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by

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Abstract

Standard Data Envelopment Analysis (DEA) approach is used to evaluate the efficiency of DMUs and treats its internal structures as a "black box". The aim of this paper is twofold. The first task is to survey and classify supply chain DEA models which investigate these internal structures. The second aim is to point out the significance of these models for the decision maker of a supply chain. We analyze the simple case of these models which is the two-stage models and a few more general models such as network DEA models. Furthermore, we study some variations of these models such as models with only intermediate measures between first and second stage and models with exogenous inputs in the second stage. We define four categories: *typical, relational, network* and *game theoretic* DEA models. We present each category along with its mathematical formulations, main applications and possible connections with other categories. Finally, we present some concluding remarks and opportunities for future research.

Keywords: Supply chain; Data envelopment analysis; Two-stage structures; Network structures

JEL codes: C60, C67, C71, C72

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1. Introduction

Data envelopment analysis (DEA) is an approach based on linear programming and is used to assess the relative efficiency among a set of decision making units (DMUs). Specifically, DEA measures the efficiency of the i-th DMU under evaluation relative with the other DMUs of the set. DMUs use multiple inputs to produce multiple outputs, which can be measured in different units. The multiple advantages of DEA make it the appropriate tool for the evaluation of supply chain efficiency (Ross and Droge, 2004). DEA aims to generate the maximum outputs or use the minimum inputs. This is another attractive aspect from the managerial point of view because as Verma and Sinha (2002) point out, a successful firm is the one with the ability to produce more while using the least possible resources.

DEA makes no assumption about the procedures taking place inside the DMU. On the contrary, DEA treats a DMU as a "black box" which uses inputs to produce outputs without considering the internal procedures, a usually sufficient assumption (Sexton and Lewis, 2003). However in some cases, like in supply chain systems, DEA models consist of two or more stages and there are intermediate measures which are considered as inputs in one stage and outputs in another stage. In multistage models we can see each stage as a decision center and the overall process is managed by the corporate manager who is the overall decision maker. Internally, the decision center aims to succeed the best possible allocation of the resources according to its preferences and needs while externally aims for a bigger market share (Ross and Droge, 2002).

According to Ross and Droge (2004), the evaluation of the efficiencies of the supply chain and its individual stages is of extreme importance for the decision maker-corporate manager. An accurate assessment of the efficiency allows the manager to better understand the overall process and the subprocesses and make a better judgment about his decisions.

The aim of this paper is to survey the models which consider the internal structures inside a DMU, especially two-stage models and a few more general cases and highlight their importance for the decision maker. We can classify these models into four categories. First, models which apply typical DEA methodology separately to each stage, without considering the interaction between the two stages. Second, models which consider the relation between two or more stages. Third, network DEA in which the second stage uses exogenous inputs apart from the intermediate inputs and may consist of more than two stages. In the last category, two-stage models are analyzed based on game theory approaches.

The structure of the paper is as follows. In section 2 we present the models which apply the standard DEA approach in each stage. In section 3 we demonstrate the relational models and in section 4 we examine network DEA models. In section 5 the game theoretic models are presented. Along these sections there is a continuous discussion about the connections between the different models. The last section concludes the number with a number of interesting remarks and proposes the lines for future research.

2. Evaluation of the two-stage efficiency by applying typical DEA methodology

This type of two-stage model apply standard DEA methodology separately in first and second stage without considering possible conflicts between the two stages. Such conflicts may arise because of the intermediate measures, which this type of model does not treat in a simultaneous manner. Suppose a supply chain where the first stage is a manufacturer and the second stage is a retailer. Now, suppose that the retailer achieves maximum efficiency in contrast with the manufacturer. It is reasonable that the manufacturer would increase his outputs in order to achieve maximum efficiency. However, an increase in the manufacturer's outputs means an increase in the retailers inputs, because the first stage outputs are the second stage inputs, and as a result a decrease in the retailer's efficiency. These conflicts cannot be addressed by these models.

The first who studied these models were Seifrod and Zhu (1999). They apply this approach to evaluate the efficiency of the top commercial banks in USA. The majority of the existing studies in the banking sector use a number of variables which are sufficient in order to evaluate bank's operational performance, but they cannot capture market performance. Seifrod and Zhu (1999) evaluate the market performance by including a number of market variables in their model. They adopt a two-stage model, one stage for the operational performance and one stage for the market performance.

This two-stage procedure is presented in Figure 1. In the first stage, the banks consume inputs and produce profits while in the second stage the banks use profits to create market value. Seiford and Zhu (1999) apply an output oriented constant returns to scale (CRS) DEA model (Charnes et al., 1978) in order to measure the efficiency of the two stages as follows:

$$max\,\theta_0^t + \varepsilon \cdot \left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \right),\ t = 1,2 \tag{1}$$

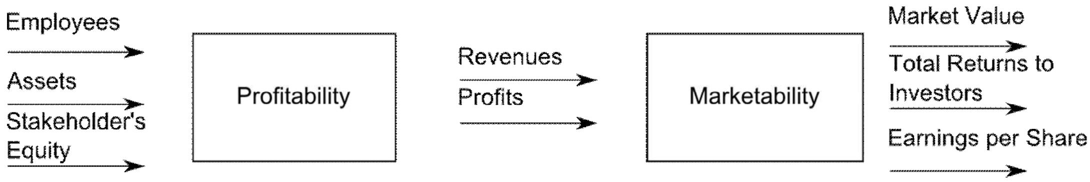
$$s.t.\quad \sum_{j=1}^{n} \lambda_j \cdot x_{ij} + s_i^- = x_{i0}\ ,\ i = 1,2,\ldots,m\,,$$

$$\sum_{j=1}^{n} \lambda_j \cdot y_{rj} - s_r^+ = \theta_0^t \cdot y_{r0}\ ,\ r = 1,2,\ldots,s\ ,$$

$$\lambda_j, s_i^-, s_r^+ \geq 0$$

where θ_0^1 and θ_0^2 are the CRS efficiencies from the first and second stage respectively, x_{ij} is the i-th input of the j-th DMU, y_{rj} is the r-th output of the j-th DMU and s_i^- and s_r^+ are the slack variables. If $\theta_0^1 = 1$ and all slack variables are zero, then the j-th DMU is efficient in the first stage. If $\theta_0^2 = 1$ and all slack variables are zero, then the j-th DMU is efficient in the second stage.

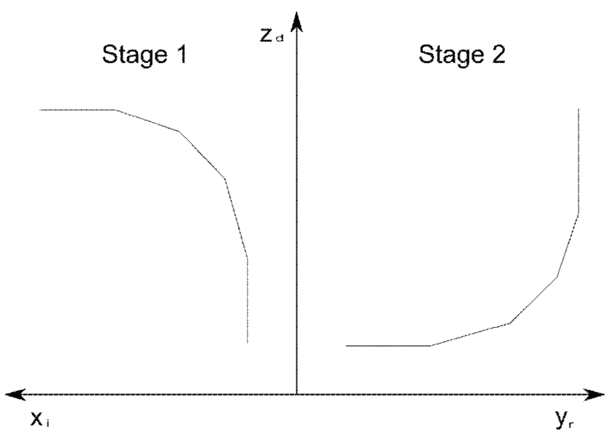
Figure 1: A two-stage procedure (Seiford and Zhu, 1999).



Zhu (2000) applies the same methodology in order to evaluate the efficiency of 500 firms of Fortune Global. Sexton and Lewis (2003) study a similar model to measure the efficiency of the teams of Major League Baseball. In the first stage, teams use money and produce talent as an intermediate measure. In stage two, teams convert the talent into victories which is the final output. Thus, in the first stage the authors assess the ability of the team to utilize the money efficiently to acquire talented players while in the second stage they evaluate the ability to exploit the talent and convert it into victories in the field.

Sexton and Lewis (2003) use an input oriented model for each of the stages. For a specific DMU_0, $x_i \ (i = 1,\ldots,m)$ are the inputs in the first stage, $z_d \ (d = 1,\ldots,D)$ are the intermediate measures and $y_r \ (r = 1,\ldots,s)$ are the final outputs in the second stage. As we can see in figure 2, in the first stage if we increase inputs x_i then intermediate measures z_d would increase as well. However, when we treat z_d as inputs in the second stage then outputs y_r would suffer a decrease.

Figure 2: An output oriented two-stage DEA model (Sexton and Lewis, 2003).



In addition, Sexton and Lewis (2003) introduce reverse variables context in two-stage DEA models. Commonly, a grater amount of an input means more consumption of this input and a greater amount of an output means more production of this output. In reverse variables context exactly the opposite occurs. Thus, a greater amount of a reverse input means less use of this input while a greater amount of a reverse output means less production of this output. The authors use “total bases lost” as a reverse intermediate measure.

Chilingerian and Sherman (2004) apply this type of model at hospitals. In the first stage, the decisions are made by the administration of the hospital while in the second stage the decisions are made by the doctors. Among others, the first stage inputs are the number of staff, the medical suppliers and the expenses, while intermediate measures are the quantity and the quality of the treatment and final outputs are the number of patients who were treated successfully, income form research activities and publications.

Narasimah et al. (2004) introduce a two stage model to study the “flexibility competence” in the first stage which is the ability of the firm to transform resources into manufacturing advances and the “execution competence” in the second stage which is the ability of the firm to transform the “flexibility competence” into a competitive advantage against its competitors.

3. Relational DEA models

As we already noted, the main drawback of the previous models is that they assess the efficiencies of the two stages independently. Thus, they do not treat intermediate measures z_d in a coordinated manner (Cook et al., 2010). For example, if a DMU under assessment decides to increase first stage outputs in order to become efficient, then second stage inputs will be increased as well and as a result second stage will become less efficient. Again if we consider the first stage as the manufacturer and the second stage as the retailer, O’Leary-Kelly and Flores (2002) note that the decision of the one component of this simple supply chain has a direct impact on the other. Consequently, it is important to incorporate this impact in the model.

Furthermore, the previous models may consider a DMU as efficient but the individual stages as inefficient. Chen and Zhu (2004) develop a model which ensures that the overall efficiency of a DMU requires all the individual stages to be efficient. Alternative models that address these drawbacks are the multiplicative model of Kao and Hwang (2008) and the additive model of Chen et al. (2009a), which are applied at general insurance companies in Taiwan.

