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Abstract: In an incomplete market with two sided altruistic agents and default. We show equilibrium existence if members of a dynasty act in an individualistic way by maximizing their own intergenerational utility functions. We also illustrate that a dynasty may end doing Ponzi schemes if its members act in a collectivistic way by maximizing a dynasty's collectivistic utility. We also prove that Ponzi schemes are ruled out and equilibrium existence is restored if there exist, always in the future, some agents who are not too altruistic either towards their parents or their offspring.

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1 Introduction

Finite horizon economies have been extended to an infinite horizon via two main approaches. The first one considers a finite number of infinitely-lived agents.[1] The second approach assumes that agents are finitely lived and succeeded by their offspring form-

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[1]This approach was used in General Equilibrium Theory (GET) by Bewley (1972) for complete markets and by Magill and Quinzi (1994, 1996), Florenzano and Gourdel (1996), Hernandez and Santos (1996), Arujo, Páscoa and Torrez-Martínez (2002) and Páscoa and Seghir (2009), among others, for incomplete markets.

ing a sequence of Overlapping Generations Models (OLG).[2] This paper focuses on the latter approach to investigate intergenerational transfers and altruistic motives from a general equilibrium perspective.

In the last two decades, the importance of non economic motives behind economic actions, such as intergenerational transfers and altruistic behavior, on public policy, fiscal policy or social security systems has been extensively emphasized in economics.[3] Generally speaking, intergenerational transfers among family members can be based on two different grounds: Altruism models and Exchange models. On one hand, altruism models assume that family members have interdependent utility functions where an increase in consumption of one member increases the utility of other members as in Bergstrom (1999) and Horri (1989). On the other hand, Exchange models are concerned with economic motives behind intergenerational transfers. This paper uses a combination of these two approaches.

In a recent paper, Seghir and Torres-Martínez (2008) study, from a general equilibrium perspective, a model of wealth transfer in the presence of uncertain lifetimes and default when agents have only forward altruistic motives through bequests. They prove the existence of equilibrium in an OLG economy of finitely lived agents and in an OLG economy when some agents may have at least one infinite life time path through the event tree. Our aim in this paper is to provide a more general notion of kinship altruism, namely two-sided (forward and backward) altruism and study the effect of altruism and dynastic/individualistic behaviors on the possibility of doing Ponzi schemes and the equilibrium existence. To this end, we consider an exchange economy with incomplete financial markets of real assets when default is allowed. As in Geanakoplos and Zame (1995), borrowers are required to constitute collateral in terms of durable goods. This collateral is seized and given to the lenders in case of default. Agents may have backward altruistic motives towards their ancestors through inter-vivos transfers.[4] Moreover, individuals care about their offspring (forward altruism) by not leaving a debt

[2]This approach was used in GET by Balasko and Shell (1980) for complete markets and by Geanakoplos and Polemarchakis (1991), Floenzano, Gourdel and Páscoa (2001), Schmachlenberg (1988), Seghir (2006) and Seghir and Torres-Martínez (2008) among others, for incomplete markets.

[3]See for instance, among recent contributions, the importance of altruism into economic research on public policy (Philippe and Thibault, 2007), fiscal policy (Lambrecht et al. 2006), family economics (Bergstrom 1997) or social security systems (Fuster 2000).

[4]Many authors have emphasized the importance of inter vivos transfers. For example, Gale and Scholz (1994) notice that inter vivos transfers constitute roughly 20 percent (or more) of U.S welath. Similar figures are found in France as statistics show that the total amount of declared inter vivos transfer represents one quarter of total transfers in the 1970's and 1980's (see Arrondel and Laferrére (2001)).

that their backward altruistic descendants may choose to pay. More precisely, agents' backward altruistic motives are represented by a disutility from not paying their parents' debt when the latter cannot honor their promises. Descendants may decide to pay their parents' debt, when the latter are not able to honor their promises, either because they have backward altruistic motives or in order to increase their parents' estate and therefore increase the inheritance they will receive when their parents pass way. A descendant may pay the debt of his ancestor either by getting a loan[5]or by using his available resources (the returns of his long positions or his initial endowments).[6] On the other hand, an ancestor's forward altruistic motives are represented by (i) a disutility proportional to his own amount of default, (ii) a disutiltiy for letting his descendants pay his debt in case he cannot pay it.[7] The presence of durable goods and financial assets allow us to introduce physical and financial inheritances (i.e.: intergenerational transfers) in our model. In our model, the utility function of each agent depends *not* only on his own decision variables but also on the decision variables of other members of his dynasty towards whom he has backward or forward altruism (e.g. how much debt transfer each agent is going to carry from her parents, the level of inter vivos transfer from her offspring, and so on). Besides the classical rational expectations hypothesis that agents perfectly anticipate future prices when making their decisions, we assume that, individuals can perfectly predict the behavior of their offspring based on past experiences and observations about these members of their family.

The objective of this paper is twofold. In the first part, we consider a model with two-sided altruistic (finitely-lived) agents in which each agent maximizes his own intergenerational utility function (Individualstic structure). We prove that equilibrium exists for such a structure under standard assumptions, regardless of agents' degree of altruism. In the second part, we are interested in the case when members of each dynasty act in a collectivistic way, i.e. a sequence of members of a dynasty acts as if they are maximizing the utility of the dynasty (Collectivistic structure). We show

[5]A descendant may be able to get a loan while his ancestor cannot as the former has better financial stability and longer lifetime expectancy.

[6]A descendant may not be able to get a loan if, for instance, he cannot constitute the required collateral. Moreover, a descendant may be able to get a loan but he may still prefer to use the returns of his long positions to pay his ancestor's debt if the interest rate on a new credit is higher than the interest rate on his ancestor's debt.

[7]Utility penalties proportional to the real amount of default have been introduced by Dubey, Geanakoplos and Shubik (2005) for a finite-horizon model and also used by Páscoa and Seghir (2009) for an infinite-horizon model with a finite number of infinitely-lived agents. Dubey, Geanakoplos and Shubik (2005) interpret utility penalties, among other things, as unmodeled reputation loss or third party punishment.

that, similarly to the case of infinitely-lived agents, members of a dynasty may end up doing Ponzi schemes and, therefore, equilibrium may fail to exist, provided that there exists a *benevolent dictator* whose objective is to maximize a collective utility function by solving for the allocation of all members of the dynasty. Moreover, we show that Ponzi schemes are ruled out and equilibrium existence is restored if there are some selfish agents, always in the future. In other words, we prove that an Individualistic structure always lead to equilibrium existence while a Collectivistic structure leads to equilibrium existence only if some members of a dynasty are selfish.

Our Collectivistic structure is consistent with the idea in Bergstrom (1997) that there is a complete class of models (Unitary models) where single members of the family act as if they are maximizing the family's utility function. To capture this structure, some models define agents to be altruistic if their preferences are represented by a weakly separable utility function and each agent prefers a change in consumption of another family member if it makes the latter better off. These papers show that each agent preferences can be represented by the Bergson-Samuelson social utility function, implying that each agent maximizes the collective utility function of the whole family (see for instance Winter (1969) and Bergstrom (1971a,b)). Other models like in Samuelson (1956) and Varian (1984) assume the existence of a benevolent dictator who solves for the allocation of all agents in the family to maximize his collective utility function. We refer to Bergstrom (1997) for more details about Unitary models.

The paper proceeds as follows. The next section presents the model. Section 3 is devoted to equilibrium existence for an individualistic structure. In Section 4, we illustrate the possibility of doing Ponzi schemes in a collectivistic (i.e. dynastic) environment. Then, we show that Ponzi schemes are ruled out and equilibrium existence is restored if there exist, always in the future, some agents who are not *too altruistic* either towards their parents or their offspring. Finally, an appendix is devoted to technical proofs.

2 The Model

2.1 Stochastic structure

We consider a discrete time economy with infinite horizon and uncertainty. The stochastic structure is described by an infinite event tree. We denote by $T = \{0, 1, ...\}$ the countable set of periods. We denote the history of realization of uncertainty by $\overline{s}_t = (s_0, \ldots, s_{t-1})$ where s_0 is the unique state of nature at first period and there exists a finite set $S(\overline{s}_t)$ of states of nature at period t. A node of the economy is an information set $\xi = (t, \overline{s}_t, s)$, with $t \geq 1$ and $s \in S(\overline{s}_t)$ where the initial node at $t = 0$ is represented by ξ_0. The set of nodes in the economy (i.e.: the event–tree) is denoted by D. Given two nodes $\xi = (t, \bar{s}_t, s)$ and $\mu = (t', \bar{s}_{t'}, s')$, we say that μ is a successor of ξ, and we write $\mu \geq \xi$, if both $t' \geq t \geq 1$ and $(\bar{s}_{t'}, s') = (\bar{s}_t, s, \ldots)$. For each node $\xi = (t, \bar{s}_t, s)$, the set of immediate successors of ξ is denoted by $\xi^+ := \{\mu \in D : \mu \geq \xi, t(\mu) = t(\xi) + 1\}$. Moreover, the unique predecessor, ξ^-, of node ξ, satisfies both $\xi^- \leq \xi$ and $t(\xi^-) = t(\xi) - 1$. Finally, we define $D(\xi) = \{\mu \in D : \mu \geq \xi\}$ as the subtree of nodes which succeed ξ and by $D_T(\xi) := \{\mu \in D(\xi) : t(\mu) \leq t(\xi) + T\}$ the subset of nodes of $D(\xi)$ at date T.

2.2 Commodity, financial and demographic structure.

At each node $\xi \in D$, a finite number G of physical goods, indexed by $g = 1, \ldots, G$, are traded on spot markets by alive consumers. Commodity price process is denoted by $p = (p(\xi), \xi \in D) \in \mathbb{R}^{G \times D}$, where $p(\xi) = (p(\xi, g); g \in G) \in R^G_+ \setminus \{0\}$ denotes the spot price of commodities at ξ. Commodities may depreciate from one node to the other. The structure of depreciation is given by a collection of $G \times G$ matrices $Y := \{Y(\xi)\}_{\xi \in D}$ with non negative entries.

