellers offer homogenous products, where consumers know their value for the product in advance of search, and where all consumers have a positive cost of searching for an additional price. In this situation, the "Diamond paradox" applies and the market can fail to operate at all: if consumers anticipate some equilibrium price P from sellers, then to search for a seller they must be willing to pay at least $P + s$ for the product, which gives a seller an incentive to charge at least $P + s$. Thus, there can be no equilibrium price which induces consumers to enter the market.

Second, Stahl (1989) modifies Diamond's model so that a fraction of consumers do not have search costs, and always investigate all options in the market. The presence of these "shoppers" gives firms an incentive to set low prices, and the market is active. In equilibrium, firms choose prices according to a mixed strategy, and there is price dispersion in the market. The shoppers buy the cheapest available product, while prices are low enough that those consumers with positive search costs buy from the first firm they find. Third, Wolinsky (1986) proposes a model with product differentiation, so that consumers need to search for a suitable product as well as a low price. Because consumers do not know their match utility from a firm until they visit that firm, the Diamond paradox need not arise even though all consumers have positive search costs. In equilibrium all firms choose the same deterministic price, and consumers keep searching until they find a product with match utility above a threshold. (If no product's utility is above the threshold, consumers go back to buy from the "least bad" option if that option yields a positive surplus.)

In the latter two search models, some consumers will return to buy from a previously sampled seller after investigating other sellers.[2] In this paper we discuss how firms may wish to discriminate against these return buyers. Of course, to do this a seller needs to be able to distinguish potential customers it meets for the first time from those who have returned after a previous visit. In the majority of circumstances this is not possible. (A supermarket, for instance, keeps no track of a consumer's entry and exit from the store.) Nevertheless, in many markets such discrimination is feasible. A sales assistant may tell from a potential customer's questions or demeanor whether she has paid a previous visit or not, or may simply recognize her face. In online markets, a retailer using tracking software may be able to tell if a visitor using the same computer has visited the site before. Sometimes—as with job offers, automobile sales, tailored financial products, medical insurance, doorstep sales, or home improvements—a consumer needs to interact with a seller to discuss specific requirements, and this process reveals the consumer's identity.

In such situations, there are two reasons why a firm might wish to discriminate against those consumers who buy later. First, there is a *strategic* reason, which is to deter a

[2] De los Santos (2008) presents an empirical study of consumer search behaviour prior to making a purchase, using data from online book purchases. De los Santos (2008, section 4) finds that of those consumers who search at least twice, approximately two-thirds buy from the final firm searched and one-third go back to a firm searched earlier.

potential consumer from going on to investigate rival offers. If a consumer cannot return to a seller once she leaves, this increases the opportunity cost of onward search, as the consumer then has fewer options remaining relative to the situation in which return is costless. Second, the observation that a consumer has come back to a seller after sampling other options reveals relevant *information* about a consumer's tastes or the prices she has been offered elsewhere, and this may provide a profitable basis for price discrimination. A seller may charge a higher price to those consumers who have already investigated other sellers, because their decision to return indicates they are unsatisfied with rival products.[3] As we will see, the former motive is most relevant when firms can commit to their buy-later policies (or at least when some consumers believe that announced buy-later policies will be used), while the latter is more important when firms have less commitment power.

A simple example may help fix some of the ideas used in this paper. A principal (a seller or an employer, say) makes an offer to a risk-neutral agent (a consumer or worker), knowing that the agent will receive another offer from a second principal subsequently. The first principal aims to maximize the probability that the agent accepts the offer. Suppose that the agent's payoff from the first principal is u_1 and her payoff from the second principal is u_2, where u_1 and u_2 are identical and independent random variables with mean $\bar{u}$. When the agent receives her first offer, she (but not the first principal) observes u_1 but does not yet know the realization of u_2. Suppose the agent incurs no search or discounting costs to obtain the second offer, and always wishes to accept one offer or the other. If the first principal allows the agent to return freely after she receives her second offer, the agent will wait for the second offer and choose the better option, so that the agent accepts the first principal's offer with probability equal to a half. However, if the first principal commits to an exploding offer so that the agent cannot come back if she waits for the second offer, then the agent accepts the exploding offer if $u_1 \geq \bar{u}$. Thus, the exploding offer increases the probability of acceptance if and only if the mean of the distribution is below the median, so that the distribution is negatively skewed. The basic trade-off involved is as follows. When the first principal uses an exploding offer, this makes the agent more likely to accept the offer immediately if she likes it, but it prevents the agent, in the event that she has only a moderate payoff from the offer, from coming back if she receives a worse offer from the second principal. When the distribution is negatively skewed, the first effect dominates. Of course, the agent is harmed when the first principal makes an exploding offer, since she obtains her ideal outcome when free recall is allowed while an exploding offer leads to inefficient matching for some realizations of (u_1, u_2).

[3] This contrasts with the substantial literature on dynamic pricing, which examines how firms can use the information of consumer *purchase* history to refine their prices. These models often predict that a firm will price low to a customer who previously purchased from a rival (or consumed the outside option in the case of monopoly), since such a customer has revealed she has only a weak preference for the firm's product. See Coase (1972) for the original monopoly analysis, and Fudenberg and Villas-Boas (2006) for a survey of this literature in the context of oligopoly.

In this paper we extend this illustrative example to allow for positive search costs and price competition between an arbitrary number of sellers, to allow sellers to set higher prices to return visitors (rather than merely to ban their return), to relax the assumption that sellers can commit to their buy-later policies, and to consider situations in which it is uncertainty about price rather than match utility which is relevant. For most of the paper we conduct the analysis in Wolinsky's framework with product differentiation, although at the end of the paper we verify that the main insight carries over to the alternative Stahl model with homogenous products and price dispersion. In section 2 we suppose that firms can employ one of just two "buy-later" policies: consumers can freely return after leaving the firm (and buy at the same price), or exploding offers are used and first-time visitors are forced to buy immediately or never. We show that firms wish to use exploding offers when the consumer demand curve is concave (which is akin to the negative skewness needed in the simple example above), while when demand is convex firms choose to allow free recall. Beyond cases of convex or concave demand, we show that exploding offers are typically an equilibrium sales technique when search frictions are large and there are many suppliers. We derive the equilibrium price when all firms use exploding offers, and find that this price can be higher or lower than the corresponding price with free recall.

In section 3 we assume firms have a richer set of buy-later policies from which to choose, and rather than simply banning return they can charge their returning visitors a higher price. We first analyze the case where firms can commit to their buy-later price. Starting from a situation in which firms treat first-time and returning consumers symmetrically, we show under mild conditions that a firm has an incentive to offer a buy-now discount. We derive the equilibrium prices for immediate and returning purchase in a duopoly example, and because of the extra search frictions introduced by the buy-now discount, even the discounted buy-now price can be higher than the non-discriminatory price.

An alternative method of discriminating against prospective buyers who leave and then return is to implement an *unannounced* price hike. When searching for air-tickets online, a consumer may find a quote on one website, go on to investigate a rival seller, only to return to the original website to find the price has mysteriously risen. Or a consulting firm may be approached by a company wanting antitrust advice and a fee is chosen, but if the company returns some weeks later after trying rival consultants (who are too expensive, or perhaps turn out to be conflicted), it may find the fee has increased. To analyze such cases, in section 3.3 we relax the assumption that firms can commit to their buy-later price when consumers make their first visit. Then, for reasons akin to Diamond's paradox, the only credible outcome when firms have no commitment power at all is that firms make exploding offers and the return market collapses. An inability to commit to its buy-later policy will therefore amplify a firm's incentive to discriminate against those consumers who buy later. However, if a firm has *limited* commitment power, in the sense that it can commit to an upper bound on the prices paid by returning visitors—this upper bound might simply be the displayed price of the item in the store, for instance—then an equilibrium exists which is identical to the full commitment outcome.

In section 4 we discuss alternative reasons why firms may wish to encourage quick decision making. We examine a model with homogenous products and price dispersion as in Stahl's search model. Here, a consumer's uncertainty about future options concerns price rather than match utility. The results are more clear-cut relative to the setting where products are differentiated, and starting from Stahl's free-recall equilibrium we show that a firm *always* has a unilateral incentive to make an exploding offer. We also discuss how consumer risk aversion makes it more likely that exploding offers are a profitable strategy, how an incumbent firm can employ exploding offers so as to deter a more efficient entrant, and why a firm may force a quick decision in order to prevent consumers from comprehending the current product (as opposed to the products offered by rival sellers).

As far as we know, this paper is the first to study the use of exploding offers (or, more generally, buy-now discounts) in consumer markets. In the alternative setting of matching markets, however, there are a number of studies in which exploding offers play a role. Exploding offers are often used in specialized labor markets, such as those for law clerks, sport players, medical staff, and student college allocations. In such markets, firms make offers to which applicants must respond quickly, and these markets often clear very fast, with firms as well as applicants having little opportunity to consider their alternatives. (See Roth and Xing, 1994, for an account of a number of such markets.) This literature often studies exploding offers, together with early contracting, in a setting where match quality information (e.g., information about workers' productivity) is revealed over time (see Li and Rosen, 1998, for instance). When exploding offers are used, these markets have a tendency to "unravel": employers compete to make and workers are willing to accept ever earlier offers. Early contracting can provide an insurance gain for agents, but it causes inefficient matching. Niederle and Roth (2009) conducted an experimental study on the use of exploding offers in a laboratory matching market. They find that firms do tend to use exploding offers when they are permitted to do so, and the result is that matching occurs inefficiently early and match quality is poor, relative to the situation in which using exploding offers is banned. Our model is more suitable for consumer markets where the interaction between buyers and sellers often occurs through individual search processes instead of through a synchronized matching market as often seen in labor markets. But both early contracting in labor markets and early buying in our model lead to an inefficient use of information available in the market.

Our paper also relates to several strands of the industrial organization literature, beyond the early papers on consumer search already discussed. Janssen and Parakhonyak (2010) extend Stahl's model so that consumers incur an exogenous cost to return to a previous firm. The optimal stopping rule with costly recall is significantly more complicated than when return is costless. When there are more than two firms, a consumer's stopping rule is non-stationary and her reservation surplus level depends on her previous offers. They further show that equilibrium prices do not depend on the recall cost.[4]

Firms often benefit from a reduction in consumer search intensity, since this usually softens price competition. In our model, the exploding offer or buy-now discount serves this purpose. Alternatively, Ellison and Wolitzky (2008) extend Stahl's model so that a consumer's incremental search cost increases with her cumulative search effort. If a firm increases its in-store search cost (say, by making its tariff harder to comprehend), this will make further search less attractive. They show that if the exogenous component of search costs falls, firms will unilaterally increase their self-determined element of search costs, with the result that equilibrium prices are unchanged. Though otherwise very different, our model and theirs study how search frictions are determined endogenously: even if intrinsic search frictions are negligible, a market may suffer from substantial search frictions—and high prices—in equilibrium.

Finally, our analysis of buy-now discounts is related to the literature on auctions with a "buy now" price (see Reynolds and Wooders, 2009, for instance). Online auctions sometimes offer bidders the option to buy the item immediately at a specified price rather than enter an auction against other bidders. In these situations, a seller has one item to sell to a number of potential bidders, and so a bidder needs to pay a *high* buy-now price in order to induce the seller from going on to search for other bidders by running an auction, whereas our model involves sellers offering a *low* buy-now price so as to induce a buyer from going on to search for other sellers. Common rationales for buy-now prices in auctions are impatience or risk-aversion on the part of bidders or the seller, neither of which is needed in our model.

2 Exploding Offers

Our underlying market is based on Wolinsky's model with differentiated products.[5] There are $2 \leq n < \infty$ symmetric firms in the market, each supplying a single horizontally differentiated product at a constant marginal cost which is normalized to zero.[6] There are a large number of consumers with idiosyncratic preferences, and their measure is normalized to one. A consumer's valuation of product i, u_i, is a random draw from some common

[4]Daughety and Reinganum (1992) make the point that the extent of consumer recall may be endogenously determined by firms' equilibrium strategies. In their model, the instrument that a firm can use to influence consumer recall is the length of time that it will hold the good for consumers at the quoted price. In contrast to our assumption that a consumer can discover a seller's buy-later policy only after investigating that seller, Daughety and Reinganum suppose that sellers can announce their recall policies to the population of consumers before search begins.

[5]See Anderson and Renault (1999) for analysis of a variant of Wolinsky (1986) in which there is no outside option and consumers always buy a product in the market.

[6]Note that if there were unlimited firms in the market ($n = \infty$), banning or discouraging return has no impact on a firm's demand or profit. As is well known, with unlimited options, consumers would not choose to return to a previously sampled option even if it was free for them to do so.

distribution with support $[0, u_{\max}]$ and with cumulative distribution function $F(\cdot)$ and continuously-differentiable and bounded density $f(\cdot)$. We suppose that the realization of match utility is independent across consumers and products. In particular, there are no systematic quality differences across the products. Each consumer wishes to buy one item, provided an item can be found with a positive surplus. We sometimes refer to the function $1 - F(\cdot)$ as the consumer demand curve.

Consumers initially have imperfect information about the deals available in the market. They gather this information through a sequential search process, and by incurring a search cost $s \geq 0$, a consumer can visit a firm and find out (i) its price, (ii) its "buy-later" policy, and (iii) the realized match value. (If the search cost is zero, we require that consumers nevertheless consider products sequentially.) In this section, the only two buy-later policies available to a firm are to use an exploding offer or to allow free recall. (If a firm allows free recall, it sets the same price to first-time visitors and returning visitors.) To implement an exploding offer, firms are assumed to be able to distinguish first-time visitors from returning customers and to have the ability to commit not to serve a returning customer. After visiting one firm, a consumer can choose to buy at this firm immediately or to investigate another firm. If permitted, she can costlessly return to a previous firm after sampling subsequent firms.[7] Since firms are *ex ante* symmetric, we consider situations with random search, so that a consumer is equally likely to investigate any of the remaining unsampled firms when she searches, and we also focus on symmetric equilibria.

The timing of the game is as follows. At the first stage, firms set prices and buy-later policies simultaneously. The strategy space of each firm is then $\mathbb{R}^+ \times \{$free recall, exploding offer$\}$. At the second stage, consumers search sequentially and make their purchase decision after search is terminated. Consumers do not observe firms' actual choices before they start searching, but hold rational expectations of equilibrium prices and buy-later policies. Information unfolds as the search process goes on, but consumers' beliefs about the offers made by unsampled firms are unchanged, even if they observe off-equilibrium offers from some firms. Both consumers and firms are assumed to be risk neutral. We use the concept of perfect Bayesian equilibrium, and focus on symmetric pure strategy equilibria in which firms set the same price and buy-later policy based on their expectation of consumers' search behavior, and at each firm consumers hold equilibrium beliefs about unsampled firms' strategies and make their search decisions accordingly.

A piece of notation which summarizes the distribution of match utilities and the extent of search frictions is

$$V(p) \equiv \int_{p}^{u_{\max}} (u - p)\, dF(u) - s \ . \tag{1}$$

[7] In most search markets, even if firms allow free return, consumers face some intrinsic cost of returning to a previously visited firm. In most of our analysis, introducing a small intrinsic returning cost does not affect results qualitatively, but only complicates the analysis, and we assume it away for analytical convenience. However, when we come to discuss buy-now discounts without commitment in section 3.3, whether an intrinsic returning cost exists or not will make an important difference.

Thus, $V(p)$ is the expected surplus of sampling a product if a consumer expects that the price will be p, the cost of sampling the product is s, and this is the only product available. Note that $V(p)$ is decreasing but $p + V(p)$ is increasing in p. Throughout this paper we assume that the search cost s is relatively small, so that

$$V(p_M) > 0 \ , \tag{2}$$

where p_M is the monopoly price which maximizes $p[1 - F(p)]$. This condition means that consumers are willing to sample a product sold even at the monopoly price. In the example where u is uniformly distributed on $[0, 1]$, which we use for illustration in the following analysis, condition (2) requires $s < \frac{1}{8}$.

