On Mental Transformations

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Abstract

This paper presents an alternative interpretation of the experimental data published by Kahneman and Tversky in their 1992 paper describing Cumulative Prospect Theory. It was assumed that mental transformations such as mental adaptation, prospect scaling, and logarithmic perception should be considered when analyzing the experimental data. This led to the design of a solution that did not require the probability weighting function. The double S-type function obtained (the decision utility) resembles the utility curve specified by the Markowitz hypothesis (1952) and substitutes the fourfold pattern of risk attitudes introduced by Cumulative Prospect Theory.

Keywords: *Prospect/Cumulative Prospect Theory, Markowitz Utility Hypothesis, Mental Processes, Adaptation & Attention Focus, Aspiration Level*

JEL classification: *D03, D81, C91*

1 Introduction

The first approach to describe economic behavior in terms of utility was proposed by Daniel Bernoulli as early as 1738. However, it was von Neumann and Morgenstern (1944) who proved that rational decision making can be described using a utility function. Friedman and Savage (1948) argued that the curvature of the utility function varies in order to explain why people buy lottery tickets and insurances. Markowitz (1952) was the first to consider the shape of the utility function around a "customary" level of wealth.

A growing body of experimental data indicated, however, that no utility function could satisfactorily explain human behavior. The best known counterexample was the Allais paradox

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(1953). This led to the creation of several theories collectively referred to as Non-Expected Utility Theories. Cumulative Prospect Theory (Kahneman, Tversky, 1979, 1992) gave rise to the concepts of the value function and the probability weighting function. The value function is supposed to evidence risk aversion for gain prospects and risk seeking for loss prospects, as well as a general aversion to loss. The probability weighting function is meant to show the non-linear transformation of probabilities when making decisions. This, in turn, is meant to explain people's willingness to participate in lotteries as well as their tendency towards less risky investments in the case of average probabilities (Camerer and Ho, 1994; Wu and Gonzalez, 1996, 1999; Prelec, 1998; Tversky and Wakker, 1995).

The theory was successfully used to explain several phenomena but has also met with criticism. Nwogugu (2006) has compiled a large collection of objections and draws on a bibliography of no fewer than 131 titles to support his claims. The author asserts that Prospect Theory was derived using improper methods and calculations and that it is not consonant with natural mental processes. Shu (1995) shows that it is wrong to assume the existence of probability weights. Neilson and Stowe (2002) demonstrate that Cumulative Prospect Theory cannot simultaneously explain participation in lotteries and the original Allais paradox. Blavatskyy (2005) shows that the theory does not explain the St. Petersburg paradox. Levy and Levy (2002) state that their experimental results negate Prospect Theory and confirm the Markowitz hypothesis.

The present paper, too, is critical of Prospect Theory. However, it is not criticizing individual components or individual methodological assumptions, but is rather focused on analyzing the entire process of how the end results of the 1992 study were obtained from the experimental data. It has been stated that any analysis of the experimental data should include mental transformations such mental adaptation, prospect scaling, and logarithmic perception (Point 2). This assumption finds its explanation in psychology, in particular cognitive psychology, and in research at the sensory and neuronal levels. On the other hand, probability weighting should be excluded from this list as it is a mathematical, rather than psychological, concept.

On the basis of the assumptions stated above and using exactly the same experimental data that were used to derive Cumulative Prospect Theory, a solution was obtained without the use of the probability weighting function (Point 3). This solution describes a direct relationship between probability and the relative certainty equivalent. The resulting curve (named the decision utility function) has a double S-shape (Point 4) consistent with the Markowitz hypothesis (1952)

(Point 5). More importantly, the decision utility function explains how people's attitude towards risk depends on their aspiration levels. The explanation of risk attitudes given by the convex-concave-convex-concave shape of the decision utility function substitutes the fourfold pattern introduced by CPT.

The essential feature of the decision utility model is that lotteries are valued and compared by their certainty equivalents rather than by hypothetical "utils" as postulated by Expected Utility and Prospect Theory (Point 6). A logarithmic perception of financial stimuli should be implemented for wider outcome ranges (Point 7). This is done using a perception function. As the solution presented is based on well documented mental transformations, the results provide a basis of negating Prospect Theory as a theory that correctly describes how decisions are made under conditions of risk.