At each node $\xi \in D$, there is a set $J(\xi)$ consisting of a finite number, $\iota(\xi)$, of one-period real assets. Each asset $j \in J(\xi)$ is characterized by a vector of real promises $A_j(\mu) \in \mathbb{R}^G_+$, at each $\mu \in \xi^+$. Let $q = (q(\xi); \xi \in D) \in \prod\limits_{\xi \in D} \mathbb{R}^{J(\xi)}_+$ be the financial price process, where $q(\xi) = (q_j(\xi))_{j \in J(\xi)}$ denotes the asset price vector at ξ.

We denote by I the set of agents in the economy. In our model, agents' lifetime is affected by uncertainty. For each agent $i \in I$, we define a finite subset $D^i \subset D$ at which agent i can trade on the spot markets. For each agent $i \in I$, we also define $\underline{D}^i := \{\mu \in D^i \setminus \{\xi_0\} : \mu^- \notin D^i\} \cup (\{\xi_0\} \cap D^i)$ the set of agent i's initial nodes. Agent $i \in I$ is said to be financially active at a node $\xi \in D^i$ if he can exchange assets and he is able to pay, partially or fully, his debt at node ξ.[8] We denote by $D'^i \subset D^i$ the subset of nodes at which agent i is financially active. We assume that agents cannot exit the economy at a node and reappear afterward. Formally, for each $i \in I$, $D(\mu) \cap D^i = \emptyset, \forall \mu \in D \setminus D^i$. Let $I(\xi) := \{i \in I : \xi \in D^i\}$ be the non-empty set of agents who are alive at node $\xi \in D$. We suppose that the number, $n(\xi) =: \#I(\xi)$, of agents who are alive at ξ, is finite.

For each agent $i \in I$, we denote by:

- $w^i = \Big(w^i(\xi, g), (\xi, g) \in D^i \times G\Big) \in \mathbb{R}^{D^i \times G}_+$ agent i's initial endowment process. To simplify notations, agent i's accumulated endowment up to node $\xi \in D^i$ will

[8] An agent i who is not financially active at a node $\xi \in D^i \setminus \underline{D}^i$ can still receive returns on his long positions made at node ξ^-, if any.

be denoted by $W^i(\xi) := w^i(\xi) + Y_\xi W^i(\xi^-)$, with $W^i(\xi^-) = 0$ if $\xi \in \underline{D}^i$.

- A consumption plan $x^i := \big(x^i(\xi); \xi \in D^i\big) \in \mathbb{R}_+^{G\times D^i}$, where $x^i(\xi) \in \mathbb{R}_+^G$ denotes agent i's consumption bundle at node $\xi \in D^i$. Note that, when agent i uses the services of a bundle $x \in \mathbb{R}_+^G$, at $\xi \in D^i$, he receives, at each immediate successor $\mu \in \xi^+ \cap D^i$, a bundle $Y_\mu x$.

- $\theta^i = (\theta^i_j(\xi))_{\substack{\xi\in D'^i \\ j\in J(\xi)}}$ agent i's long positions.

- $\varphi^i = (\varphi^i_j(\xi))_{\substack{\xi\in D'^i \\ j\in J(\xi)}}$ agent i's short positions.

As in Geanakoplos and Zame (1995), at each node $\xi \in D'^i$, borrower i of one unit of asset $j \in J(\xi)$ has to constitute a collateral $C^j(\xi) \in \mathbb{R}_+^G \setminus \{0\}$ in terms of durable goods. This collateral will be seized and given to the lenders in case of default.

For each agent $i \in I$ and for each node $\xi \in D'^i$, agent i's debt at node $\mu \in \xi^+$ (induced by his sales of $\varphi^i_j(\xi)$ units of asset $j \in J(\xi)$ at node ξ) is given by $p(\mu)A_j(\mu)\varphi^i_j(\xi)$ and his effective payment (in units of account), at node $\mu \in (\xi^+ \cap D^i)$, is denoted by $\Delta^i_j(\mu)$. A borrower i of $\varphi^i_j(\xi)$ units of asset $j \in J(\xi)$, at node $\xi \in D'^i$, has a real default $\frac{p(\mu)A_j(\mu)\varphi^i_j(\xi)-\Delta^i_j(\mu)}{p(\mu)b(\mu)}$ at each node $\mu \in (\xi^+ \cap D^i)$, where $b(\mu)=(b(\mu,g), g \in G) \in \mathbb{R}_{++}^G$ is a fixed reference bundle.

2.3 Two–sided altruism

In the current paper, we are interested in the case when agents have altruistic motives towards both their ancestors (backward altruism) and descendants (forward altruism). This generalizes a model by Seghir and Torres-Martínez (2008) where agents have only altruistic motives towards their descendants through bequests. In our model, agents' altruism is represented by utility penalties. More precisely, agents who have forward altruistic motives towards their descendants suffer utility penalties proportional to their real amount of default. Similarly, agents who have backward altruistic motives towards their ancestors suffer utility penalties proportional to their ancestors' real amount of default. Formally, for each agent $i \in I$, we denote by $D^i_F \subset D^i$ the set of nodes at which agent i has *forward* altruistic motives and by $D^i_B \subset D^i$ the set of nodes at which agent i has *backward* altruistic motives. For each $i \in I$ and $\xi \in D^i_B$, we denote by $I^i_B(\xi) \subset I(\xi^-)$ the set of agents for whom agent i has backward altruistic motives at node ξ. Moreover, for each $i \in I$ and $\xi \in D^i_F$, we denote by $I^i_F(\xi) \subset \Big(I(\xi) \cup \bigcup_{\mu\in\xi^+} I(\mu)\Big)$ the set of agents for whom agent i has forward altruistic motives at node ξ.[9]

[9]For ease of notations, we consider one-period altruism, that is agents have backward altruistic motives towards their immediate predecessors and forward altruistic motives towards their immediate

Due to backward altruistic motives towards his ancestor k, agent i may choose to pay, at node $\xi \in D'^i \setminus D'^k$ an amount $\Delta_j^{i,k}(\xi)$, of his ancestor k's debt induced by the sales of asset $j \in J(\xi^-)$ by ancestor k. That is:

$$0 \leq \Delta_j^{i,k}(\xi) \leq p(\xi)A_j(\xi)\varphi_j^k(\xi^-) - \min\{p(\xi)Y(\xi)C^j(\xi^-), p(\xi)A_j(\xi)\}\, \varphi_j^k(\xi^-). \qquad (1)$$

Note that $\Delta_j^{i,k}(\xi) = 0$ for $\xi \in D_B^i \setminus (D'^i \cup D'^k)$. On the other hand, due to forward altruistic motives, agent i's effective payment, $\Delta_j^i(\xi)$, at node $\xi \in D'^i \setminus \underline{D}^i$, on his (own) debt induced by the sales of asset $j \in J(\xi^-)$ may exceed the minimum between the original promise and the value of the depreciated collateral. That is:

$$\min\{p(\xi)Y(\xi)C^j(\xi^-), p(\xi)A_j(\xi)\}\, \varphi_j^i(\xi^-) \leq \Delta_j^i(\xi) \leq p(\xi)A_j(\xi)\varphi_j^i(\xi^-). \qquad (2)$$

Note that $\Delta_j^i(\mu) = \min\{p(\mu)A^j(\mu)\varphi_j^i(\xi), p(\mu)Y(\mu)C^j(\xi)\varphi_j^i(\xi)\}$ for $\mu \in \xi^+ \cap (D_B^i \setminus D'^i)$. Before defining agents' utility functions, we need to introduce the following notations for each agent $i \in I$:

$$\Gamma^i(\xi) := \left\{k \in I : \; k \in I_F^i(\xi) \text{ and } i \in I_B^k(\xi)\right\}, \text{ for each node } \xi \in D^i,$$

$$\varphi^{-i} := (\varphi_j^k(\xi^-))_{\substack{\xi \in D_B^i \\ k \in I_B^i(\xi)}}, \qquad \Delta^{k,i} := (\Delta_j^{k,i}(\mu))_{\substack{\xi \in D_F^i,\ \mu \in \xi^+ \setminus D'^i, \\ k \in \Gamma^i(\xi)}}$$

$$\Delta^{h,k} := (\Delta_j^{h,k}(\xi))_{\substack{\xi \in D_B^i \\ k \in (I_B^i(\xi) \cap I_B^h(\xi)) \setminus \{i\}}}, \qquad \eta^{-i} := \left(\varphi^{-i},\ \Delta^{k,i},\ \Delta^{h,k}\right).$$

Agent i's preferences are represented by an additively time-node separable utility function U^i defined as follows:

$$\begin{aligned}
U^i \quad & \left((p, \eta^{-i}), (x^i, \theta^i, \varphi^i, \Delta^i, \Delta^{i,k})\right) \\
:= \quad & v^i(x^i) - \sum_{\xi \in D_F^i \setminus \underline{D}^i} \sum_{j \in J(\xi^-)} \lambda_j^i(\xi) \frac{\left[p(\xi)A_j(\xi)\varphi_j^i(\xi^-) - \Delta_j^i(\xi)\right]}{p(\xi)b(\xi)} \\
+ \quad & \sum_{\substack{\{\xi \notin D'^i: \\ \xi^- \in D_F^i\}}} \sum_{j \in J(\xi^-)} \frac{p(\xi)A_j(\xi)\varphi_j^i(\xi^-) - \sum_{k \in \Gamma^i(\xi^-)} \gamma_j^{i,k}(\xi)\Delta_j^{k,i}(\xi)}{p(\xi)\, b(\xi)} \\
- \quad & \sum_{\xi \in D_B^i} \sum_{k \in I_B^i(\xi)} \sum_{j \in J(\xi^-)} \delta_j^{i,k}(\xi) \frac{\left[p(\xi)A_j(\xi)\varphi_j^k(\xi^-) - \sum_{h \neq i:\ k \in I_B^h(\xi)} \Delta_j^{h,k}(\xi))\right] - \Delta_j^{i,k}(\xi)}{p(\xi)b(\xi)},
\end{aligned}$$

descendants. Our results remain unaffected if we extend this altruism structure to a more general one in which agents have altruistic motives towards more subsequent generations.

where $v^i : \mathbb{R}_+^{G\times D^i} \longrightarrow \mathbb{R}_+$ for all $i \in I$, $\xi \in D$. The first term on the right hand side of the equation above represents the utility agent i gets from consuming x^i. The second and third terms represents agent i forward altruism while the fourth term represents i's backward altruism. More precisely, the second term is a disutility that agent i suffers for transferring his debt (or his bad default history) to his descendants. The third term is agent i's disutility if his altruistic descendants $k \in \Gamma^i(\xi)$ pays, fully or partially, his debt at a node $\xi \in D'^i$. Finally, the fourth term represents agent i's disutility proportional to his financially inactive ancestors' default. Note hat the higher the payment of agent i or his siblings on their ancestors' debt the higher is agent i's utility.