According to Chen and Zhu (2004) the standard CRS DEA model (1) and the typical VRS DEA model are unable to assess the efficiency of a two stage procedure because of the intermediate measures. The authors propose the following VRS model in order to address this problem:

$$\min_{\alpha,\beta,\lambda_j,\mu_j,\tilde{z}} \xi_1 \cdot \alpha - \xi_2 \cdot \beta \tag{2}$$

$$s.t. \quad \left(1^{st}\, stage\right)$$

$$\sum_{j=1}^{n} \lambda_j \cdot x_{ij} \leq \alpha \cdot x_{i0}\ ,\ i = 1,2,\ldots,m\,,$$

$$\sum_{j=1}^{n} \lambda_j \cdot z_{dj} \geq \tilde{z}_{d0}\ ,\ d = 1,2,\ldots,D\,,$$

$$\sum_{j=1}^{n} \lambda_j = 1,\ \lambda_j \geq 0\,,\ j = 1,2,\ldots,n\,,$$

$$\left(2^{nd}\, stage\right)$$

$$\sum_{j=1}^{n} \mu_j \cdot z_{dj} \leq \tilde{z}_{d0}\ ,\ d = 1,2,\ldots,D\,,$$

$$\sum_{j=1}^{n} \mu_j \cdot y_{rj} \geq \beta \cdot y_{r0}\ ,\ r = 1,2,\ldots,s\,,$$

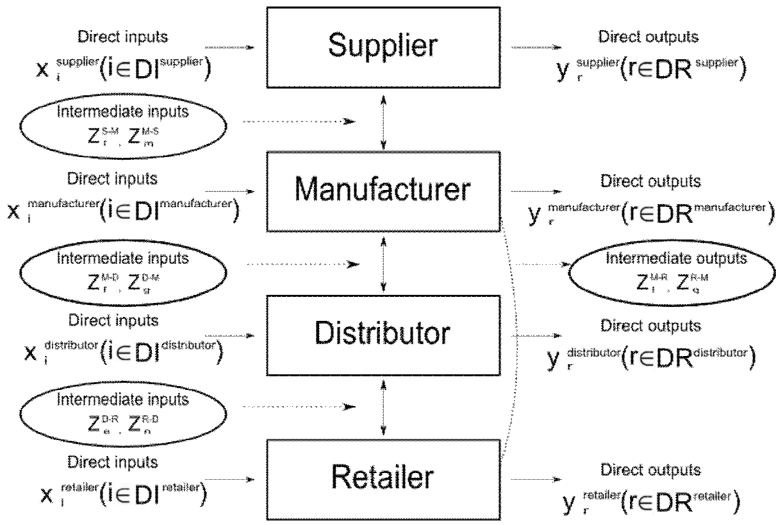
$$\sum_{j=1}^{n} \mu_j = 1 \, , \; \mu_j \geq 0 \, , \; j = 1,2,\ldots,n$$

where ξ_1 and ξ_2 are the weights of the two stages and are defined in an exogenous manner by the decision maker based on the preferences over the two stages and the symbol "~" stands for the unknown decision variables. The authors point out that the inclusion of additional constraints is possible because their model treats intermediate measures as unknown decision variables. Chen and Zhu (2004) apply model (2) at the banking sector and measure the indirect impact of information technology on the efficiency of a firm, based on Wang et al. (1997) data set.

According to Zhu (2003) the general case of model (2) can be used to determine the efficiency of a supply chain. A supply chain is the most appropriate case study for this type of models because every single member of the supply chain applies its own strategy in order to become efficient. From a general point of view, the efficiency of a single member does not ensure the efficiency of another member. In fact, it is reasonable that most of the times the inefficiency of a member is caused by someone else's efficiency. The author presents an example of a supplier and a manufacturer. The supplier increases the price of the raw materials in order to increase his income and become more efficient. From the manufacturer's point of view, the increase in raw materials price means an increase in his overall costs and as a result a decrease in his efficiency, with all others constant.

Figure 3 presents a typical supply chain (Zhu, 2003) where there are four members, namely supplier, manufacturer, distributor and retailer. Moreover, Zhu (2003) marks the significance to assess the efficiency of the supply chain and its individual members. This analysis assists the decision maker to better comprehend the procedures inside the supply chain and to identify the best practices in order to monitor, manage and improve the performance of the supply chain.

Figure 3: A typical supply chain (Zhu, 2003).



Zhu (2003) proposes the following model to evaluate the efficiency of j supply chains, which is the general form of Chen and Zhu's (2004) model (2). ξ_i are the weights of each member of the supply chain and are defined in an exogenous manner by the decision maker based on the preferences over the individual stages.

$$E^* = \min_{E_i,\lambda_j,\beta_j,\delta_j,\gamma_j,\tilde{z}} \frac{\sum_{i=1}^{4} \xi_i \cdot E_i}{\sum_{i=1}^{4} \xi_i} \tag{3}$$

$s.t.\quad (supplier)$

$$\sum_{j=1}^{J} \lambda_j \cdot x_{ij}^{supplier} \le E_1 \cdot x_{i0}^{supplier} \quad i \in DI^{supplier},$$

$$\sum_{j=1}^{J} \lambda_j \cdot y_{rj}^{supplier} \ge y_{r0}^{supplier} \quad r \in DR^{supplier},$$

$$\sum_{j=1}^{J} \lambda_j \cdot z_{tj}^{S-M} \ge z_{t0}^{S-M} \quad t = 1,\ldots,T,$$

$$\sum_{j=1}^{J} \lambda_j \cdot z_{mj}^{M-S} \le \tilde{z}_{m0}^{M-S} \quad m = 1,\ldots,M,$$

$$\lambda_j \ge 0,\ j = 1,\ldots,J$$

$(manufacturer)$

$$\sum_{j=1}^{J} \mu_j \cdot x_{ij}^{manufacturer} \le E_2 \cdot x_{i0}^{manufacturer} \quad i \in DI^{manufacturer} ,$$

$$\sum_{j=1}^{J} \mu_j \cdot y_{rj}^{manufacturer} \ge y_{r0}^{manufacturer} \quad r \in DR^{manufacturer} ,$$

$$\sum_{j=1}^{J} \mu_j \cdot z_{tj}^{S-M} \le z_{t0}^{S-M} \quad t = 1,\ldots,T ,$$

$$\sum_{j=1}^{J} \mu_j \cdot z_{mj}^{M-S} \ge \tilde{z}_{m0}^{M-S} \quad m = 1,\ldots,M ,$$

$$\sum_{j=1}^{J} \mu_j \cdot z_{fj}^{M-D} \ge \tilde{z}_{f0}^{M-D} \quad f = 1,\ldots,F ,$$

$$\sum_{j=1}^{J} \mu_j \cdot z_{gj}^{D-M} \le \tilde{z}_{g0}^{D-M} \quad g = 1,\ldots,G ,$$

$$\sum_{j=1}^{J} \mu_j \cdot z_{lj}^{M-R} \ge \tilde{z}_{l0}^{M-R} \quad l = 1,\ldots,L ,$$

$$\sum_{j=1}^{J} \mu_j \cdot z_{qj}^{R-M} \le \tilde{z}_{q0}^{R-M} \quad q = 1,\ldots,Q ,$$

$$\mu_j \ge 0 ,\ j = 1,\ldots,J$$

$(distributor)$

$$\sum_{j=1}^{J} \delta_j \cdot x_{ij}^{distributor} \le E_3 \cdot x_{i0}^{distributor} \quad i \in DI^{distributor} ,$$

$$\sum_{j=1}^{J} \delta_j \cdot y_{rj}^{distributor} \ge y_{r0}^{distributor} \quad r \in DR^{distributor} ,$$

$$\sum_{j=1}^{J} \delta_j \cdot z_{fj}^{M-D} \le \tilde{z}_{f0}^{M-D} \quad f = 1,\ldots,F ,$$

$$\sum_{j=1}^{J} \delta_j \cdot z_{gj}^{D-M} \le \tilde{z}_{g0}^{D-M} \quad g = 1,\ldots,G ,$$

$$\sum_{j=1}^{J} \delta_j \cdot z_{ej}^{D-R} \geq \tilde{z}_{e0}^{D-R} \; e = 1, \ldots, E,$$

$$\sum_{j=1}^{J} \delta_j \cdot z_{nj}^{R-D} \leq \tilde{z}_{n0}^{R-D} \; n = 1, \ldots, N,$$

$$\delta_j \geq 0, \; j = 1, \ldots, J$$

$$(retailer)$$

$$\sum_{j=1}^{J} \zeta_j \cdot x_{ij}^{retailer} \leq E_4 \cdot x_{i0}^{retailer} \quad i \in DI^{retailer},$$

$$\sum_{j=1}^{J} \zeta_j \cdot y_{rj}^{retailer} \geq y_{r0}^{retailer} \quad r \in DR^{retailer},$$

$$\sum_{j=1}^{J} \zeta_j \cdot z_{lj}^{M-R} \leq \tilde{z}_{l0}^{M-R} \quad l = 1, \ldots, L,$$

$$\sum_{j=1}^{J} \zeta_j \cdot z_{qj}^{R-M} \geq \tilde{z}_{q0}^{R-M} \quad q = 1, \ldots, Q,$$

$$\sum_{j=1}^{J} \zeta_j \cdot z_{ej}^{D-R} \leq \tilde{z}_{e0}^{D-R} \; e = 1, \ldots, E,$$

$$\sum_{j=1}^{J} \zeta_j \cdot z_{nj}^{R-D} \geq \tilde{z}_{n0}^{R-D} \; n = 1, \ldots, N,$$

$$\zeta_j \geq 0, \; j = 1, \ldots, J \;\; \text{x}$$

where DI and DR are the direct inputs and direct outputs respectively, the symbol "~" stands for the unknown decision variables, where the first letter represents its production and the second letter represents its consumption. For example, z^{S-M} represents the intermediate measure which produced by supplier and consumed by manufacturer. Thus, "s" represents the supplier, "m" represents the manufacturer, "d" represents the distributor and "r" represents the retailer. As noted in model (2), the inclusion of additional constraints is possible because the intermediate measures are treated as unknown decision variables. Zhu (2003) points out that if

$E^* = 1$, then there is an optimal solution that ensures $\lambda_0^* = \beta_0^* = \delta_0^* = \gamma_0^* = 1$, where symbol "*" represents an optimal value in model (3). Furthermore, if $E^* = 1$ then the supply chain is rated as efficient and E_i^* is the optimal efficiency for $i = 1,2,3,4$ members of the supply chain.

