Crisis in the Euro area: coopetitive game solutions as new policy tools

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Abstract

The crisis within the euro area have become frequent during 2010. First was the Greek economy to face a default problem of its sovereign debt, in November it was Ireland who has been in a serious financial situation at the verge of collapse causing difficulties to the euro. In this contribution we focus on the Greek crisis and we suggest, through a model of coopetition based on game theory and conceived at a macro level, feasible solutions in a cooperative perspective for the divergent interests which drive the economic policies in Germany and Greece, with the aim of improving the position of Greece, Germany and the whole euro area, also making a contribution to expand the set of macroeconomic policy tools. By means of our general analytical framework of coopetition, we show the strategies that could bring to feasible solutions in a cooperative perspective for Germany and Greece, where these feasible solutions aim at offering a win-win outcome for both countries, letting them to share the pie fairly within a growth path represented by a non-zero sum game. A remarkable analytical result of our work consists in the determination of the win-win solution by a new selection method on the transferable utility Pareto boundary of the coopetitive game.

Keywords: European Monetary Union, Coopetitive Games, Macroeconomic Policy, Bargaining solutions.
JEL Classification: F40, E6, D7, C71, C72, C78
Introduction*.

In the present work we analyze the crisis in the euro area and we suggest, through a model of coopetition based on game theory and conceived at a macro level, feasible solutions in a cooperative perspective for the divergent interests which drive the economic policies in Germany and Greece, with the object of improving the position of Greece, Germany and the whole euro area. Our model is also aimed at making a contribution to expand the set of macroeconomic policy tools.

The work is organized as follows: the first section examines the general topic of the crisis in the euro area focusing on the Greek crisis and concentrating on the real aspects of the crisis. The second section suggests a possible way out to reduce the intra-eurozone imbalances through a new macroeconomic tool based on coopetitive solutions within a growth path. The third section introduces a game theory framework of coopetition. The fourth section provides two specific and original models of coopetitive games applied to the eurozone context and shows their solutions. Conclusions end up the paper.

1. The Euro and the Crisis.

The crisis within the euro area have become frequent during 2010. First was the Greek economy to face a default problem of its sovereign debt last spring, in November it was Ireland who has been in a serious financial situation at the verge of collapse, due mainly to poor quality bank regulation, causing difficulties to the euro. So the eurozone Governments and international

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institutions are continually trying to solve these problems that create instability and jeopardize the very existence of European Monetary Union. Many of the countries of the European Monetary Union have accumulated large budget deficit to GDP ratios in 2009, which are in turn caused by the global crisis that since 2008 developed itself in the United States for then spreading around the world.

The global crisis has inevitably burdened the public debt of countries such as Greece, Ireland, Portugal, Spain and even Italy, which have found difficult to finance their debt in the financial markets, creating a problem of instability and cohesion of the European Monetary Union. But this is only one aspect of the crisis, the other one, related to the former, is the recession hitting the EMU economies, which are suffering of high unemployment, slowing down of production, difficulties in exporting, further crisis of the welfare state.

In this contribution we focus on the Greek crisis, we know that EMU Governments and IMF agreed to provide Greece with enough financing to cover its refinancing needs for three years, while the Greek government commits to an additional tough austerity program. We also know that Germany is the country of the euro area which has a large trade surplus with Greece and other euro partners, hence strong trade imbalances occur within the eurozone economy.

Germany is the country who has profited most from the euro since the start of the European Monetary Union, according to Adam Posen (2010). Because the benefits received from the German economy has been possible thanks to a cooperative economic system, the main purpose of our paper is to explore win-win solutions for Greece and Germany, involving improvements in domestic demand in Germany.

We do not analyze the causes of the financial crisis in Greece and its relevant political and institutional effects on the European Monetary Union. Rather we focus on some crucial aspects of the Greek economy, with their implications on the euro area. Specifically we concentrate on stability and growth, which should drive the economic policy of Greece, Germany and the other euro countries.