The coefficients $\lambda_j^i(\xi)$ and $\gamma_j^{i,k}(\xi)$ represent agent i's degree of forward altruism towards descendants $k \in I_F^i(\xi)$ while the coefficients $\delta_j^{i,k}(\xi)$ represent agent i's degree of backward altruism towards ancestors $k \in I_B^i(\xi)$.

Due to markets anonymity and borrowers' (possible) altruism, lenders will have a degree of uncertainty about the returns of their long positions. As in Dubey, Geanakoplos and Shubik (2005), we denote by $\left(K^j(\xi) \in [0,1], \xi \in D, j \in J(\xi^-)\right)$ the expected delivery rates, at node $\xi \in D$, on asset $j \in J(\xi^-)$. This variable is taken as given by the agents and will be determined endogenously in equilibrium.

Since goods may be durable in our model and agents may have access to the financial markets even at the last period of their lifetime, a part of their (financial and/or physical) wealth may be left over when they pass away. In order to regulate the sharing of these physical and financial allocations amongst legitimate beneficiaries, we introduce exogenous intestacy rules that regulate wealth distribution. When an agent passes away, his estate is equal to the depreciated value of his collateral along with the net returns of his portfolios. In Seghir and Torres-Martínez (2008), borrowers deliver the minimum between the original promise and the depreciated collateral, as the unique punishment in case of default is the seizure of the depreciated collateral. In such a case, the net returns of a portfolio are equal to the minimum between the debt and the depreciated collateral times the net positions (i.e.: the long positions less the short positions). In our model, due to altruistic motives, borrowers will deliver at least the depreciated value of the collateral but may choose to deliver more than that. Therefore, the net returns on agents' portfolios when they pass away must take into account that the returns on their long positions may be greater than the minimum between borrowers' original promise and the depreciated value of the collateral. More precisely, when an agent passes away, we define the net returns on his portfolios as the total expected deliveries from his long positions at the previous node less the minimum between his original debt induced by his short positions at the previous node and the depreciated value of his collateral. Formally, for each agent $i \in I$ and each node $\xi \in D^i$, we define the value of agent i's estate at $\mu \in \xi^+ \setminus D^i$ as follows:

$$e^i_\mu \left(p(\mu), K(\mu), \left(x^i, \theta^i, \varphi^i\right)\right) = p(\mu)Y_\mu x^i(\xi) + \sum_{j \in J(\xi)} \left(K_j(\mu)p(\mu)A_j(\mu)\theta^i_j(\xi) - \beta_j(\mu)\,\varphi^i_j(\xi)\right) \tag{3}$$

where $\beta_j(\mu) = \min\{p(\mu)A_j(\mu),\, p(\mu)Y(\mu)C^j(\xi)\}$. The first term on the right hand side of Equation (3) represents the value of depreciated consumption of agent i that served as collateral or not while the second term represents the net returns on his portfolios. As in Seghir and Torres-Martínez (2008), we assume that (i) contracts can be enforced by markets when borrowers pass away and (ii) lenders are paid back before the distribution of the testamentary rights among the legitimate beneficiaries. Formally, for each $i \in I$, for each node $\xi \in D^i$ and for each immediate successor $\mu \in \xi^+ \setminus D^i$, we define $\overline{I}_i(\mu) \subset I(\mu)$ as the set of agent i's legitimate beneficiaries. Moreover, for each $\mu \in \xi^+ \setminus D^i$ and for each $k \in \overline{I}_i(\mu)$, we define $\alpha^i_k(\mu) \in [0, 1]$ as the proportion of agent i's positive estate that agent k will receive at μ, where $\sum_{k \,\in\, \overline{I}_i(\mu)} \alpha^i_k(\mu)e^i_\mu = e^i_\mu$.
Due to market anonymity, agents may not know their legal forced shares guaranteed by civil laws in case of their ancestors' death. Hence, for each $i \in I$ and each $\xi \in D^i$, we denote by $s^i(\xi) \in \mathbb{R}_+$ the amount of anonymous nominal transfer that agent i expects to receive as inheritance from his ancestors at node ξ via civil law jurisdictions. This variable is taken as given by agents and will be determined endogenously in equilibrium. Finally, we define our economy $\mathcal{E}$ as follows:

$$\mathcal{E} := \left(\left(v^i, \omega^i, (\lambda^i(\xi))_{\xi \in D^i_F}, (\gamma^i_k(\xi))_{\substack{\xi \in D^i_F \\ k \in I^i_F(\xi)}}, (\delta^i_k(\xi))_{\substack{\xi \in D^i_B \\ k \in I^i_B(\xi)}}, (\alpha^i_k)_{k \in \overline{I}_i}\right)_{i \in I}, A, \left(C^j(\xi)\right)_{\substack{\xi \in D \\ j \in J(\xi)}}, Y\right).$$

3 Individualistic Structure.

Definition 3.1 *[Budget sets].*
Given prices, expected deliveries and anonymous nominal transfers (p, q, K, s^i), *the budget set* $B^i(p, q, K, s^i)$ *of an agent* $i \in I$, *is the set of* $\left(x^i, \theta^i, \varphi^i, \Delta^i, (\Delta^{i,k}(\xi))_{\substack{\xi \in D^i_B \\ k \in I^i_B(\xi)}}\right)$ *in* $\mathbb{R}^{G \times D^i} \times \prod_{\xi \in D'^i} \mathbb{R}^{i(\xi)} \times \prod_{\xi \in D'^i} \mathbb{R}^{i(\xi)} \times \prod_{\xi \in D'^i} \mathbb{R}^{i(\xi^-)} \times \prod_{\xi \in D^i_B} \mathbb{R}^{i(\xi^-)}$ *such that:*

- *At each initial node* $\xi \in \underline{D}^i$,

$$p(\xi)x^i(\xi)+p(\xi)C(\xi)\varphi^i(\xi)+q(\xi)\left(\theta^i(\xi)-\varphi^i(\xi)\right)+\sum_{j\in J(\xi^-)}\sum_{k\in I_B^i(\xi)}\Delta_j^{i,k}(\xi)\le p(\xi)w^i(\xi)+s^i(\xi);\tag{4}$$

- *At each* $\xi \in D'^i \setminus \underline{D}^i$,

$$\begin{aligned} & p(\xi)x^i(\xi)+p(\xi)C(\xi)\varphi^i(\xi)+q(\xi)\left(\theta^i(\xi)-\varphi^i(\xi)\right)+\sum_{j\in J(\xi^-)}\Delta_j^i(\xi)+\sum_{j\in J(\xi^-)}\sum_{k\in I_B^i(\xi)}\Delta_j^{i,k}(\xi) \\ \le \quad & p(\xi)\left(w^i(\xi)+Y_\xi x^i(\xi^-)\right)+s^i(\xi)+\sum_{j\in J(\xi^-)}K^j(\xi)p(\xi)A_j(\xi)\theta_j^i(\xi^-) \end{aligned}\tag{5}$$

- *At each* $\xi \in D^i \setminus (D'^i \cup \underline{D}^i)$,

$$p(\xi)x^i(\xi)\le p(\xi)\left(w^i(\xi)+Y_\xi x^i(\xi^-)\right)+s^i(\xi)+\sum_{j\in J(\xi^-)}K^j(\xi)p(\xi)A_j(\xi)\theta_j^i(\xi^-)\tag{6}$$

Definition 3.2 *[Individualistic Equilibrium].*
An individualistic equilibrium of $\mathcal{E}$ *is a vector* $\left(\overline{p},\overline{q},\overline{K},(\overline{s}^i)_{i\in I},\left(\overline{x}^i,\overline{\theta}^i,\overline{\varphi}^i,\overline{\Delta}^i,(\overline{\Delta}^{i,k}(\xi))_{\substack{\xi\in D_B^i\\ k\in I_B^i(\xi)}}\right)_{i\in I}\right)$ *such that* $\overline{p}(\xi)>0$, *for all* $\xi\in D$ *and satisfying:*

(i) For each agent $i\in I$, $\left(\overline{x}^i,\overline{\theta}^i,\overline{\varphi}^i,\overline{\Delta}^i,\overline{\Delta}^{i,k}(\xi)\right)_{\substack{\xi\in D_B^i\\ k\in I_B^i(\xi)}}$ *maximizes the objective function* U^i *over* $B^i(\overline{p},\overline{q},\overline{K},\overline{s}^i)$.