In the remainder of section 2, we compare market performance when firms allow free recall with the situation where firms make exploding offers (sections 2.1 and 2.3), and we discuss the incentive firms have to make exploding offers (section 2.2).

2.1 The market with free recall and with exploding offers

Here, we examine the market when all firms allow free recall and compare this to the less familiar situation where all firms make exploding offers. If all firms allow free recall, the situation is as in Wolinsky (1986), and for reference later we recapitulate part of his analysis. In a symmetric equilibrium in which all firms set the same price p_0, consumers have a stationary stopping rule whereby they buy a product immediately if they obtain a match utility u greater than a threshold a, and if no product yields that level of utility, the consumer samples all products and buys from the best of the n options provided that option generates a positive surplus. Here, the reservation utility a is determined by the formula

$$V(a) = 0 \ . \tag{3}$$

The expression $\int_a^{u_{\max}} (u - a)\, dF(u)$ in $V(a)$ is the incremental benefit of searching once more if the best current utility is a and the consumer has free recall. So the optimal threshold makes the consumer indifferent between searching on, which incurs cost s, and purchasing this product with utility a. Since $V(\cdot)$ is a decreasing function, (3) has a unique solution and a decreases with s, and condition (2) is equivalent to $a > p_M$. Note that in this case there is efficient matching of consumers to products in the sense that a consumer will always buy her most preferred product from those products she sees.

The following result describes the equilibrium price in the market with free recall.[8]

Lemma 1 *In the market where all firms allow free recall, the first-order condition for p_0 to be the equilibrium price is*

$$\frac{1 - F(p_0)^n}{p_0} + n \int_{p_0}^{a} F(u)^{n-1} f'(u) du = f(a) \frac{1 - F(a)^n}{1 - F(a)} \ . \tag{4}$$

[8]The first-order condition (4) was derived in Wolinsky (1986), while the results about existence and sufficiency of the first-order condition have apparently not been stated in the literature.

If the demand curve $1 - F$ is strictly logconcave, then in the interval $0 < p_0 < a$ the first-order condition (4) has a solution and any such solution lies in the range

$$\frac{1 - F(a)}{f(a)} < p_0 < p_M \ .$$

If the monopoly profit function $p[1 - F(p)]$ is concave, then the first-order condition (4) is sufficient for p_0 to be the equilibrium price.

(Unless otherwise stated, all omitted proofs can be found in the appendix.)

As the number of suppliers becomes large, the equilibrium price in (4) converges to $p_0 = \frac{1-F(a)}{f(a)}$. As the search cost tends to its upper bound in (2) (i.e., as a tends to p_M), consumers stop searching whenever they find a product with positive surplus and each firm acts as a monopolist, so the price converges to $p_0 = p_M$ (which then also equals $\frac{1-F(a)}{f(a)}$).

Suppose next that all firms force their first-time visitors to buy immediately or not at all. Suppose consumers anticipate that each firm sets the price p. What is a consumer's optimal search strategy? As is intuitive, consumers become less choosy as they run out of options, and their reservation utility for purchasing decreases the fewer firms remain to be searched. Indeed, if they are unfortunate enough to reach the final firm they will have to accept any offer which leaves them non-negative surplus. In particular, since a consumer may end up buying a product which is inferior to products rejected earlier in her search process, the matching of products to consumers is less efficient than in the market with free recall. The precise stopping rule is derived in the following result.[9]

Lemma 2 *Suppose consumers face a search market with m firms, each of which makes an exploding offer and sets price p. Then a consumer will enter the market if and only if $p < a$, where a is given in (3), in which case she obtains expected surplus*

$$W_m \equiv a_m - p \geq 0 \ ,$$

where a_m solves the recursive equation

$$a_{m+1} = a_m + V(a_m) \tag{5}$$

with initial value $a_0 = p$ and where $V(\cdot)$ is defined in (1). If $p < a$, a consumer who has $l \geq 0$ firms remaining unsampled will buy from her current firm if match utility is greater than a_l, and the sequence $a_0, a_1, ...$ is increasing.

Note that, unlike the case with free recall, each a_m depends on the price p since the starting value a_0 does so. Note also that when $p < a$ the sequence a_m in (5) converges to the free-recall reservation utility a as $m \to \infty$.

[9] Related analysis of the optimal stopping rule for search without recall has been derived by, for example, Lippman and McCall (1976).

We next derive the symmetric equilibrium price when firms use exploding offers. Suppose $n-1$ firms set the price p and the remaining firm is considering its choice of price, say $\tilde{p}$. (Of course, when choosing their search strategy consumers anticipate that this firm has set the equilibrium price p.) Suppose this deviating firm happens to be in the k_{th} position of a consumer's search process, so $n-k$ firms remain unsampled. Then the probability that the consumer will visit this firm is $h_1 \equiv 1$ if $k=1$, and if $k>1$ this probability is

$$h_k \equiv \prod_{i=1}^{k-1} F(a_{n-i}) \ . \tag{6}$$

From Lemma 2, she will then buy from this firm if $u - \tilde{p} > a_{n-k} - p$, which occurs with probability $1 - F(a_{n-k} - p + \tilde{p})$, and so the firm's demand given it is in the consumer's k_{th} search position is

$$h_k[1 - F(a_{n-k} - p + \tilde{p})] \ . \tag{7}$$

Since the firm is in any position $1 \le k \le n$ with equal probability, its total demand with price $\tilde{p}$ when all other firms are expected to set price p is

$$Q(\tilde{p}) = \frac{1}{n} \sum_{k=1}^{n} h_k[1 - F(a_{n-k} - p + \tilde{p})] \ ,$$

and its profit is $\tilde{p}Q(\tilde{p})$. The following result characterizes the equilibrium price when exploding offers are used.[10]

Lemma 3 *In the market where all firms make exploding offers, the first-order condition for p to be the equilibrium price is*

$$p = \frac{\sum_{k=1}^{n} h_k[1 - F(a_{n-k})]}{\sum_{k=1}^{n} h_k f(a_{n-k})} \ . \tag{8}$$

If the demand curve $1-F$ is strictly logconcave, then in the relevant interval $0 < p < a$ the first-order condition (8) has a solution and any such solution lies in the range

$$\frac{1 - F(a)}{f(a)} < p < p_M \ .$$

If the monopoly profit function $p[1 - F(p)]$ is concave, the first-order condition (8) is sufficient for p to be the equilibrium price.

Since each a_{n-k} depends on p, equation (8) defines p only implicitly. Notice that the numerator in the right-hand side of (8) equals $1 - \prod_{k=1}^{n} F(a_{n-k})$, the total output in equilibrium.

[10] Lemmas 1 and 3 do not discuss the uniqueness of equilibria in the respective regimes. However, one can show that if the demand curve is strictly logconcave then there is a unique solution to the free-recall first-order condition (4), while if the demand curve is concave there is a unique solution to the exploding-offer first-order condition (8).

One can show that as the number of firms tends to infinity, this equilibrium price converges to the same lower bound $\frac{1-F(a)}{f(a)}$ as in the free-recall case. Intuitively, when the number of firms is unlimited, a consumer would never choose to return to a previously sampled firm, even if she could freely do so, and so the use of exploding offers has no effect on the equilibrium price. Finally, as the search cost tends to its upper bound (i.e., as a tends to p_M), p converges to the monopoly price p_M as in the free-recall regime.

2.2 Incentives to make an exploding offer

Before we compare the outcome when all firms use exploding offers with the benchmark model with free recall, we first investigate a more fundamental issue: when will firms use exploding offers in equilibrium? That is, if all its rivals make exploding offers and set the price p in (8), does a firm have an incentive to deviate and allow free recall (and, possibly, set a different price as well)? Before pursuing the analysis in general, consider this simple duopoly example with fixed prices which yields the main insight.

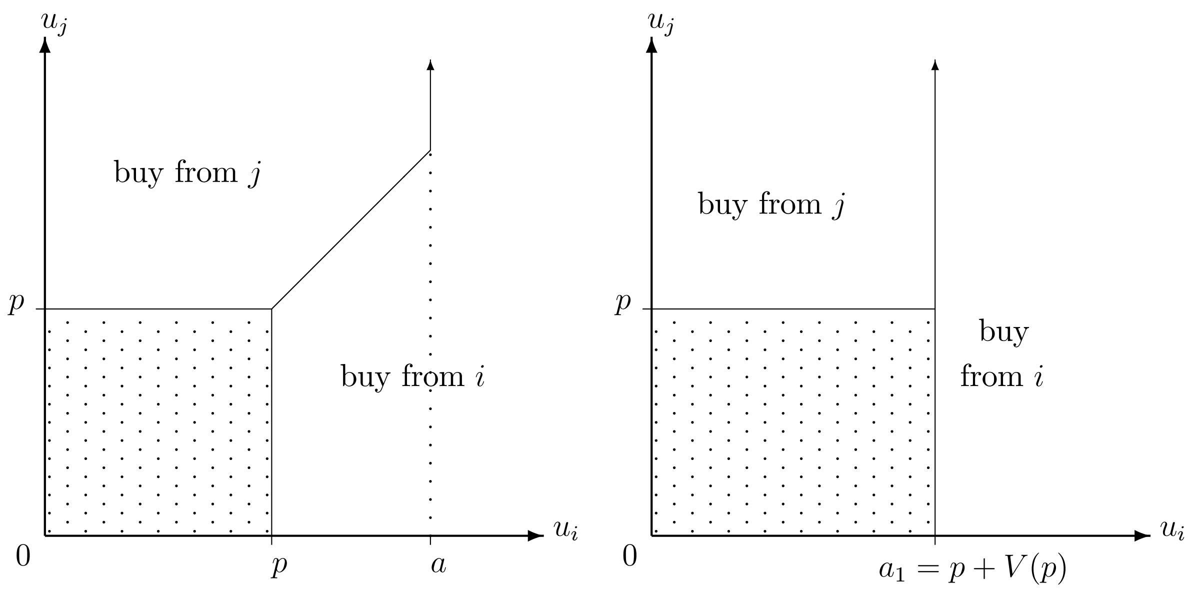

Figure 1a: Demand with free recall   Figure 1b: Demand with exploding offer

Suppose there are two firms, each of which sets the exogenous price $p < a$. Is a firm's demand boosted or reduced if it decides to force its first-time visitors to buy immediately or not at all? First, for those consumers who first sample its rival, firm i's decision whether or not to use an exploding offer has no impact on its demand. Therefore, the only impact on the firm's demand comes from that half of the consumer population who sample it first. If firm i allows free recall, such a consumer will buy from it immediately whenever $u_i > a$, and a consumer will return to buy from it whenever $p < u_i < a$ and $u_i > u_j$. (This is true regardless of whether or not the rival firm makes an exploding offer.) This pattern of demand is depicted in Figure 1a. If, instead, firm i uses an exploding offer, expression (5) implies that a consumer will buy from it if and only if $u_i > a_1 = p + V(p)$. This pattern of demand is depicted in Figure 1b.

As we have shown, $a_1 \in (p, a)$ and so the use of an exploding offer makes a consumer more likely to buy immediately, but it eliminates the possibility that the consumer comes back after finding an inferior product elsewhere. One can calculate that when u is uniformly distributed on $[0, 1]$, firm i's demand in the two figures is identical, and when a firm forces immediate sale this has no net impact on its demand. More generally, as in the illustrative example discussed in the introduction, the impact of using an exploding offer is to eliminate the firm's demand from "low u_i" consumers, who have match utility close to price p and might otherwise come back, and to boost its demand from "high u_i" consumers, who do not wish to risk losing the existing desirable option by going on to sample the rival. If u has an increasing density (i.e., the demand curve $1 - F$ is concave), the latter effect dominates the former, and the net impact of forcing immediate sale is to boost a firm's demand. Similarly, if the demand curve is convex, then the former effect dominates and the probability of a consumer accepting its offer is reduced when an exploding offer is used.

The next result shows that this argument is valid with any number of firms and endogenous prices.

Proposition 1 *(i) If the demand curve $1 - F$ is strictly concave then every symmetric equilibrium involves firms using exploding offers;*
(ii) if the demand curve $1 - F$ is strictly convex then every symmetric equilibrium involves firms allowing free recall;
(iii) if the demand curve $1 - F$ is linear, i.e., u is uniformly distributed, then an equilibrium with exploding offers and an equilibrium with free recall both exist.

This result only covers situations with concave or convex demand (i.e., where the density for the match utility is monotonic). The reason why results are then clear-cut is that the impact of exploding offers on a firm's demand does not depend on the prevailing price. With a non-monotonic density function, whether exploding offers are an equilibrium sales technique may depend on price. In particular, it may depend on both the number of firms in the market and the magnitude of search frictions. Results with non-monotonic densities can be obtained if there are many suppliers, as described in the next result.[11]

Proposition 2 *Suppose f is a hump-shaped density with mode u^* (i.e., f is strictly increasing for $u < u^*$ and strictly decreasing for $u > u^*$). Then for sufficiently large n:*
(i) if $a < u^*$ it is an equilibrium for all firms to use exploding offers;
(ii) if $a > u^*$ it is an equilibrium for all firms to allow free recall.

[11] From the proof of this result, one can also see that if $a < u^*$, then for large n free recall cannot emerge in any symmetric equilibrium, which implies that every symmetric equilibrium involves all firms using exploding offers. Unfortunately, we cannot prove the counterpart of this result when $a > u^*$.

Loosely speaking, when n is large only the behavior of f around the threshold point a matters for the incentives to make an exploding offer. (Proposition 1 by contrast showed that exploding offers are used when the density is everywhere increasing.) This result implies that when the density is hump-shaped and the number of firms is large, the size of search frictions determines the equilibrium sales policy. When the search cost is high enough that a is smaller than the mode, firms use exploding offers; otherwise, firms allow free recall. In particular, in "competitive" markets (in the sense that search frictions are small and the number of suppliers is large), we anticipate that allowing free recall is an equilibrium policy. The other useful comparative statics exercise is to consider the impact of the number of firms on the equilibrium sales policy. To illustrate this in an example, consider the case where match utilities follow the hump-shaped Weibull distribution with $F(u) = 1 - e^{-u^3}$ and support $[0, \infty)$, which has mode $u^* \approx 0.87$ and monopoly price $p_M \approx 0.69$. For a search cost such that $a = 1 > u^*$, one can verify that when $n = 2, 3$ and 4 the only symmetric equilibrium involves firms using exploding offers, while for $n = 5$ and 6 all firms allow free return.[12]

Our analysis presumed that consumers search through market options in a purely random order. In some markets, however, a prominent seller may attract a disproportionate share of initial consumer searches. (De los Santos (2008) showed this was so in the online book market.) Indeed, the examples of doorstep selling mentioned in the introduction do not fit the random search assumption well since such a seller is relatively likely to be the first seller for that product encountered by the consumer over the relevant time horizon. Nevertheless, prominence does not affect a firm's incentive to adopt exploding offers, at least when the demand curve is concave or convex, and Proposition 1 applies regardless of the fraction of first-time visitors a given seller receives. The reason can be understood by looking at Figure 1. The decision about whether or not to use an exploding offer only affects a firm's demand from those consumers who sample it first, and this demand effect is positive (negative) if the demand curve is concave (convex), independent of the proportion of such consumers. Thus, much of the analysis in this paper applies equally to situations where some sellers are more prominent than others.

Our analysis to this point relies on a firm's ability to commit to an exploding offer. However, if a consumer does come back to a firm after sampling a rival, the firm will have an incentive to sell to that consumer.[13] This credibility problem is enhanced by the fact

[12] See our online appendix for the details of these calculations. A second factor which could arise with non-monotonic densities is that firms may choose intermediate buy-later policies, which make return costly for their first-time visitors but not prohibitively so. For example, online sellers can ask customers to log on to their accounts or input information again, or firms can ask consumers to queue again or make another appointment if they want to come back. With a monotonic density, a firm wishes either to make return impossible or free, even if it could impose intermediate returning costs.