The deep financial crisis of Greece, which was almost causing the default of its sovereign debt, determining also financial instability in the European markets and the devaluation of the euro, has revealed the weaknesses of
Greek economy. This crisis has also showed the contradictions that have been characterized the EMU and the euro since their start. Furthermore, EMU is featured by the presence of two countries, Germany and France, which have a major and increasing political and economic role. Greece is a country with a total population of 11 million and it represents 2.6% of the eurozone’s GDP. This country adopted the euro in 2001, then interest rates fell to near German levels, the lowest in the euro area, fueling consumer spending and house prices. Since joining EMU, Greece has lost competitiveness and, because of that, Greek’s unit labor cost rose 34 percent from 2000 to 2009. Thus, Greece relied on state spending to drive growth. With the outbreak of the crisis, debt in Greece has surged as in the other countries, but in 2009 Greece recorded a deficit/GDP ratio of 13.6%, one of the highest of the eurozone economies. This has created deep concerns about its fiscal sustainability.

Greece has also accumulated a huge debt of about 310 billion euros, thus its financial exposition prevents the Greek government to find capital in the financial markets. The country, therefore, has become at risk of sovereign default. In the meantime the other EMU countries, after a period of uncertainty which raises the cost of the bailout, have decided to help Greece financially also with the support of IMF. This financial contribution is likely to be given until 2012 and it will be very substantial. But tough austerity conditions are requested in return for the emergency loans, which are to be paid with interest rates below the market rates, the Greek Government is required to take courageous and specific actions that will lastingly and credibly consolidate the public budget. The EMU-IMF package also

1. One feature of the institutional setting of the European Monetary Union is the Stability and Growth Pact that guards against the emergence of public deficits and debt, but actually there isn’t a true and effective mechanism of enforcement in the Pact. Therefore the budget policy in each country of the eurozone is not under control. Yet the European Commission has, just after the Greek crisis, proposed tougher rules to enforce fiscal discipline in the eurozone and to set up a permanent crisis management mechanism to prevent sovereign debt disasters.

2. An agreement has been reached on May 2nd, between the Eurogroup, the IMF and the Greek Government.

3. The total sum given to Greece in three years should be of 110 billion euros.

4. First, to recover from the budget disequilibrium, Greece is expected to improve the primary balance of 10 percent of GDP over the next three years (This is an heavy task, but other
includes measures to enhance competition in many sectors which are still protected; thus the country is expected to reduce its budget deficit from 13.6 per cent of gross domestic product to below 3 per cent by 2014.

However, although a restrictive fiscal policy and budget austerity are necessarily implemented by the Greek Government, they could be insufficient for Greece to overcome its crisis. The austerity measures are likely to hit hard the Greek economy, since its growth is expected to be negative this year and the next year, making the financial recovery even more problematic. Furthermore, exports are much less than imports, so the trade balance shows a deficit around 10%. Therefore, the focus of economic policy of Greece should be on its productive system and growth must become the major goal for the Greek economy. This surely would help its reequilibrium process.

On the other hand, Germany is considered the soundest European economy. First of all, it accounts for about one-third of the eurozone economy. Secondly, it is the world’s second biggest exporter, but its wide commercial surplus is originated mainly by the exports in the euro area, that accounts for about two thirds. Furthermore, since 2000 its export share has gradually increased vis-à-vis industrial countries. Thirdly, its government has not allowed itself the extraordinary budget deficits that are threatening economies like Ireland, Italy, Greece, Portugal and Spain. Despite these positive records, the contribution of domestic demand to real GDP from 1999 onwards in Germany has been weak. It is clear, from such a context, that the Germany’s growth path has been driven by exports. We do not discuss in this work the factors explaining Germany’s increase in export share, but we observe that its international competitiveness has been improving, with the unit labor cost which has been kept fairly constant, since wages have essentially kept pace with productivity. Therefore the prices of economies like Lettonia and Hungary have succeeded in the recent past with the help and the assistance of the IMF and EU). Moreover, the package includes measures to reduce the size of Greece’s public sector, cuts in public sector salaries and pensions, a rise in value added tax and other tax increases.