Next, we present the multiplicative model of Kao and Hwang (2008). Model (1) calculates the optimal solution in the dual CRS DEA problem and apparently it is in linear form. The efficiency E_0 of the primal problem of model (1) in fractional form is calculated below.

$$E_0 = max \frac{\sum_{r=1}^{s} u_r \cdot y_{r0}}{\sum_{i=1}^{m} v_i \cdot x_{i0}} \tag{4}$$

$$s.t. \quad \frac{\sum_{r=1}^{s} u_r \cdot y_{rj}}{\sum_{i=1}^{m} v_i \cdot x_{ij}} \leq 1, \; j = 1,2,\ldots,n,$$

$$u_r, v_i \geq \varepsilon, \; i = 1,2,\ldots,m, \; r = 1,2,\ldots,s$$

The efficiencies E_0^1 and E_0^2 for stages 1 and 2 respectively, are calculated in the same manner.

$$E_0^1 = max \frac{\sum_{r=1}^{s} w_d \cdot z_{d0}}{\sum_{i=1}^{m} v_i \cdot x_{i0}} \tag{5}$$

$$s.t. \quad \frac{\sum_{d=1}^{D} w_d \cdot z_{dj}}{\sum_{i=1}^{m} v_i \cdot x_{ij}} \leq 1, \; j = 1,2,\ldots,n,$$

$$u_r, v_i \geq \varepsilon, \; i = 1,2,\ldots,m, \; d = 1,2,\ldots,D$$

$$E_0^2 = max \frac{\sum_{r=1}^{s} u_r \cdot y_{r0}}{\sum_{d=1}^{D} w_d \cdot z_{d0}} \tag{6}$$

$$s.t. \quad \frac{\sum_{r=1}^{s} u_r \cdot y_{rj}}{\sum_{d=1}^{D} w_d \cdot z_{dj}} \leq 1,\ j = 1,2,\ldots,n\ ,$$

$$u_r, v_i \geq \varepsilon\ ,\ d = 1,2,\ldots,D\ ,\ r = 1,2,\ldots,s$$

According to Kao and Hwang (2008) from models (4), (5) and (6) the overall and individual efficiencies, E_0 , E_0^1 and E_0^2 , for the DMU under assessment are calculated as follows:

$$E_0 = \frac{\sum_{r=1}^{s} u_r^* \cdot y_{r0}}{\sum_{i=1}^{m} v_i^* \cdot x_{i0}} \leq 1,\ E_0^1 = \frac{\sum_{d=1}^{D} w_d^* \cdot z_{d0}}{\sum_{i=1}^{m} v_i^* \cdot x_{i0}} \leq 1 \text{ and } E_0^2 = \frac{\sum_{r=1}^{s} u_r^* \cdot y_{r0}}{\sum_{d=1}^{D} w_d^* \cdot z_{d0}} \leq 1 \tag{7}$$

where u_r^* , v_i^* and w_p^* are the optimal weights. Thus, the overall efficiency is the product of the two individual efficiencies: $E_0 = E_0^1 \times E_0^2$. In order to incorporate the interaction between the two stages, Kao and Hwang (2008) include constraints (7) in model (4). Also, they consider the weights of intermediate measures as the same regardless if intermediate measures are considered as outputs in stage 1 or as inputs in stage 2. This assumption links the two stages and allows the authors to convert the fractional program into a linear one (Chen et al., 2009a). That is

$$E_0 = max \frac{\sum_{r=1}^{s} u_r \cdot y_{r0}}{\sum_{i=1}^{m} v_i \cdot x_{i0}} \tag{8}$$

$$s.t.\quad \frac{\sum_{r=1}^{s} u_r \cdot y_{rj}}{\sum_{i=1}^{m} v_i \cdot x_{ij}} \le 1,\ \frac{\sum_{d=1}^{D} w_d \cdot z_{dj}}{\sum_{i=1}^{m} v_i \cdot x_{ij}} \le 1,\ \frac{\sum_{r=1}^{s} u_r \cdot y_{rj}}{\sum_{d=1}^{D} w_d \cdot z_{dj}} \le 1,\ j = 1,2,\ldots,n,$$

$$u_r, v_i, w_p \ge \varepsilon,\ i = 1,2,\ldots,m,\ r = 1,2,\ldots,s,\ d = 1,2,\ldots,D$$

Kao and Hwang (2008) transform fractional program (8) into the linear program (9) as follows:

$$E_0 = max \sum_{r=1}^{s} u_r \cdot y_{r0} \tag{9}$$

$$s.t.\quad \sum_{i=1}^{m} v_i \cdot x_{i0} = 1,$$

$$\sum_{r=1}^{s} u_r \cdot y_{rj} - \sum_{i=1}^{m} v_i \cdot x_{ij} \le 0, \sum_{p=1}^{q} w_p \cdot z_{pj} - \sum_{i=1}^{m} v_i \cdot x_{ij} \le 0, \sum_{r=1}^{s} u_r \cdot y_{rj} - \sum_{p=1}^{q} w_p \cdot z_{pj} \le 0,$$

$$j = 1,2,\ldots,n,\ u_r, v_i, w_p \ge \varepsilon,\ i = 1,2,\ldots,m,\ r = 1,2,\ldots,s,\ d = 1,2,\ldots,D$$

Optimal weights in model (9) may not be unique and as a result the decomposition of the overall efficiency E_0 into the efficiencies of each stage, E_0^1 and E_0^2 respectively, may not be unique either. Kao and Hwang (2008) propose the maximization of one of the individual efficiencies, say E_0^1, while maintaining the overall efficiency at E_0 as calculated in model (9). The other individual efficiency E_0^2 is calculated as $E_0 = E_0^1 \times E_0^2 \Rightarrow E_0^2 = \frac{E_0}{E_0^1}$. For example, if we wish to maximize the individual efficiency of the second stage E_0^2 while maintaining the overall efficiency at E_0 as calculated in model (9), the model will be the following:

$$E_0^2 = max \sum_{r=1}^{s} u_r \cdot y_{r0} \tag{10}$$

$$s.t.\quad \sum_{d=1}^{D} w_d \cdot z_{d0} = 1,\ \sum_{r=1}^{s} u_r \cdot y_{r0} - E_0 \cdot \sum_{i=1}^{m} v_i \cdot x_{i0} = 0,$$

$$\sum_{r=1}^{s} u_r \cdot y_{rj} - \sum_{i=1}^{m} v_i \cdot x_{ij} \leq 0, \sum_{d=1}^{D} w_d \cdot z_{dj} - \sum_{i=1}^{m} v_i \cdot x_{ij} \leq 0, \sum_{r=1}^{s} u_r \cdot y_{rj} - \sum_{d=1}^{D} w_d \cdot z_{dj} \leq 0,$$

$$j = 1,2,\ldots,n,\ u_r, v_i, w_p \geq \varepsilon,\ i = 1,2,\ldots,m,\ r = 1,2,\ldots,s,\ d = 1,2,\ldots,D$$

and the other individual efficiency E_0^1 will be $E_0^1 = \frac{E_0}{E_0^2}$. As noted by Cook et al. (2010) this decomposition is not available either on typical DEA or network DEA models, which will be discussed later.

Chen et al. (2009b) prove that Chen and Zhu's (2004) model (2) transformed in CRS is equivalent with Kao and Hwang's (2008) model (10). The advantage of model (10) is the assessment of individual efficiencies for the two stages. In contrast, model (2) of Chen and Zhu (2004) fail to do so, because when transformed in CRS, α and β do not represent the efficiencies of each stage. On the other hand, the drawback of Kao and Hwang's (2008) model is that can be used only in CRS. Chen et al. (2009a) overcome this drawback by proposing the additive efficiency decomposition model, which allows the VRS assumption.

According to Chen et al. (2009a) the overall efficiency is evaluated as follows:

$$E_0 = \xi_1 \cdot \frac{\sum_{d=1}^{D} w_d \cdot z_{d0}}{\sum_{i=1}^{m} v_i \cdot x_{i0}} + \xi_2 \cdot \frac{\sum_{r=1}^{s} u_r \cdot y_{r0}}{\sum_{d=1}^{D} w_d \cdot z_{d0}}$$

And the maximization problem will be expressed as:

$$max \left[\xi_1 \cdot \frac{\sum_{d=1}^{D} w_d \cdot z_{d0}}{\sum_{i=1}^{m} v_i \cdot x_{i0}} + \xi_2 \cdot \frac{\sum_{r=1}^{s} u_r \cdot y_{r0}}{\sum_{d=1}^{D} w_d \cdot z_{d0}} \right] \tag{11}$$

$$s.t.\ \frac{\sum_{d=1}^{D} w_d \cdot z_{dj}}{\sum_{i=1}^{m} v_i \cdot x_{ij}} \leq 1,\ \frac{\sum_{r=1}^{s} u_r \cdot y_{rj}}{\sum_{d=1}^{D} w_d \cdot x_{wj}} \leq 1,\ j = 1,2,\ldots,n,\ u_r, v_i, w_d \geq 0$$

Problem (11) cannot be converted in linear form. In order to surpass this problem, Chen et al.