[13] Indeed, the quote from Cialdini in the introduction immediately goes on to say: "This, of course, is nonsense; the company and its representatives are in the business of making sales, and any customer who called for another visit would be accommodated gladly."

that consumers often *will* wish to return to previous firms, since their stopping rule is such that their remaining option may have lower utility than previously rejected options. Even if firms lack any ability to commit, though, firms may wish to claim to employ exploding offers if a fraction of consumers are "credulous". When some consumers mistakenly believe a seller's claim that they must buy immediately or not at all, then Proposition 1 still applies. To see this, notice that the other, rational, consumers will ignore what the sellers say about their buy-later policies and behave as in the free-recall case. So the decision about whether to make an exploding offer depends only on the credulous consumers who behave just as the consumers do in the full commitment case.[14] In addition, as we discuss in section 3.3, when the more flexible sales policy with buy-now discounts is available, the lack of commitment power can *strengthen* a firm's incentive to discriminate against consumers who buy later.

2.3 The impact of exploding offers

It is hard in general to compare market performance with and without the use of exploding offers, and the comparison between the prices in (4) and (8) is opaque. To gain further insights consider first the case of a uniform distribution for match utility, so that u is uniformly distributed on $[0, 1]$ and the demand curve is linear. In this example, the first-order condition for the free-recall equilibrium price in (4) simplifies to

$$\frac{1}{p_0} = p_0^{n-1} + \frac{1-a^n}{1-a} \ , \tag{9}$$

while (8) implies that the equilibrium price with exploding offers satisfies

$$\frac{1}{p} = h_n + \sum_{k=1}^{n} h_k \tag{10}$$

where $h_k = \prod_{i=1}^{k-1} a_{n-i}$ is the probability in the exploding-offer equilibrium that a consumer will visit the k_{th} firm in her search order.[15] (Lemmas 1 and 3 imply that these first-order conditions are sufficient for p_0 and p to be the equilibrium price in each regime.) Expression (5) implies that the reservation utility thresholds in the exploding offer regime satisfy

$$a_{m+1} = \frac{1}{2}(a_m^2 + 1) - s$$

starting with $a_0 = p$. As m becomes large a_m converges to $a = 1 - \sqrt{2s}$, the free-recall threshold. (Recall that in this uniform example condition (2) requires $s < \frac{1}{8}$, i.e., $a > \frac{1}{2}$.)

[14] While the proportion of credulous consumers does not affect the incentive to use exploding offers (at least when the demand curve is convex or concave), this proportion will affect the equilibrium price when exploding offers are (claimed to be) used. A conceptual issue arising in such a model with both rational and naive consumers is how they form their expectation of equilibrium prices. Our discussion here implicitly assumes that all consumers somehow hold the correct expectation about prices.

[15] To derive (10), note that the numerator in (8) is equal to market demand, which is given by $1 - ph_n$.

The following result shows that the price rises when exploding offers are used in this example.

Proposition 3 *Suppose u is uniformly distributed on $[0,1]$ and $s < \frac{1}{8}$. Then price is higher when firms use exploding offers than when they allow free recall.*

The solid curve in Figure 2a depicts how the exploding-offer price p varies with the number of firms for the case $s = 0$, while the dashed curve depicts the free-recall price p_0. Both prices converge to zero for large n, but it seems that prices with exploding offers are approximately double those which prevail with free recall. (This figure includes the monopoly case $n = 1$, in which case the monopolist charges the price $p_M = \frac{1}{2}$ and the use of exploding offers has no impact.) As the search cost gets larger, the difference between the exploding-offer and free-recall prices decreases (and if $s = \frac{1}{8}$, the difference vanishes).

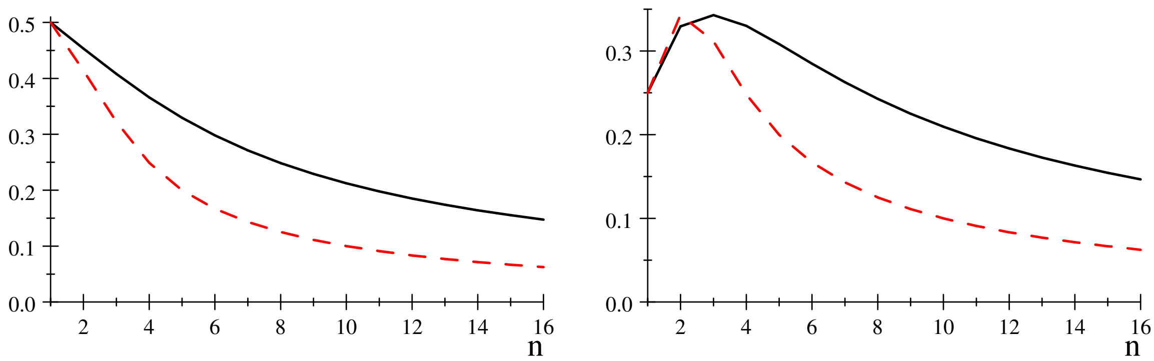

Figure 2a: Prices with exploding offers Figure 2b: Profits with exploding offers

In general, though, it is possible that prices remain unchanged or even fall when exploding offers are used, as the following examples demonstrate.[16]

- Consider the exponential distribution with a c.d.f. $F(u) = 1 - e^{-u/\mu}$ defined on $[0, \infty)$, where μ is the expected value of match utility. Then one can verify that $p = p_0 = \mu$, i.e., the use of exploding offers has no impact on equilibrium prices.[17]

[16]In the first two of the following examples, although the demand curve is well-behaved in the sense that $1 - F$ is logconcave, the monopoly profit function $p[1 - F(p)]$ is not concave as required by Lemmas 1 and 3. But one can numerically verify that a firm's profit function is nevertheless single-peaked, so that first-order condition is sufficient for p_0 and p to be the equilibrium prices.

[17]The special feature of the exponential distribution is that a monopoly firm facing this population of consumers, where each consumer has an outside option with utility $z \geq 0$, will choose the same price $p = \mu$ regardless of z. With price p, the monopolist will sell to a consumer if $u - p \geq z$, and so will choose p to maximize $pe^{-(p+z)/\mu}$, a choice which does not depend on z. (Perloff and Salop (1985), which is the antecedent of Wolinsky's model without search frictions, noted these properties of the exponential distribution.) When firms use exploding offers, this immediately implies that each firm will choose $p = \mu$, regardless of the number of firms and the search cost. When there is free recall, it is also straightforward to check that $p_0 = \mu$ solves expression (4) in this example.

- Consider the Weibull distribution with a c.d.f. $F(u) = 1 - e^{-u^2}$ defined on $[0, \infty)$, where the monopoly price is $p_M \approx 0.71$. When $n = 2$ and $a = 10$, one can show that $p \approx 0.63 < p_0 \approx 0.64$.
- Finally, consider the distribution with density function $f(u) = \frac{1}{2} + \frac{1}{1+e^{-k(u-1/2)}}$ defined on $[0, 1]$ (which is a truncated logistic function). For $k > 0$, this density is increasing and so the demand curve $1-F$ is concave. When $k = 50$ (in which case the monopoly price is $p_M \approx 0.53$), $n = 2$ and $a = 0.6$, one finds that $p \approx 0.4997 < p_0 \approx 0.5054$.

Therefore, the comparison of prices in the free-recall and the exploding-offer regimes is ambiguous, even with reasonable regularity conditions placed on the demand function. The reason why it is hard to obtain clear-cut results about the impact of exploding offers on price is that there are a number of distinct effects at work. On one hand, using exploding offers makes first-time visitors less likely to search on, which tends to reduce a firm's demand elasticity since fewer consumers can compare prices across firms. On the other hand, using exploding offers excludes potential returning consumers, the demand from whom is typically rather inelastic, and which therefore may raise demand elasticity.[18] On top of these two potentially conflicting effects, for a given price the use of exploding offers excludes more consumers from the market, which also affects the demand elasticity. As a result, the net impact of the use of exploding offers on price depends on the shape of the demand curve in a complex way.

Whenever $p \geq p_0$ (such as in the uniform or exponential examples), aggregate consumer surplus and total welfare (measured by the sum of consumer surplus and profit) fall when all firms use exploding offers, relative to the situation when all firms allow free recall. Consumer surplus falls since the price rises compared to the free-recall situation *and* consumers are prevented from returning to a product which yields positive surplus. (Even if $p = p_0$, i.e., if using exploding offers did not change the market price, consumers would obtain lower surplus in the exploding-offer case due to the no-return restriction. The higher price $p > p_0$ only adds to their loss.) As far as total welfare is concerned, relative to the free-recall situation, the use of exploding offers not only induces suboptimal matching (i.e., consumers on average cease their search too early due to the "buy now or never" requirement), but also excludes more consumers from the market since $p \geq p_0$, both of which harm efficiency.

However, it is ambiguous whether the exploding-offer equilibrium has a higher profit level than the free-recall equilibrium even if $p > p_0$. Figure 2b above compares industry profit between the two cases in the uniform example with $s = 0$ (the solid curve represents the exploding-offer case). The profit is lower with exploding offers when $n = 2$. For a

[18] To understand the elasticity of the returning customers, consider the final integral term in (19) in the appendix, which represents a firm's return demand. If u is uniform, so that f is constant, this demand does not depend at all on the firm's price $\tilde{p}$, while if the density is increasing, this demand actually increases with the firm's price.

higher search cost, this can happen with a greater number of firms (e.g., when $s = 0.05$, it is true for $n \leq 4$). In the above logistic example, one can also check that firms earn less in the exploding-offer equilibrium than in the free-recall equilibrium. Together with Proposition 1, these examples indicate that firms may end up playing a prisoner's dilemma when they are able to use exploding offers: each firm has a unilateral incentive to use an exploding offer, but when all firms do so their profits fall.

3 Buy-Now Discounts

An alternative framework allows a firm to charge a higher price to returning visitors instead of the drastic measure of banning return. When this more flexible sales policy is available, we will show that a firm's incentive to discriminate against returning customers is present under more general conditions than needed for Proposition 1. Consider the same model as before, except that instead of choosing the extreme policies of either allowing free return or no return, each firm can choose two distinct prices: p is the price for first-time visitors and $\hat{p}$ is the price for returning customers, and the strategy space of each firm becomes $\mathbb{R}^+ \times \mathbb{R}^+$. (Neither price is observable to consumers before they start searching.) Whenever $\hat{p} > p$, returning to a previous firm is costly.[19] Indeed, when $\hat{p}$ is sufficiently high, the firm in effect makes an exploding offer. One interpretation of this discriminatory pricing is that a firm sets a regular (or "buy-later") price $\hat{p}$ and offers first-time visitors a "buy-now" discount $\tau \equiv \hat{p} - p$. Until section 3.3, we assume that a firm can commit to $\hat{p}$ when it offers new visitors the buy-now price p.

3.1 Incentives to offer a buy-now discount

We first analyze when a firm has an incentive to offer a buy-now discount τ, starting from the situation in which all firms offer the equilibrium uniform price p_0 in expression (4). First, we observe that the impact of offering a small buy-now discount on a firm's profit is just as if the firm levies a small buy-later premium.[20]

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[19] If $\hat{p} < p$, then a consumer has an incentive to leave a firm and then return, even if she has no intention of investigating other firms. If this kind of consumer arbitrage behavior—of stepping out the door and then back in again—cannot be prevented, then setting $\hat{p} < p$ is equivalent to setting a uniform price $\hat{p}$, and so without loss of generality we assume firms are constrained to set $\hat{p} \geq p$.

[20] Suppose all firms but one choose the uniform price p_0 in (4). If the remaining firm offers the buy-now price p and buy-later price $p + \tau$, denote this firm's profit by $\pi(p, \tau)$. If $p \approx p_0$ and $\tau \approx 0$, then

$$\pi(p, \tau) \approx \pi(p_0, 0) + (p - p_0)\pi_p(p_0, 0) + \tau\pi_\tau(p_0, 0) = \pi(p_0, 0) + \tau\pi_\tau(p_0, 0) \ ,$$

where the equality follows from the assumption that p_0 is the equilibrium uniform price and subscripts denote partial derivatives. Thus, the impact on the firm's profit is captured by the term $\tau\pi_\tau(p_0, 0)$, which is independent of p.

Lemma 4 *Starting from the situation in which all firms offer the equilibrium uniform price p_0 in (4), the impact on a firm's profit of offering a small buy-now discount τ (so its buy-now price is $p_0 - \tau$ and its buy-later price is p_0) is approximately equal to the impact of levying a buy-later premium τ (so its buy-now price is p_0 and its buy-later price is $p_0 + \tau$).*

Intuitively, the fact that p_0 is the equilibrium uniform price implies that a firm's profit is not significantly affected by small changes in its uniform price, and the only first-order impact on a firm's profit comes from its buy-now discount τ (regardless of whether this is interpreted as a discount for immediate purchase relative to the buy-later price p_0, or as a premium for later purchase relative to the buy-now price p_0).

To illustrate the pros and cons of offering a discount most transparently, consider initially the case of duopoly. It is somewhat more straightforward to consider the incentive to charge a buy-later premium, and then to invoke Lemma 4. If firm i introduces a buy-later premium, this has no impact on its demand and profit from those consumers who first sample the rival given they hold equilibrium beliefs, and so we can restrict attention to that portion of consumers who sample firm i first. A buy-later premium not only discourages consumers from searching on, as the exploding offer did in the earlier analysis, but also generates extra revenue from returning consumers.

How does τ affect a consumer's decision whether to buy immediately from firm i? Denote by $a(\tau)$ the reservation utility which leads the consumer to buy immediately, i.e., if she finds match utility $u_i \geq a(\tau)$ at the firm she will buy without investigating the rival. Clearly if no premium is levied ($\tau = 0$) then $a(0) = a$, the free-recall reservation level in (3). By definition, if a consumer discovers utility $u_i = a(\tau)$ at firm i she is indifferent between buying immediately (thus obtaining surplus $a(\tau) - p_0$) and going on to investigate firm j, which yields expected utility

$$\underbrace{\int_{a(\tau)-\tau}^{u_{\max}} (u_j - p_0) dF(u_j)}_{\text{utility when she buys from } j} + \underbrace{F(a(\tau) - \tau)[a(\tau) - p_0 - \tau]}_{\text{utility when she returns to buy from } i} - s \ . \tag{11}$$

To understand expression (11), note that if the consumer finds utility u_j at the rival, she will buy from that firm if $u_j - p_0 \geq a(\tau) - p_0 - \tau$, and otherwise she will return to buy from firm i (but at the higher price $p_0 + \tau$). Equating $a(\tau) - p_0$ with expression (11) yields the following formula for $a(\tau)$:

$$V(a(\tau) - \tau) = \tau \ . \tag{12}$$

(Remember $V(\cdot)$ is defined in (1), and given τ this equation has a unique solution $a(\tau)$.) The pattern of demand for the consumers who first sample firm i is depicted in Figure 3.[21]

[21] This analysis and Figure 3 presume that some consumers do return to firm i after sampling firm j, which requires that the premium τ is not too large. By examining the figure, one sees that the condition is $a(\tau) > p_0 + \tau$. From (12), and noting that $V(\cdot)$ is a decreasing function, this is equivalent to $\tau < V(p_0)$. When the discount exceeds $V(p_0)$, the returning cost is so great that a consumer never returns to a firm once she leaves it (i.e., the firm in effect uses an exploding offer).

Note that $a(\tau)$ decreases with τ, and by differentiating (12) we obtain

$$a'(\tau) = \frac{-F(a(\tau) - \tau)}{1 - F(a(\tau) - \tau)} < 0 \ . \tag{13}$$

This is intuitive, as raising the cost of returning makes a consumer more likely to buy immediately (just as in the extreme case of exploding offers).

Using Figure 3, the fraction of those consumers who sample firm i first and who actually buy from the firm is

$$1 - F(a(\tau)) + \underbrace{\int_{p_0+\tau}^{a(\tau)} F(u - \tau) f(u) du}_{\text{firm } i\text{'s returning consumers}} \ .$$

By using (13), the derivative of firm i's demand with respect to τ is equal to

$$\int_{p_0+\tau}^{a(\tau)} F(u - \tau) f'(u) du \ . \tag{14}$$

In particular, the firm's demand is boosted with a buy-later premium whenever the density is increasing, as we saw when we discussed exploding offers in section 2.2.