This view, of course, is not shared by the economists who believe that fiscal adjustments not always cause recessions (Giavazzi,Pagano, 1990; Von Hagen, Strauch, 2001; Alesina, Adagna, 2009).
the German products have been relatively cheap, favouring the export of German goods towards the euro countries and towards the markets around the world, especially those of the emerging economies (China, India, Brasil, Russia). Finally, just during 2010 Germany has recovered very well from the 2008-2009 global crisis and is growing at a higher rate than the others euro partners. Thus, we share the view that Germany (and the other surplus countries of the euro area, i.e. Netherlands) should contribute to overcome the crisis of the EMU economies and of Greece in particular stimulating its domestic demand and relying less on exports towards the euro area. Germany, as Adam Posen (2010) underlined\(^6\), has benefited from being the anchor economy for the eurozone over the last 11 years. In fact, it enjoyed a wider and deep range of trade in the euro currency than it had under the Deutch Mark. For instance, in 2009, during a time of global contraction, Germany has been a beneficiary, being able to run a sustained trade surplus with its European neighbours. Germany exported, in particular, 6.7 billions euros worth of goods to Greece, but imported only 1.8 billion euros worth in return. Clearly a policy which aims at growth in Greece, Germany and the whole euro area is very important, specially if we take a medium-long term perspective and if we consider that the rate of unemployment in the euro area has reached 10.1\(^7\), the highest rate in almost 12 years\(^8\).

We believe that a policy that aims at adjusting budget and trade imbalances and looks at improving the growth path of the real economy in the medium and long term in Greece is the only possible one to assure a stable re-

\(^6\) See also Abadi (2010).
\(^7\) Source: Eurostat. The figure refers to April 2010.
\(^8\) Another aspect to highlight is that despite the new huge rescue plan of 750 billion euros supported by the EU and IMF to avoid the contagion of the Greec ek crisis to the other EMU countries, the recent turmoil in the financial markets and the consequent weakening of the euro seem to confirm the poorly optimistic expectations of the financial markets on the future of the Greek economy. Investors are looking for a credible plan that indicated public finances in Greece but also in whole euro area could be kept at a sustainable level. In this context the view that a partial debt restructuring by the Greek government might become a sensible and realistic solution.
balancing of the Greek economy and to contribute to the stability of the euro area. As we have already argued, German modest wage increases and weak domestic demand favoured the export of German goods towards the euro countries. This is why Posen, as reported by Business Week on March 31, 2010, said that Germany should boost domestic demand and increase wages to ease the lopsided euro-region trade flows that restrict growth in economies like Greece and Portugal. Therefore he suggests a “win-win solution” for the EMU countries, which entails that Germany, which still represents the leading economy, should stimulate domestic demand, increase wages in its own country, so that to make its own people better off, and thereby ease some of the pressure on the southern countries of the euro area. In Posen’s proposal there is a clear suggestion to Germany to rebalance its trade surplus.

Of course, we are aware that this is a mere hypothesis. Although Germany has been pursuing a strategy of competitiveness based on investments in technology and R&D on the one hand, and on industrial relations, which are featured by cooperative behaviors between labor and capital, on the other since 2003. We believe that this cooperative attitude, which is an hallmark of German capitalism, can be also taken with respect to its euro partners and the Greece in particular. Thus we pursue our hypothesis and suggest a game theory coopetitive model as an innovative instrument to analyze possible solutions to obtain a win-win outcome for Greece and Germany, which would also help the whole EMU economy.

Giving that Greece must fulfil the conditions of the agreement signed with the eurozone Governments and the IMF for their financial help and, for this reason, it must implement a fiscal policy of government budget consolidations, with current spending cuts and tax increases, to reduce its public and private debt, these changes in current variables (taxes, 

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9 A win-win solution is the outcome of a game which is designed in a way that all participants can profit from it in one way or the other. In conflict resolution a win-win strategy is a process that aims to accommodate all disputants.

10 After the Greek crisis, because of the turmoil in the financial markets, the German government have decided to take austerity fiscal measures, which consists of a seven years plan of government budget consolidations of 70 billion euros (10 billion euros for each year), based mainly on structural spending cuts to welfare payments and reduction in the public sector. This plan, however, will also favour investment in education and research to improve Germany’s capacity to compete at a global level.
incentives, provision of public services) would probably also change the expectations about future fiscal policy\textsuperscript{11}. In our view, Greece must keep its wages and salaries under control and, at the same time, focus on investments and exports as the two main strategic variables to improve the structure of production and to shift the aggregate demand towards a higher growth path. However, aiming at exports for a country like Greece that has a low “extra euro area” export share on GDP (about 4\%) does not mean to rely on the external demand, for instance through the devaluation of the euro, rather to follow an appropriate medium term strategy. In this medium term strategy, Greece should focus on innovative investments, specially investments in knowledge\textsuperscript{12}, to change and improve its production structure and to increase its production capacity and its productivity, which is made possible by the structural change process. As a result of that its competitiveness will raise. An economic policy that focuses on investments and exports, instead of consumptions, will address Greece towards a sustainable growth and, consequently, its financial reputation and stability will get improved.