(2009a) try to find the best possible method to specify the exogenous weights ξ_1 and ξ_2, which represent the significance of each stage in the overall process. The authors state that a proper measure for the significance of each stage is their size, which can be proxied by the total inputs of each stage. Thus, the overall size is $\sum_{i=1}^{m} v_i \cdot x_{i0} + \sum_{d=1}^{D} w_d \cdot z_{d0}$ which is the sum of the first stage size $\sum_{i=1}^{m} v_i \cdot x_{i0}$ and the second stage size $\sum_{d=1}^{D} w_d \cdot z_{d0}$. Therefore, the significance of each stage is calculated as:

$$\xi_1 = \frac{\sum_{i=1}^{m} v_i \cdot x_{i0}}{\sum_{i=1}^{m} v_i \cdot x_{i0} + \sum_{d=1}^{D} w_d \cdot z_{d0}} \text{ and } \xi_2 = \frac{\sum_{d=1}^{D} w_d \cdot z_{d0}}{\sum_{i=1}^{m} v_i \cdot x_{i0} + \sum_{d=1}^{D} w_d \cdot z_{d0}} \tag{12}$$

Next, the authors include the exogenous weights (12) in model (11). That is

$$max \frac{\sum_{d=1}^{D} w_d \cdot z_{d0} + \sum_{r=1}^{s} u_r \cdot y_{r0}}{\sum_{i=1}^{m} v_i \cdot x_{i0} + \sum_{d=1}^{D} w_d \cdot z_{d0}} \tag{13}$$

$$s.t. \quad \frac{\sum_{d=1}^{D} w_d \cdot z_{dj}}{\sum_{i=1}^{m} v_i \cdot x_{ij}} \le 1, \quad \frac{\sum_{r=1}^{s} u_r \cdot y_{rj}}{\sum_{d=1}^{D} w_d \cdot z_{dj}} \le 1, \; j = 1,2,\ldots,n, \; u_r, v_i, w_d \ge 0$$

which can now be converted into a linear problem, as

$$max \sum_{d=1}^{D} w_d \cdot z_{d0} + \sum_{r=1}^{s} u_r \cdot y_{r0} \tag{14}$$

$$s.t. \quad \sum_{i=1}^{m} v_i \cdot x_{i0} + \sum_{d=1}^{D} w_d \cdot z_{d0} = 1$$

$$\sum_{d=1}^{D} w_d \cdot z_{dj} - \sum_{i=1}^{m} v_i \cdot x_{ij} \le 0, \quad \sum_{r=1}^{s} u_r \cdot y_{rj} - \sum_{d=1}^{D} w_d \cdot z_{dj} \le 0,$$

$$j = 1,2,\ldots,n, \; u_r, v_i, w_d \ge 0$$

The optimal overall efficiency for the process is evaluated by model (14). The individual efficiencies are calculated by the authors in the same manner as in Kao and Hwang (2008) model (2). As already noted, the advantage of Chen et al. (2009a) model (14) over Kao and Hwang (2008) model (2) is that the first model can be applied under the VRS assumption.

4. Network DEA

Network-DEA is not a specific type of model but a group of models which share some common features. Färe and Grosskopf (1996a), based on Shephard (1970) and Shephard and Färe (1975), developed a series of models in order to deal with special cases that typical DEA fail to manage.

4.1. Structure of Network DEA

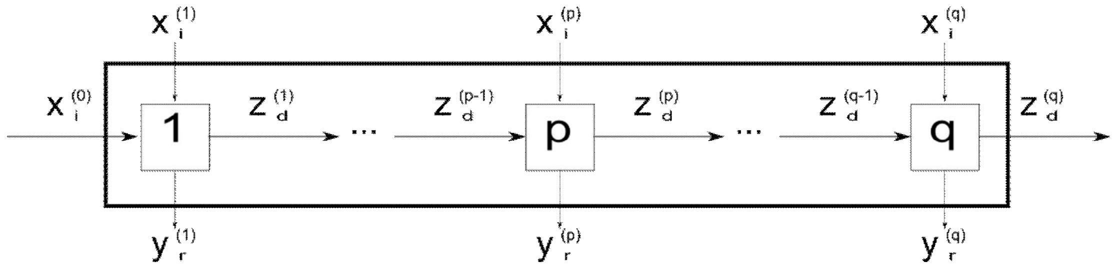
There are two types of structure in a Network DEA model, the serial and the parallel (Kao, 2009).

4.1.1. Serial structure

The two stage models that already have been presented in our paper are in the simple form of a serial network DEA model. Specifically, a serial network DEA model includes DMUs with two or more internal procedures which are linked with intermediate measures. In the simple form, a set of inputs is used by the first stage and a set of intermediate measures is produced, while the second stage uses the intermediate measures that stage 1 produce and generates a set of final outputs. In the simple form there are no exogenous inputs in stage 2 and the entire intermediate measures are used by the second stage. Furthermore, final outputs are produced only by the second stage. A general form of a serial network DEA model is presented in figure 4 (Kao and Hwang, 2008).

The differences between the simple and the general form lie on the number of internal procedures (in the general form there are more than two stages), inputs may enter in any stage, final outputs may be produced in any stage and intermediate measures may not be consumed entirely.

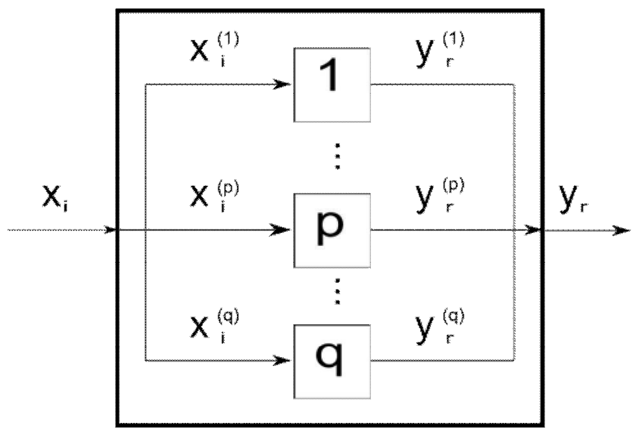
Figure 4: Serial Network DEA (Kao and Hwang, 2010).



4.1.2. Parallel structure

In this type of network DEA models the individual stages operate parallel and separately to each other. An extension of this type of model is the shared flows systems where the inputs are shared among the individual stages (Kao and Hwang, 2010). According to Kao and Hwang (2010) university is a perfect paradigm to describe a parallel system, where the individual stages are the departments which operate parallel and separately inside the university. In addition, the authors point out that a parallel model is a special case of a serial model without intermediate measures. Parallel model is presented in figure 5.

Figure 5: Parallel Network DEA (Kao and Hwang, 2010).



4.2. Types of network DEA models

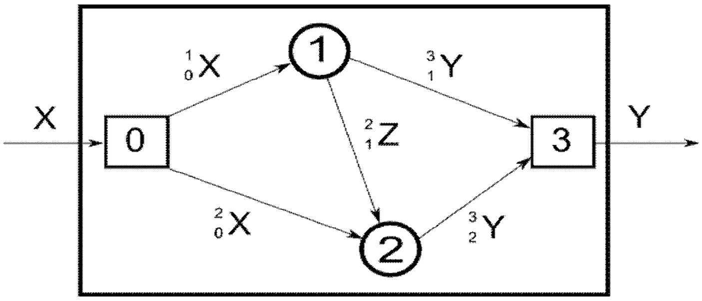
The main types of network DEA models as described by Färe and Grosskopf (2000) and Färe et al. (2007) are static, dynamic and technology adoption models.

4.2.1. Static model

Static models are applied when the individual stages are linked with intermediate measures. Two stage DEA models are special cases of static models. In the general form there may exist multiple stages which are linked with intermediate measures. In addition, exogenous inputs and final outputs may exist in any stage. Färe and Whittaker (1995) apply static model at rural production, Lewis and Sexton (2004) evaluate the efficiency of the American baseball teams and Prieto and Zofio (2007) assess the efficiency of OECD countries. Färe and Whittaker (1995) investigate a two stage model, where “1” stands for stage 1 and “2” stands for stage 2, “0” is the stage where exogenous inputs enter the system and “3” is the stage where final outputs are produced.

The vector of inputs is denoted as ${}_{0}^{ic}X$ where “ic” is the individual stage which consumes the input and 0 represent the stage where the input enters the system. For example, ${}_{0}^{2}X$ is the vector of inputs for stage 2. Also, overall inputs must be equal or greater than the sum of inputs of individual stages, $X \geq {}_{0}^{1}X + {}_{0}^{2}X$. The vector of outputs is denoted as ${}_{ip}^{ic}Y$ where “ip” is the individual stage which produces the output and “ic” is the individual stage which uses the output. For example, ${}_{1}^{2}Y$ is produced in stage 1 and consumed in stage 2. Furthermore, this output is the only intermediate measure in figure 6 and can be denoted as ${}_{1}^{2}Z$. Also, overall outputs must be equal with the sum of outputs of individual stages. s^1 is the number of outputs that comes from stage 1 and s^2 is the number of outputs that comes from stage 2.

Figure 6: Static Network DEA.



The above network model for $j = 1,2,\ldots,n$ DMUs can be written as a linear problem. That is

$$Y = \left({}^{3}_{1}Y,\ {}^{3}_{2}Y\right) \tag{15}$$

$$s.t.\quad \sum_{j=1}^{n} \lambda_j \cdot {}^{3}_{2}y_{rj} \geq {}^{3}_{2}y_r, \quad r = 1,\ldots,s^2\ ,$$

$$\sum_{j=1}^{n} \lambda_j \cdot {}^{2}_{0}x_{ij} \leq {}^{2}_{0}x_i, \quad i = 1,\ldots,m,$$

$$\sum_{j=1}^{n} \lambda_j \cdot {}^{2}_{1}z_{dj} \leq {}^{2}_{1}z_d, \quad d = 1,\ldots,D,$$

$$\lambda_j \geq 0, \quad \sum_{j=1}^{n} \lambda_j = 1,$$

$$\sum_{j=1}^{n} \mu_j \cdot \left({}^{2}_{1}z_{dj} + {}^{3}_{1}y_{rj}\right) \geq \left({}^{2}_{1}z_d + {}^{3}_{1}y_r\right), \quad d = 1,\ldots,D, \quad r = 1,\ldots,s^1\ ,$$

$$\sum_{j=1}^{n} \mu_j \cdot {}^{1}_{0}x_{ij} \leq {}^{1}_{0}x_i, \quad i = 1,\ldots,m\ ,$$

$$\mu_j \geq 0, \quad \sum_{j=1}^{n} \mu_j = 1,$$

$${}^{1}_{0}x_i + {}^{2}_{0}x_i \leq x_i, \quad i = 1,\ldots,m$$

where λ_j and μ_j are the weights of DMUs for stages 2 and 1 respectively. From constraints $\sum_{j=1}^{n} \lambda_j = 1$ and $\sum_{j=1}^{n} \mu_j = 1$, it is clear that the model adopts the VRS assumption. Constraints

$\sum_{j=1}^{n}\lambda_j \cdot {}_0^2x_{ij} \le {}_0^2x_i$ and $\sum_{j=1}^{n}\lambda_j \cdot {}_1^2z_{dj} \le {}_1^2z_d$ are the input constraints for stage 2 and constraint $\sum_{j=1}^{n}\mu_j \cdot {}_0^1x_{ij} \le {}_0^1x_i$ is the input constraint for stage 1. Constraints $\sum_{j=1}^{n}\lambda_j \cdot {}_2^3y_{rj} \ge {}_2^3y_r$ and $\sum_{j=1}^{n}\mu_j \cdot \left({}_1^2z_{dj} + {}_1^3y_{rj}\right) \ge \left({}_1^2z_d + {}_1^3y_r\right)$ are the output constraints where the second constraint includes the intermediate measures. Last, constraint ${}_0^1x_i + {}_0^2x_i \le x_i$ ensures that the sum of inputs of each stage will not exceed the total available inputs.