(ii) Physical and financial markets clear at each node $\xi\in D$, *that is:*

$$\sum_{i\in I(\xi)}\overline{\theta}^i(\xi)=\sum_{i\in I(\xi)}\overline{\varphi}^i(\xi)\ \text{ and }\ \sum_{i\in I(\xi)}\overline{x}^i(\xi)=\sum_{i\in I(\xi)}\omega^i(\xi)+\sum_{i\in I(\xi)}Y(\xi)\overline{x}^i(\xi^-).$$

(iii) For each agent $i\in I$ *and for each node* $\xi\in D^i$, *expected anonymous transfers are equal to effective transfers agent i receives from his ancestors, that is:*

$$\overline{s}^i(\xi)=\sum_{k\in I:i\in\overline{I}_k(\xi)}\alpha_i^k\,e_\xi^k\left(\overline{p}(\xi),(\overline{x}^k,\overline{\theta}^k,\overline{\varphi}^k)\right).$$

(iv) For each node $\xi\in D\setminus\{\xi_0\}$, *for each asset* $j\in J(\xi^-)$, *total expected deliveries to lenders is equal to the total effective deliveries made by the sellers and/or their descendants, that is:*

$$\overline{K}^j(\xi) \sum_{i \in I(\xi^-)} \overline{p}(\xi) A^j(\xi) \overline{\theta}_j^i(\xi^-) = \sum_{i \in I(\xi)} \overline{\Delta}_j^i(\xi) + \sum_{i \in I(\xi^-) \setminus I(\xi)} \sum_{\{k \in I(\xi):\, i \in I_B^k(\xi)\}} \overline{\Delta}_j^{k,i}(\xi).$$

Remark 3.1 As pointed out by Dubey et al. (2005), Páscoa and Seghir (2009), Steinert and Torres-Martínez (2007), among others, the existence of an (pure spot market) equilibrium in a trivial way can be easily shown when returns from asset purchases are endogenous. Indeed, if for any $\xi \in D$ and $j \in J(\xi)$, asset prices and expected delivery rates $\Big(q(\xi, j), (K^j(\mu))_{\mu \in \xi^+}\Big)$ are zero, then any spot market equilibrium constitutes an equilibrium for the economy. That is, if agents are over–pessimistic (i.e.: borrowers are expected to make zero payments), there will be no financial transaction (see Steinert and Torres-Martínez (2007) for more details). To overcome the absence of financial trade as a consequence of zero delivery rates, the existence of an equilibrium in which expected delivery rates are strictly positive needs to be guaranteed. That is an equilibrium in which either there is financial trade or delivery rates are nonnull needs to be secured. On the other hand, if the credit constraint functions have nonpositive values, then there will be no financial trade in equilibrium. This brings about the following definition.

Definition 3.3 *[Non–trivial individualistic equilibrium]*
A non–trivial individualistic equilibrium $\Big(\overline{p}, \overline{q}, \overline{K}, (\overline{s}^i)_{i \in I}, (\overline{x}^i, \overline{\theta}^i, \overline{\varphi}^i, \overline{\Delta}^i)_{i \in I}, \overline{\Delta}^{i,k}(\xi))_{\substack{\xi \in D_B^i \\ k \in I_B^i(\xi)}}\Big)$ *of* $\mathcal{E}$ *is an individualistic equilibrium such that for any* (ξ, j)*, we have* $\Big(\overline{\theta}_j(\xi), \overline{\varphi}_j(\xi)\Big) \neq 0$ *or* $\overline{K}^j(\xi) > 0$.

Theorem 3.1 *Assume that the following assumptions hold:*

[A.1] For each node $\xi \in D$ *and for each asset* $j \in J(\xi)$, $C^j(\xi) \neq 0$.

[A.2] For each agent $i \in I$ *and for each node* $\xi \in D^i$, $W^i(\xi) \gg 0$.

[A.3] For each agent $i \in I$, v^i *is continuous, monotone and concave.*

Then, a non–trivial individualistic equilibrium exists.

Proof. See Appendix.

Assumptions [A.1]-[A.3] are standard in finite–horizon economies with default and collateral requirement. The non-nullity of the required collateral assumed in [A.1] guarantees that short–sales are bounded node by node. The survival assumption [A.2] ensures that the interior of the budget correspondences are lower semicontinuous. Assumption [A.3] assures that individuals' maximization problems have a solution.

4 Collectivistic Structure.

In the previous section, we proved that an equilibrium exists, in an individualistic setting *regardless of agents' degree of altruism.* In fact, when borrowers are required to constitute collateral in terms of durable goods, the feasibility condition (ii) in Definition 3.2 guarantees that short-sales are endogenously bounded node by node which is sufficient to guarantee equilibrium existence when finitely-lived agents act in an individualistic way. In this section, we show that a dynasty of finitely–lived agents may end up doing Ponzi schemes in it acts collectively in order to maximize a total utility function. However, we show that occasional selfishness rules out Ponzi schemes.

4.1 Equilibrium Definition.

Definition 4.1 *Let $\tilde{\imath} \in I$. We define $N^{\tilde{\imath}}$ as the set of agents who have backward or forward altruistic motives towards agent $\tilde{\imath}$ and agents for whom agent $\tilde{\imath}$ has backward or forward altruistic motives. Formally, for each agent $\tilde{\imath} \in I$, we define:*

$$N^{\tilde{\imath}} := \Big\{ k \in I : \ \exists \xi \in D \ \textit{such that} \ k \in I_B^{\tilde{\imath}}(\xi) \cup I_F^{\tilde{\imath}}(\xi) \ \textit{or} \ \tilde{\imath} \in I_B^k(\xi) \cup I_F^k(\xi) \Big\}.$$

We define the dynasty $d(\tilde{\imath})$ of agent $\tilde{\imath}$ as follows:

$$d(\tilde{\imath}) := \bigcup_{k \in N^{\tilde{\imath}}} N^k.$$

The preferences of each dynasty $d(\tilde{\imath})$ are represented by the following collectivistic utility function:

$$V^{d(\tilde{\imath})}\Big(\Big(U^i((p,\eta^{-i}),(x^i,\theta^i,\varphi^i,\Delta^i,\Delta^{i,k}{}_{\substack{\xi\in D_B^i\\ k\in I_B^i(\xi)}}))\Big)_{i\in d(\tilde{\imath})}\Big) = \sum_{i\in d(\tilde{\imath})} U^i\Big((p,\eta^{-i}),(x^i,\theta^i,\varphi^i,\Delta^i,\Delta^{i,k}{}_{\substack{\xi\in D_B^i\\ k\in I_B^i(\xi)}})_{i\in I}\Big).$$

Definition 4.2 *[Collectivistic Equilibrium].*
A collectivistic equilibrium of $\mathcal{E}$ is a vector $\Big(\overline{p},\overline{q},\overline{K},(\overline{s}^i)_{i\in I},\Big(\overline{x}^i,\overline{\theta}^i,\overline{\varphi}^i,\overline{\Delta}^i,(\overline{\Delta}^{i,k}(\xi))_{\substack{\xi\in D_B^i\\ k\in I_B^i(\xi)}}\Big)_{i\in I}\Big)$ such that $\overline{p}(\xi) > 0$, for all $\xi \in D$ and satisfying items (ii), (iii) and (iv) in Definition 3.2 and the following optimality condition:

(i') For each dynasty $d(\tilde{\imath})$, the allocation $\Big(\overline{x}^i,\overline{\theta}^i,\overline{\varphi}^i,\overline{\Delta}^i,(\overline{\Delta}^{i,k}(\xi))_{\substack{\xi\in D_B^i\\ k\in I_B^i(\xi)}}\Big)_{i\in d(\tilde{\imath})}$ maximizes the collectivistic objective function $V^{d(\tilde{\imath})}$ over $\prod_{i\in d(\tilde{\imath})} B^i(\overline{p},\overline{q},\overline{K},\overline{s}^i)$.

Definition 4.3 *[Non–trivial collectivistic equilibrium]*
A non–trivial collectivistic equilibrium $\left(\overline{p}, \overline{q}, \overline{K}, (\overline{s}^i)_{i\in I}, \left(\overline{x}^i, \overline{\theta}^i, \overline{\varphi}^i, \overline{\Delta}^i, (\overline{\Delta}^{i,k}(\xi))_{\substack{\xi\in D_B^i \\ k\in I_B^i(\xi)}}\right)_{i\in I}\right)$ *of* $\mathcal{E}$ *is a collectivistic equilibrium such that for any* (ξ, j)*, we have* $\left(\overline{\theta}_j(\xi), \overline{\varphi}_j(\xi)\right) \neq 0$ *or* $\overline{K}^j(\xi) > 0$.

4.2 Assumptions.

We make the following assumptions on $\mathcal{E}$:

[A.4] For each dynasty $d(\tilde{\imath})$, one has:

(i) The collectivistic utility function $V^{d(\tilde{\imath})}$ is time–node additively separable, in the sense that, $\sum_{i\in d(\tilde{\imath})} v^i(x^i) = \sum_{i\in d(\tilde{\imath})} \sum_{\xi\in D^i} v^i_\xi(x^i(\xi))$.

(ii) For each plan $x^{d(\tilde{\imath})} := (x^i)_{i\in d(\tilde{\imath})} \in \ell^\infty_+(\mathbb{R}^{G\times D})$, the (possibly) infinite sum $\sum_{i\in d(\tilde{\imath})} \sum_{\xi\in D^i} v^i_\xi(x^i(\xi))$ is finite.[10]

[A.5] The depreciation structure is given by: $[Y(\xi)] = [\mathrm{diag}[a(\xi, g)]]_{g\in G}$ and there exists $\varpi \in (0,1)$ such that for each node $\xi \in D$, $\max_{g\in G}\{a(\xi, g)\} \leq \varpi$.

[A.6] There exists $\overline{W} \in \mathbb{R}^G_{++}$ such that for each node $\xi \in D$, $\sum_{i\in I(\xi)} W^i(\xi) \leq \overline{W}$.

[A.7] The sequence $(n(\xi))_{\xi\in D}$ belongs to $\ell^\infty_+(\mathbb{R}^D)$.