The idea which is driving our model to face the Greek crisis is based on a notion of coopetition where the cooperative aspect will prevail. Thus we are not talking about a situation in which Germany and Greece are competing in the same European market for the same products, rather we are assuming a situation in which Germany stimulates its domestic demand and, in doing so, will create a larger market for products from abroad, but also we are envisaging the case in which Germany purchases a greater quantity of

\textsuperscript{11} Regarding the indirect positive effect on aggregate demand see Hellwig, Neumann (1987) that merge the Keynesian view and the expectations view or “German view” on budget cutting. See also Giavazzi, Pagano (1990).

\textsuperscript{12} Schilirò (2010b).
Greek products, in this case Greece increases its exports, selling more products in Germany. The final results will be that Greece will find in a better position, but also Germany will get an economic advantage determined by the higher growth in the two countries and, finally, because it will prevail a greater stability within the EMU system. Therefore we provide, in the present work, a new set of tools based on the notion of coopetition, that could be fruitful for the setting of the Greek policy issues.

The concept of coopetition has been devised following different theoretical approaches. Essentially the literature on coopetitive games has a microeconomic origin and has an important point of reference in the seminal paper of Brandenburger and Nalebuff (1995), who studied the strategic behaviour of firms applying some basic notion of game theory and elaborated their theoretical original concept of coopetition within a competitive environment. Brandenburger and Nalebuff suggest the term coopetition (a situation in which the firm must cooperate and compete at the same time) to indicate a situation in which the firm thinks about both cooperative and competitive ways to change the game (1995, p.59). Another approach to coopetition represents the synthesis between the competitive paradigm (Porter, 1985) and the cooperative paradigm (Gulati, Nohria, Zaheer, 2000), a sort of integrative framework between the two, like that offered by Padula and Dagnino (2007), who define coopetition as the intrusion of competitive elements into a cooperative environment, because of the partially divergent interests among the partners. Thus coopetition is a complex construct and it is the result of the interplay between competition and cooperation.

Our model of coopetition is closer to the approach that regards coopetition as a complex construct rooted in a cooperative environment. Thus we suggest a model of coopetitive games, applied at a macroeconomic level, which intends to offer possible solutions to the partially divergent interests of Germany and Greece in a perspective of a cooperative attitude that should drive their policies. Another important goal of the model is to provide a new tool of macroeconomic policy for the crisis in the euro area, thus enriching the toolbox of economic policy.
3. The general definition of coopetitive game.

In this section we provide an original recent definition of coopetitive game, which we shall use, in the next section, to build up two economic models feasible to represent the interaction of the two EMU countries, Germany and Greece. The two above coopetitive models will show possible new solutions reasonable in a particular coopetitive context, defined by the set of strategy profiles at disposal of the two countries and by a set of possible convenient \textit{ex ante} agreements. This suggested analytical framework enables us to widen the set of possible solutions from a purely competitive into a coopetitive context and moreover it allows “to share the pie fairly” in a win-win scenario. At the same time, it permits to examine the range of possible economic outcomes along a coopetitive dynamic path. Finally, we propose a rational way to limit the space within which the coopetitive solutions can be determined.

\textbf{Remark.} The basic original definition we propose and apply of coopetitive game is that introduced in 2010 by D. Carfì in [11] and [3]; the method that we shall use to study the payoff space of a normal form game can be found in [8] and [12]; the complete study of a normal form game is presented and applied in [7], [9] and [10]; for a general definition and basic properties of Pareto boundaries see [6].

\textbf{Definition (of coopetitive game).} Let \( E, F \) and \( C \) be three nonempty sets. We define \textbf{two person coopetitive gain game carried by the strategic triple} \((E, F, C)\) any pair of the form \( G = (f, >) \), where \( f \) is a function from the Cartesian product \( E \times F \times C \) into the real Cartesian plane \( \mathbb{R}^2 \) and \( > \) is the usual strict upper order of the Cartesian plane, defined, for every couple of points \( p, q \), by \( p > q \) if and only if \( p_i > q_i \), for each index \( i \).