2 Review of Mental Transformations

2.1 Transformation of Probabilities

That perception of probabilities is distorted is simultaneously one of the key assumptions and key results of Prospect Theory. The concept of decision weights was introduced into the first version of Prospect Theory in 1979. Even at that early stage, Kahneman and Tversky were stating that decision weights were not probabilities and did not comply with the axioms of probability. This led to serious mathematical objections (failure to comply with the First Order Stochastic Dominance). As a result, Rank-Dependent Expected Utility Theory (Quiggin, 1982) was developed to remedy the shortcomings of its predecessor. The key concepts of that theory were later adopted by Cumulative Prospect Theory (Tversky, Kahneman, 1992). The axiomatization is based on pretty complex topological models and Choquet integrals (Schmeidler, 1989, Wakker 1989, 1990; Kahneman and Tversky, 1992 and appendix to their publication).

It is important to note that Kahneman and Tversky (1979) distinguish *overestimation* (often encountered when assessing the probability of rare events) and *overweighting* (as a feature of decision weights). The latter phenomenon lacks psychological justification to the extent that the former has it (for instance by dint of insufficient knowledge). It is difficult to explain in psychological terms how a decision regarding an event whose probability is known seems to assume a different probability value. Furthermore no mechanism was posited to explain why this effect of probability transformation only manifests itself at the moment a decision is made. A failure to distinguish between *overestimation* (which can be referred to as a kind of *subjective* view of events whose probabilities are not known) and *overweighting* (an artificial concept to explain the results of experiments regarding events whose probabilities are known) leads to the commonly accepted view that the probability weighting function has a profound psychological justification.

Despite the extensive research that has been conducted since Prospect Theory was introduced, the basic question, however, remains unanswered: *Why and how do people overweight small probabilities and underweight high probabilities when the probabilities are known?* Until an answer to this question is given, and a psychological explanation of this phenomenon presented, probability weighting cannot be regarded as the psychological explanation of the decision making behaviors observed in lottery experiments.

An important evidence against the "psychology" of the probability weighting concept was given by Birnbaum (2004). In several studies subjects were assigned to receive different prospect formats: text displays, pie charts, tickets, list and semi-split lists, marbles in urns, and finally the decumulative probabilities. The last case is the way how probabilities are used in Cumulative Prospect Theory. If the theory were psychologically true, this format should facilitate calculations and would result in lower percentage of violations. It turned out, however, that violations of stochastic dominance were the greatest in the decumulative probability condition. This shows that this representation is not a natural way of how people use probabilities in risky choice decisions. Birnbaum concludes his research: "*The weighting functions of RDU/RSDU/CPT are merely artifacts of wrong theory*".

2.2 Mental Adaptation

Evolutionary adaptation was first described by British natural theologians John Ray (1627–1705) and William Paley (1743–1805). The theory was later refined by Charles Darwin (1809–82). Peter Medawar, winner of the Nobel Prize for Medicine and Physiology in 1960, describes the term as "*a process allowing organisms to change to become better suited for survival and reproduction in their given habitat"*. The Oxford Dictionary of Science defines adaptation as "*any change in the structure or functioning of an organism that makes it better suited to its environment"*. More definitions can be found in Rappaport (1971) and Williams (1966). Summarizing *"adaptation can refer to a trait that confers some fitness on an animal, but it also represents the process by which that trait has come about"* (Greenberg, 2010).

"*Neural or sensory adaptation is a change over time in the responsiveness of the sensory system to a constant stimulus. More generally, the term refers to a temporary change of the neural response to a stimulus as the result of preceding stimulation*". This Wikipedia definition is close to those met in academic texts: "*Adaptation in the context of sensation refers to the fact that a prolonged and uniform sensory stimulus eventually ceases to give rise to a sensory message*" (Medawar, 1983, more in Laughlin, 1989 and Hildebrandt, 2010). The best example of neural adaptation is eye adaptation. Similar mechanisms are well attested for smell, temperature, taste, pain and touch (Gregory, Colman, 1995, Medawar, 1983).

The definitions presented so far all assume that it is the living organism which adapts to changing environmental conditions. However, from the standpoint of a human being, adaptation may be seen as a process of changing the external world to suit its requirements. This was best expressed by Leakey (1981) as follows: *"Animals adapt themselves to environment, hominids adapt environment to themselves using tools, language and complex cooperative social structures*". Mutual human - environment interaction was described by the famous Swiss psychologist Jean Piaget, who *"considers in fact intelligence rising from mental adaptation, where the adaptation is the equilibration of the action of an organism on the environment (assimilation) and of the action of the environment on the organism (accommodation)"* (Maniezzo, Roffilli, 2005).