4.3 Opportunity of doing Ponzi schemes in a collectivistic environment.

This section illustrates how Ponzi schemes may reappear in a collectivistic environment. More precisely, we show that a dynasty can always improve upon any budgetary feasible plan by changing the short-sales and the payments of its members. In such a case, the problem of maximizing the collectivistic utility function of a dynasty has no solution. Formally, let $(p, q, K, (s^h)_{h\in I})$ be a system of prices, expected deliveries and expected monetary transfers. Let $\tilde{\imath} \in I$ and consider an allocation $\left(\left(x^h, \theta^h, \varphi^h, \Delta^h, (\Delta^{h,k}(\xi))_{\substack{\xi\in D_B^h \\ k\in I_B^h(\xi)}}\right)_{h\in d(\tilde{\imath})}\right)$ in $\prod_{h\in d(\tilde{\imath})} B^h(p, q, K, s^i)$. Let $\xi \in D$ such that $\xi \in \underline{D}^{\tilde{\imath}}$. Consider the following changes, from node ξ onwards, for each agent $k \in d(\tilde{\imath})$ such that $D'^k \cap D(\xi) \neq \emptyset$:

[10] Given a set S, $\ell^\infty_+(\mathbb{R}^S) = \{x = (x(s);\, s\in S) \in \mathbb{R}^S_+ :\ \max_{s\in S} x(s) < +\infty\}$.

$$\forall\sigma \in D(\xi)\cap {D'}^k,\ \ \tilde{\varphi}_j^k(\sigma)=\varphi_j^k(\sigma)+\varepsilon,\ \varepsilon>0,$$

$$\forall\sigma \in D^+(\xi)\cap {D'}^k,\ \tilde{\Delta}_j^k(\sigma)=\Delta_j^k(\sigma)+p(\sigma)A_j(\sigma)\varepsilon,$$

$$\forall\sigma \in D^+(\xi)\backslash {D'}^k,\ \tilde{\Delta}_j^{h,k}(\sigma)=\Delta_j^{h,k}(\sigma)+\frac{p(\sigma)A_j(\sigma)\,\varepsilon}{\gamma_j^{k,h}(\sigma^-)},\ \text{for some agent } h \text{ such that } k\in I_B^h(\sigma).$$

That is, we change the short positions and effective payments of all members of a dynasty from some ancestor ĩ onwards. More precisely, from node ξ onwards, the short-sales of each agent $k\in d(\tilde{\imath})$ are increased by $\varepsilon>0$ at each node $\sigma\in D(\xi)\cap {D'}^k$. Moreover, the additional debt induced by these additional sales will be fully paid, at each node $\mu\in\sigma^+$, either by agent k if he is financially active (i.e.: if $\sigma\in {D'}^k$) or by a descendant h if k is not financially active (i.e.: if $\sigma\notin {D'}^k$).
It is easy to verify that for each agent $h\in d(\tilde{\imath})$, $\left(x^h,\theta^h,\tilde{\varphi}^h,\tilde{\Delta}^h,(\Delta^{h,k})_{k\in I_B^h}\right)$ satisfies the budget constraints (4) and (5) at nodes $\sigma\in D(\xi)\cap D^h$ if the following conditions are satisfied:

$$\exists j_\xi\in J(\xi):\ p(\xi)C^j(\xi)-q_j(\xi)<0. \tag{7}$$

$$\forall\sigma\in D^+(\xi),\ \exists j_\sigma\in J(\sigma):\ p(\sigma)C^{j_\sigma}(\sigma)-q_{j_\sigma}(\sigma)<p(\sigma)[Y(\sigma)C^{j_{\sigma^-}}(\sigma^-)-A^{j_{\sigma^-}}(\sigma)]. \tag{8}$$

In addition, the plans $\left((\tilde{\varphi}^k)_{k\in I_B^h},\tilde{\varphi}^h,\tilde{\Delta}^h,(\tilde{\Delta}^{h,k})_{k\in I_B^h}\right)$ and $\left((\varphi^k)_{k\in I_B^h},\varphi^h,\Delta^h,(\Delta^{h,k})_{k\in I_B^h}\right)$ lead to the same total default level for the dynasty $d(\tilde{\imath})$. That is, by choosing this new plan, agent ĩ can consume strictly more at node ξ and there will be no change on the utility of the other members of dynasty $d(\tilde{\imath})$. Therefore, this new allocation leads to a higher total utility for dynasty $d(\tilde{\imath})$ and it is budgetary feasible, provided that conditions (7) and (8) are satisfied. Then, dynasty $d(\tilde{\imath})$ can always improve upon any budgetary feasible allocation in order to increase the utility of ancestor ĩ and, therefore, the maximization problem of dynasty $d(\tilde{\imath})$ has no solution.

4.4 Occasional selfishness and equilibrium existence.

As illustrated in the previous paragraph, a dynasty may end up doing Ponzi schemes in a collectivistic setting in the presence of altruistic motives. In this section, we introduce an assumption on the degree of agents' selfishness that rules out Ponzi schemes.

[A.8] For each dynasty $d(\tilde{\imath})\subset I$, for each node $\xi\in D$, at least one of the following conditions is satisfied:

(i) There exists an agent $i \in d(\tilde{\imath})$ and there exists a date $t_{(i)} > t(\xi)$ such that at any node σ : $t(\sigma) = t_{(i)}$ such that $\sigma \in (D^i_F \setminus \underline{D}^i)$, if $\varphi^i_j(\sigma^-) \leq \frac{1}{\|C^j(\sigma^-)\|_1} \frac{\overline{W} \cdot n(\sigma^-)}{1-\varpi}$, then:

$$\sum_{j \in J(\sigma^-)} \lambda^i_j(\sigma) \frac{p(\sigma)A_j(\sigma)\varphi^i_j(\sigma^-) - D_j(\sigma)\varphi^i_j(\sigma^-)}{p(\sigma)\, b(\sigma)} \leq v^i_\sigma(\omega^i(\sigma)), \tag{9}$$

(ii) There exists an agent $i \in d(\tilde{\imath})$ and there exists a date $t_{(i)} > t(\xi)$ such that at any node σ : $t(\sigma) = t_{(i)}$ such that $\sigma \in D^i \setminus (D'^i \cup \underline{D}^i)$, if $\varphi^i_j(\sigma^-) \leq \frac{1}{\|C^j(\sigma^-)\|_1} \frac{\overline{W} \cdot n(\sigma^-)}{1-\varpi}$, then

$$\sum_{j \in J(\sigma^-)} \frac{p(\sigma)A_j(\sigma)\varphi^i_j(\sigma^-)}{p(\sigma)\ b(\sigma)} \Big(\sum_{k \in \Gamma^i(\sigma^-)} \gamma^{i,k}_j(\sigma) - 1 \Big) \leq v^i_\sigma(\omega^i(\sigma)), \tag{10}$$

(iii) There exists an agent $i \in d(\tilde{\imath})$ and there exists a date $t_{(i)} > t(\xi)$ such that at any node σ : $t(\sigma) = t_{(i)}$ such that $\sigma \in D^i_B$, if $\varphi^k_j(\sigma^-) \leq \frac{1}{\|C^j(\sigma^-)\|_1} \frac{\overline{W} \cdot n(\sigma^-)}{1-\varpi}$, for each $k \in I^i_B(\sigma)$, then

$$\sum_{k \in I^i_B(\sigma)} \sum_{j \in J(\sigma^-)} \delta^{i,k}_j(\sigma) \frac{p(\sigma)A_j(\sigma)\varphi^k_j(\sigma^-) - D_j(\sigma)\varphi^k_j(\sigma^-)}{p(\sigma)\, b(\sigma)} \leq v^i_\sigma(\omega^i(\sigma)), \tag{11}$$

where $D_j(\sigma) := \min\{p(\sigma)A^j(\sigma)), p(\sigma)Y(\sigma)C^j(\sigma^-)\}$.

Item (i) assumes that there exists, always in the future, an agent whose utility from consuming his current endowment when he is financially active is higher than the penalty associated with the maximum default. This assumption is satisfied for instance if an agent does not care about a bad reputation he may transfer to his descendants. Item (ii) requires, always in the future, the existence of an agent whose utility from consuming his initial endowment when he is financially inactive is higher than the disutility he suffers if his descendants pay his full debt. Such an assumption is satisfied for instance when an agent is totally selfish towards his descendants. Finally, Item (iii) assumes that there exists, always in the future, an agent whose utility from consuming his current endowment when he is financially active is higher than the disutility he suffers from not paying his ancestors' debt when the latter is financially inactive. This assumption is satisfied for instance when an agent is totally selfish towards his ancestors.

Theorem 4.1 *Under assumptions [A.1]–[A.8], the economy $\mathcal{E}$ has a non-trivial collectivistic equilibrium.*

Proof. See Appendix.

Appendix

Proof of Theorem 3.1.

To prove Theorem 3.1, we first prove that each truncated (finite–horizon) economy has an equilibrium. Then, we prove that the cluster point of the equilibrium sequence is an equilibrium of the original (infinite–horizon) economy.

Let $\mathcal{E}^T$ be the truncated economy associated with the original economy $\mathcal{E}$, which has the same characteristics as $\mathcal{E}$, but where we suppose that agents cannot access spot markets after period T and cannot exchange assets after period $T-1$. Formally, for each $T > 0$, let us define the following sets:

$$D^T = \{\xi \in D : \ t(\xi) \leq T\}, \ I^T = \{i \in I : \ D^i \subset D^T\}, \ \mathcal{K}^T := [0,1]^{(\sum_{\xi \in D^T} \iota(\xi))},$$

$$\Pi^{T-1} := \left\{ (p,q) \in \mathbb{R}_+^{D^T \times G} \times \prod_{\xi \in D^T} \mathbb{R}^{\iota(\xi)} \ \middle| \ \begin{array}{l} \forall \xi : \ t(\xi) < T, \ \|p(\xi)\|_1 + \|q(\xi)\|_1 = 1, \\ \forall \xi : \ t(\xi) = T, \ \|p(\xi)\|_1 = 1. \end{array} \right\},$$

and for each $i \in I^T$,

$$D^{iT} = D^i \cap D^T,$$

$$X^{iT} = \{(x^i(\xi), \ \xi \in D^i) \in X^i \mid \forall \xi : t(\xi) > T, \ x^i(\xi) = 0\},$$

$$Z^{iT} = \left\{ \left(z^i(\xi) := \theta^i(\xi) - \varphi^i(\xi) \right)_{\xi \in D^i} \in Z^i \mid \forall \xi : t(\xi) \geq T, \ \theta^i(\xi) = \varphi^i(\xi) = 0 \right\}.$$