\textbf{Remark.} The difference among a two person normal-form gain game and a two person coopetitive gain game is simply the presence of the third strategy Cartesian factor \( C \).
**Terminology and notation.** Let $G = (f, >)$ be a two person coopetitive gain game carried by the strategic triple $(E, F, C)$. We will use the following terminologies:

- the function $f$ is called the **payoff function of the game** $G$;

- the first component $f_1$ of the payoff function $f$ is called the **payoff function of the first player** and analogously the second component $f_2$ is called the **payoff function of the second player**;

- the set $E$ is said the **strategy set of the first player**, the set $F$ the **strategy set of the second player**;

- the set $C$ is said **the cooperative strategy set of the two players**.

- the Cartesian product $E \times F \times C$ is called the **coopetitive strategy space of the game** $G$.

**Memento.** The first component $f_1$ of the payoff function $f$ of a coopetitive game $G$ is the function of the strategy space of the game $G$ into the real line defined by $f_1(x,y,z) = \text{pr}_1(f(x,y,z))$, where $\text{pr}_1$ is the usual first projection of the Cartesian plane; analogously we proceed for the second component $f_2$.

**Strategic interpretation.** We have two players, each of them has a strategy set in which to choose his own strategy; moreover, the two players can **cooperatively choose a strategy** $z$ in a third set $C$. The two players will choose their cooperative strategy $z$ to maximize (in some sense) the gain function $f$.

**Bargaining solutions of a coopetitive game.** The payoff function of a two player coopetitive game is (as in the case of normal-form game) a vector valued function with values belonging to the Cartesian plane $\mathbb{R}^2$; so that we should consider the maximal Pareto boundary of the payoff space $\text{im}(f)$ as an appropriate zone for the bargaining solutions (by $\text{im}(f)$ we denote the image of the function $f$).
The family of normal form games associated with a coopetitive game.
For any cooperative strategy \( z \), selected in the cooperative strategy space \( C \), there is a corresponding normal form game

\[ G_z = (f_z, >) \]

upon the strategy pair \( (E, F) \) and with payoff function the section

\[ f(., z) : E \times F \to \mathbb{R}^2, \]

of the payoff function \( f \) of the coopetitive game, where the section of \( f \) is defined, as usual, on the competitive strategy space \( E \times F \) by

\[ f(., z)(x) = f(x, z), \]

for every bi-strategy \( x \) in the bi-strategy space \( E \times F \).

General solution. The two players should choose the cooperative strategy \( z \) in order that, for instance:

- the Nash equilibria of \( G_z \) are “better” than the Nash equilibria in each other game \( G_{z'} \);

- the supremum of \( G_z \) is greater than the supremum of any other game \( G_{z'} \);

- the Pareto maximal boundary of \( G_z \) is “higher” than that of any other game \( G_{z'} \);

- the Nash bargaining solution is better in \( G_z \) than that in \( G_{z'} \);

- and so on, fixed a common standard kind of solution for any game \( G_z \), say \( S(z) \) the set of these kind of solutions, we can consider the problem to find the optimal solutions in set valued path \( S \), defined on the cooperative strategy set \( C \);
we note the fundamental circumstance that in general the above criteria are multi-criteria and so they generate multi-criteria optimization problems.

Let us formalize the concept of game-family associated with a coopetitive game.

**Definition (the family of normal form games associated with a coopetitive game).** Let $G = (f, >)$ be a two players coopetitive gain game carried by the strategic set triple $(E, F, C)$. We call family of normal-form games associated with the coopetitive game $G$ the family of normal form games

$$\mathcal{G} = (G_z)_{z \in C},$$

which we will denote by the symbol $\mathcal{G}$, having, for any cooperative strategy $z$ selected in the cooperative strategy space $C$, as $z$-member the normal form game

$$G_z = (f_z, >),$$

upon the strategy pair $(E, F)$, with payoff function the section

$$f(. , z) : E \times F \rightarrow \mathbb{R}^2,$$

of the payoff function $f$ of the coopetitive game $G$.