In the author's opinion, the term "mental adaptation" is best expressed as *"the state of not thinking about certain phenomena"*. This definition follows the Sulavik (1997) paper on mental adaptation to death in the case of professional rescuers, although it can easily be extended to cover many other situations like stress, major illness, bereavement, financial loss, immigration (Jasinskaja-Lahti, 2006), disasters (Leon, 2004), and even space travel (NASA). It should be borne in mind that mental adaptation occurs in positive situations as well – financial windfalls, professional achievements, falling in love etc. "Hedonic treadmill" is another term for mental adaptation coined by Brickman and Campbell (1971) *"to describe the now widely accepted notion that though people continue to accrue experiences and objects that make them happy – or unhappy – their overall level of well-being tends to remain fairly static."* (Mochon et al., 2008, also Kahneman, 1999). There are several other meanings of adaptation encountered in the literature (e.g. social adaptation). A wide coverage of hedonic adaptation examples is given by Frederick and Loewenstein (1999). Nevertheless, most of them have a common feature, viz. they signify a shift of either the organism's structure or its perception system to a new level. As a result, people (and animals) become better suited to external conditions, do not sense any more external stimuli, and cease to think about certain phenomena.

2.3 Prospect Scaling

Prospect Scaling is of key significance for deriving the solution presented in this study. The springboard for discussion is the Weber law[3], one of the fundamental laws of psychophysics. The law states that the Just Noticeable Difference (JND) is a constant proportion of the initial stimulus magnitude. It follows from the Weber law that the same change in stimulus (for instance 0.2 kg) can be strongly felt, slightly noticed or not perceived at all depending on the magnitude of the initial stimulus. It further follows that an unambiguous and absolute perception level of a specific stimulus change cannot be determined, as this depends on the situational context.

This applies equally to financial stimuli. The human sensory system adapts itself to financial quantities, just as it does to physical ones. This means that when looking at financial prospects (projects, investments, lotteries etc.), the reference value (size of the investment, major lottery prize) becomes a value of reference in the entire mental process, causing an absolute amount of money (say 10 USD) to be relevant (for instance when shopping) or irrelevant (when buying a house), i.e. depending on the context. This conclusion constitutes a fundamental difference to Prospect Theory, which regards profits and losses in absolute terms, and tries to draw a value function as a function of absolute amounts of money.

Although not implemented by Prospect Theory, the phenomenon is well known to behavioral researchers. Thaler (1999) considers the example that "*most people will travel to save the $5 when the item costs $15 but not when it costs $125*". Thaler (1980) proposes that "*search for any purchase will continue until the expected amount saved as a proportion of the total price equals some critical value. This hypothesis is a simple application of the Weber-Fechner law of psychophysics*". Kahneman and Tversky (1985) define minimal, topical, and comprehensive accounts, where: *"a topical account relates the consequences of possible choices to a reference level that is determined by the context within which the decision arises*". They suggest *"that people spontaneously frame decisions in terms of topical account"* and that *"the topical organization of mental accounts leads people to evaluate gains and losses in <u>relative</u> rather than in absolute terms"* (emphasis added). Baltussen, Post and Van den Assem (2008) used an extensive sample of choic-

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[3] Not to be confused with the Weber-Fechner Law discussed in 2.4.

es from ten different editions of the high stakes TV game show "*Deal or No Deal*": *"In each sample, contestants respond in a similar way to the stakes relative to their initial level, even though the initial level differs widely across the various editions. Amounts therefore appear to be primarily evaluated relative to a subjective frame of reference rather than in terms of their absolute monetary value"*.

The mechanism responsible for this mental transformation is attention – one of the most thoroughly examined concepts in cognitive psychology. According to one definition, attention is the process of selectively concentrating on a single perceived object, source of stimulation, or topic from among the many available options (Nęcka, 2007). The existence of attention is indispensable on account of a living organism's need to adapt to the demands of the environment (Broadbent, 1958) and on account of the finite ability of the brain to process information (Duncan, Humphreys, 1989). Several models of attention division are discussed, especially in relation to Focused Attention. The entire mechanism can be explained by such aspects of attention as Selection and Gain (Amplification) Control, the existence of which is evidenced by attention research at the neuronal level. Hillyard et al. (1998) state that attention has a gain (amplification) control character which aims to increase the signal to noise ratio of the stimuli on which attention is focused. The signal of most interest to the brain is maintained at a stable and optimal level as a result. It may well be assumed that the amplification control mechanism operates at a higher mental level as well. This leads to problems differing in scale being perceived as equally significant when attention is focused. Very clearly, the mathematical equivalent of amplification is homothety (scaling).