An interesting case of the above model is the simple case of the two stages as presented in figure 1. According to Färe and Grosskopf (1996b) the simple case of the two stage network DEA is the following:

$$\min_{E,\lambda_j,\mu_j,\tilde{z}} E \tag{16}$$

$$s.t. \quad (stage1)$$

$$\sum_{j=1}^{n}\lambda_j \cdot x_{ij} \le E \cdot x_{i0}, \quad i = 1,\ldots,m,$$

$$\sum_{j=1}^{n}\lambda_j \cdot z_{dj} \ge \tilde{z}_{d0}, \quad d = 1,\ldots,D,$$

$$\lambda_j \ge 0, \quad j = 1,\ldots,n$$

$$(stage2)$$

$$\sum_{j=1}^{n}\mu_j \cdot z_{dj} \le \tilde{z}_{d0}, \quad d = 1,\ldots,D,$$

$$\sum_{j=1}^{n}\mu_j \cdot y_{rj} \ge y_{r0}, \quad r = 1,\ldots,s,$$

$$\mu_j \ge 0, \quad j = 1,\ldots,n$$

where $\tilde{z}_{d0}$ are set as unknown decision variables. Model (16) can be written as follows:

$$\min_{E,\lambda_j,\mu_j,\tilde{z}} E \tag{17}$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_j \cdot x_{ij} \leq E \cdot x_{i0}, \quad i = 1,\ldots,m,$$

$$\sum_{j=1}^{n} \left(\lambda_j - \mu_j\right) \cdot z_{dj} \geq 0, \quad d = 1,\ldots,D,$$

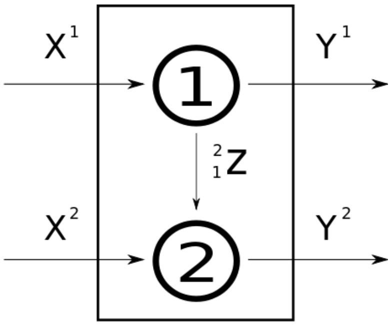
$$\sum_{j=1}^{n} \mu_j \cdot y_{rj} \geq y_{r0}, \quad r = 1,\ldots,s,$$

$$\lambda_j, \mu_j \geq 0, \quad j = 1,\ldots,n$$

According to Cook et al. (2010) model (16) is equivalent to model (2) of Chen and Zhu (2004) and model (17) is equivalent to model (9) of Kao and Hwang (2008) and the cooperative model which will be discussed later. As we have already mentioned, typical DEA models cannot evaluate the efficiency of individual stages in a coordinated manner while Kao and Hwang's (2008) two stage model address this problem. Kao (2009) develop a relational model which assesses the efficiency of more general systems with more than two stages. Hsieh and Lin (2010) apply Kao's (2009) model at tourist hotels in Taiwan while Kao and Hwang (2010) investigate the impact of information technology on a firm's efficiency.

Another special case is a system with two stages which are linked with intermediate measures but final outputs are generated from both stages. Färe et al. (2004) use this model to study property rights. In their model there are two stages and each stage represents a firm. Firm 1 generates two outputs, a good one and a bad one. The good output is a final output while the bad output is an intermediate output which is used as an input by firm 2. Then, firm 2 converts the bad output into a good final output. This model is presented in figure 7.

Figure 7: Static Network DEA with first stage final outputs and bad intermediate outputs (Färe et al., 2004).



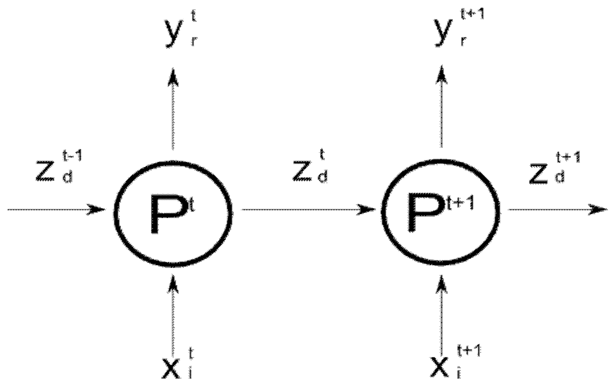
The static network DEA model can be used to evaluate the efficiency of a supply chain by incorporating game theory aspects. This alternative method will be presented later.

4.2.2. Dynamic model

In dynamic model the outputs of the procedure in a specific time period are used as inputs in the next period and can be treated as intermediate measures in time. Dynamic model is widely applied in the literature. Färe and Grosskopf (1997) investigate countries' inefficiency which occurs from misallocation of resources in time. Nemota and Gota (1999) study the dynamic inefficiency based on Hamilton-Jacobi-Bellman equation. Jaenicke (2000) apply a dynamic model in rural production while Nemota and Gota (2003) use it in the case of electricity production. Chen (2009) proposes a unified framework for efficiency assessment in a dynamic production network system.

In figure 8 (Färe and Grosskopf, 2000) a DMU is presented with two stages, P^t and P^{t+1} which take place in time t and t+1 respectively. Stage P^t produces $y_r^t \left(r=1,\ldots,s^t\right)$ as a final output and $z_d^t \left(d=1,\ldots,D^t\right)$ as an intermediate output in time. Inputs $x_i^t \left(i=1,\ldots,m^t\right)$ and $x_i^{t+1} \left(i=1,\ldots,m^{t+1}\right)$ are exogenously entering the system. The terms $z_d^{t-1} \left(d=1,\ldots,D^{t-1}\right)$ and $z_d^{t+1} \left(d=1,\ldots,D^{t+1}\right)$ are used to generalize the system with more stages. If we are interested only in periods t and t+1, we exclude these terms. It is obvious that dynamic and static models are both consisted by multiple stages linked with intermediate measures, but in a dynamic model the individual stages function in a different time period.

Figure 8: Dynamic Network DEA (Färe and Grosskopf, 2000).

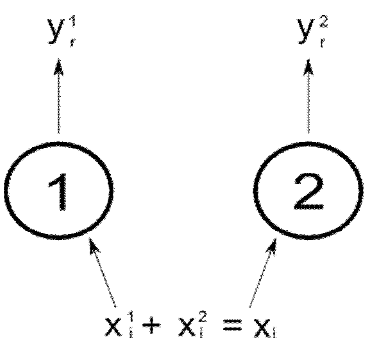


4.2.3. Shared flow or technology adoption model

This model is used in order to allocate the resources properly among the different stages of production technologies. Färe et al. (1997) apply this model to study the allocation of rural land. Lothgren and Tambour (1999) investigate the allocation of labor time among production and customer service while Färe et al. (2007) examine the use of technology adoption model to allocate pollution permits.

The simple case of technology adoption model is presented in figure 9 (Färe et al., 2007). Inputs x_i are allocated among two production technologies. x_i^1 are the inputs of the first production technology and x_i^2 are the inputs of the second production technology. The sum of individual inputs must not exceed the overall inputs x_i , $x_i \geq x_i^1 + x_i^2$. Then, the two production technologies produce the final outputs y_r^1 and y_r^2 respectively.

Figure 9: Shared flow model (Färe et al., 2007)



As we have previously presented, Chen and Zhu (2004) study the impact of information technology on the efficiency of firms. Chen et al. (2006) argue that the disadvantage of Chen and Zhu (2004) model is that information technology in their model has an impact only in the first stage, ignoring the possible impact in the second stage. Chen et al. (2006) address this problem by proposing a technology adoption model where the impact of information technology is decomposed and allocated among all stages.

5. Game theory models

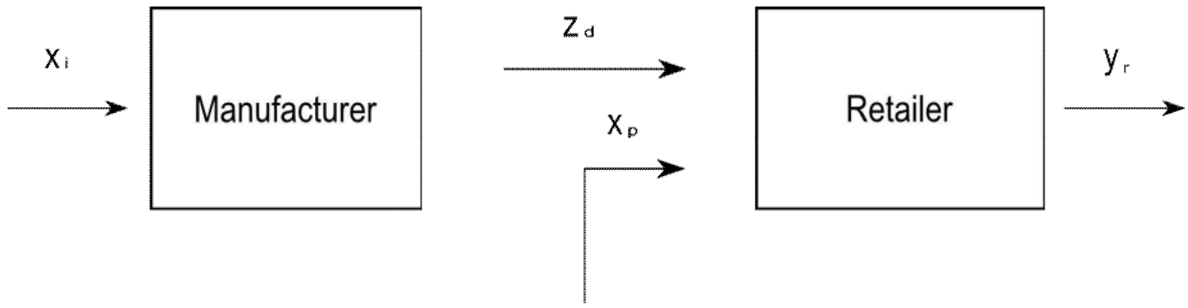
We have presented in previous section the models of Zhu (2003) and Chen and Zhu (2004) which evaluate the efficiency of a supply chain, considering the overall and individual efficiencies of each stage simultaneously. A typical supply chain is presented in figure 3 (Zhu, 2003) which is consisted by a supplier, a manufacturer, a distributor and a retailer.