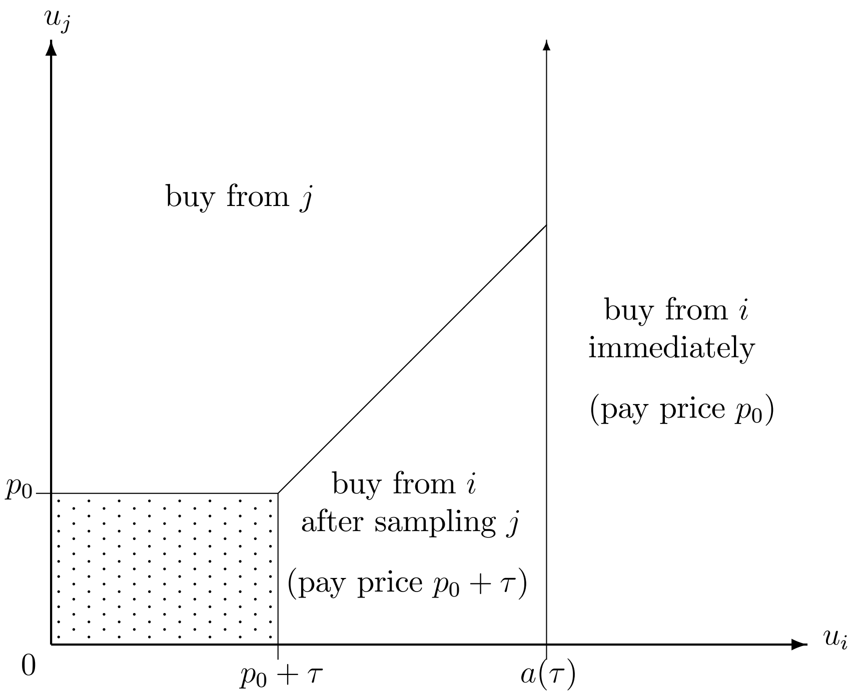


Figure 3: Pattern of demand when firm i levies buy-later premium τ

Firm i makes revenue p_0 from each of its customers, plus an additional τ from each of its customers who buy later. It follows that the derivative of firm i's profits with respect to τ evaluated at $\tau = 0$ is

$$\int_{p_0}^{a} F(u) \left[f(u) + p_0 f'(u) \right] du \ . \tag{15}$$

Here, $\int_{p_0}^{a} Ff du$ is the extra revenue generated from the returning customers while $\int_{p_0}^{a} Ff' du$ is the extra (maybe negative) demand generated by increasing the cost of return.

From (15), the firm has an incentive to introduce a buy-now discount whenever the demand curve is concave. But it has an incentive to introduce a discount much more generally, and the incentive is present whenever p_0 in (4) is strictly above $\frac{1-F(a)}{f(a)}$, which we know from Lemma 1 is the case with strictly logconcave demand. To see this, use (4) to obtain

$$\begin{aligned} p_0 \int_{p_0}^{a} F(u) f'(u) du &= \frac{1}{2}\left[\frac{p_0 f(a)}{1-F(a)}(1-F(a)^2) - (1-F(p_0)^2)\right] \\ &> -\frac{1}{2}\left[F(a)^2 - F(p_0)^2\right] = -\int_{p_0}^{a} F(u) f(u) du \ , \end{aligned}$$

where the inequality follows from the assumption that $p_0 > \frac{1-F(a)}{f(a)}$. Thus, expression (15) is positive and a firm has a unilateral incentive to offer a buy-now discount.

The next proposition shows that this result holds for an arbitrary number of firms.

Proposition 4 *(i) Starting from the free-recall equilibrium with price p_0 in (4), a firm has a unilateral incentive to offer first-time visitors a buy-now discount if the demand curve $1 - F$ is strictly logconcave;*
(ii) starting from the exploding-offer equilibrium with price p in (8), a firm has a unilateral incentive to offer a buy-later price low enough to induce some first-time visitors to return.

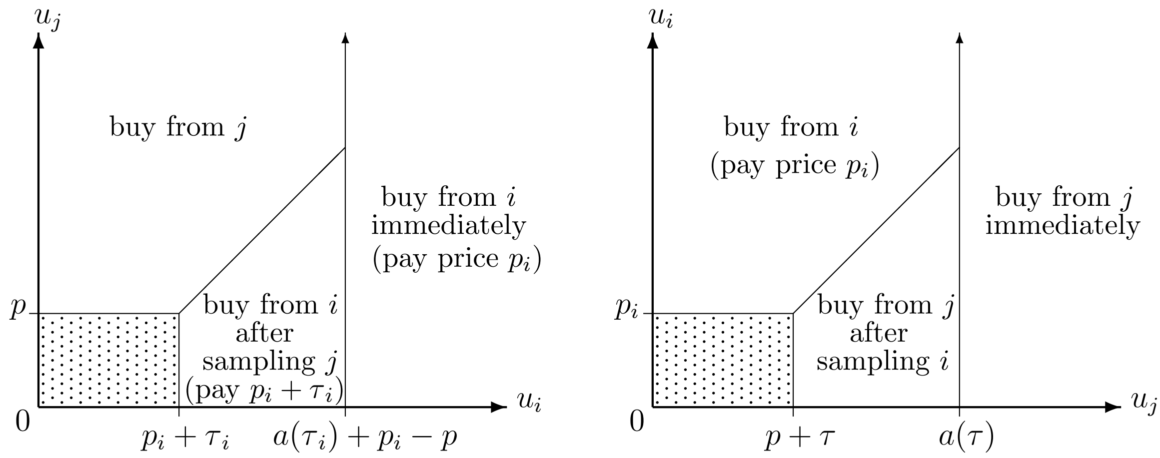
An implication of Proposition 4 is that if a symmetric equilibrium exists when firms choose a buy-now and a buy-later price, it must involve an intermediate buy-now discount such that some consumers do return in equilibrium. Part (i) of Proposition 4 indicates that a seller typically has an incentive to offer a first-time visitor a discount on the regular price if the consumer buys immediately, so that uniform pricing is not an equilibrium outcome when firms can distinguish new from returning visitors.[22] The intuition for this result is as follows. As Lemma 4 shows, the impact of a small buy-now discount is the same as a small buy-later premium. A small buy-later premium has two effects: the extra revenue effect (every returning consumer now pays a premium) and the demand effect (first-time visitors become more likely to buy immediately, but potential returning consumers are less likely to come back). The second effect is similar to the impact of exploding offers, and it is positive if the demand curve is concave. However, the first revenue effect must be positive. Part (i) shows that this first effect is powerful enough for the overall effect to be positive under a much milder condition on the demand curve. Part (ii) shows that a firm prefers to set a “moderate” buy-later price, rather than such a high buy-later price that none of

[22] In the example discussed in section 2.3 where match utility is exponentially distributed, a firm does not have an incentive to offer a buy-now discount, and uniform prices are an equilibrium even when firms have the ability to discriminate against those consumers wishing to buy later. (The demand curve is not strictly logconcave in this example.)

its initial visitors return. The intuition is that a firm can enjoy the strategic benefits of exploding offers but also generate some additional revenue if it charges returning visitors a high price instead of banning return altogether.[23]

3.2 Equilibrium buy-now discounts in duopoly

In this section we investigate the equilibrium buy-now discount in the case of duopoly.[24] We first report how to derive the equilibrium prices and discuss the existence of equilibrium. We then illustrate the equilibrium outcome in the example in which match utility u_i is uniformly distributed on $[0, 1]$. For convenience, we analyze the model in terms of the buy-now price p and the buy-now discount $\tau = \hat{p} - p$ (rather than in terms of p and $\hat{p}$).



a: firm i is first in search order　　b: firm i is second in search order

Figure 4: Pattern of demand when firm i offers (p_i, τ_i)

Suppose a symmetric equilibrium outcome is (p, τ) and consumers expect both firms to offer this tariff. Suppose instead that firm i deviates and offers an alternative tariff (p_i, τ_i). It is without loss of generality that we consider deviations restricted to $\tau_i \leq V(p)$.[25]

[23] Thus, if firms can commit to distinct prices for their first-time visitors and those consumers who buy later, we do not expect to see exploding offers used in equilibrium. Nevertheless, our analysis of exploding offers in section 2 is still worthwhile. For instance, the simplicity of the exploding offer policy may be easier to get across to consumers in a sales context, and some of the claimed "excuses" forcing quick decisions, such as the salesman being the area for that day only, make better sense for exploding offers. Finally, as we discuss in section 3.3, when sellers cannot commit to their buy-later prices, exploding offers emerge once more as the equilibrium sales policy.

[24] It appears to be hard to characterize the buy-now discount equilibrium for an arbitrary number of firms, as we were able to do in our discussion of exploding offers. As is also discussed by Janssen and Parakhonyak (2010), when there are more than two firms the consumer stopping rule with buy-now discounts is non-stationary and depends on the history of realized match utilities, and this makes the equilibrium analysis complex. (When exploding offers are used, by contrast, the stopping rule does not depend on previous offers, since the consumer has no ability to return.)

[25] As can be seen from Figure 4 and expression (12), when $\tau_i > V(p)$, returning demand disappears and the firm's profit is independent of τ_i. Hence, our restriction to $\tau_i \leq V(p)$ is without loss of generality.

Similarly to Figure 3, firm i's demand from those consumers who sample it first is as depicted on Figure 4a (they buy at firm i immediately if $u_i - p_i > a(\tau_i) - p$). (Recall that $a(\cdot)$ is defined above in (12).) Firm i's demand from those consumers who first encounter the rival is shown on Figure 4b (they will come to firm i if $u_j < a(\tau)$ since they hold equilibrium beliefs).

Then firm i's deviation profit is

$$p_i Q_T + \tau_i Q_R \ , \tag{16}$$

where Q_T is firm i's total demand and Q_R is the portion of demand from its returning customers. (The firm obtains revenue p_i from each of its customers, plus the incremental revenue τ_i from each of its returning customers.) By calculating the measures of the regions in Figure 4 one can check that

$$\begin{aligned} 2Q_T = 1 - F\left(a(\tau_i) - p + p_i\right) + \underbrace{\int_p^{a(\tau_i)-\tau_i} F\left(u\right) f\left(u - p + p_i + \tau_i\right) du}_{2Q_R} \\ + F(a(\tau))\left[1 - F(a(\tau) - \tau - p + p_i)\right] + \int_p^{a(\tau)-\tau} F\left(u + \tau\right) f\left(u - p + p_i\right) du \ . \end{aligned}$$

(The first line above reflects the demand depicted in Figure 4a, while the second line captures the demand in Figure 4b.) Using (13), one can verify that the first-order conditions for (p, τ) to be equilibrium prices are

$$\begin{aligned} \frac{1 - F(p)F(p+\tau)}{p} &= f(a(\tau)) + F(a(\tau))f(a(\tau) - \tau) \\ &\quad - \int_p^{a(\tau)-\tau} \left[F(u+\tau)f'(u) + (1 + \frac{\tau}{p})F(u)f'(u+\tau)\right] du \ , \end{aligned} \tag{17}$$

and

$$\frac{\tau f(a(\tau))F(a(\tau) - \tau)}{1 - F(a(\tau) - \tau)} = \int_p^{a(\tau)-\tau} F(u)[(p+\tau)f'(u+\tau) + f(u+\tau)]du \ . \tag{18}$$

If $\tau = 0$, expression (17) degenerates to the first-order condition (4) in Wolinsky's model with $n = 2$. If $\tau = V(p)$ (i.e., $p = a(\tau) - \tau$) so there are no returning consumers, expression (17) degenerates to the first-order condition (8) in the exploding-offer regime with $n = 2$. In particular, when u is uniformly distributed on $[0, 1]$, so that $a(\tau) - \tau = 1 - \sqrt{2(s+\tau)}$ and $s < \frac{1}{8}$, the above two first-order conditions become

$$\frac{1}{p} - p = 1 + a(\tau) + \tau; \quad \frac{2\tau[a(\tau) - \tau]}{1 - a(\tau) + \tau} = [a(\tau) - \tau]^2 - p^2 \ .$$

In our online appendix, we show that under some conditions (e.g., when the density function f is weakly increasing) the system of (17) and (18) has a solution (p, τ) with $0 < \tau < V(p)$ (note that the above demand analysis is predicated on $\tau < V(p)$ which ensures the existence of returning consumers in equilibrium), and in the uniform example, the first-order conditions are also sufficient for (p, τ) to be the equilibrium prices.

In the following, we report some properties of the uniform example. First, as with the use of exploding offers in Proposition 3, we observe that the use of buy-now discounts leads to higher prices, i.e., $p_0 < p < \hat{p}$. That is, even the discounted buy-now price in the discriminatory case is higher than the uniform price, and the ability to offer such discounts drives up *both* prices.[26] The intuition is that the buy-now discount adds to the intrinsic search frictions in the market, and this allows firms to charge a higher price. Figure 5a below depicts how the three prices vary with the search cost s, where from the bottom up the three curves represent p_0, p and $\hat{p}$, respectively.

Second, the equilibrium buy-now discount τ (the distance between the upper curve and the middle curve in Figure 5a) decreases with the search cost s. In particular, when $s = 0$, we have $p \approx 0.45$ and $\hat{p} \approx 0.51$, and so $\tau \approx 0.06$. In this case, although the market has no intrinsic search frictions, firms in equilibrium generate search frictions on consumers via the buy-now discount, which here is about 12% of the buy-later price. By contrast, in a market with $s = \frac{1}{8}$, which is the highest intrinsic search cost which induces consumers to participate, we have $p = \hat{p} = \frac{1}{2}$ and $\tau = 0$, so that there is no buy-now discount. (When $s = \frac{1}{8}$, search costs are so high that consumers will accept the first offer which yields a non-negative surplus. In particular, there are no returning consumers even with costless recall.)

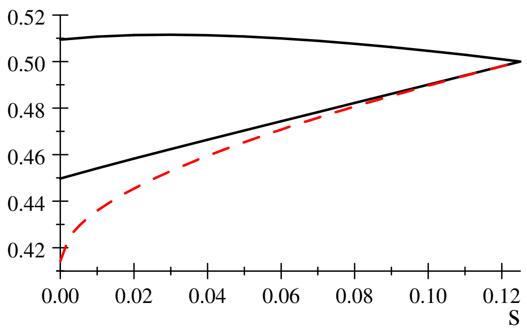


Figure 5a: Prices and search cost

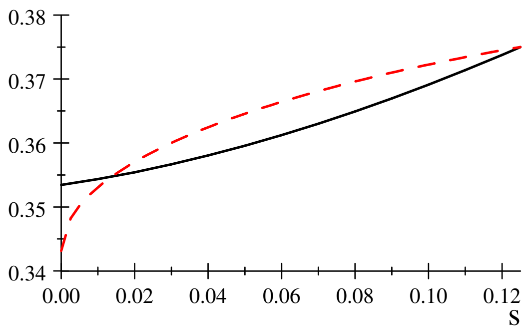


Figure 5b: Profits and search cost

Since both prices rise, the buy-now discount equilibrium excludes more consumers from the market. In addition, as expected, the use of buy-now discounts boosts the demand from consumers who buy immediately and reduces demand from those who buy later. This is illustrated for the case $s = 0$ in Table 1 (including for reference the case where exploding offers are used).

[26] It is not unusual that the ability to price discriminate in oligopoly leads to a fall in all prices, but cases where all prices rise are less familiar.

	p	$\hat{p}$	buy immediately	buy later	excluded
free recall	0.41	0.41	41%	41%	17%
buy-now discount	0.45	0.51	66%	11%	23%
exploding offer	0.45	n/a	73%	0%	27%

Table 1: The impact on prices and demand of buy-now discounts and exploding offers

However, whether the use of buy-now discounts leads to higher profit depends on the magnitude of the search cost. Figure 5b shows how industry profits with uniform pricing (the dashed curve) and profits with buy-now discounts (the solid curve) vary with the search cost s. We see that price discrimination leads to higher profit only if the search cost is relatively small. When the search cost is relatively high, price discrimination leads to prices which exclude too many consumers. In these cases, firms are engaged in a prisoner's dilemma: when feasible an individual firm wishes to offer a buy-now discount, but when both do so industry profits fall. Finally, for similar reasons as in the exploding-offer case, aggregate consumer surplus and total welfare fall when firms use buy-now discounts in this example.