The arguments cited indicate that the attention focused on a specific payment in the conducted experiments seems to be a natural effect that has to be factored into any analysis of the results. This is especially the case under experimental conditions as those surveyed are remunerated for their participation; it means they are paid to focus their whole attention on the analyzed problems. The assumption that the value of a prospect payment becomes a reference value in the conducted experiments leads to a completely different solution than that which Prospect Theory proposes.

2.4 Logarithmic Perception

Logarithmic perception is the last mental transformation necessary to derive the results presented in the final part of the study. Here, the reference point for discussion is also a fundamental psychophysical law, viz. the Weber-Fechner law. "*Fechner assumed that JNDs correspond to equal increments in subjective intensity and that JNDs are proportional to the physical variable being studied (Weber's law). These assumptions led to the famous, logarithmic Weber-Fechner law of subjective intensity*" (Johnson, 2002). Hearing, as measured using the decibel scale, is an example of this sort of perception. The law applies to many other stimuli. Zauberman et al. (2008) argue that "*error in time estimation following the Weber-Fechner law can explain both sub-additive and hyperbolic discounting*".

Bernoulli stated that the St. Petersburg Paradox could be explained using logarithmic utility as early as 1738. Logarithmic utility expresses the diminishing marginal value of money. Interestingly, Sinn (2002) proves that "*expected utility maximization with logarithmic utility is a dominant preference in the biological selection process in the sense that a population following any other preference will disappear as time goes to infinity*".

The Weber-Fechner law, however, has been severely criticized by Stevens (1957), who claimed that the power function better describes human perception. The power function was included in Prospect Theory to describe the value function. Surprisingly, the difference in approach turns out to be insignificant as both functions (logarithmic and power) have an almost identical shape for low x-coordinate values.[4] This leads to the conclusion that the perception of monetary amounts used in Kahneman and Tversky's experiments could be equally well described using a logarithmic curve. Some arguments in favor of a logarithmic perception are provided by other results presented in CPT, which states that mixed prospects are accepted when gains are at least twice as high as losses. This effect can be easily explained by noticing that in logarithmic terms, a 100% profit corresponds to a 50% loss. More arguments in support of a logarithmic, rather than a power, perception of monetary amounts are given in Point 7 of this paper.

3 Solution Using Mental Adaptation & Prospect Scaling Transformations

This section contains the alternative analysis of the experimental data presented by Kahneman and Tversky in their 1992 paper. This analysis is based on the assumption that Mental Adaptation and Prospect Scaling Transformations should be considered when analyzing the ex-

[4] Within the range [0, 0.6], $x^{0.88}/1.34$ is the best approximation of the $ln(1+x)$ function using a power function. The coefficient 0.88 is exactly the same as the power coefficient of the value function in Prospect Theory.

perimental data. During the experiment conducted by Kahneman and Tversky, certainty equivalents CE were collected for the prospects of payment $\$P_{min}$ with probability 1 - p or payment $\$P_{max}$ with probability p, where:

$$|P_{min}| < |P_{max}| \tag{3.1}$$

The payment $\$P_{min}$ should be interpreted as the riskless component. The experimental results are presented in Table 3.3 of the original CPT publication (1992). It is assumed that there is a function F such that:

$$CE = F(P_{min}, P_{max}, p) \tag{3.2}$$

The variables CE' and P_{max}' are now introduced to account for the mental adaptation process. These are a P_{min} translation of CE and P:

$$CE' = CE - P_{min} \tag{3.3}$$

$$P' = P_{max} - P_{min} \tag{3.4}$$

If $P_{min} = 0$, we refer to the prospect as having no riskless component and then $CE' = CE$ and $P' = P_{max}$. Introducing these new variables presupposes the existence of a function G such that:

$$CE' = G(P', p) \tag{3.5}$$

At this point (3.5) is transformed in such a way that probability p, and not CE', becomes the value to be determined. Due to the fact that CE' is monotonic with respect to p, it may be assumed that there is an inverse function H such that:

$$p = H(CE', P') \tag{3.6}$$

In order to take Prospect Scaling into account, it is assumed that the value of payment P' becomes the reference value for the certainty equivalent CE' and that the equivalent values are scaled by a coefficient $1/P'$. As a result, a variable $r = CE' / P'$ is introduced as the relative certainty equivalent with a value in the range [0,1]. This also supports the existence of the following D function defined over the range [0,1]:

$$p = D(CE' / P') = D(r) \tag{3.7}$$

For example, for the specific values listed in Table 3.3 of Kahneman and Tversky's paper, the relationships $D(9/50) = 0.10$, $D(21/50) = 0.50$, and $D(37/50) = 0.90$ are obtained for the prospect (0, 50), and the relationships $D(14/100) = 0.05$, $D(25/100) = 0.25$ are obtained for the prospect (0, 100). For the prospect with the riskless component (50, 150), the relationships $D((64-50)/100) = D(14/100) = 0.05$, $D((72.5-50)/100) = D(22.5/100) = 0.25$, and $D((86-50)/100) = D(36/100) = 0.5$, are obtained after the Mental Adaptation Transformation.

The obtained values are plotted on the graph $p = D(r)$ and approximated using the least squares method with the assistance of the Cumulative Beta Distribution $I_r(\alpha, \beta)$ (i.e. regularized incomplete beta function). This particular function was selected because it is defined in the domain [0, 1] and because of the extraordinary flexibility the two parameters α and β give its shape. Approximations were made separately for the loss (P < 0)[5] and gain (P > 0) prospects. The results are presented in Figure 3.1.

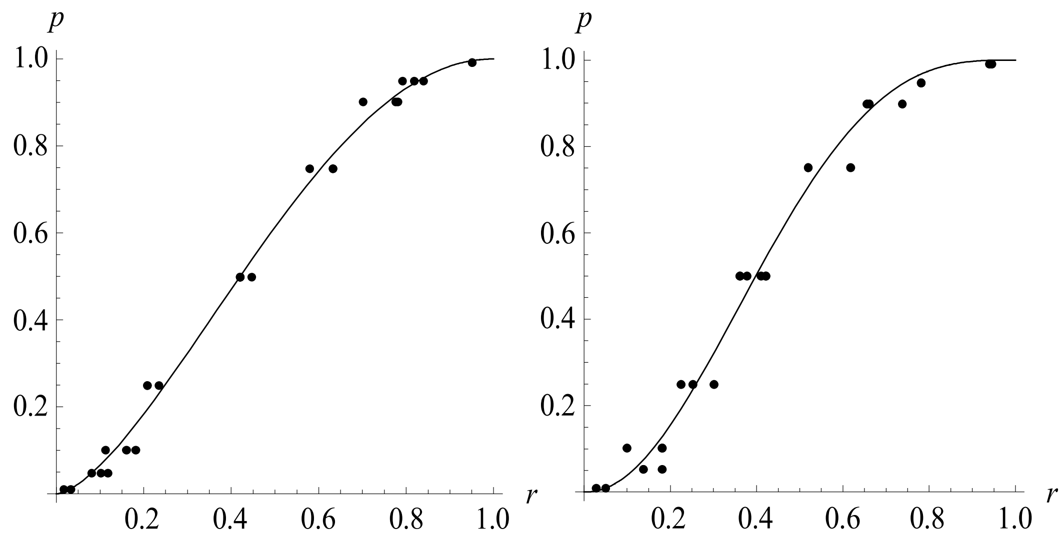

Figure 3.1 Transformed experimental points and approximation $p = D(r)$ using cumulative beta distribution function for loss prospects (left) and for gain prospects (right).

The approximations obtained for the function $p = D(r)$ for loss and gain prospects allow the following conclusions to be drawn:

1. The function $p = D(r)$ is S-shaped for both loss and gain prospects.
2. The respective values of the parameters α and β are 2.03 and 2.83 for gain prospects and 1.60 and 2.09 for loss prospects. The disparity between the parameters α and β in both cases confirms the asymmetry of the function $D(r)$ with respect to the center point $(p, r) = (1/2, 1/2)$.
3. The intersection of the approximation functions $p = D(r)$ with the straight line $p = r$ occurs when r has a value of approximately 0.27 for gains and 0.25 for losses. This value is called the aspiration level, as (in case of gains) the risk seeking attitude is present for lower values of the

[5] It should be noted that for the loss prospects, the value of relative certainty equivalent r is also positive, as *CE'* and *P'* are both negative in this case.

relative outcome r, and risk aversion is present for greater values of r. This implies a change of attitude to risk at the aspiration level, which is in accordance with generally accepted interpretations of this term. The pattern for losses is reversed.