Liang et al. (2006) investigate the supply chain as a seller-buyer game under non-cooperative and cooperative assumptions. A common type of non-cooperative game is the leader-follower model, also known as Stackelberg model. A simpler supply chain consisting by only two members, a manufacturer and a retailer, is presented in figure 10. The manufacturer is considered the leader and the retailer is the follower. In this type of model, the efficiency of the leader (manufacturer) is evaluated first by applying a typical DEA model and then the efficiency of the follower (retailer) is calculated subject to the leader's efficiency. The game considers the maximization of leader's efficiency as more significant for the overall supply chain compared to the follower's efficiency (Liang et al., 2008).

Under the cooperative assumption, both stages are considered as equally important for the overall supply chain. Both parties cooperate with each other and wish to jointly maximize the overall and their individual efficiencies. The key point of the cooperation is found at the intermediate measures. The individual efficiencies are evaluated simultaneously and the overall efficiency is equal with the mean efficiency of the individual stages.

The simple form of the supply chain is presented in figure 10 (Zhu, 2009), where stage 1 is the manufacturer and stage 2 is the retailer. The model consists of $j = 1,\ldots,n$ DMUs. The manufacturer consumes x_i $(i = 1,\ldots,m)$ inputs and generates z_d $(d = 1,\ldots,D)$ intermediate outputs. The retailer uses z_d $(d = 1,\ldots,D)$ intermediate inputs from the manufacturer and x_p $(p = 1,\ldots,P)$ exogenous inputs and produces y_r $(r = 1,\ldots,s)$ final outputs.

Figure 10: A simple supply chain with exogenous inputs in second stage (Zhu, 2009).



5.1. Non-cooperative game

Let's assume a seller-buyer game, where the manufacturer is the seller and the retailer is the buyer. Also, the manufacturer is considered the leader while the retailer is the follower. Then, according to Liang et al. (2006) we evaluate the leader's efficiency by applying a typical DEA model formulated as

$$max\ E_1 = \frac{\sum_{d=1}^{D} w_d \cdot z_{d0}}{\sum_{i=1}^{m} v_i \cdot x_{i0}} \qquad (18)$$

$$s.t.\ \frac{\sum_{d=1}^{D} w_d \cdot z_{dj}}{\sum_{i=1}^{m} v_i \cdot x_{ij}} \leq 1\ ,\ j = 1,2,\ldots,n\ ,$$

$$w_d, v_i \geq 0, \quad d = 1,\ldots,D, \quad i = 1,\ldots,m$$

which can be easily transformed into a typical CRS DEA model as

$$max\ E_1 = \sum_{d=1}^{D} \mu_d \cdot z_{d0} \tag{19}$$

$$s.t. \quad \sum_{i=1}^{m} \omega_i \cdot x_{ij} - \sum_{d=1}^{D} \mu_d \cdot z_{dj} \geq 0\,, \; j = 1,2,\ldots,n\,,$$

$$\sum_{i=1}^{m} \omega_i \cdot x_{i0} = 1\,,$$

$$\mu_d, \omega_i \geq 0, \quad d = 1,\ldots,D, \quad i = 1,\ldots,m$$

Model (19) assesses the manufacturer's maximized efficiency E_1^* and the optimal weights μ_d^* and ω_i^*. Subject to these optimal values Liang et al. (2006) evaluate the follower's efficiency as

$$max\ E_2 = \frac{\sum_{r=1}^{s} u_r \cdot y_{r0}}{Q \times \sum_{d=1}^{D} \mu_d \cdot z_{d0} + \sum_{p=1}^{P} v_p \cdot x_{p0}} \tag{20}$$

$$s.t. \quad \frac{\sum_{r=1}^{s} u_r \cdot y_{rj}}{Q \times \sum_{d=1}^{D} \mu_d \cdot z_{dj} + \sum_{p=1}^{P} v_p \cdot x_{pj}} \leq 1\,, \; j = 1,2,\ldots,n\,,$$

$$\sum_{d=1}^{D} \mu_d \cdot z_{d0} = E_1^*\,,$$

$$\sum_{i=1}^{m} \omega_i \cdot x_{ij} - \sum_{d=1}^{D} \mu_d \cdot z_{dj} \geq 0\,, \; j = 1,2,\ldots,n\,,$$

$$\sum_{i=1}^{m} \omega_i \cdot x_{i0} = 1\,,$$

$$\mu_d, \omega_i, u_r, v_p, Q \geq 0\,, \quad d = 1,\ldots,D, \quad i = 1,\ldots,m, \quad r = 1,\ldots,s, \quad p = 1,\ldots,P$$

which can be transformed into the following non-linear problem:

$$max\ E_2 = \sum_{r=1}^{s} \gamma_r \cdot y_{r0} \tag{21}$$

$$s.t. \quad q \times \sum_{d=1}^{D} \mu_d \cdot z_{dj} + \sum_{p=1}^{P} \omega_p \cdot x_{pj} - \sum_{r=1}^{s} \gamma_r \cdot y_{rj} \geq 0, \; j = 1,2,\ldots,n,$$

$$q \times \sum_{d=1}^{D} \mu_d \cdot z_{d0} + \sum_{p=1}^{P} \omega_p \cdot x_{p0} = 1,$$

$$\sum_{d=1}^{D} \mu_d \cdot z_{d0} = E_1^*,$$

$$\sum_{i=1}^{m} \omega_i \cdot x_{ij} - \sum_{r=1}^{s} \mu_d \cdot z_{dj} \geq 0, \; j = 1,2,\ldots,n,$$

$$\sum_{i=1}^{m} \omega_i \cdot x_{i0} = 1,$$

$$\mu_d, \omega_i, \gamma_r, \omega_p, q \geq 0, \quad d = 1,\ldots,D, \quad i = 1,\ldots,m, \quad r = 1,\ldots,s, \quad p = 1,\ldots,P$$

where the first two constraints refer to the retailer while next three constraints refer to the manufacturer and ensure his optimal efficiency.

Model (21) is non-linear because of the "q" term. As we can see from the constraints of model (21):

$$q \times \sum_{d=1}^{D} \mu_d \cdot z_{d0} + \sum_{p=1}^{P} \omega_p \cdot x_{p0} = 1 \text{ and } \sum_{d=1}^{D} \mu_d \cdot z_{d0} = E_1^*.$$

Thus: $$q = \frac{\left(1 - \sum_{p=1}^{P} \omega_p \cdot x_{p0}\right)}{\sum_{d=1}^{D} \mu_d \cdot z_{d0}} \Rightarrow q = \frac{\left(1 - \sum_{p=1}^{P} \omega_p \cdot x_{p0}\right)}{E_1^*} \tag{22}$$

The constraint $q \times \sum_{d=1}^{D} \mu_d \cdot z_{d0} + \sum_{p=1}^{P} \omega_p \cdot x_{p0} = 1$ shows that $\sum_{p=1}^{P} \omega_p \cdot x_{p0}$ can take values from 0 to 1 because both terms, ω_p and x_{p0}, are non-negative quantities. If $\sum_{p=1}^{P} \omega_p \cdot x_{p0}$ takes the value 0, the numerator in (22) will become 1 and the overall fraction will become 0, otherwise if $\sum_{p=1}^{P} \omega_p \cdot x_{p0}$ takes the value 1, the numerator in (22) will become 0 and the overall fraction will become $\frac{1}{E_1^*}$. Therefore, we can determine an upper and a lower bound for q term, $0 \le q < \frac{1}{E_1^*}$. Thus, we can treat q as a parameter and model (21) can be solved as a parametric linear program.

According to Liang et al. (2006) in order to solve the problem, we set an initial value to q term, $q_0 = \frac{1}{E_1^*}$ and solve the resulting linear problem. Then, we decrease the q term each time by a small number ε until we reach the lower bound and we name the resulting values of q as q_t. We solve each resulting linear problem for every q_t and we name the solutions as $E_2^*(q_t)$. The optimal solution is $E_2^* = max\, E_2^*(q_t)$ which is the retailer's efficiency and the optimal q associated with this solution is q^*.

With the individual efficiencies evaluated, we can calculate the overall efficiency of the supply chain as $E = \frac{1}{2}\left(E_1^* + E_2^*\right)$ (Liang et al., 2006). Similarly, model (21) can assess the efficiency of the overall supply chain by considering the retailer as the leader, in the same manner.

5.2. Cooperative game

In the cooperative model we try to find the optimal weights for intermediate measures that maximize the leader's efficiency. In the cooperative model the seller and the buyer have the same bargaining power and cooperate to jointly maximize their efficiency. Therefore, they now treat the intermediate measures in a coordinated manner by setting their optimal weights as equal treating them either as outputs at the seller's stage or as inputs at the buyer's stage.