Moreover, given $(p, q, K, (s^i)_{i \in I^T})$, the budget set, $B^{iT}(p, q, K, (s^i)_{i \in I^T})$, of an agent $i \in I^T$ for the truncated economy is defined as the set of $\left(x^i, \theta^i, \varphi^i, \Delta^i, (\Delta^{i,k}(\xi))_{\substack{\xi \in D^i_B \\ k \in I^i_B(\xi)}} \right)$ satisfying the budget constraints (4), (5) and (6) at nodes $\xi \in D^T$. In addition, for each agent $i \in I^T$, the utility function U^{iT} for each truncated economy $\mathcal{E}^T$ is defined as follows:

$$\begin{aligned}
U^{iT} & \Big((p,\eta^{-i\,T}),(x^i,\theta^i,\varphi^i,\Delta^i,\Delta^{i,k})\Big) \\
:= & \; v^i(x^{iT}) - \sum_{\xi\in(D^i_F\setminus\underline{D}^i)\cap D^T}\;\sum_{j\in J(\xi^-)}\lambda^i_j(\xi)\frac{\Big[p(\xi)A_j(\xi)\varphi^i_j(\xi^-)-\Delta^i_j(\xi)\Big]}{p(\xi)b(\xi)} \\
+ & \sum_{\substack{\{\xi\notin D'^i:\\ \xi^-\in D^i_F\cap D^T\}}}\;\sum_{j\in J(\xi^-)}\frac{p(\xi)A_j(\xi)\varphi^i_j(\xi^-)-\sum_{k\in\Gamma^i(\xi^-)}\gamma^{i,k}_j(\xi)\Delta^{k,i}_j(\xi)}{p(\xi)\,b(\xi)} \\
- & \sum_{\xi\in D^i_B\cap D^T}\;\sum_{k\in I^i_B(\xi)}\;\sum_{j\in J(\xi^-)}\delta^{i,k}_j(\xi)\frac{\Big[p(\xi)A_j(\xi)\varphi^k_j(\xi^-)-\sum_{h:\;k\in I^h_B(\xi)}\Delta^{h,k}_j(\xi))\Big]-\Delta^{i,k}_j(\xi)}{p(\xi)b(\xi)},
\end{aligned}$$

Definition 4.4 *[Equilibria of the truncated economies]*
An individualistic equilibrium of the truncated economy $\mathcal{E}^T$ is a vector $\Big(\overline{p}^T,\overline{q}^T,\overline{K}^T,(\overline{s}^{iT})_{i\in I^T},\Big(\overline{x}^{iT},\overline{\theta}^{iT},\overline{\varphi}^{iT},\overline{\Delta}^{iT},(\overline{\Delta}^{iT,k}(\xi))_{\substack{\xi\in D^i_B\\ k\in I^i_B(\xi)}}\Big)_{i\in I^T}\Big)$ satisfying:

(a) For each agent $i\in I^T$, $(\overline{x}^{iT},\overline{\theta}^{iT},\overline{\varphi}^{iT},\overline{\Delta}^{iT},\overline{\Delta}^{iT,k})$ maximizes U^{iT} over $B^{iT}(\overline{p}^T,\,\overline{q}^T,\,\overline{K}^T,\overline{s}^T)$, where $\overline{s}^T := (\overline{s}^{iT})_{i\in I^T}$,

(b) Conditions (ii)–(iv) of Definition 3.2 hold at $(\overline{x}^{iT},\overline{\theta}^{iT},\overline{\varphi}^{iT},\overline{\Delta}^{iT},\overline{\Delta}^{iT,k})_{i\in I^T}$ for $\xi\in D^T$, with $\overline{\varphi}^T(\xi)=0$ when $t(\xi)=T$.

An individualistic equilibrium of $\mathcal{E}^T$ is said to be non–trivial if it satisfies the following condition:

(c) For any (ξ,j), either $\Big(\overline{\theta}_j(\xi),\overline{\varphi}_j(\xi)\Big)$ is different from 0 or $\overline{K}^j(\xi)>0$.

Lemma 4.1 *Under Assumption [A2], for each $\xi\in D^T$, there exists $M^T(\xi)$ such that for each allocation $\Big(x^i,\theta^i,\varphi^i,\Delta^i,(\Delta^{i,k}(\xi))_{\substack{\xi\in D^i_B\\ k\in I^i_B(\xi)}}\Big)_{i\in I^T}$ satisfying the conditions of Definition 4.4, one has: for each agent $i\in I(\xi)$,*

$$\left\|\Big(x^i,\theta^i,\varphi^i,\Delta^i,(\Delta^{i,k}(\xi))_{\substack{\xi\in D^i_B\\ k\in I^i_B(\xi)}}\Big)_{i\in I^T}\right\|_{max} < M^T(\xi).$$

Proof. The upper bound $M^T(\xi)$ can be obtained using similar arguments to Páscoa-Seghir (2009). Indeed, the feasibility conditions guarantee that individual consumption allocations are bounded, node by node, by the aggregate initial endowments. Assumption [A1] ensures then that short–sales are bounded, node by node. Financial market

feasibility implies that long-positions are bounded, node by node, as well. Finally, Conditions (1) and (2) guarantee that payments are bounded, node by node
Note that the feasibility conditions (iii) ensure that for each node $\xi \in D^T$, for each agent $i \in I^T$, anonymous nominal transfers $s^i(\xi)$ are also bounded node by node, by some $\zeta^i(\xi)$. For each $\xi \in D^T$, let us define $\zeta(\xi) = \max_{i \in I^T} \zeta^i(\xi)$. Finally, let us define $S^T(\xi) = \{s^T(\xi) : \; s^T(\xi) \leq \zeta(\xi)\}$ and $S^T = \prod_{\xi \in D^T} S^T(\xi)$.
For each node $\xi \in D^T$, let us define $M^T = \max_{\xi \in D^T} M^T(\xi)$. Now, for each $i \in I$, let us define:

$$B^{iT}(p,q,K,s,M) = \left\{ (x^i, \theta^i, \varphi^i, \Delta^i, \Delta^{i,k}) \in B^{iT}(p,q,K,s) \;\middle|\; \begin{array}{l} x^i(\xi,g) \leq M^T \\ \theta^i_j(\xi) \leq M^T \\ \varphi^i_j(\xi) \leq M^T \\ \Delta^i_j(\xi) \leq M^T \\ \Delta^{i,k}_j(\xi) \leq M^T \end{array} \right\}.$$

Let $\mathcal{E}^T(M)$ be the compactified economy which has the same characteristics as $\mathcal{E}^T$ except for the budget constraints which are now defined by the sets $B^{iT}(p,q,K,s,M)$.

Definition 4.5 *A non–trivial individualistic equilibrium of the compactified economy* $\mathcal{E}^T(M)$ *is a vector* $\left(\overline{p}^T, \overline{q}^T, \overline{K}^T, (\overline{s}^{iT})_{i \in I^T}, \left(\overline{x}^{iT}, \overline{\theta}^{iT}, \overline{\varphi}^{iT}, \overline{\Delta}^{iT}, (\overline{\Delta}^{iT,k}(\xi))_{\substack{\xi \in D^i_B \\ k \in I^i_B(\xi)}}\right)_{i \in I^T}\right)$ *verifying conditions (b) and (c) of Definition 4.4 and such that:*

(a') $\forall i \in I^T$, $(\overline{x}^{iT}, \overline{\theta}^{iT}, \overline{\varphi}^{iT}, \overline{\Delta}^{iT}, \overline{\Delta}^{iT,k})$ maximizes U^{iT} over $B^{iT}(\overline{p}^T, \overline{q}^T, \overline{K}^T, \overline{s}^T, M^T)$.

It is easy to show that each compactified economy $\mathcal{E}^T(M)$ has a non-trivial equilibrium.[11] Equilibrium existence for each truncated economy can then be guaranteed

[11] A straightforward adaptation of the arguments in Páscoa and Seghir (2009) can be used to prove equilibrium existence for each truncated economy. The arguments differs from Páscoa and Seghir (2009) only by introducing fictitious agents in the generalized game who choose $s^i(\xi)$ in order to minimize $\left(s^i(\xi) - \sum_{k \in I^T:\, i \in \overline{I}_k(\xi)} \alpha^k_i \, e^k_\xi\big(p(\xi), (x^k, \theta^k, \varphi^k)\big)\right)^2$. The arguments in Páscoa and Seghir (2009) can also be easily adapted to prove the non-triviality of equilibrium. Indeed, one can easily show that the delivery rates can be set greater or equal to $\frac{\min\{p(\xi)A^j(\xi),\, p(\xi)Y(\xi)C^j(\xi^-)\}}{p(\xi)A^j(\xi)}$ (where p is different from zero at equilibrium as preferences are monotone), which is bounded from below by $\min\{1, \frac{y(\xi)\, c^j(\xi^-)\, \delta_j(\xi)}{\overline{A}^j(\xi)}\}$, with: $y(\xi) = \min\{a(\xi,g) :\; a(\xi,g) > 0\}$, $c^j(\xi) = \min\{C^j_g(\xi) :\; C^j_g(\xi) > 0\}$, $\overline{A}^j(\xi) = \max\{A^j_g(\xi), g \in G\}$ and $\delta_j(\xi) = \sum_{g \in S(\xi,j)} p(\xi,g)$, where $S(\xi,j) = \{g \in G :\; a(\xi,g) > 0 \text{ and } C^j_g(\xi) > 0\}$. We omit the proof of non–triviality of the equilibrium as the similarities with the proof in Páscoa and Seghir (2006) are substantial.

using classical convexity arguments.