3.3 Buy-later prices without commitment

The preceding analysis has assumed that a firm can commit to its buy-later price when consumers first visit. We discuss here whether buy-now discounts are used if we relax this assumption. That is to say, we investigate whether firms wish to implement an unannounced price rise when consumers return to buy. The basic game structure is the same as before, except that now when a consumer discovers a firm's buy-now price, she can only form some belief about its buy-later price (the belief is of course required to be correct in equilibrium). The actual buy-later price can be learned only after she returns to the firm.

Here, unlike the rest of the paper, it makes an important difference whether or not consumers face an *intrinsic* returning cost when they come back to a previously-visited firm. Since in most situations such a returning cost does exist, we focus on this case. (In the previous analysis with commitment, the presence of a small intrinsic return cost makes no qualitative difference, and for simplicity we assumed this cost was precisely zero.) Proposition 5 describes the outcome when firms cannot fully commit to their buy-later price.[27]

[27] If, by contrast, consumers face no intrinsic returning cost, there is often an equilibrium in which uniform pricing (as in the free-recall benchmark model in section 2.1) is a credible strategy, so that no buy-now discount is offered. That is to say, (i) consumers do not anticipate that they will face a higher price if they return to buy from a previously sampled firm and plan their search strategy accordingly, and (ii) when a consumer does return to a firm, that firm has no *ex post* incentive to surprise the consumer with an unexpected price hike. This is easy to understand in the extreme case with $s = 0$. When search costs are zero, consumers sample all firms before they purchase (given their belief that there is no buy-later surcharge), and so all buyers are returning customers. Thus, we are just in the situation of Wolinsky's

Proposition 5 *Suppose consumers face an intrinsic returning cost.*
(i) If firms cannot commit to their buy-later price, in equilibrium no consumers return to a previously-visited firm and the equilibrium price is as described in Lemma 3;
(ii) if firms can commit to an upper bound on their buy-later price, then firms in any equilibrium will choose their buy-later price to equal the upper bound, and the outcome is as if firms can fully commit to their buy-later prices.

Thus, part (i) shows that if firms cannot commit to their buy-later price and if there is an intrinsic returning cost (no matter how small), rational consumers anticipate that buy-later prices will be so high that it is never worthwhile to return to a previous firm after leaving it. In effect, because of the informational motive to raise prices to those consumers who buy later, firms are forced to make exploding offers, and consumers have just one chance to buy from any firm. Thus, the lack of commitment power strengthens a firm's temptation to exploit returning consumers. This result is analogous to Diamond's (1971) paradox, showing how a small search cost can cause a market to shut down. Diamond's result relies on consumers knowing their match utility in advance, and a central advantage of Wolinsky's formulation with *ex ante* unknown match utilities is that this paradox can be avoided. But even in Wolinsky's framework, the *returning* consumers know their match utilities, and so the returning market fails for the same reason as the primary market failed in Diamond's framework.

Of course, as shown in part (ii) of Proposition 4, a firm would like to avoid this complete shut down of the return market if possible. One method, when feasible, is to commit to a buy-later price cap. For instance, in most retailing markets the price printed on the price label in the store usually has this commitment power, and a sales person has no authority to increase the price above the displayed price. Similarly, as discussed in the introduction, the firm in Bone's (2006) study offered its potential customers a regular price (in the form of a written quote) if they decided to buy later. Whenever this form of partial commitment is feasible, part (ii) of Proposition 5 shows that the equilibrium is the same as that in the full commitment situation analyzed in sections 3.1 and 3.2. Thus, a cap on the buy-later price can be used as a full commitment device.

4 Alternative Motives for Exploding Offers

Factors other than those discussed in our main model may also play a role in giving firms an incentive to use high-pressure sales tactics, and in this section we discuss additional motivations for making exploding offers.

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model with zero search costs, and the incentive to set the price to returning consumers is exactly the same as the incentive to set the uniform price p_0 in the $s = 0$ version of expression (4). If search is costly ($s > 0$) one can prove that this uniform price equilibrium also exists whenever the demand curve $1 - F$ is convex. (See our online appendix for the details.)

4.1 Exploding offers with homogenous products

The analysis to this point has used Wolinsky's framework with differentiated products and no equilibrium price dispersion. In many search markets, however, consumers do not face significant uncertainty about the utility they obtain from a seller's product, but rather from the price they will pay. In this alternative situation, does a seller also have an incentive to use an exploding offer? We explore this issue in the context of Stahl's (1989) model with homogenous products and endogenous price dispersion.

Consider Stahl's model with n firms and unit consumer demand. Let v be each consumer's willingness to pay for the product. All consumers are risk neutral and sample firms sequentially and randomly to gather price information. A fraction λ of consumers are "shoppers" who have zero search cost (so they will be fully informed of market prices if sellers put no restrictions on their ability to return to a previously sampled firm), and a fraction $1-\lambda$ of consumers have a positive search cost $s>0$ for sampling an additional firm. (These costly searchers are assumed to be able to sample the first firm for free.) Stahl's model assumed free recall, so that each firm allowed consumers to return to buy after they have investigated its rivals. As explained by Stahl, in equilibrium firms choose prices according to a mixed strategy, and there is price dispersion in the market. In more detail, in symmetric equilibrium each firm chooses its price according to a c.d.f. $G(p)$ with support $[p_{\min}, r]$, the shoppers investigate all sellers and buy from the cheapest seller, while the costly searchers stop searching whenever they find a price no greater than r and so buy from the first firm they encounter.[28] In order for a firm to be indifferent between all prices in the interval $[p_{\min}, r]$, a firm's profit must be constant in this range.

Starting from Stahl's free-recall equilibrium, we show that a firm *always* has a strict incentive to make an exploding offer. This stands in contrast to the more ambiguous situation with product differentiation (see Proposition 1 above). This result is relatively easy to understand in the case of duopoly. Suppose that one firm is considering whether to make an exploding offer at some price $p \in [p_{\min}, r]$, given that its rival allows free recall and charges a stochastic price according to the c.d.f. $G(\cdot)$. A costly searcher will buy from the firm (given $p \le r$), regardless of whether it makes an exploding offer or not. Likewise, those shoppers who first encounter the rival seller will be unaffected by the firm's use of an exploding offer. Thus, to determine whether making an exploding offer is profitable for the firm, we need only consider its demand from those shoppers who visit it first. If the firm sets price $p \in (p_{\min}, r)$ and allows free recall, it competes against a rival offering a stochastic price and so will make the sale with probability less than one. (Recall that each such price generates the same expected profit in the free-recall equilibrium.) If it instead uses an exploding offer, it competes against a rival offering the (known) expected price $\bar{p} = \int_{p_{\min}}^{r} \tilde{p} dG(\tilde{p})$. Hence, using an exploding offer with any price in the range $p_{\min} < p < \bar{p}$

[28] A costly searcher's reservation price r is endogenously determined in equilibrium. (When the search cost s is large, the equilibrium involves $r = v$.)

will boost demand, and so profit, compared to the free-recall equilibrium.

A similar result holds with an arbitrary number of firms:

Proposition 6 *Starting from Stahl's free-recall equilibrium, a firm has a unilateral incentive to make an exploding offer.*

4.2 Risk aversion

If consumers are risk-averse, this will amplify a firm's incentive to use an exploding offer. To illustrate this, consider again the simple example introduced in the introduction but where the agent is risk-averse. When the first principal uses an exploding offer, the agent will accept whenever u_1 is greater than the *certainty equivalent* of the gamble from the second principal, and with risk aversion this certainty equivalent is lower than the mean $\bar{u}$. (In the limit of extreme risk aversion, the agent will always accept the exploding offer, as she is unwilling to risk the chance of a low payoff with the second principal.) Hence, exploding offers will boost the probability of acceptance in more cases than with risk-neutrality.

4.3 Exploding offers as a barrier to entry

Aghion and Bolton (1987) discuss how an incumbent seller may wish to contract with a potential buyer in such a way that entry is discouraged. In their basic model, products are homogeneous, and there is a single buyer who wishes to purchase a single unit which she values at v. The incumbent has known cost $c_I < v$ for supplying the product, while a single potential entrant has cost c_E which is unknown to the incumbent and buyer (but known to the entrant). If there is no contract in place between the incumbent and buyer, entry occurs whenever $c_E \leq c_I$ and the equilibrium price is c_I. If $c_E > c_I$ then entry does not take place and the incumbent can set the monopoly price $p = v$.

An important issue is whether the incumbent has an incentive to offer an exclusive contract to the buyer in which the buyer agrees to buy from the incumbent at some price P before entry takes place. (If the buyer refuses to sign the contract, then the subsequent outcome is the game without contracts just described.) Aghion and Bolton show that the incumbent *never* has this incentive: in order to persuade the buyer to buy in advance of entry, and thus to forego the potential benefits of competition, the incumbent must offer such a low price that its profits are no higher than if it waited for the entrant's decision.[29] Clearly, though, these exclusive contracts are not exploding offers in our sense, since if the buyer refuses to accept the incumbent's contract, she has the ability to return to buy from the firm in the no-contract interaction which follows.

[29] However, they show that the incumbent always has an incentive *partially* to discourage the entrant, in the sense that the buyer agrees to a penalty payment made to the incumbent if she decides to buy from the entrant. Such contracts are somewhat akin to buy-now discounts in our model, in that buy-now discounts merely discourage rather than prevent a consumer from investigating a rival firm.

However, if the incumbent *does* make a publicly-observed exploding offer to the buyer, and thus commits not to serve the buyer later if she refuses to sign the contract, entry can profitably be deterred. The key point is that if the buyer refuses the exploding offer, she obtains zero surplus: she then either has no supplier at all (if the entrant's cost turns out to be very high), or she faces an entrant who knows the buyer cannot return to the incumbent and so charges the monopoly price $p = v$. In effect, rejecting the incumbent's exploding offer forces the entrant to set the monopoly price, and this leaves the buyer no surplus should she decide to wait for entry to occur. The buyer will therefore accept an exploding offer at any price $P < v$, and a sufficiently high price will yield higher profit than the interaction without contracts. Thus, if their use is credible, an exploding offer from the incumbent can deter more efficient entry and harm consumers. The mechanism at work is quite different from that in our main model: for an exploding offer to deter entry it needs to be observed by the second firm so that that firm is induced to set a high price, whereas in section 2 firms choose their sales policies simultaneously.

4.4 Preventing learning about a firm's own product

Lastly, a motive for forcing quick decisions might be to prevent consumers from properly understanding the *current* product rather than the deals offered by rival firms. If a seller forces consumers to decide quickly, a consumer might have to decide whether or not to purchase before she has worked out how much she actually wants the product. Without accurate information about the realized match utility, suppose that a (risk-neutral) consumer then bases her purchase decision on the expected match utility, $\bar{u}$. Unlike our main model, this issue can be analyzed within a simple monopoly framework. Suppose the monopolist has marginal cost c for supplying the product. If the seller gives the consumer time to calculate her match utility u, the seller's profit with price p is $(p - c)[1 - F(p)]$, and the optimal price maximizes this expression. If instead the seller forces the consumer to buy immediately, knowing only her expected utility, the seller can charge $p = \bar{u}$ and obtain profit $\bar{u} - c$. Since $\bar{u} > p[1 - F(p)]$ for all p, it follows that the latter strategy is more profitable whenever c is sufficiently close to zero. By contrast, if c is sufficiently large (above $\bar{u}$, for instance), then the monopolist prefers to give consumers enough time to understand the realized match utility.[30]

5 Conclusions

This paper has explored the incentives firms have to discriminate against those consumers who buy from a firm after investigating rival offers. There are two broad reasons why firms

[30] For further details of the monopolist's incentives to reveal or conceal match-specific information, see Lewis and Sappington (1994). They show that the monopolist typically will choose to reveal all information or none.

wish to do this if they can: a strategic motive to discourage consumers from investigating rivals, and an informational motive which reflects the fact that returning buyers prefer the firm's offer to rival offers. The strategic motive is more important when firms can commit to their sales policy, while the informational motive applies when firms have less commitment power. When firms can commit to their sales policy, the use of exploding offers is individually profitable for firms when products are differentiated and the demand curve is concave (Proposition 1), when there are many suppliers and search costs are significant (Proposition 2), or when products are homogenous (Proposition 6). A less extreme policy is to offer first-time visitors a buy-now discount, and firms have an incentive to offer such discounts under the mild condition that the demand curve is strictly logconcave (Proposition 4(i)). If firms cannot commit at all to their buy-later price, the information motive forces firms to make exploding offers (Proposition 5(i)). This outcome is suboptimal for firms (Proposition 4(ii)). However, a little commitment power solves this problem, and if firms can commit to an upper bound on their buy-later price, then the outcome is as if firms can commit to their buy-later price when they first meet prospective customers (Proposition 5(ii)). When firms use exploding offers, this often, but not always, causes market prices to increase (e.g., see Proposition 3), as well as causing a poor match between products and consumers when products are differentiated.

As demonstrated in this paper, inducements to make a quick decision can limit a consumer's ability to make a well-informed decision, which in turn can harm market performance. Public policy has attempted to address this problem. For instance, the *Unfair Commercial Practices Directive,* adopted in 2005 across the European Union, prohibits in all circumstances "Falsely stating that a product will only be available for a very limited time, or that it will only be available on particular terms for a very limited time, in order to elicit an immediate decision and deprive consumers of sufficient opportunity or time to make an informed choice." However, the enforcement of such laws is often difficult. A more efficient method to tackle the issue may involve less direct means. For example, exploding offers could in essence be prohibited by mandating a "cooling off period", so that consumers have the right to return a product within some specified time after agreeing to purchase. (They could then return a product if they subsequently find a preferred option.) Many jurisdictions impose cooling off periods for some products, especially those sold in the home.

To end, we point out reasons why sales tactics which disadvantage returning visitors are not seen in many markets, even when their use is permitted. A "behavioral" reason why firms do not surcharge their returning customers or force their customers to decide quickly is that consumers could be antagonized by those sales techniques, and decide to buy elsewhere. But first and foremost, many retailers, especially in the traditional bricks-and-mortar sector, cannot distinguish first-time from returning visitors. Shopping at the supermarket, say, is unlikely to involve much contact with sales personnel at all, and there is currently no mechanism by which the firm can detect first-time from returning visitors.

More generally, consumers may be able to conceal their search history (e.g., by deleting cookies on their computer). Thus, if firms discriminate against return visitors and if it is easy to pretend to be a new visitor, consumers will do this, and the market will operate as a standard search market with each firm offering uniform prices.