*Applicative remark.* It is clear that with any family of normal form games

$$\mathcal{G} = (G_z)_{z \in C}$$

we can associate

- a family of payoff spaces
(im(f_z))_{z \in C},

- a family of Pareto maximal boundary

(bd^* G_z)_{z \in C};

- a family of suprema

(sup G_z)_{z \in C};

and so on.

And we can interpret any of the above families as set-valued paths in the strategy space $E \times F$.

It is just the study of these induced set-valued paths of solutions which becomes of great interest in the study of a coopetitive game $G$.

4. Two models of coopetitive games

In our analysis Germany is the first exporting country among the EMU countries, which has also experienced a weak domestic demand due to a modest wage increases. Thus our hypothesis is to stimulate Germany's domestic demand and to re-balance its trade surplus in favour of Greece.

On the other hand, Greece is the country that showed a high and rising public debt, which determined its sovereign debt at risk of default. Given that Greece must pursue a budget austerity program externally imposed by the euro area Governments and by IMF in exchange of their financial help, this country has anyway experienced a declining competitiveness of its products. Therefore our hypothesis is that Greece aims at growth by undertaking
innovative investments and by increasing its exports primarily towards Germany and also towards the other euro countries\textsuperscript{13}.

The coopetitive models that we propose hereunder must be interpreted as normative models, in the sense that they will show the more appropriate solutions of a win-win strategy chosen within a cooperative perspective.

The main variables of the two models are:

\textbf{strategies x} of Germany (\textit{the consumptions of Germany}), which directly influence only Germany pay-off;

\textbf{strategies y} of Greece (\textit{the investments of Greece}) which increase only Greece pay-off function;

\textbf{a shared strategy z} which is determined \textit{ex ante} together by the two countries, Germany and Greece (z is a given amount of Greek exports imported by Germany).

Therefore, in the two models we assume that Germany and Greece define the set of coopetitive strategies.

\subsection*{4.1 First coopetitive model.}

\textbf{Main Strategic assumptions.} We assume that a real number x, in the unit interval $U = [0,1]$, is the consumption of Germany and a real number y, in the same unit interval $U$, is the investment of Greece, moreover a real number z, again in the unit interval $U$, is the amount of Greek exports which is imported by Germany.

We also consider as payoff function of Germany \textbf{its domestic demand}, that we represent in our model as the algebraic sum of the two strategies x and z, and also of the exports of Germany as a reaction function with respect to

\textsuperscript{13} The potential benefit coming from a better trade balance can also contribute to ease the government budget constraint and improve its public debt.
its domestic consumption.

4.1.1 Payoff function of Germany

We assume that the payoff function of Germany is the function $g$ of the unit square $U \times U$ into the real line $\mathbb{R}$, defined by

$$g(x, z) = x + (x + 1)^{-1} - z,$$

for every pair $(x, z)$ in the square $U \times U$; where the reaction function $E$, of $U$ into the real line, defined by

$$E(x) = 1/(x+1),$$

for every consumption $x$ of Germany in $U$, is the export of Germany corresponding to the level $x$ of consumption. The reaction function $E$ is a decreasing function, randomly chosen, and within certain limits, this choice does not diminish the generality of the model.

4.1.2 Payoff function of Greece

We consider as payoff function of Greece the algebraic sum of the economic strategies $y$ and $z$ and of two linear reaction functions $M$ and $N$.

We assume that the payoff function of Greece is the function $e$ of the square $U \times U$ into the real line, defined by

$$e(y, z) = y + z + my + nz = (1+m) y + (1+n) z,$$

for every pair $(y, z)$ in the Cartesian square $U \times U$, where $m$ and $n$ are two real numbers strictly greater than 1. We note that the function $e$ does not
depend upon the strategy $x$ chosen by Germany and that $e$ is a linear function.

**Reaction function** $M$. The term $my$ represents the quantitative effect of investments on the exports. In fact, the investments, especially innovative investments, contribute at improving the competitiveness of Greek goods, favoring the exports.

**Reaction function** $N$. The term $nz$ is the cross-effect of the competitive variable $z$ that represents the additive level of investment required to support the production of $z$.

### 4.1.3 Payoff function of the game

We so have build up a gain game with payoff function given by

$$p(x, y, z) = (x + 1/(x+1) - z, (1+m) y + z) =$$
$$= (x + 1/(x+1), (1+m) y) + z (-1,1+n),$$

with $x,y,z$ in the unit interval $[0,1]$.