Assuming Mental Adaptation and Prospect Scaling Transformation thus led to a different solution than that presented by Prospect Theory. The entire description has been reduced to the relationship:

$$p = D(r) \tag{3.8}$$

where

$$r = \frac{CE - P_{min}}{P_{max} - P_{min}} \tag{3.9}$$

The function D is called here "decision utility" because it describes how decisions are made under conditions of risk. The value function and the probability weighting function have disappeared altogether as they are not needed to describe the experimental results. Decision utility is defined in the [0, 1] interval. Transformation (3.9) combines Mental Adaptation and Prospect Scaling, and normalizes the certainty equivalent within the [0, 1] interval. Therefore, it can be named "framing" for short. This notion of framing is slightly different than the one normally discussed in the Prospect Theory context. In the latter case, framing is usually understood as considering the problem as a prospective gain or loss. In the present case, the problem is bounded from both sides, which is closer to the dictionary definition of frames. According to the present model, problems are always framed before making decisions.

Please note that the decision utility function was derived using linearly assessed outcomes. Despite this, the curve approximation is very good for lotteries in the ranges from \$50 to \$400. However, a nonlinear perception utility function might be needed for wider outcome ranges. This is discussed in Point 7.

4 Combining Gains and Losses

The solutions obtained so far comprise two $p = D(r)$ functions with one describing losses, the other gains. The two functions need to be scaled before they can be used together. The simplest assumption has been adopted with the use of the loss aversion coefficient λ. This is similar to the Prospect Theory approach when defining the value function, and allows both parts of decision utility to be presented on a single graph (Figure 4.1). The derivation of the loss aversion coefficient λ is presented later in this paper (see Point 6.3).

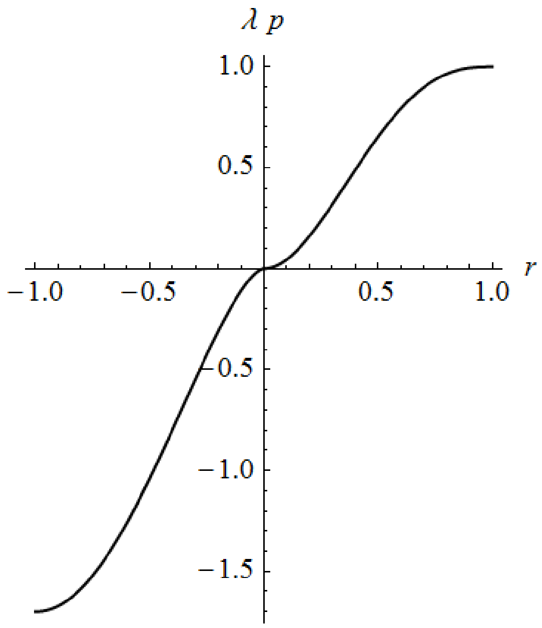

Figure 4.1 Functions $D_l(r)$ for losses and $D_g(r)$ for gains presented together on a single graph. Function $D_l(r)$ is multiplied by the negative loss aversion coefficient λ.

This decision utility curve presents the sum total of all the knowledge that has come out of Prospect Theory and its cumulative version.

1. The fourfold pattern of risk attitudes, which was presented by CPT and confirmed in other studies, is evident:

   a). in case of gain prospects, the curve is convex for probabilities below 27% (corresponding to risk taking), and becomes concave for probabilities above 27% (corresponding to risk aversion);

   b). in case of loss prospects, the curve is concave for probabilities below 25% (corresponding to risk aversion), and becomes convex for probabilities above 25% (corresponding to risk seeking).

2. The convex-concave-convex-concave shape of the decision utility substitutes therefore the fourfold pattern of risk attitudes described by CPT.

3. The function's more linear shape for loss prospects confirms the results of other studies that people's attitude to risk for losses is rather neutral in nature[6].

4. Both parts of the curve (for loss and gain prospects) describe the results of experiments without having to resort to the probability weighting function.

5. Both parts of the curve are scaled, which means that mixed prospects can also be analyzed.

[6] See Wakker (2003) for reference, who confirms that the pattern for losses is less clear than in the case of gains.

5 The Markowitz Utility Function Hypothesis

In 1952, Markowitz published an article "The Utility of Wealth" presenting his hypothesis on the shape of the utility function. While this article was known to Kahneman and Tversky, they believed that neither this nor any other utility function could explain certain psychological experiments. This led to the development of Prospect Theory as an alternative to classical economic theories based on utility functions. That the decision utility curve so closely resembles the curve presented in the Markowitz article (Figure 5.1) is highly surprising given the result was obtained using the same experimental data used to derive Cumulative Prospect Theory.