APPENDIX

Proof of Lemma 1: As shown by Wolinsky, given that other firms are charging the price p_0, if firm i deviates and charges $\tilde{p}$, its demand is[31]

$$Q_0(\tilde{p}) = \frac{1}{n}\left[1 - F(a - p_0 + \tilde{p})\right] \frac{1 - F(a)^n}{1 - F(a)} + \int_{p_0}^{a} F(u)^{n-1} f(u - p_0 + \tilde{p}) du \; . \qquad (19)$$

In equilibrium, firm i maximizes $\tilde{p}Q_0(\tilde{p})$ by choosing $\tilde{p} = p_0$, and so expression (19) implies the first-order condition for p_0 to be the equilibrium price is as given in expression (4). Recall p_M is the monopoly price and it solves $p_M = \frac{1-F(p_M)}{f(p_M)}$, which has a unique solution if $1 - F$ is logconcave (i.e., if $\frac{1-F}{f}$ is decreasing). Also notice that the logconcavity of $1 - F$ implies

$$\frac{f\left(u\right)^2}{1 - F\left(u\right)} + f'\left(u\right) > 0 \; . \qquad (20)$$

We first show that in the relevant interval $0 < p_0 < a$, equation (4) has a solution in the range

$$\frac{1 - F(a)}{f(a)} < p_0 < p_M \; .$$

First, for $p_0 \leq \frac{1-F(a)}{f(a)}$ we show that the left-hand side of (4) is greater than the right-hand side. If $p_0 \leq \frac{1-F(a)}{f(a)}$, we have

$$\frac{1 - F\left(p_0\right)^n}{p_0} = \frac{1 - F\left(a\right)^n}{p_0} + \frac{F\left(a\right)^n - F\left(p_0\right)^n}{p_0} \geq f\left(a\right) \frac{1 - F(a)^n}{1 - F(a)} + \frac{F\left(a\right)^n - F\left(p_0\right)^n}{p_0} \; .$$

[31] To understand this expression, consider the two sources of firm i's demand. First, if firm i's deal generates consumer surplus greater than $a - p_0$ then a consumer who visits it will buy from it immediately. Suppose firm i is in the k_{th} position in a consumer's search order, so that to reach firm i the consumer has already sampled, and rejected, $k - 1$ firms, an event which occurs with probability $F(a)^{k-1}$ (since a consumer will buy immediately if $u_j \geq a$). The consumer who reaches it will buy immediately at firm i if $u_i - \tilde{p} \geq a - p_0$, which occurs with probability $1 - F(a - p_0 + \tilde{p})$. (If the firm is in the final position, i.e., $k = n$, then she will surely buy from firm i if $u_i - \tilde{p} \geq a - p_0$, since her surplus $u_i - \tilde{p}$ is positive and higher than all other firms.) Since a firm is in the k_{th} position with probability $1/n$, summing over k leads to the first term on the right-hand side of (19). Second, if a consumer searches through all sellers and does not find any product with net surplus greater than $a - p_0$, she will then buy from firm i if it offers the highest surplus and this surplus is positive. The probability of this event is

$$\Pr(\max_{j \neq i}\{0, u_j - p_0\} < u_i - \tilde{p} < a - p_0) = \int_{\tilde{p}}^{a - p_0 + \tilde{p}} F(u_i - \tilde{p} + p_0)^{n-1} dF(u_i) \; ,$$

which equals the second term in (19) after changing variables from u_i to $u = u_i + p_0 - \tilde{p}$.

So it suffices to show that

$$\frac{F(a)^n - F(p_0)^n}{p_0} + n\int_{p_0}^{a} F(u)^{n-1} f'(u)du > 0 \Leftrightarrow \int_{p_0}^{a} F(u)^{n-1}\left[\frac{f(u)}{p_0} + f'(u)\right] du > 0\ . \tag{21}$$

Since $p_0 \le \frac{1-F(a)}{f(a)}$ and $\frac{1-F}{f}$ is decreasing, we have $p_0 < \frac{1-F(u)}{f(u)}$ for any $u \in (p_0, a)$, and so

$$\frac{f(u)}{p_0} + f'(u) > \frac{f(u)^2}{1-F(u)} + f'(u) > 0\ ,$$

where the final inequality follows from (20), which proves (21).

Second, for $p_M \le p_0 \le a$, we show that the left-hand side of (4) is smaller than the right-hand side. From the definition of p_M and that $\frac{1-F}{f}$ is decreasing, we know that $p_M \le p_0$ implies $p_0 \ge \frac{1-F(p_0)}{f(p_0)}$, and so

$$\frac{1-F(p_0)^n}{p_0} \le f(p_0)\frac{1-F(p_0)^n}{1-F(p_0)}\ .$$

Then we only need to show that

$$f(a)\frac{1-F(a)^n}{1-F(a)} - f(p_0)\frac{1-F(p_0)^n}{1-F(p_0)} > n\int_{p_0}^{a} F(u)^{n-1}f'(u)du\ ,$$

or equivalently

$$\int_{p_0}^{a}\frac{d}{du}\left[f(u)\frac{1-F(u)^n}{1-F(u)} - n\int_0^u F(x)^{n-1}f'(x)dx\right]du > 0\ .$$

The term inside the bracket $[\cdot]$ is strictly increasing in u if and only if $1-F$ is strictly logconcave,[32] and so the above inequality holds.

Finally, we show that if the monopoly profit function $p[1-F(p)]$ is concave, then a firm's profit function in Wolinsky's model is also concave, which implies the sufficiency of the first-order condition. Define $\alpha(u) \equiv nF(u)^{n-1}/[1+F(a)+\cdots+F(a)^{n-1}]$, so that $\alpha(u) < 1$ and $\alpha'(u) > 0$. Then from (19) a firm's profit when it charges price $\tilde{p}$ is proportional to

$$\tilde{p}\left[1-F(a-p_0+\tilde{p}) + \int_{p_0}^{a}\alpha(u)f(u-p_0+\tilde{p})du\right]\ .$$

One can check that the second-order derivative of this profit with respect to $\tilde{p}$ is

$$\begin{aligned} & -[2f(a-p_0+\tilde{p}) + \tilde{p}f'(a-p_0+\tilde{p})] + \int_{p_0}^{a}\alpha(u)[2f'(u-p_0+\tilde{p}) + \tilde{p}f''(u-p_0+\tilde{p})]du \\ = & -[1-\alpha(u)][2f(a-p_0+\tilde{p}) + \tilde{p}f'(a-p_0+\tilde{p})] - \alpha(p_0)[2f(\tilde{p}) + \tilde{p}f'(\tilde{p})] \\ & \qquad - \int_{p_0}^{a}\alpha'(u)[2f(u-p_0+\tilde{p}) + \tilde{p}f'(u-p_0+\tilde{p})]du\ . \end{aligned} \tag{22}$$

[32] The derivative of the bracket term is

$$\left(\frac{f}{1-F}\right)'(1-F^n) - \frac{f}{1-F}nF^{n-1}f - nF^{n-1}f' = \left(\frac{1-F^n}{1-F} - nF^{n-1}\right)\left(\frac{f^2}{1-F} + f'\right) > 0.$$

(The equality follows by integrating by parts.) Notice that

$$2\frac{f(u-p_0+\tilde{p})}{\tilde{p}}+f'(u-p_0+\tilde{p})\geq 2\frac{f(u-p_0+\tilde{p})}{u-p_0+\tilde{p}}+f'(u-p_0+\tilde{p})$$

for any $u \geq p_0$. Hence, a sufficient condition for (22) to be negative is $2f(p)+pf'(p)\geq 0$ which is equivalent to $p[1-F(p)]$ being concave.

Proof of Lemma 2: First note that if $p < a$ then $V(p) > 0$, and so (5) implies that $\{a_m\}$ is an increasing sequence as claimed. In particular, $a_m - p \geq 0$ for all $m \geq 0$. Second, if a consumer's expected surplus from entering a no-recall search market with m firms is W_m, for $m \geq 0$, then when the consumer has $l \geq 0$ firms remaining unsampled she will accept the product from her current firm if and only if $u - p \geq W_l = a_l - p$, i.e., if $u \geq a_l$ as stated in the result. It remains to prove that a consumer's expected surplus from entering a no-recall search market with m firms is indeed W_m.

We prove this by means of an inductive argument. If $m = 0$, then clearly the consumer obtains zero surplus from participating in the market, and the result applies as stated. Suppose now that the result holds for $m \geq 0$ firms, and consider the situation when there are $m+1$ firms in the market. If the consumer chooses to enter the market and sample the first firm, she will buy from the first firm if and only if $u \geq a_m$, since her surplus from searching beyond the firm is $a_m - p$ by the inductive assumption. Therefore, her expected surplus from entering the market is

$$\begin{aligned}\int_{a_m}^{u_{\max}}(u-p)dF(u)+(a_m-p)F(a_m)-s &= \int_{a_m}^{u_{\max}}(u-a_m)dF(u)+a_m-p-s\\ &= V(a_m)+a_m-p\\ &= a_{m+1}-p\geq 0\end{aligned}$$

where the second step uses the definition of $V(\cdot)$ in (1) and the final equality follows from the recursive relation (5). Thus, when $p < a$, the consumer's expected surplus from entering the no-recall search market with $m+1$ firms is $W_{m+1} \geq 0$ as stated. Finally, suppose that $p > a$. Then when the consumer has only one firm to sample ($m = 1$) her expected surplus from entering the market is $V(p) < 0$. By induction, if she does not enter the market with $m \geq 1$ firms when $p > a$, she will also not enter when there are $m+1$ firms. This completes the proof of the result.

Proof of Lemma 3: In the relevant range $0 < p < a$, we have $a_{n-k} \in (p, a)$ for every $k \leq n-1$. (Recall $a_0 = p$, and whenever $p < a$, a_m increases with m and converges to a as $m \to \infty$.) Under the logconcavity condition, $\frac{1-F}{f}$ is a strictly decreasing function. This implies that each $\frac{1-F(a_{n-k})}{f(a_{n-k})}$ lies between $\frac{1-F(a)}{f(a)}$ and $\frac{1-F(p)}{f(p)}$ (when $k = n$, it is equal to the latter), and therefore so is the right-hand side of (8), which establishes the existence of a solution in the range $\frac{1-F(a)}{f(a)} < p < p_M$.

We next show that if $p[1-F(p)]$ is concave, then $\tilde{p}[1-F(a_{n-k}-p+\tilde{p})]$ is concave in $\tilde{p}$ for any k (which then implies that the profit function $\tilde{p}Q(\tilde{p})$ is concave). Notice that the second-order derivative is negative if $2f(a_{n-k}-p+\tilde{p})+\tilde{p}f'(a_{n-k}-p+\tilde{p})\geq 0$. Since $a_{n-k}\geq p$, we have

$$2\frac{f(a_{n-k}-p+\tilde{p})}{\tilde{p}}+f'(a_{n-k}-p+\tilde{p})\geq 2\frac{f(a_{n-k}-p+\tilde{p})}{a_{n-k}-p+\tilde{p}}+f'(a_{n-k}-p+\tilde{p})\ .$$

Hence, a sufficient condition for a concave profit function is $2f(x)+xf'(x)\geq 0$, which is equivalent to $p[1-F(p)]$ being concave.

Proof of Proposition 1: Part (i): Our proof of this part consists of two steps. First, we show that if the match utility density f is strictly increasing, then all firms using exploding offers is an equilibrium. Second, we exclude the possibility that all firms allowing free recall is also an equilibrium.

The hypothesis is that all firms choose to use exploding offers and to set the price p in (8). Suppose a deviating firm chooses price $\tilde{p}$ and allows free recall, while other firms follow the proposed equilibrium strategy. Suppose that the deviating firm is in the k_{th} position of a consumer's search process and $k<n$. (If $k=n$ then allowing free recall or not does not affect the firm's demand.) Then the probability that this consumer will visit the firm is still h_k in (6), since consumers hold equilibrium beliefs. However, her incentive to search beyond the firm is now altered. Since she can return to this firm whenever she wants, she becomes more willing to continue searching. If at the deviating firm she finds utility u such that $u-\tilde{p}\leq 0$, she will never buy from the firm (either immediately or later). So consider the situation where $u-\tilde{p}>0$. Then if she leaves the deviating firm, she will enter a no-recall search market with $n-k$ products each being sold at price p, but now with an outside option $u-\tilde{p}$.

We first describe a consumer's expected surplus and optimal stopping rule in a no-recall search market with m products each being sold at price p and with an outside option $z>0$ (which she can consume without starting to search). This result is a generalization of Lemma 2, which applied to the situation with $z=0$, and its proof is essentially the same.

Claim 1 *Suppose consumers face a search market with m firms, each of which use exploding offers and set price p. Suppose that if consumers do not buy in the market, their outside option is $z\geq 0$. Then a consumer obtains expected surplus equal to*

$$W_m(z)=\begin{cases} z & \text{if } z\geq a-p \\ a_m(z)-p & \text{if } z<a-p \end{cases} \tag{23}$$

where $a_l(z)$ solves the recursive equation

$$a_{l+1}(z)=a_l(z)+V(a_l(z)) \tag{24}$$

with $a_0(z)=z+p$ and where $V(\cdot)$ defined in (1). A consumer enters the market if and only if $z<a-p$, in which case a consumer who has $l\geq 0$ firms remaining unsampled

will buy from her current firm if match utility is greater than $a_l(z)$. If she finds no such product after visiting all firms, she will consume the outside option.

Note that when $z < a - p$, we have $z < a_l(z) - p < a - p$ for $l \geq 1$, so the consumer will consume the outside option only if she samples all products and finds that the last one has a match utility lower than $a_0(z)$. If we denote by $r_m(z)$ the probability that the consumer will consume the outside option, then $r_m(z) = 1$ if $z \geq a - p$ and $r_m(z) = F(a_{m-1}(z)) \cdots F(a_0(z)) < 1$ if $z < a - p$. Properties from (24) which will be used below are

$$a_m(a-p) = a; \;\; a_m(0) = a_m; \;\; a'_m(z) = r_m(z) \tag{25}$$

for $z < a - p$, where a_m is defined in (5).

When the deviating firm occupies the k_{th} position in a consumer's search order, the consumer will buy from it immediately if and only if $u - \tilde{p} \geq a - p$. If $u - \tilde{p} \in (0, a-p)$, then she will come back to buy after sampling all $n-k$ subsequent firms, which occurs with probability $r_{n-k}(u - \tilde{p})$. Thus, the firm's demand when it is in the k_{th} position, charges price $\tilde{p}$ and permits free return, is

$$\begin{aligned} & h_k \left[1 - F(a - p + \tilde{p}) + \int_{\tilde{p}}^{a-p+\tilde{p}} r_{n-k}(u - \tilde{p}) f(u) du \right] \\ & = h_k \left[1 - F(a - p + \tilde{p}) + \int_{p}^{a} r_{n-k}(u - p) f(u - p + \tilde{p}) du \right] \; , \end{aligned} \tag{26}$$

where the equality follows after changing variables in the integral. Compared to the demand generated with an exploding offer given in (7), it now has reduced immediate demand since $a > a_{n-k}$, but has positive returning demand comprised of the integral term.

Claim 2 *Demand in (26) is smaller than that in (7) if f is strictly increasing.*

Proof. We need to show

$$\begin{aligned} \int_p^a r_{n-k}(u-p) f(u - p + \tilde{p}) du \;\; & < \;\; F(a - p + \tilde{p}) - F(a_{n-k} - p + \tilde{p}) \\ & = \;\; \int_p^a r_{n-k}(u - p) f(\phi(u)) du \; , \end{aligned} \tag{27}$$

where

$$\phi(u) \equiv a - p + \tilde{p} - \int_u^a r_{n-k}(x - p) dx \; .$$

The equality comes from noting that $\phi'(u) = r_{n-k}(u - p)$, $\phi(a) = a - p + \tilde{p}$, and $\phi(p) = a_{n-k} - p + \tilde{p}$ (which follows from (25)). Since $\phi(u) > u - p + \tilde{p}$ (because $r_{n-k}(x - p) < 1$ for $x < a$), the inequality (27) holds if f is an increasing function. ■

Therefore, for any price $\tilde{p}$, unilaterally allowing free recall causes the deviating firm's demand (and hence profit) to fall when f is increasing. (This is true regardless of the firm's position in a consumer's search order, except when it is in the final position in which case the use of exploding offers makes no difference to the firm's demand.) It follows that an equilibrium in which all firms use exploding offers exists.

The second step is to exclude the possibility of a free-recall equilibrium when f is strictly increasing. We show that, starting from the hypothetical free-recall equilibrium with price p_0, a firm has a unilateral incentive to use an exploding offer no matter what position it is in the consumer's search process (except when it is in the final position).

As in expression (19) and footnote 7, firm i's demand, if it is in the k_{th} position of the consumer's search process with $k < n$ and if it sets a price $\tilde{p}$ and allows free recall, is

$$F(a)^{k-1}[1 - F(a - p_0 + \tilde{p})] + \int_{p_0}^{a} F(u)^{n-1} f(u - p_0 + \tilde{p}) du \ . \tag{28}$$

Suppose now that the firm instead uses an exploding offer with a price $\tilde{p}$. We will show the firm's demand with this deviation is higher than (28) for any $\tilde{p}$ when the density is increasing, and hence the hypothetical equilibrium is not valid.[33] Define $\delta \equiv \max\{0, u_1 - p_0, \cdots, u_{k-1} - p_0\}$. Then the consumer will visit the firm if and only if $\delta < a - p_0$. If she finds match utility u at the firm, she will buy (immediately) if $u - \tilde{p}$ is greater than the expected surplus from searching further.