### 4.1.4 Study of the game $G = (p, >)$.

Note that, fixed a cooperative strategy $z$ in the unit interval $U$, the normal form game $G(z) = (p(z), >)$ with payoff function $p(z)$, defined on the square $U^2$ by

$$p(z)(x, y) = p(x, y, z),$$

is the translation of the game $G(0)$ by the vector

$$v(z) = z(-1,1+n),$$
so that we can study the game $G(0)$ and then we can translate the various informations of the game $G(0)$ by the vector $v(z)$.

### 4.1.5 Study of the game $G(0)$.

So let us consider the game $G(0)$. Let the strategic square $S = U^2$ be with vertices $A$, $B$, $C$, $D$, where $A$ is the origin $(0,0)$, $B$ is the first canonical vector $(1,0)$, $C = (1,1)$, the sum of the two canonical vectors, and $D$ be the second canonical vector $(0,1)$.

The transformation of the side $[A, B]$ is the trace of the parametric curve $c$ defined by

$$c(x) = \rho(x,0,0) = (x + (x+1)^{-1}, 0),$$

that is the segment

$$[A', B'] = [(1,0), (3/2,0)].$$

The transformation of the segment $[A, D]$ is the trace of the curve $c$ defined by

$$c(y) = \rho(0,y,0) = (1, (1+m)y),$$

that is the segment

$$[A', D'] = [(1,0), (1,1+m)].$$

The transformation of the segment $[B, C]$ is the trace of the curve $c$ defined by

$$c(y) = \rho(1, y,0) = (1 + 1/2, (1 + m)y),$$
that is the segment

$$[B', C'] = [(3/2,0), (3/2,1 + m)].$$

So that the payoff space of the game $G(0)$ is the rectangle with vertices $A'$, $B'$, $C'$, $D'$.

**Figure 1. The payoff space of the initial game $G(0)$.**

**Remark.** It is easily seen that the critical zone of the game is irrelevant (in this case) to determine the payoff space of the game.
4.1.6 Payoff space of the coopetitive game $G$.

The image of the payoff function $p$, is the union of the family of payoff spaces

$$(\text{im} \ p(z))_z,$$

that is the convex envelope of the of the four points $A', B', C', D'$ and of their translations by the vector

$$v(1) = (-1, 1+n).$$

The Pareto maximal boundary of the payoff space $f(S)$ is the segment

$$[P', Q'],$$

where $P' = C'$ and

$$Q' = C' + v(1) = (3/2, 1+m) + (-1, 1+n) = (1/2, 2 + m + n).$$

It is important to note that the absolute slope of the coopetitive Pareto boundary is the absolute slope of the vector $(-1, 1+n)$, that is the real number

$$\text{abslope}(-1, 1+n) = 1 + n,$$

this real number is strictly greater than 1 since the factor $n$ is strictly positive.

In the following figure we see the payoff space of the coopetitive game $G$ as the trace of the path of payoff spaces corresponging to the path of normal form games $G_i$. This path of payoff spaces is nothing but a path of translations of a rectangle, namely the payoff space of the “initial” game $G(0)$. 
Figure 2. The payoff space of the coopetitive game $G$.

Thus the collective payoff function

$$g + e$$

of the game is not constant on the coopetitive Pareto boundary and, therefore, the game implies possibility of growth, because the minimum value of the aggregate payoff $g + e$ is attained exactly at the supremum of the game $G(0)$.

4.1.7 Compromise solutions and the coopetitive compromise

The Nash bargaining solution and the Kalai-Smorodinsky bargaining
solution, with respect to the infimum of the Pareto boundary, coincide with the medium point of the segment \([P', Q']\)^{14}.

Transferable utility solution. In this coopetitive context it is more convenient to adopt a transferable utility solution: indeed the point of maximum collective gain is the point

\[ Q' = (1/2, 2 + m + n), \]

that is the supremum of the game \(G(1)\).

Thus we have to propose a new kind of coopetitive compromise solution to “share the pie fairly”.