Now, let $\left(\overline{p}^T, \overline{q}^T, \overline{K}^T, (\overline{s}^{iT})_{i\in I^T}, \left(\overline{x}^{iT}, \overline{\theta}^{iT}, \overline{\varphi}^{iT}, \overline{\Delta}^{iT}, (\overline{\Delta}^{iT,k}(\xi))_{\substack{\xi\in D_B^i \\ k\in I_B^i(\xi)}}\right)_{i\in I^T}\right)$ be a non–trivial individualistic equilibrium of $\mathcal{E}^T$. In view of Lemma 4.1, one can assume, without loss of generality, that $\left(\overline{p}^T, \overline{q}^T, \overline{K}^T, (\overline{s}^{iT})_{i\in I^T}, \left(\overline{x}^{iT}, \overline{\theta}^{iT}, \overline{\varphi}^{iT}, \overline{\Delta}^{iT}, (\overline{\Delta}^{iT,k}(\xi))_{\substack{\xi\in D_B^i \\ k\in I_B^i(\xi)}}\right)_{i\in I^T}\right)$ converges to $\overline{\eta} := \left(\overline{p}, \overline{q}, \overline{K}, (\overline{s}^i)_{i\in I}, \left(\overline{x}^i, \overline{\theta}^i, \overline{\varphi}^i, \overline{\Delta}^i, (\overline{\Delta}^{i,k}(\xi))_{\substack{\xi\in D_B^i \\ k\in I_B^i(\xi)}}\right)_{i\in I}\right)$. Moreover, $\overline{\eta}$ clearly satisfies conditions (ii)–(iv) of Definition 3.2. Let us show that for each agent $i\in I$, $\overline{\Sigma}^i := \left(\overline{x}^i, \overline{\theta}^i, \overline{\varphi}^i, \overline{\Delta}^i, (\overline{\Delta}^{i,k}(\xi))_{\substack{\xi\in D_B^i \\ k\in I_B^i(\xi)}}\right)$ is optimal in $B^i\left(\overline{p}, \overline{q}, \overline{K}, (\overline{s}^i)_{i\in I^T}\right)$. Assume that there exists an agent $i\in I$, there exists $\Sigma^i := \left(x^i, \theta^i, \varphi^i, \Delta^i, (\Delta^{i,k}(\xi))_{\substack{\xi\in D_B^i \\ k\in I_B^i(\xi)}}\right)$ in $B^i\left(\overline{p}, \overline{q}, \overline{K}, (\overline{s}^i)_{i\in I^T}\right)$ such that $U^i(\Sigma^i) > U^i(\overline{\Sigma}^i)$. Without loss of generality, we can suppose that, the budget constraints are strictly satisfied. Let us choose T large enough to have $U^{iT}(\Sigma^i) > U^{iT}(\overline{\Sigma}^{iT})$ and all the budget constraints are strictly verified with $\left(\overline{p}^T, \overline{q}^T, \overline{K}^T, (\overline{s}^{iT})_{i\in I^T}\right)$ which contradicts the optimality of $\overline{\Sigma}^{iT}$. □

Proof of Theorem 4.1. To prove Theorem 4.1, we also prove that each truncated (finite–horizon) economy has an equilibrium and, then, we prove that the cluster point of the equilibrium sequence is an equilibrium of the original (infinite–horizon) economy. However, the techniques are quite different due to the infinite lifetime of the dynasties as opposed to finitely-lived individuals in Theorem 3.1. We define the truncated economy $\mathcal{E}^T$ and the associated sets as in the proof of Theorem 3.1. We define a dynasty $d^T(\tilde{\imath})$ for each agent $\tilde{\imath}\in I^T$ as follows: $d^T(\tilde{\imath}) = d^T(\tilde{\imath})\cap I^T$. Moreover, for each dynasty $d^T(\tilde{\imath})\subset I^T$, the collectivistic utility function $V^{d^T(\tilde{\imath})}$ for each truncated economy $\mathcal{E}^T$ is defined as

$$V^{d^T(\tilde{\imath})}\left(\left(U^i((p,\eta^{-i}),(x^i,\theta^i,\varphi^i,\Delta^i,\Delta^{i,k}))\right)_{i\in d^T(\tilde{\imath})}\right) = \sum_{i\in d^T(\tilde{\imath})} U^i\left((p,\eta^{-i}),(x^i,\theta^i,\varphi^i,\Delta^i,\Delta^{i,k})\right).$$

Definition 4.6 *[Collectivistic equilibria of the truncated economies]*
A non–trivial collectivistic equilibrium of the truncated economy $\mathcal{E}^T$ is a vector $\left(\overline{p}^T, \overline{q}^T, \overline{K}^T, (\overline{s}^{iT})_{i\in I^T}, \left(\overline{x}^{iT}, \overline{\theta}^{iT}, \overline{\varphi}^{iT}, \overline{\Delta}^{iT}, (\overline{\Delta}^{iT,k}(\xi))_{\substack{\xi\in D_B^i \\ k\in I_B^i(\xi)}}\right)_{i\in I^T}\right)$ satisfying conditions (b) and (c) of Definition 4.4 and such that:

(a") For each dynasty $d^T(\tilde{\imath})$, the allocation $\left(\overline{x}^{iT}, \overline{\theta}^{iT}, \overline{\varphi}^i, \overline{\Delta}^{iT}, (\overline{\Delta}^{iT,k}(\xi))_{\substack{\xi\in D_B^i \\ k\in I_B^i(\xi)}}\right)_{i\in d^T(\tilde{\imath})}$ maximizes the collectivistic objective function $V^{d^T(\tilde{\imath})}$ over $\prod_{i\in d^T(\tilde{\imath})} B^i(\overline{p}^T, \overline{q}^T, \overline{K}^T, \overline{s}^{iT})$.

One can easily reconstruct the generalized game in the proof of Theorem 3.1 taking into account the new objective functions and dynasties' characteristics to prove that each truncated economy has a non–trivial collectivistic equilibrium which will be denoted by $\left(\overline{p}^T, \overline{q}^T, \overline{K}^T, (\overline{s}^{iT})_{i\in I^T}, \left(\overline{x}^{iT}, \overline{\theta}^{iT}, \overline{\varphi}^{iT}, \overline{\Delta}^{iT}, (\overline{\Delta}^{iT,k}(\xi))_{\substack{\xi\in D_B^i \\ k\in I_B^i(\xi)}}\right)_{i\in I^T}\right) := \overline{\eta}^T$.

Lemma 4.2 *Under the assumptions of Theorem 4.1, the sequence* $\left(\overline{\eta}^T(\xi)\right)_{T\geq t(\xi)}$ *is bounded, for each node* $\xi \in D$.

Proof. The feasibility conditions together with assumptions [A1] and [A5]-[A7] guarantee that each decision variable is uniformly bounded along the event–tree. Since the bounds depend on the node but not on the horizon of the truncation, one gets that the sequence of truncated equilibrium variables at each node is bounded.

Given that the event-tree D is a countable set, it follows from Tychonov's theorem that the equilibrium variables, passing to a subsequence if necessary, converge node by node. That is, $\left(\overline{p}^T, \overline{q}^T, \overline{K}^T, (\overline{s}^{iT})_{i\in I^T}, \left(\overline{x}^{iT}, \overline{\theta}^{iT}, \overline{\varphi}^{iT}, \overline{\Delta}^{iT}, (\overline{\Delta}_B^{iT,k})\right)_{i\in I^T}\right)$ converges to $\left(\overline{p}, \overline{q}, \overline{K}, (\overline{s}^i)_{i\in I}, \left(\overline{x}^i, \overline{\theta}^i, \overline{\varphi}^i, \overline{\Delta}^i, (\overline{\Delta}_B^{i,k})\right)_{i\in I}\right)$.

Let us introduce the following notations:

$$\overline{\Sigma}^{iT} := \left(\overline{x}^{iT}, \overline{\theta}^{iT}, \overline{\varphi}^{iT}, \overline{\Delta}^{iT}, (\overline{\Delta}_B^{iT,k})\right),\ \overline{\Omega}^T := \left(\overline{p}^T, \overline{q}^T, \overline{K}^T, (\overline{s}^{iT})_{i\in I^T}\right),$$

$$\overline{\Sigma}^i := \left(\overline{x}^i, \overline{\theta}^i, \overline{\varphi}^i, \overline{\Delta}^i, (\overline{\Delta}_B^{i,k})\right) \text{ and } \overline{\Omega} := \left(\overline{p}, \overline{q}, \overline{K}, (\overline{s}^i)_{i\in I}\right).$$

The cluster point $\left(\overline{\Omega}, (\overline{\Sigma}^i)_{i\in I}\right)$ clearly satisfies conditions (ii), (iii), (iv) of Definition 3.2 . It remains to show that $(\overline{\Sigma}^i)_{i\in d(\tilde{\imath})}$ is optimal for each dynasty $d(\tilde{\imath})$, if condition (i), (ii) or (iii) of Assumption [A.8] is satisfied.

Let us fix a dynasty $d(\tilde{\imath})$ and assume, by contradiction, that there exists $\varepsilon > 0$ and $\hat{\Sigma}^{d(\tilde{\imath})} := (\hat{\Sigma}^i)_{i\in d(\tilde{\imath})} := (\hat{x}^i, \hat{\theta}^i, \hat{\varphi}^i, \hat{\Delta}^i, (\hat{\Delta}^{i,k})_{k\in I_B^i})_{i\in d(\tilde{\imath})}$ in $\prod_{i\in d(\tilde{\imath})} B^i(\overline{p}, \overline{q}, \overline{K}, \overline{s})$ such that:

$$V^{d(\tilde{\imath})}\left((U^i((\overline{p}, \hat{\eta}^{-i}), \hat{\Sigma}^i)))_{i\in d(\tilde{\imath})}\right) - V^{d(\tilde{\imath})}\left((U^i((\overline{p}, \overline{\eta}^{-i}), \overline{\Sigma}^i)))_{i\in d(\tilde{\imath})}\right) > \epsilon.$$

Then, there exists $T_1 \in \mathbb{N}$ such that for each $T > T_1$, one has:

$$V^{d^T(\tilde{\imath})}\left((U^i((\overline{p}, \hat{\eta}^{-i}), \hat{\Sigma}^i)))_{i\in d^T(\tilde{\imath})}\right) - V^{d(\tilde{\imath})}\left((U^i((\overline{p}, \overline{\eta}^{-i}), \overline{\Sigma}^i)))_{i\in d(\tilde{\imath})}\right) > \epsilon.$$

(a) Let us show that Item (i) of Assumption [A.8] guarantees the optimality of the cluster point. Fix $T_2 > T_1$ such that for each node $\sigma :\ t(\sigma) = T_2+1$, item (i) of Assumption

[A.8] is satisfied for some agent $i \in d(\tilde{\imath})$. Note that $\sigma \in D'^i$, for all $\sigma: \quad t(\sigma) = T_2+1$. For each plan $a := (a(\xi),\ \xi \in D)$ and for each (p, q, K, s), let us define the correspondence: $R^{d(\tilde{\imath})}(a) := \Psi^{T_2}(a) \cap \prod_{i \in d(\tilde{\imath})} B^{i\,T_2}(p, q, K, s)$, where

$\Psi^{T_2}(a) := \left\{ (b^i)_{i \in d^{T_2}(\tilde{\imath})} : V^{d^{T_2}(\tilde{\imath})}((U^i((p, b^{-i}), (b^i)))_{i \in d^{T_2}(\tilde{\imath})}) > V^{d(\tilde{\imath})}((U^i((p, a^{-i}), (a^i)))_{i \in d(\tilde{\imath})}) \right\}$.