Denote by $Y_m(z)$ the expected surplus from participating in a free-recall search market with $m \geq 0$ products offered at price p_0 and an outside option $0 < z < a - p_0$. Then[34]

$$Y_m(z) = z + \int_{z+p_0}^{a} [1 - F(u)^m] du \ . \tag{29}$$

One can check that $z \leq Y_m(z) < a - p_0$.

The consumer will buy from firm i if and only if $u - \tilde{p} \geq Y_{n-k}(\delta)$. Here, δ is the consumer's outside option if the consumer leaves the firm and continues searching (since the firm is using an exploding offer). The c.d.f. of δ defined on $[0, u_{\max} - p_0]$ is $G(\delta) \equiv F(\delta + p_0)^{k-1}$, which has a mass point at zero. Therefore, the deviating firm's demand when

[33] We only need to show that the firm's deviation demand when it uses an exploding offer with price p_0 is greater than that in the free-recall equilibrium. We consider a more general deviation for the purpose of proving the result in part (ii) where, to check a free-recall equilibrium can be sustained, we need consider both the buy-later policy and the price deviation.

[34] The consumer will stop searching before she runs out of options if and only if she finds a product with match utility greater than a. (This is true regardless of z since $z < a - p_0$.) This also implies that the consumer will consume the outside option if and only if she has sampled all firms and each of them offers $u_i - p_0 < z$, which occurs with probability $F(z+p_0)^m$. Since z does not affect the consumer's stopping rule, it affects her welfare only when she consumes the outside option. Hence, we deduce $Y_m'(z) = F(z + p_0)^m$. Our formula follows by noting that $Y_m(a - p_0) = a - p_0$, i.e., when the outside option is $a - p_0$ the consumer will consume it immediately without searching.

it is in the k_{th} position is

$$
\begin{aligned}
& \Pr\left(\delta < a - p_0 \text{ and } u - \tilde{p} > Y_{n-k}(\delta)\right) \qquad (30)\\
= \; & G(0)[1 - F(\tilde{p} + Y_{n-k}(0))] + \int_0^{a-p_0} [1 - F(\tilde{p} + Y_{n-k}(\delta))] dG(\delta) \\
= \; & F(a)^{k-1}[1 - F(\tilde{p} + Y_{n-k}(a - p_0))] + \int_{p_0}^{a} f(\tilde{p} + Y_{n-k}(u - p_0)) Y'_{n-k}(u - p_0) F(u)^{k-1} du \; ,
\end{aligned}
$$

where the second equality follows after integrating by parts and changing the integral variable from δ to $u = \delta + p_0$. According to the definition of $Y_m(\cdot)$ in (29), we have $Y_{n-k}(a - p_0) = a - p_0$ and

$$
Y_{n-k}(u - p_0) = u - p_0 + \int_u^a [1 - F(x)^{n-k}] dx \; , \; Y'_{n-k}(u - p_0) = F(u)^{n-k} \; .
$$

Substituting these into (30) shows that the firm's deviation demand is

$$
F(a)^{k-1}[1 - F(a - p_0 + \tilde{p})] + \int_{p_0}^{a} F(u)^{n-1} f\left(u - p_0 + \tilde{p} + \int_u^a [1 - F(x)^{n-k}] dx\right) du \; . \quad (31)
$$

The second term in (31) reflects the increased probability that the consumer will buy immediately.

One can see that if f is strictly increasing, demand in (31) is strictly greater than demand in (28). Therefore, the firm does have an incentive to deviate from the supposed free-recall equilibrium. This completes the proof of part (i). Parts (ii) and (iii) can be proved in a similar manner.

Proof of Proposition 2: (i) Consider first the case where each firm's price is fixed at $p < a$. From (27), we know that starting from each firm using exploding offers, no firm has a unilateral incentive to allow free recall if

$$
\int_p^a r_{n-k}(u - p) f(u) du < \int_p^a r_{n-k}(u - p) f\left(a - \int_u^a r_{n-k}(x - p) dx\right) du \; .
$$

Notice that a hump-shaped density and $a < u^*$ imply that $f(u)$ is increasing at any $u < a$. Hence, the above inequality holds since $a - \int_u^a r_{n-k}(x - p)dx > u$ for $u < a$.

We now allow for price deviations as well. Then we need to show that (27) holds with the most profitable deviation price $\tilde{p}$. It is easy to see that if any $\tilde{p}$ is allowed, the condition that f is increasing at $u < a$ is not enough for (27) to hold. However, when n is large, we know that the exploding-offer equilibrium price is $p \approx \frac{1-F(a)}{f(a)}$, and we can also show that the optimal deviation price when firm i unilaterally allows free recall is close to this price too. Specifically, firm i's deviation demand consists of two parts: the returning demand

$$
Q_R = \frac{1}{n} \sum_{k=1}^{n} h_k \int_p^a r_{n-k}(u - p) f(u - p + \tilde{p}) du
$$

(which is the sum over k of the second term in (26) divided by n), and the immediate demand

$$Q_I = \frac{1}{n}\sum_{k=1}^{n} h_k[1 - F(a - p + \tilde{p})]$$

(which is the sum over k of the first term in (26) divided by n). We have

$$Q_R < \frac{1}{n}\sum_{k=1}^{n} F(a)^{k-1}F(a)^{n-k}\int_p^a f(u - p + \tilde{p})du = F(a)^{n-1}\int_p^a f(u - p + \tilde{p})du$$

(the inequality used $a_m < a$ and $r_m(z) = F(a_{m-1}(z))\cdots F(a_0(z)) < F(a)^m$ for $z < a - p$), and

$$Q_I > \frac{1}{n}\sum_{k=1}^{n} F(p)^{k-1}[1 - F(a - p + \tilde{p})] = \frac{1 - F(p)^n}{n(1 - F(p))}[1 - F(a - p + \tilde{p})]$$

(the inequality used $a_m > p$). Thus,

$$\frac{Q_R}{Q_I} < \frac{nF(a)^{n-1}[1 - F(p)]}{1 - F(p)^n}\frac{\int_p^a f(u - p + \tilde{p})du}{1 - F(a - p + \tilde{p})}$$

which converges to zero as $n \to \infty$. Therefore, when n is large, the optimal deviation price should be close to the price which maximizes $\tilde{p}Q_I$, i.e., it is close to $p \approx \frac{1-F(a)}{f(a)}$. Then we can focus on deviation prices $\tilde{p} = p + \varepsilon(n)$, where $\varepsilon(n) \to 0$ as $n \to \infty$. From (27), we need to show that

$$\int_p^a r_{n-k}(u - p)f(u + \varepsilon(n))du < \int_p^a r_{n-k}(u - p)f\left(a + \varepsilon(n) - \int_u^a r_{n-k}(x - p)dx\right)du .$$

Since $\varepsilon(n) \to 0$, the previous argument with a fixed price carries over.

(ii) Again, we first consider the case where the price is fixed at p_0. From (28) and (31), we can see that starting from all firms allowing free recall, no firm has a unilateral incentive to introduce exploding offers if

$$\int_{p_0}^a F(u)^{n-1}f\left(a - \int_u^a F(x)^{n-k}dx\right)du < \int_{p_0}^a F(u)^{n-1}f(u)du .$$

When n is large, this condition holds if

$$f\left(a - \int_u^a F(x)^{n-k}dx\right) < f(u) \text{ for } u \approx a .$$

Since $a - \int_u^a F(x)^{n-k}dx > u$ for any $u < a$, this inequality must hold if f is strictly decreasing at a. For a hump-shaped density, this is implied by $a > u^*$.

We now allow for price deviations as well and need to show that

$$\int_{p_0}^a F(u)^{n-1}f\left(a - p_0 + \tilde{p} - \int_u^a F(x)^{n-k}dx\right)du < \int_{p_0}^a F(u)^{n-1}f(u - p_0 + \tilde{p})du$$

for any deviation price $\tilde{p}$. However, when n is sufficiently large, we knew that the equilibrium price in the free-recall case is $p_0 \approx \frac{1-F(a)}{f(a)}$, and again we can show that the optimal deviation price when firm i unilaterally makes an exploding offer is close to $\frac{1-F(a)}{f(a)}$. From (31), firm i's returning demand is now

$$Q_R = \frac{1}{n}\sum_{k=1}^{n}\int_{p_0}^{a} F(u)^{n-1} f\left(u - p_0 + \tilde{p} + \int_u^a [1-F(x)^{n-k}]dx\right) du$$

(which is the sum over k of the second term in (31) divided by n), and the immediate demand is

$$Q_I = \frac{1-F(a)^n}{n(1-F(a))}[1 - F(a - p_0 + \tilde{p})]$$

(which is the sum over k of the first term in (31) divided by n). If M is the upper bound of f, we have

$$Q_R < M \int_{p_0}^{a} F(u)^{n-1} du \ .$$

It follows that Q_R/Q_I converges to zero as $n \to \infty$, and so the optimal deviation price should be close to the price which maximizes $\tilde{p}Q_I$, i.e., it is close to $p_0 \approx \frac{1-F(a)}{f(a)}$. Then following a similar logic as in part (i), one can show that the above argument with a fixed price carries over.

Proof of Proposition 3: Suppose to the contrary that $p \le p_0$, where these two prices are given in (9)–(10) and which we know lie in the interval $(1-a, 1/2)$. Then

$$0 \le \frac{1}{p} - \frac{1}{p_0} = h_n + \sum_{k=1}^{n} h_k - p_0^{n-1} - \frac{1-a^n}{1-a} \le h_n + \sum_{k=1}^{n} h_k - p^{n-1} - \frac{1-a^n}{1-a} \equiv J_n(p) \ .$$

We will show below that $J_n(p) < 0$ for $p \in (1-a, a)$ and any $n \ge 2$. (Note that $a_m = a$ if $p = a$ and so $J_n(a) = 0$.) Then the above inequality implies $p \le 1-a$ or $p \ge a$. This is a contradiction to $p \in (1-a, 1/2)$ and so we can conclude that $p > p_0$.

We use an inductive argument to show $J_n(p) < 0$ for $p \in (1-a, a)$. First, we have

$$J_2(p) = 1 + 2a_1 - p - (1+a) = a - a^2 - (p - p^2) < 0 \ .$$

The second step used $a_1 = p + V(p) = a - (a^2 - p^2)/2$, and the final step follows because $p \in (1-a, a)$. Now suppose for $n \ge 2$ that

$$J_n(p) < 0 \Leftrightarrow h_n + \sum_{k=1}^{n} h_k < p^{n-1} + \frac{1-a^n}{1-a}$$

for $p \in (1-a, a)$. We aim to show $J_{n+1}(p) < 0$ in the same interval. Using the inductive assumption, we have

$$\begin{aligned} J_{n+1}(p) &= h_{n+1} + \sum_{k=1}^{n+1} h_k - p^n - \frac{1-a^{n+1}}{1-a} \\ &= 1 + a_n\left(h_n + \sum_{k=1}^{n} h_k\right) - p^n - \frac{1-a^{n+1}}{1-a} \\ &< 1 + a_n\left(p^{n-1} + \frac{1-a^n}{1-a}\right) - p^n - \frac{1-a^{n+1}}{1-a} \ . \end{aligned}$$

As we show below,

$$a_n \left(p^{n-1} + \frac{1-a^n}{1-a} \right) < p^n + a\frac{1-a^n}{1-a} \tag{32}$$

for $p \in (1-a, a)$. Hence, after some rearranging $J_{n+1}(p) < 0$ follows.

As the final step, we now prove inequality (32). Let

$$\gamma_n \equiv \frac{p^n + a\frac{1-a^n}{1-a}}{p^{n-1} + \frac{1-a^n}{1-a}} < a \ .$$

We want to prove $a_n < \gamma_n$. Again we use the induction method, starting at $n = 1$. First, one can verify that $a_1 < \gamma_1$. Now suppose $a_n < \gamma_n$, and we aim to show $a_{n+1} < \gamma_{n+1}$. Using the inductive assumption, we have $a_{n+1} = a_n + V(a_n) < \gamma_n + V(\gamma_n)$ since $x + V(x)$ is increasing in x. So a sufficient condition for $a_{n+1} < \gamma_{n+1}$ is that

$$\gamma_n + V(\gamma_n) < \gamma_{n+1} \ . \tag{33}$$

Notice that in the uniform case $x + V(x) = a - (a^2 - x^2)/2 < a - ax + x^2$ for $x < a$. Hence, (33) holds if

$$a - a\gamma_n + \gamma_n^2 < \gamma_{n+1} \Leftrightarrow a - \gamma_{n+1} < \gamma_n(a - \gamma_n) \Leftrightarrow \frac{a - \gamma_{n+1}}{\gamma_n(a-\gamma_n)} < 1 \ .$$

One can verify that

$$\frac{a - \gamma_{n+1}}{\gamma_n(a-\gamma_n)} = \frac{p(1-a^n) + p^n(1-a)}{a(1-a^n) + p^n(1-a)} \times \frac{p^{n-1}(1-a) + 1 - a^n}{p^n(1-a) + 1 - a^{n+1}} \ .$$

The first term on the right-hand side above is less than 1 since $p < a$. The second term is less than 1 if and only if $p^{n-1}(1-p) < a^n$, which must be true for $p \in (1-a, a)$. This completes the proof.

Proof of Proposition 4: (i) We will show that a firm has an incentive to introduce a small buy-later premium, and then invoke Lemma 4 to show that the firm also has an incentive to offer a small buy-now discount. Compared to the duopoly case analyzed in the main text, the additional analysis needed for the general n-firm case involves the extra complexity of a consumer's stopping rule. In particular, the consumer's stopping rule at a firm which offers a buy-later premium will depend on the history of offers she sees before she encounters the firm, and this feature was absent in the duopoly analysis.

Let p_0 be the price in the free-recall equilibrium defined by (4). Assumption (2) implies that $p_0 < a$. We first consider the following hypothetical search problem:

A search problem: *Suppose a consumer encounters firm i first, and is offered match utility u_i, the buy-now price p_0, and a buy-later premium $\tau > 0$ (so the buy-later price at firm i is $\hat{p} = p_0 + \tau$). Suppose she expects that m remaining firms charge price $p_0 < a$ and allow free recall, and suppose the consumer has an outside option $\delta < a - p_0$. When will this consumer buy from firm i?*

It is clear that (a) if $u_i \geq a$, the consumer will surely stop searching and buy at firm i immediately (this is even true when $\tau = 0$); and (b) if $u_i - p_0 \leq \delta$, then firm i's offer is dominated by the outside option and the consumer will never buy from the firm.

Now consider the intermediate case with $u_i - p_0 \in (\delta, a - p_0)$. If the consumer buys immediately at firm i, her payoff is $u_i - p_0$. If she leaves firm i, she will enter a free-recall search market with m firms and an outside option

$$z = \max\{\delta, u_i - \hat{p}\} < a - p_0 \ .$$

(Recall she will pay the higher price $\hat{p} > p_0$ if she returns to buy from firm i.) As before, her expected surplus $Y_m(z)$ from entering this search market is given by (29). Given δ, z is a function of u_i and we can therefore regard $Y_m(z)$ as a function of u_i: it is flat until u_i reaches $\delta + \hat{p}$ and then increases with u_i with slope less than one. (Note that we are considering the case with $u_i < a$, so the slope cannot be equal to one.) Recall from (29) that for $z < a - p_0$, we have $z < Y_m(z) < a - p_0$.

Clearly, the consumer will buy immediately from firm i if and only if

$$u_i - p_0 \geq Y_m(\max\{\delta, u_i - \hat{p}\}) \ . \tag{34}$$

Given the properties of $Y_m(\cdot)$, the value of u_i which achieves equality in (34) has a unique solution which we denote by $a_m(\tau) \in (\delta + p_0, a)$. We conclude that the consumer will buy immediately from firm i if and only if $u_i \geq a_m(\tau)$.