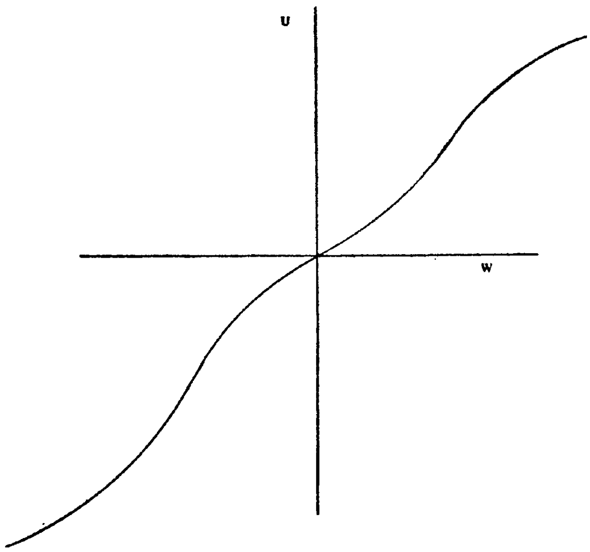

Figure 5.1 The shape of the utility function according to the Markowitz hypothesis of 1952.

Markowitz specified the utility function as follows: "*The utility function has three inflection points. The middle inflection point is defined to be at the "customary" level of wealth. The first inflection point is below customary wealth and the third inflection point is above it. The distance between the inflection points is a non-decreasing function of wealth. The curve is monotonically increasing but bounded from above and from below; it is first concave, then convex, then concave, and finally convex. We may also assume that* $|U(-X)| > U(X)$*,* $X > o$ *(where* $X = o$ *is customary wealth)*[7] ".

It is clear that all but one of the requirements of the utility curve expressed by Markowitz in his hypothesis are met by the curve presented in Figure 4.1. The decision utility curve has three

[7] One more requirement was important for Markowitz: in the case of recent windfall gains or losses the second inflection point may, temporarily, deviate from present wealth. This requirement does not influence the shape of the curve but is important when considering the dynamics of people's behavior.

inflection points right where Markowitz predicted they would be. The function is monotonically increasing and is limited from the top and from the bottom. Convexities and concavities occur in the order assumed by Markowitz. The condition related to the function value for X values having opposite signs is also met. The only condition not met is that the distances between the inflection points depend on people's wealth. Markowitz noted: "*If the chooser were rather rich, my guess is that he would act as if his first and third inflection points were farther from the origin. Conversely, if the chooser were rather poor, I should expect him to act as if his first and third inflection points were closer to the origin*". In the Markowitz hypothesis the position of inflection points changes because the *w* (wealth)-axis is expressed in absolute terms. This differs from the decision utility curve, where the *r*-axis is expressed relative to the size of the prospect.

There is no reason to regard this approach as being in any way anomalous. People commonly say "I have gained 15% on my stock investments" rather than "I have gained 5% *of my wealth* on my stock investments". It is clear enough that the former sentence assumes the value of the stock investment as the reference for gain/losses considerations. Moreover, according to Thaler (1985), people keep mentally separate accounts, so that investments and expenditures are considered as separate parts rather than as a whole. As a result people typically consider "I have lost 5% on my house but I have gained 15% on stocks" despite the fact that the absolute values of stock and house investments may differ substantially. It follows that the decision utility applies for each separate account, albeit with different reference values established by the attention focus. This may mean that people could be risk seeking and risk averse at the same time depending on the status and prospects of each account.

Markowitz's assumption that the shape of the utility curve corresponds with the value of wealth precluded his curve (however tempting its shape) from being able to explain experiments on financial payments which were not directly related to the wealth of the people being studied. This is what led Kahneman and Tversky to reject the Markowitz hypothesis and develop Prospect Theory. The result presented here, however, may signal a return to an approach based on the utility-like function and lead to a negation of Prospect Theory. Accepting that gains and losses need not be considered in relation to wealth, but to any other value depending on where a person's attention is focused, is all that it would take to come back to this earlier concept. The payoff is a simpler and more accurate description of people's behavior.