We proceed as it follows (in the case \(m = 0\)):

First, we consider the coopetitive rectangle \(R\) having:

\begin{enumerate}
  \item two sides on the straight lines of equations
  \[ Y = 1 \text{ and } Y = 2 + n; \]
  \item two vertices in \((1/2, 1)\) and \((1/2, 2 + n)\);
  \item the diagonal on the straight line \(S\) of equation
  \[ Y + X = 2.5 + n. \]
\end{enumerate}

^14 The classic Kalai-Smorodinsky solution, that we applied in both models, coincides with the solution on the coopetitive Nash path; this result allows us to provide a construct of coopetition which is only “weakly” cooperative, in the sense that it is not necessary to cooperate at every stage of the decision process.
Second, we consider the segment $S'$ of vertices $(3/2,1)$ (supremum of the game $G(0)$) and the supremum of the rectangle $R$, the point $(3/2+n,2+n)$, this segment is the set

$$S' = (3/2,1) + (n,1+n)[0,1].$$

Third, our best payoff coopetitive compromise $K$ is the intersection of the two segments $S$ and $S'$.

This compromise payoff $K$ represents a win-win solution with respect to the initial supremum $(3/2,1)$, since $K$ is a payoff strongly greater than the initial supremum.
Remark. In some way, the choices of the coopetitive bargaining interval (rectangle) and of the coopetitive solution are the only reasonable. Indeed:

1) the constraint where we should search for a bargaining solution is the transferable utility Pareto boundary (segment of the line with equation \( Y + X = 2.5 + n \) determined by the positive cone of the plane);

2) the possible bargaining Greece’s outcomes should belong to the interval \([1, 2+n]\), with end-points the minimum and maximum value of the Greece’s payoff function in the coopetitive game;
3) points 2 and 3 determine the coopetitive bargaining rectangle;
4) the solution we propose is nothing but a best compromise solution
   with utopia point the supremum of this rectangle and threat point
   the maximum of the initial game G(0).

4.2 Second coopetitive model

*Let us consider now that a fraction ax of Germany consumption comes from*
*consumption of Greek goods, apart from the given amount of Greek exports*
*that Germany has already determined through an ex ante agreement with*
*Greece (z).*

**Payoff function of Greece**

\[ e(x,y,z) = by + z + ax \]

**Payoff function of the game**

\[ p(x,y,z) = (x + 1/(x+1) - z, ax + by + cz) =
   = (x + 1/(x+1), ax + by) + z (-1, c) \]

with a, x, y, z in [0,1] and b, c > 1.

Similarly to the previous coopetitive model, but through a more complex
procedure, we deduce that the Pareto boundary of the new coopetitive game
G = (p, >) – in the payoff space - is the above segment [P', Q'] translated by
the vector (0, a).

*The Nash bargaining solution and the Kalai-Smorodinsky bargaining solution,*
with respect to the infimum of the Pareto boundary, coincide with
the medium point of the segment

\[ [P', Q'] + (0, a), \]

which is the optimum of the game G_{1/2}. 
Analogously the coopetitive compromise solution of this new game is the that of the first model translated by the vector \((0,a)\).

**Conclusions**

This contribution has tried to provide, through a game theory model of coopetition, feasible solutions in a cooperative perspective to the problems of Greek economy after its crisis. In particular, it has focused on stability and growth as the primary goals, which should drive Greece and Germany economic policy with their positive effects on the whole euro area.

The idea underlying the present work was that of contributing to expand the set of macroeconomic policy tools available to face the economic crisis in Greece, and more generally in the European Monetary Union, where a cooperative attitude should prevail.

In this work have underlined two aspects which emerged from the crisis. First, the necessity of government budget consolidation of Greece; second, the opportunity to re-balance the trade surplus of Germany with respect to Greece (and also with respect to the other euro countries that have a deficit trade balance).

By means of two coopetitive models derived by an original general analytical framework of coopetition, we have showed the strategies that could bring to feasible solutions in a cooperative perspective for Germany and Greece, where these feasible solutions aim at offering a win-win outcome for both countries, letting them to share the pie fairly within a growth path represented by a non-zero sum game. In fact, our analytical results allow us to find a “fair” amount of Greek exports which Germany must import, in order to re-balance the trade surplus of Germany, as well as the investments necessary to improve the Greek economy, thus contributing to growth and to the stability of the Greek economy and, indirectly, of the whole European Monetary Union.

Finally, a remarkable analytical result of our work consists in the determination of the win-win solution by a new selection method on the transferable utility Pareto boundary of the coopetitive game.
References