There are then two cases, depending on the size of the premium τ:

(a) If $u_i - p_0$ crosses $Y_m(z)$ at the flat portion, which occurs when $\delta + \hat{p} - p_0 > Y_m(\delta)$, i.e., when $\tau > Y_m(\delta) - \delta$, then

$$a_m(\tau) = p_0 + Y_m(\delta) \ , \tag{35}$$

which does not depend on τ. In this case, the consumer will leave firm i if $u_i < a_m(\tau)$ and then will never return to firm i because $u_i - \hat{p} < a_m(\tau) - \hat{p} < \delta$.

(b) If $u_i - p$ crosses $Y_m(z)$ at the increasing portion, which occurs when $\tau \leq Y_m(\delta) - \delta$, then $a_m(\tau)$ is implicitly determined by $a_m(\tau) - p_0 = Y_m(a_m(\tau) - p_0 - \tau)$, which from (29) implies that $a_m(\tau)$ satisfies

$$\tau = \int_{a_m(\tau)-\tau}^{a} [1 - F(u)^m] du \ , \tag{36}$$

which does not depend on p_0 or δ. In particular, $a_m(0) = a$. Expression (36) is the generalization beyond duopoly of our earlier formula (12). In this case, the consumer will initially reject firm i's offer if $u_i < a_m(\tau)$, but will come back to the firm after sampling the remaining m firms if $u_i - \hat{p} > \max_{1 \leq j \leq m}\{\delta, u_j - p_0\}$. Note that the assumption $\delta < a - p_0$ implies that $Y_m(\delta) - \delta > 0$, and so case (b) is relevant for all sufficiently small $\tau > 0$.

In sum, we deduce the following result:

Claim 3 *In this hypothetical search problem, the consumer will buy from firm i immediately if and only if $u_i \geq a_m(\tau)$, where $a_m(\tau)$ is defined in (35) if $\tau > Y_m(\delta) - \delta$, and otherwise $a_m(\tau)$ is defined in (36).*

Finally, since $Y_m(\delta) - \delta$ is decreasing in δ, the condition $\tau > Y_m(\delta) - \delta$ is equivalent to $\delta \in (\delta_\tau, a - p_0)$, where δ_τ solves

$$\tau = Y_m(\delta_\tau) - \delta_\tau = \int_{\delta_\tau + p_0}^{a} [1 - F(u)^m] du \tag{37}$$

if $\tau < Y_m(0)$, and $\delta_\tau = 0$ otherwise. In particular, $Y_m(\delta_0) = \delta_0 = a - p_0$.

We now prove Proposition 4. Starting from the free-recall equilibrium with price p_0, suppose firm i unilaterally introduces a returning purchase premium $\tau > 0$ but keeps the buy-now price unchanged at p_0. Suppose firm i happens to be in the k_{th} position of the consumer's search process. If $k = n$, then τ has no impact on firm i's profit. In the following, we show that for any $k < n$, introducing a small premium $\tau > 0$ is profitable for the firm.

As in the proof of Proposition 1, let $\delta \equiv \max\{0, u_1 - p_0, \cdots, u_{k-1} - p_0\}$ be the best offer from the previous $k-1$ firms. A consumer will visit firm i if $\delta < a - p_0$. If the consumer arrives at firm i and discovers match utility u_i and the buy-later premium τ (but still holds the equilibrium belief about the remaining $n-k$ firms' policies), she faces the search problem we have just analyzed with $m = n - k$, and her stopping rule will depend on her best previous offer δ. Let us focus on a relatively small τ such that $\tau < Y_{n-k}(0)$ and define δ_τ as in (37) with $m = n - k$. Then if $\delta \in (\delta_\tau, a - p_0)$, the reservation utility according to (35) is $a_{n-k}(\tau) = p_0 + Y_{n-k}(\delta)$. In this case, the consumer will buy immediately if $u_i \geq a_{n-k}(\tau)$, and otherwise she will keep searching and never come back. Alternatively, if $\delta \leq \delta_\tau$ the reservation utility $a_{n-k}(\tau)$ is as given in (36) with $m = n-k$. In this case, even if the consumer leaves firm i first (i.e., if $u_i < a_{n-k}(\tau)$), she will eventually come back after sampling all remaining firms if $u_i - p_0 - \tau$ is greater than their offered surplus *and* the outside option δ which represents the best offer among the previous $k-1$ firms. Explicitly, firm i's returning demand in this case is

$$\begin{aligned} & \Pr(\max_{j>k}\{\delta, u_j - p_0\} < u_i - p_0 - \tau < a_{n-k}(\tau) - p_0 - \tau) \\ = & \int_{p_0+\tau}^{a_{n-k}(\tau)} F(u_i - \tau)^{n-1} dF(u_i) = \int_{p_0}^{a_{n-k}(\tau)-\tau} F(u) f(u+\tau) du \ . \end{aligned}$$

(Note δ is also a random variable with c.d.f. $G(\delta) = F(\delta + p_0)^{k-1}$, and the second step follows after changing the integral variable.) Therefore, firm i's profit if it is in the k_{th} search position and charges the buy-later premium τ is

$$p_0 \int_{\delta_\tau}^{a-p_0} [1 - F(p_0 + Y_{n-k}(\delta))] \, dG(\delta) + p_0 G(\delta_\tau)[1 - F(a_{n-k}(\tau))]$$

$$+ (p_0 + \tau) \int_{p_0}^{a_{n-k}(\tau)-\tau} F(u)^{n-1} f(u+\tau) du \ . \tag{38}$$

Note from (36) that

$$(1 - a'_{n-k}(0))(1 - F(a)^{n-k}) = 1 \ . \tag{39}$$

By using the observations $Y_{n-k}(\delta_0) = \delta_0 = a - p_0$ and (39), the derivative with respect to τ of firm i's profit in (38) when it is in the k_{th} position (with $k < n$), evaluated at $\tau = 0$, is

$$\int_{p_0}^{a} F(u)^{n-1} [f(u) + p_0 f'(u)] du \ , \tag{40}$$

which generalizes the duopoly expression (15). Here, $\int_{p_0}^{a} F^{n-1} f du$ is the extra revenue generated from the returning customers, while $\int_{p_0}^{a} F^{n-1} f' du$ is the extra demand generated by increasing the cost of return. That (40) is positive when $p_0 > \frac{1-F(a)}{f(a)}$ follows the argument given in the main text for duopoly. Since (40) is positive (and the same) for all $k < n$, the proof of part (i) is complete.

(ii) Suppose by contrast that there is an equilibrium without returning demand. Let $\bar{\tau}$ be the minimum buy-later premium needed for such an equilibrium. Then $\bar{\tau}$ satisfies

$$a_{n-1} = p + \bar{\tau} \ , \tag{41}$$

where p is the equilibrium exploding-offer price defined in (8). (Recall that $\{a_m\}$ is the sequence of reservation utilities with exploding offers.) To see this, notice that a_m is increasing in m and so if a consumer never wants to go back to the first sampled firm (at which she was the most choosy), she also does not want to go back to any other firm. That is, (41) implies

$$a_m < p + \bar{\tau} \tag{42}$$

for any $m \leq n - 2$.

Starting from the hypothetical equilibrium in which each firm sets a buy-now price p and a buy-later premium $\bar{\tau}$,[35] suppose firm i deviates and sets a buy-later premium $\bar{\tau} - \varepsilon$ where $\varepsilon > 0$ is small enough that (42) continues to hold (but keeps its buy-now price p unchanged). Note that this small deviation will not affect the search behavior of consumers who sample any other firm first because of (42), and so we focus on those consumers who sample firm i first.

Given a buy-later premium smaller than $\bar{\tau}$, the consumer will become more likely to search on at firm i. Let $\tilde{a}_{n-1} > a_{n-1}$ be the new reservation utility. For a small deviation, $\tilde{a}_{n-1}$ must satisfy

$$\tilde{a}_{n-1} - p = W_{n-1}(\tilde{a}_{n-1} - p - \bar{\tau} + \varepsilon) \ , \tag{43}$$

where W_{n-1} is as given in (23). The right-hand side is the expected surplus from participating a "no-recall" market with $n-1$ firms and a positive outside option $\tilde{a}_{n-1} - p - \bar{\tau} + \varepsilon$

[35] The same argument applies if firms charge buy-later premia greater than $\bar{\tau}$.

which is available if the consumer comes back to firm i.[36] Let $\tilde{a}_{n-1} \approx a_{n-1} + \theta\varepsilon$ be the first-order approximation of $\tilde{a}_{n-1}$, where θ is to be determined. Then (41) and (43) imply

$$\begin{aligned} a_{n-1} - p + \theta\varepsilon &\approx W_{n-1}((1+\theta)\varepsilon) \\ &\approx W_{n-1}(0) + (1+\theta)\varepsilon W'_{n-1}(0) \\ &= a_{n-1} - p + (1+\theta)\varepsilon r_{n-1}(0) \ . \end{aligned}$$

The equality used (23) and (25), and $r_{n-1}(0)$ is the probability that the consumer will purchase nothing when firms make exploding offers. Thus, θ satisfies

$$\theta = (1+\theta) r_{n-1}(0) \ . \tag{44}$$

For a small ε, firm i's fresh demand from those who visit firm i first will be reduced by

$$f(a_{n-1})\theta\varepsilon \ . \tag{45}$$

On the other hand, the reduction of the buy-later premium will generate new returning demand. Those consumers who find $u \in [p + \bar{\tau} - \varepsilon, \tilde{a}_{n-1}]$ at firm i will search on first and eventually come back with a probability approximately equal to $r_{n-1}(0)$. Since the length of the above interval is (approximately) $(1+\theta)\varepsilon$, the returning demand is

$$f(a_{n-1})(1+\theta) r_{n-1}(0)\varepsilon \ . \tag{46}$$

From (44), one can see that (45), the decrease of the fresh demand is actually equal to (46), the increase of the returning demand, and so total demand is unchanged to first order with this deviation. But each returning consumer pays more than each first-time visitor $(p + \bar{\tau} - \varepsilon > p)$. Hence, the deviation is profitable.

Proof of Proposition 5: (i) Denote by $r > 0$ the intrinsic returning cost. Suppose in some equilibrium that each consumer forecasts that a firm's buy-later price is $\hat{p}(p_i)$ when its buy-now price is p_i, where $\hat{p}(\cdot)$ can take any form. Suppose that the buy-now price in this equilibrium is p^*, say, and suppose—contrary to the claim—there is some returning demand in this equilibrium. But if a consumer returns to firm i after sampling other firms, her match utility must satisfy $u_i \geq \hat{p}(p^*) + r$, since the consumer needs to pay the returning cost r. Since all its returning customers have match utility at least as great as $\hat{p}(p^*) + r$, the firm's optimal price for these customers must be at least $\hat{p}(p^*) + r$. This is because charging returning consumers $\hat{p}(p^*) + r$ will not induce any of them to leave this firm again and buy from others (since going back to any other firm also involves a returning cost r), while charging them a price below that cannot increase demand (since the deviation is not public). We thus obtain a contradiction to the assumption that $\hat{p}(p^*)$

[36] The consumer's reservation utility at the subsequent firms may also change, but (42) still holds for a sufficiently small ε. That is why we can regard the subsequent market as a no-recall market.

was the correctly anticipated buy-later price. Therefore, in any equilibrium there are no returning consumers. The unique equilibrium outcome is then that firms charge first-time visitors a price as described in the exploding-offer equilibrium in Lemma 3, and charge returning consumers a sufficiently high price such that consumers never come back to previously sampled firms.

(ii) Suppose now that firms can commit to an upper bound on the price they will charge returning visitors. Suppose that firm i charges the buy-now price p_i and commits to an upper bound on its buy-later price given by $\hat{p}_i$. Then any consumer who returns to buy from firm i must expect that the firm will actually charge price $\hat{p}_i$. (Suppose to the contrary that a returning consumer anticipates that the firm will actually charge price $\tilde{p} < \hat{p}_i$. Then, following the same logic as in part (i) of this proof, firm i then has an incentive to increase its buy-later price above $\tilde{p}$ since it knows that the consumer is willing to pay at least $\tilde{p} + r$ for the product. Therefore, the only equilibrium belief can be that returning consumers anticipate that firms will set their buy-later price equal to their announced upper bound.) The firm has an incentive to raise the price above $\hat{p}_i$, as in the proof to part (i), but that is not feasible given that the firm commits to its cap. Hence, firm i will charge its returning customers exactly $\hat{p}_i$. We deduce that announcing an upper bound to the buy-later price is equivalent to committing to an actual buy-later price at the level of the cap, and so the analysis of sections 3.1 and 3.2 can be applied.

Proof of Proposition 6: Suppose a firm unilaterally makes an exploding offer with price $p \leq r$. For a costly searcher who visits this firm, she will stop searching immediately as before (since the expected incremental benefit from searching on is now smaller than in the free-recall case). So the firm's demand from costly searchers does not change relative to the free-recall case with price p. In the following, therefore, we focus on the impact of such a deviation on the demand from shoppers.

When a shopper comes to this firm, she might now stop searching because of the exploding offer. But this decision depends on both p and the prices she has already observed in the previously sampled firms. Suppose the firm is in the k_{th} position of the consumer's search process with $k < n$. Let $\delta = \min\{p_1, \ldots, p_{k-1}\}$ be the minimum price observed so far. (If $k = 1$, let $\delta = r$.) If the consumer chooses to search on, then she must give up the offered price p, and the expected price she will then pay is

$$\begin{aligned} P_{n-k}(\delta) &= \delta[1 - G(\delta)]^{n-k} + \int_{p_{\min}}^{\delta} \tilde{p} \frac{d}{d\tilde{p}} \{1 - [1 - G(\tilde{p})]^{n-k}\} d\tilde{p} \\ &= p_{\min} + \int_{p_{\min}}^{\delta} [1 - G(\tilde{p})]^{n-k} d\tilde{p} \ , \end{aligned}$$

where $1 - [1 - G(\tilde{p})]^{n-k}$ is the c.d.f. of the minimum price among the $n-k$ subsequent firms. (Note that $P_{n-k}(\delta)$ is an increasing function and $P_{n-k}(\delta) < \delta$.) Thus, this shopper will buy from the deviating firm if and only if the exploding offer price satisfies $p < P_{n-k}(\delta)$.

Consider the particular exploding offer with price $p_{\min} + \varepsilon$, where $\varepsilon > 0$ is small. It suffices to show that this deviation gives rise to a higher demand from shoppers than in the free-recall offer with the same price. In the free-recall case, the demand from shoppers is

$$[1 - G(p_{\min} + \varepsilon)]^{n-1} \approx 1 - \varepsilon(n-1)g(p_{\min}) \ , \tag{47}$$

regardless of the firm's position in the consumer's search order. However, with an exploding offer, demand from shoppers when the firm is in the k_{th} position becomes

$$\Pr(p_{\min} + \varepsilon < P_{n-k}(\delta)) = \Pr(\delta > \hat{\delta}) \ ,$$

where $\hat{\delta}$ solves

$$p_{\min} + \varepsilon = P_{n-k}(\hat{\delta}) \Leftrightarrow \varepsilon = \int_{p_{\min}}^{\hat{\delta}} [1 - G(\tilde{p})]^{n-k} d\tilde{p} \ .$$

One can show $\hat{\delta} \approx p_{\min} + \varepsilon$, and so

$$\begin{aligned} \Pr(\delta > \hat{\delta}) &\approx [1 - G(p_{\min} + \varepsilon)]^{k-1} \\ &\approx 1 - \varepsilon(k-1)g(p_{\min}) \ . \end{aligned} \tag{48}$$

Comparing (47) and (48) shows that for a given price $p_{\min} + \varepsilon$, the demand from shoppers with exploding offers is indeed higher than in the free-recall case for all cases except $k = n$ (when demand is equal in the two regimes). (One can check that $g(p_{\min}) > 0$ in Stahl's equilibrium.) Thus, the deviating firm makes strictly greater profit when it makes an exploding offer with a price slightly above $p_{\min}$.

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