The cooperative game of Liang et al. (2006) is the following:

$$max\ E=\frac{1}{2}\left[\frac{\sum_{d=1}^{D} w_d \cdot z_{d0}}{\sum_{i=1}^{m} v_i \cdot x_{i0}}+\frac{\sum_{r=1}^{s} u_r \cdot y_{r0}}{\sum_{d=1}^{D} w_d \cdot z_{d0}+\sum_{p=1}^{P} v_p \cdot x_{p0}}\right] \tag{23}$$

$$s.t.\quad \frac{\sum_{d=1}^{D} w_d \cdot z_{dj}}{\sum_{i=1}^{m} v_i \cdot x_{ij}} \le 1,\ j=1,2,\dots,n\ ,$$

$$\frac{\sum_{r=1}^{s} u_r \cdot y_{rj}}{\sum_{d=1}^{D} w_d \cdot z_{dj}+\sum_{p=1}^{P} v_p \cdot x_{pj}} \le 1,\ j=1,2,\dots,n\ ,$$

$$w_d, v_i, u_r, v_p \ge 0,\quad d=1,\dots,D,\quad i=1,\dots,m,\quad r=1,\dots,s,\quad p=1,\dots,P$$

Next, the authors apply the Charnes-Cooper transformation in order to convert model (23) into a linear problem. That is

$$t_1=\frac{1}{\sum_{i=1}^{m} v_i \cdot x_{i0}},\ t_2=\frac{1}{\sum_{d=1}^{D} w_d \cdot z_{d0}+\sum_{p=1}^{P} v_p \cdot x_{p0}}, \tag{24}$$

$$\omega_i=t_1\cdot v_i\ ,\ \omega_p=t_2\cdot v_p\ ,\ \mu_d^1=t_1\cdot w_d\ ,\ \mu_d^2=t_2\cdot w_d\ ,\ \gamma_r=t_2\cdot u_r\ ,$$

$$d=1,\dots,D,\quad i=1,\dots,m,\quad r=1,\dots,s,\quad p=1,\dots,P$$

Obviously, there is a linear relation between μ_d^1 and μ_d^2, $\mu_d^2=k\times\mu_d^1$ where $k=\frac{t_2}{t_1}$ is a positive number. Therefore, we can convert model (23) into the following model:

$$max\ E=\frac{1}{2}\left[\sum_{d=1}^{D}\mu_d^1\cdot z_{d0}+\sum_{r=1}^{s}\gamma_r\cdot y_{r0}\right] \tag{25}$$

$$s.t.\quad \omega_i^M\cdot x_{ij}^M-\sum_{d=1}^{D}\mu_d^M\cdot z_{dj}\ge 0,\ j=1,2,\dots,n\ ,$$

$$\sum_{d=1}^{D}\mu_d^2\cdot z_{dj}+\sum_{p=1}^{P}\omega_p\cdot x_{pj}-\sum_{r=1}^{R}\gamma_r\cdot y_{rj}\ge 0,\ j=1,2,\dots,n\ ,$$

$$\omega_i \cdot x_{i0} = 1,$$

$$\sum_{d=1}^{D} \mu_d^2 \cdot z_{d0} + \sum_{p=1}^{P} \omega_p \cdot x_{p0} = 1,$$

$$\mu_d^2 = k \times \mu_d^1,$$

$$\omega_i, \omega_p, \mu_d^1, \mu_d^2, \gamma_r, k \geq 0, \quad i = 1,\ldots,m, \quad d = 1,\ldots,D, \quad p = 1,\ldots,P, \quad r = 1.\ldots,s$$

where the first and the third constraints refer to the manufacturer while the second and the fourth refer to the retailer. Model (25) is non-linear because in the second constraint there is the term μ_d^2 which includes a summation at the denominator as we can see in (24). However, we can replace this term by using the relation $\mu_d^2 = k \times \mu_d^1$. Thus:

$$max\ E = \frac{1}{2}\left[\sum_{d=1}^{D} \mu_d^1 \cdot z_{d0} + \sum_{r=1}^{s} \gamma_r \cdot y_{r0}\right] \tag{26}$$

$$s.t. \quad \sum_{i=1}^{m} \omega_i \cdot x_{ij} - \sum_{d=1}^{D} \mu_d^1 \cdot z_{dj} \geq 0, \ j = 1,2,\ldots,n,$$

$$\sum_{d=1}^{D} k \times \mu_d^1 \cdot z_{dj} + \sum_{p=1}^{P} \omega_p \cdot x_{pj} - \sum_{r=1}^{s} \gamma_r \cdot y_{rj} \geq 0, \ j = 1,2,\ldots,n,$$

$$\omega_i \cdot x_{i0} = 1,$$

$$\sum_{d=1}^{D} k \times \mu_d^1 \cdot z_{d0} + \sum_{p=1}^{P} \omega_p \cdot x_{p0} = 1,$$

$$\omega_i, \omega_p, \mu_d^1, \gamma_r, k \geq 0, \quad i = 1,\ldots,m, \quad d = 1,\ldots,D, \quad p = 1,\ldots,P, \quad r = 1.\ldots,s$$

The term μ_d^1 does not include a summation at the denominator and as a result μ_d^1 does not create a non-linearity problem. Now, only the k term creates the non-linearity problem. As we can see from the constraints of model (26):

$$\sum_{d=1}^{D} k \times \mu_d^1 \cdot z_{d0} + \sum_{p=1}^{P} \omega_p \cdot x_{p0} = 1 \text{ and } \sum_{d=1}^{D} \mu_d^1 \cdot z_{d0} = E_1^*$$

$$\text{Thus: } k=\frac{\left(1-\sum_{p=1}^{P}\omega_p\cdot x_{p0}\right)}{\sum_{d=1}^{D}\mu_d^1\cdot z_{dj}}\Rightarrow q=\frac{\left(1-\sum_{p=1}^{P}\omega_p\cdot x_{p0}\right)}{E_1^*} \tag{27}$$

The constraint $\sum_{d=1}^{D}k\times\mu_d^1\cdot z_{d0}+\sum_{p=1}^{P}\omega_p\cdot x_{p0}=1$ shows that $\sum_{p=1}^{P}\omega_p\cdot x_{p0}$ can take values from 0 to 1 because both terms, ω_p and x_{p0}, are non-negative quantities. If $\sum_{p=1}^{P}\omega_p\cdot x_{p0}$ takes the value 0, the numerator in (27) will become 1 and the overall fraction will become 0, otherwise if $\sum_{p=1}^{P}\omega_p\cdot x_{p0}$ takes the value 1, the numerator in (27) will become 0 and the overall fraction will become $\frac{1}{E_1^*}$. Therefore, we can determine an upper and a lower bound for k term, $0\le k<\frac{1}{E_1^*}$. Thus, we can treat k as a parameter and model (26) can be solved as a parametric linear program, using the same method as in model (21).

Liang et al. (2006) propose the above model in order to assess the overall and the individual efficiencies simultaneously. The individual efficiencies are calculated as $E_1^*=\mu_d^1\cdot z_{d0}$ and $E_2^*=\gamma_r\cdot y_{r0}$. The authors note that the cooperative efficiencies are at least equal with the non-cooperative efficiencies. The cooperative model of Liang et al. (2006) evaluates the efficiency of a simple supply chain which consists of two parties. Zhu and Cook (2007) extend the model of Liang et al. (2006) in order to include three or more parties.

5.2.1. Discussion of cooperative and non-cooperative models

The models of Liang et al. (2006) include exogenous inputs in the second stage. These exogenous inputs create non-linearity which dealt with parametric linear programming. Liang et al. (2008) investigate similar models without exogenous inputs in the second stage. The only inputs in the second stage are the intermediate measures produced in first stage. In Liang et al. (2008) models, the overall efficiency is calculated as the product of individual efficiencies, $E = E_1 \times E_2$ instead of $E = \frac{1}{2}(E_1 \times E_2)$. Exogenous inputs in the second stage do not allow this calculation in Liang et al.'s (2006) models because the transformation into a linear or parametric linear program will not be possible.

Models of Liang et al. (2006) and Liang et al. (2008) have a comparative advantage over other models, like Chen and Zhu (2004), Seiford and Zhu (1999) and network DEA because they assess both overall and individual efficiencies of the supply chain. As we have already noted, this is also true for the model of Kao and Hwang (2008) which according to Cook et al. (2010) is equivalent to the cooperative model.

Furthermore, Liang et al. (2008) prove that when there is only one intermediate measure in their models, the resulting efficiencies from cooperative and non-cooperative models are exactly the same. Also, the decomposition of the overall efficiency into individuals is unique. Additionally, individual efficiencies are the same as if we apply a typical DEA model at each stage separately. On the other hand, if there are multiple intermediate measures, then the non-cooperative model yields unique efficiency decomposition while efficiency decomposition for the cooperative model is not unique.

5.3. Nash bargaining game

Du et al. (2010) apply another form of cooperative model in two stage DEA, the Nash bargaining game. They adopt a similar supply chain with Liang et al. (2008), with no exogenous inputs in the second stage and all the first stage outputs are intermediate measures and consumed entirely by the second stage. Additionally, following the previous cooperation models of Liang et al. (2006, 2008) and Kao and Hwang (2008) they treat the intermediate measures in a coordinated manner by setting their optimal weights as equal treating them either as outputs in the first stage or as inputs in the second stage.

Du et al. (2010) consider the two stages as two players in a Nash bargaining game who bargain for a better payoff. Three main aspects must be defined in a Nash bargaining game, a) the participating players, say a manufacturer and a retailer, $N = \{1, 2\}$, b) a feasible set of payoffs, which is the set of DEA efficiencies and c) a breakdown point, which is the payoff if the participating players do not reach an agreement. The authors define as a breakdown point the efficiencies of the worst possible DMU, that is the DMU with maximum inputs and minimum outputs, thus $max\, x_i$, $min\, z_d$ in the first stage and $max\, z_d$, $min\, y_r$ in the second stage. These are the worst possible efficiencies and are denoted as θ_1^{min} and θ_2^{min} for the two stages respectively. These efficiencies are set as the breakdown point. In addition, the weights in the two stage model are considered as the possible strategies for the participating players. Nash point out that for the bargaining game there is a unique solution which can be found by applying the following maximization problem.

$$\max_{\vec{u} \in S, \vec{u} \geq b} \prod_{i=1}^{2} \left(u_i - b_i\right) \tag{28}$$

where $\vec{u}$ is the payoff vector for the two participating players, S is the feasible set of payoffs and b is the breakdown point.

After defining the above, the model of Du et al. (2010) will be the following:

$$max \quad \left[\frac{\sum_{d=1}^{D} w_d \cdot z_{d0}}{\sum_{i=1}^{m} v_i \cdot x_{i0}} - \theta_1^{min} \right] \times \left[\frac{\sum_{r=1}^{s} u_r \cdot y_{r0}}{\sum_{d=1}^{D} w_d \cdot z_{d0}} - \theta_2^{min} \right] \tag{29}$$

$$s.t. \quad \frac{\sum_{d=1}^{D} w_d \cdot z_{d0}}{\sum_{i=1}^{m} v_i \cdot x_{i0}} \geq \theta_1^{min}, \quad \frac{\sum_{r=1}^{s} u_r \cdot y_{r0}}{\sum_{d=1}^{D} w_d \cdot z_{d0}} \geq \theta_2^{min},$$

$$\frac{\sum_{d=1}^{D} w_d \cdot z_{dj}}{\sum_{i=1}^{m} v_i \cdot x_{ij}} \leq 1, \; j = 1,2,\ldots,n,$$

$$\frac{\sum_{r=1}^{s} u_r \cdot y_{rj}}{\sum_{d=1}^{D} w_d \cdot z_{dj}} \le 1,\ j = 1,2,\ldots,n,$$

$$v_i, w_d, u_r > 0, \quad i = 1,\ldots,m, \quad d = 1,\ldots,D, \quad r = 1,\ldots,s$$

where the objective function is the bargaining problem (28). The first two constraints ensure that individual efficiencies will not be less than the worst possible efficiencies θ_1^{min} and θ_2^{min}. The next two constraints are the typical constraints of a fractional DEA program.