Using the same arguments in Páscoa and Seghir (2009), one can easily prove that the correspondence $R^{d(\tilde{\imath})}$ is lower semicontinuous with respect to product topologies on $L^\infty(D)$. It follows that there exists T_3 large enough and a sequence $(\hat{\Sigma}^{iT})_{i \in d^T(\tilde{\imath})}$ that converges, node by node, to $\hat{\Sigma}^{d(\tilde{\imath})}$ such that $(\hat{\Sigma}^{iT})_{i \in d^T(\tilde{\imath})}$ in $R^{d(\tilde{\imath})}(\overline{\Sigma}^{iT})$ for all $T \geq T_3$.
Without loss of generality, one can assume that $T_3 > T_2$. Take $T = T_3$ to get that $V^{d^{T_2}(\tilde{\imath})}\left((U^i((\overline{p}^{T_3}, \hat{\eta}^{-i\,T_3}), \hat{\Sigma}^{i\,T_3}))_{i \in d^{T_2}(\tilde{\imath})}\right) > V^{d^{T_3}(\tilde{\imath})}\left((U^i((\overline{p}^{T_3}, \overline{\eta}^{-i\,T_3}), \overline{\Sigma}^{i\,T_3}))_{i \in d^{T_3}(\tilde{\imath})}\right)$ and $\hat{\Sigma}^{d^{T_3}(\tilde{\imath})}$ satisfies the budget constraints till T_2 at $(\overline{p}^{T_3}, \overline{q}^{T_3}, \overline{K}^{T_3}, \overline{s}^{T_3})$. Let us consider the following changes on agent i's plans:

$$x^{iT}(\xi) = \begin{cases} \widehat{x}^{iT}(\xi) & \text{if } \xi \in D^{T_2} \cap D^i \\ \omega^i(\xi) & \text{if } t(\xi) = T_2 + 1 \\ 0 & \text{if } t(\xi) > T_2 + 1 \end{cases}, \qquad \theta^{iT}(\xi) = \begin{cases} \widehat{\theta}^{iT}(\xi) & \text{if } \xi \in D^{T_2} \cap D'^i \\ 0 & \text{if } t(\xi) \geq T_2 + 1 \end{cases}$$

$$\varphi^{iT}(\xi) = \begin{cases} \widehat{\varphi}^{iT}(\xi) & \text{if } \xi \in D^{T_2} \cap D'^i \\ 0 & \text{if } t(\xi) \geq T_2 + 1 \end{cases}, \quad \Delta_j^{iT}(\xi) = \begin{cases} \widehat{\Delta}_j^{iT}(\xi) & \text{if } \in D^{T_2} \cap D'^i, \\ D_j(\xi)\,\widehat{\varphi}_j^{iT}(\xi^-), & \text{if } t(\xi) = T_2 + 1, \\ 0 & \text{if } t(\xi) > T_2 + 1. \end{cases}$$

$$\Delta_j^{iT,k}(\xi) = \begin{cases} \widehat{\Delta}_j^{iT,k}(\xi) & \text{if } \in D^{T_2} \cap D'^i, \\ 0 & \text{if } t(\xi) \geq T_2 + 1. \end{cases}$$

where $D_j(\xi) := \min\{\overline{p}^T(\xi)\,A^j(\xi)\,, \overline{p}^T(\xi)\,Y(\xi)\,C^j(\xi^-))\}$.
The new plan $(x^{iT}, \theta^{iT}, \varphi^{iT}, \Delta^{iT}, \Delta^{iT,k})$ satisfies the budget constraints till period T_3. Moreover, item (i) of Assumption [A.8] guarantees that dynasty $d(\tilde{\imath})$ still prefers this new allocation to $(\overline{x}^{iT}, \overline{\theta}^{iT}, \overline{\varphi}^{iT}, \overline{\Delta}^{iT}, \overline{\Delta}^{iT,k})_{i \in d(\tilde{\imath})}$, contradicting the optimality of the latter.

(b) Let us show that Item (ii) of Assumption [A.8] also ensures the optimality of the cluster point. Fix $T_2 > T_1$ such that for each node $\sigma: \quad t(\sigma) = T_2 + 1$, item (ii) of Assumption [A.8] is satisfied for some agent $i \in d(\tilde{\imath})$. Note that $\sigma \notin D'^i$, for all $\sigma: \quad t(\sigma) = T_2 + 1$. One can use the same techniques above and consider the following changes on agent i's plans:

$$x^{iT}(\xi)=\begin{cases}\widehat{x}^{iT}(\xi) & \text{if } \xi\in D^{T_2}\cap D^i\\ \omega^i(\xi) & \text{if } t(\xi)=T_2+1\\ 0 & \text{if } t(\xi)>T_2+1\end{cases}, \qquad \theta^{iT}(\xi)=\begin{cases}\widehat{\theta}^{iT}(\xi) & \text{if } \xi\in D^{T_2}\cap D'^i\\ 0 & \text{if } t(\xi)\geq T_2+1\end{cases}$$

$$\varphi^{iT}(\xi)=\begin{cases}\widehat{\varphi}^{iT}(\xi) & \text{if } \xi\in D^{T_2}\cap D'^i\\ 0 & \text{if } t(\xi)\geq T_2+1\end{cases}, \quad \Delta_j^{iT}(\xi)=\begin{cases}\widehat{\Delta}_j^{iT}(\xi) & \text{if } \in D^{T_2}\cap D'^i,\\ D_j(\xi)\,\widehat{\varphi}_j^{iT}(\xi^-), & \text{if } t(\xi)=T_2+1,\\ 0 & \text{if } t(\xi)>T_2+1.\end{cases}$$

$$\Delta_j^{iT,k}(\xi)=\begin{cases}\widehat{\Delta}_j^{iT,k}(\xi) & \text{if } \in D^{T_2}\cap D'^i,\\ 0 & \text{if } t(\xi)\geq T_2+1.\end{cases}$$

where $D_j(\xi):=\min\{\overline{p}^T(\xi)\,A^j(\xi)\,,\overline{p}^T(\xi)\,Y(\xi)\,C^j(\xi^-))\}$.

The modified plan $(x^{iT},\theta^{iT},\varphi^{iT},\Delta^{iT},\Delta^{iT,k})$ satisfies the budget constraints of agent i till period T_3. Moreover, item (ii) of Assumption [A.8] guarantees that agent i prefers this new plan *regardless of the payments of his descendants on his own debt*. Keeping the plans of all other members of the dynasty unchanged, the total utility of dynasty $d(\tilde{\imath})$ will increase, a contradiction.

(c) Let us show that the cluster point is optimal if Item (iii) of Assumption [A.8] is satisfied. Fix $T_2>T_1$ such that for each node $\sigma:\ t(\sigma)=T_2+1$, item (iii) of Assumption [A.8] is satisfied for some agent $i\in d(\tilde{\imath})$. Note that $\sigma\in D'^i$, for all $\sigma:\ \ t(\sigma)=T_2+1$. Let $k\in I_B^i(\sigma)$ and consider the following changes on agent i's plans:

$$x^{iT}(\xi)=\begin{cases}\widehat{x}^{iT}(\xi) & \text{if } \xi\in D^{T_2}\cap D^i\\ \omega^i(\xi) & \text{if } t(\xi)=T_2+1\\ 0 & \text{if } t(\xi)>T_2+1\end{cases}, \qquad \theta^{iT}(\xi)=\begin{cases}\widehat{\theta}^{iT}(\xi) & \text{if } \xi\in D^{T_2}\cap D'^i\\ 0 & \text{if } t(\xi)\geq T_2+1\end{cases}$$

$$\varphi^{iT}(\xi)=\begin{cases}\widehat{\varphi}^{iT}(\xi) & \text{if } \xi\in D^{T_2}\cap D'^i\\ 0 & \text{if } t(\xi)\geq T_2+1\end{cases}, \quad \Delta_j^{iT}(\xi)=\begin{cases}\widehat{\Delta}_j^{iT}(\xi) & \text{if } \in D^{T_2+1}\cap D'^i,\\ 0 & \text{if } t(\xi)>T_2+1.\end{cases}$$

$$\Delta_j^{iT,k}(\xi)=\begin{cases}\widehat{\Delta}_j^{iT,k}(\xi) & \text{if } \in D^{T_2}\cap D'^i,\\ 0 & \text{if } t(\xi)\geq T_2+1.\end{cases}$$

The modified plan $(x^{iT},\theta^{iT},\varphi^{iT},\Delta^{iT},\Delta^{iT,k})$ satisfies the budget constraints of agent i till period T_3. Moreover, item (iii) of Assumption [A.8] guarantees that agent i prefers this new plan, although his payments on his ancestor k's debt has decreased at period T_2+1. Therefore, this modified plan together with the original plans for the other members of the dynasty leads to a higher total utility, a contradiction.

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