Then, the authors apply the transformation (24) at model (29) in order to convert it into a linear program. That is

$$t_1 = \frac{1}{\sum_{i=1}^{m} v_i \cdot x_{i0}},\ t_2 = \frac{1}{\sum_{d=1}^{D} w_d \cdot z_{d0}}, \tag{30}$$

$$\omega_i = t_1 \cdot v_i,\ \mu_d = t_1 \cdot w_d,\ \gamma_r^1 = t_1 \cdot u_r,\ \gamma_r^2 = t_2 \cdot u_r,$$

$$d = 1,\ldots,D, \quad i = 1,\ldots,m, \quad r = 1,\ldots,s$$

Obviously, there is a linear relation between γ_r^1 and γ_r^2, $\gamma_r^1 = \alpha \times \gamma_r^2$ where $\alpha = \frac{t_1}{t_2}$ is a positive number. Therefore, we can convert model (29) into the following model:

$$max \sum_{r=1}^{s} \gamma_r^1 \cdot y_{r0} - \theta_1^{min} \cdot \sum_{r=1}^{s} \gamma_r^2 \cdot y_{r0} - \theta_2^{min} \sum_{d=1}^{D} \mu_d \cdot z_{d0} + \theta_1^{min} \cdot \theta_2^{min} \tag{31}$$

$$s.t. \quad \sum_{d=1}^{D} \mu_d \cdot z_{d0} \ge \theta_1^{min},\ \sum_{r=1}^{s} \gamma_r^2 \cdot y_{r0} \ge \theta_2^{min},$$

$$\sum_{i=1}^{m} \omega_i \cdot x_{i0} = 1,\ \sum_{d=1}^{D} \mu_d \cdot z_{d0} = \alpha,$$

$$\sum_{d=1}^{D} \mu_d \cdot z_{dj} - \sum_{i=1}^{m} \omega_i \cdot x_{ij} \le 0,\ j = 1,2,\ldots,n,$$

$$\sum_{r=1}^{s} \gamma_r^1 \cdot y_{rj} - \sum_{d=1}^{D} \mu_d \cdot z_{dj} \le 0,\ j = 1,2,\ldots,n,$$

$\gamma_r^1 = \alpha \cdot \gamma_r^2$, $r = 1,2,\ldots,s$,

$\omega_i, \mu_d, \gamma_r^1, \gamma_r^2, \alpha > 0, \quad i = 1,\ldots,m, \quad d = 1,\ldots,D, \quad r = 1,\ldots,s$

Model (31) can be transformed into the following model by applying the relation $\gamma_r^1 = \alpha \times \gamma_r^2$. That is

$$max \quad \alpha \times \sum_{r=1}^{s} \gamma_r^2 \cdot y_{r0} - \theta_1^{min} \cdot \sum_{r=1}^{s} \gamma_r^2 \cdot y_{r0} - \theta_2^{min} \sum_{d=1}^{D} \mu_d \cdot z_{d0} + \theta_1^{min} \cdot \theta_2^{min} \tag{32}$$

$$s.t. \quad \sum_{d=1}^{D} \mu_d \cdot z_{d0} \geq \theta_1^{min} , \ \sum_{r=1}^{s} \gamma_r^2 \cdot y_{r0} \geq \theta_2^{min} ,$$

$$\sum_{i=1}^{m} \omega_i \cdot x_{i0} = 1 , \ \sum_{d=1}^{D} \mu_d \cdot z_{d0} = \alpha ,$$

$$\sum_{d=1}^{D} \mu_d \cdot z_{dj} - \sum_{i=1}^{m} \omega_i \cdot x_{ij} \leq 0 , \ j = 1,2,\ldots,n ,$$

$$\alpha \times \sum_{r=1}^{s} \gamma_r^2 \cdot y_{rj} - \sum_{d=1}^{D} \mu_d \cdot z_{dj} \leq 0 , \ j = 1,2,\ldots,n ,$$

$$\omega_i, \mu_d, \gamma_r^2, \alpha > 0, \quad i = 1,\ldots,m, \quad d = 1,\ldots,D, \quad r = 1,\ldots,s$$

Model (32) is non-linear because of the "α" term. As we can see from the constraints of model (32):

$$\sum_{d=1}^{D} \mu_d \cdot z_{dj} - \sum_{i=1}^{m} \omega_i \cdot x_{ij} \leq 0 \Rightarrow \sum_{d=1}^{D} \mu_d \cdot z_{dj} \leq \sum_{i=1}^{m} \omega_i \cdot x_{ij} , \ j = 1,2,\ldots,n ,$$

$$\sum_{i=1}^{m} \omega_i \cdot x_{i0} = 1 \ , \ \sum_{d=1}^{D} \mu_d \cdot z_{d0} = \alpha \ \text{and} \ \sum_{d=1}^{D} \mu_d \cdot z_{d0} \geq \theta_1^{min} .$$

If we combine constraints $\sum_{d=1}^{D} \mu_d \cdot z_{d0} = \alpha$ and $\sum_{d=1}^{D} \mu_d \cdot z_{d0} \geq \theta_1^{min}$ then: $\sum_{d=1}^{D} \mu_d \cdot z_{d0} \geq \theta_1^{min} \Rightarrow \alpha \geq \theta_1^{min}$. Therefore, we replace constraint $\sum_{i=1}^{m} \omega_i \cdot x_{i0} = 1$ and $\alpha \geq \theta_1^{min}$ back to the first constraint: $\sum_{d=1}^{D} \mu_d \cdot z_{dj} \leq \sum_{i=1}^{m} \omega_i \cdot x_{ij} \Rightarrow \theta_1^{min} \leq \alpha \leq 1$.

Therefore, we can determine an upper and a lower bound for α term. Thus, we can treat α as a parameter and model (32) can be solved as a parametric linear program, using the same method as in model (21). Thus, according to Du et al. (2010) the efficiencies of the model are $E_1^* = \alpha^*$ from the constraint $\sum_{d=1}^{D} \mu_d \cdot z_{d0} = \alpha$, $E_2^* = \gamma_r^2 \cdot y_{r0}$ and the efficiency of the entire supply chain can be calculated as $E^* = E_1^* \times E_2^*$.

The authors point out that if there is only one intermediate measure in the supply chain then the individual efficiencies are the same as if we apply a typical DEA model at each stage separately. As a result, in this case the efficiencies of the model are equal with the efficiencies of the cooperative model of Liang et al. (2008). In addition, the model of Liang et al. (2008) is a special case of model (32) with zero breakdown point. Finally, the efficiencies of Liang et al.'s (2008) model are the best feasible efficiencies for model (32) as it is not possible to achieve further improvement.

6. Conclusion and future perspectives

In this paper we provide a thorough survey and a detailed classification of supply chain DEA models with internal structures. We concentrate on two-stage models with intermediate measures between the first and the second stage and some variations such as models with exogenous inputs in the second stage. We also extend our survey in some special cases where there are more than two stages or there are no intermediate measures.

We classify the models into four categories: 1) Models which apply standard DEA approach in each stage separately. Models in this category do not consider the possible conflicts between the two stages. 2) Relational DEA models which treat intermediate measures in a coordinated manner. 3) Network DEA models which may include more than two stages, may not have intermediate measures, may have dynamic structures and may share the inputs among the stages. 4) Game theoretic models which are divided in non-cooperative and cooperative models. Also, cooperative models include the Nash bargaining game model.

As it is clear along our survey, a model must treat the intermediate measures in a coordinated manner in order to incorporate the interrelated decisions among the stages. A reliable model can assist the decision maker in order to achieve strategic alignment among the different members of the supply chain (O'Leary-Kelly and Flores, 2002). Furthermore, a comprehensive and accurate model would assist the decision maker to monitor and control the entire supply chain and its individual components more effectively.

The models we present in this paper can be applied in various firms and organizations with simple or complex internal structures. Verma and Sinha (2002) and Swink et al. (2006) point out that a high technology firm develops multiple new products. The firm has specific resources which must be allocated among the developing products. A shared flow model as presented above may be the proper model to study the resource interdependence among the different products. Ross and Droge (2002, 2004) examine the distribution centers of a petroleum industry. The authors group the distribution centers into three categories. If we want to extend this model and examine the efficiency of $j = 1,2,\ldots,n$ petroleum industries, we could see the three groups as three individual stages and apply a supply chain DEA model which will capture the interrelations among the distribution centers.

Several studies can be extended in two or more stages structure with the proper modification. Sarkis (2000) examines the efficiency of the airports, Sun (2004) investigates the efficiency of braches of Taiwanese army and Narasimah et al. (2005) apply DEA in a government agency. All the above are complex entities and supply chain DEA models may be the appropriate tool in order to capture the possible conflicts and interrelations among the stages and evaluate the overall efficiency properly. In addition, other aspects inside a firm could be properly studied with a supply chain model, like the impact of information technology on the efficiency of the firm. Hendricks et al. (2007) and Bendoly et al. (2009)

study the use of information technology inside a firm. Chen and Zhu (2004) and Chen et al. (2006) show how this impact can be approached by supply chain models. Specifically, Chen et al. (2006) extend his model to decompose and allocate the information technology in every stage.

In conclusion, standard DEA approach is a valuable tool for efficiency evaluation however when there are more complex systems than a simple input-output procedure fails, to address the internal structures. A decision maker needs a tool which can incorporate these interrelations into the model and provide more accurate results in order to monitor the overall and individual procedures more effectively and make better decisions. In this paper we provide all the available models for a simple two-stage procedure and some extensions into more complex structures.

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