6 Lottery Valuation

6.1 Using Certainty Equivalents

The essential feature of the decision utility model is that lotteries are valued and compared by their certainty equivalents rather than by hypothetical "utils" as postulated by Expected Utility and Prospect Theory. As presented in Point 3, the relative certainty equivalent r is directly transformed into probability p (and vice versa). This means that in order to determine the certainty equivalent CE for probability p, the value of r need only be read directly from the graph and multiplied by the value of payment P'. For example, $r = 0.75$ for $p = 0.95$. Hence, $CE' = 75$ for $P' = 100$ (the value obtained experimentally was 78). In case of prospects with riskless components, e.g. (50, 150), the riskless component P_{min} (here 50) increases the value of the certainty equivalent: $CE = CE' + P_{min} = 75 + 50 = 125$ (the value 128 was obtained in the experiment).

This can be presented in a more general way. The certainty equivalent CE is derived from (3.8) and (3.9):

$$CE = P_{min} + D^{-1}(p)(P_{max} - P_{min}) \tag{6.1}$$

For $P_{min} = 0$, (6.1) simplifies to:

$$CE = P_{max} D^{-1}(p) \tag{6.2}$$

Using certainty equivalents to value and compare lotteries is a very intuitive method: lotteries are ordered according to the minimum payment a person is ready to accept for giving up playing. This method allows a cardinal comparison: the difference between lotteries having respective certainty equivalents of \$45, \$50, and \$100 is readily visible.

6.2 Multi-Outcome Lotteries

The procedure adopted here is very similar to the one used for multi-outcome lotteries except that the equivalent probability has to be calculated first. It is assumed that for each lottery having more than two outcomes, there exists an equivalent two-outcome lottery whose "equivalent" probability p_{eq} can easily be derived (Kontek, 2010):

$$p_{eq} = \sum_{i=1}^{n} p_i D(r_i) \tag{6.3}$$

The equivalent probability p_{eq} is then applied in (6.1) or (6.2) instead of the single probability p to determine the prospect certainty equivalent.

Please note the strong resemblance of the equivalent probability formula (6.3) to the Expected Utility valuation. In fact, the decision utility model follows Expected Utility Theory with a transformed outcome domain. Decision utility is expressed in terms of probability and does not require any hypothetical "utils" to describe behavior. Once accustomed to this seemingly strange notion, everything can be considered at the basic probability theory level.

A more detailed analysis of the model for multi-outcome lotteries is presented in Kontek (2010). It is enough to state here that the decision utility model presents similar results to Cumulative Prospect Theory in the case of two-outcome lotteries. However, the results differ for multi-outcome lotteries. Most importantly, the decision utility model avoids stochastic dominance violations, which is the case with Prospect Theory. It can be shown that the CPT solution to this problem not only leads to a much more complex form of the theory for multi-outcome lotteries but also to some "strange" valuations. These predictions are largely inexplicable and call the correctness and applicability of this theory into question. The result obtained using the decision utility function is not similarly disadvantaged. This is because it uses the classical notion of probability.

6.3 Mixed Lotteries

The valuation of mixed prospects is a matter of great concern. Gain-loss separability is a basic premise underlying Prospect Theory. This axiom requires that preferences for gains be independent of preferences for losses and that the valuation of a mixed lottery be the sum of the valuations of the gain and loss portions of that lottery. Unfortunately, this is more a theoretical assumption than an experimentally confirmed fact. Kahneman and Tversky (1979, 1992) generally examined gain and loss prospects separately. The only case where mixed lotteries were examined was limited to a probability of 0.5.

Gain-loss separability has recently become a subject of criticism, as its violations have been evidenced (Wu, Markle, 2008; Birnbaum, Bahra, 2007). The authors have presented several explanations of the phenomenon based on Prospect Theory and the TAX model. Kontek (2011) also put forward a hypothesis, which explains the case by assuming a perception utility with a very different shape and properties than the Prospect Theory value function (see Point 7). While this re-opens the subject of mixed prospect valuation, it is definitely too soon to give a definite answer. The decision utility function has been derived in this paper separately for gains and losses based on Tversky and Kahneman's data, and no decision utility function can be presented for mixed prospects without any further data sets or new experiments.

At this stage, a solution *assuming* separability is presented and this works well in many cases. This is because people usually first evaluate prospective gains and losses separately to assess how much they can expect to win and lose when considering mixed prospects. Their decision will then be a trade-off between these two values. This method presupposes that positive outcomes are evaluated separately as a gain prospect and negative outcomes as a loss prospect. These two valuations determine the respective certainty equivalents of $CE_g > 0$ for a gain portion and $CE_l < 0$ for a loss portion. The two certainty equivalent values are then compared and integrated into the certainty equivalent of the mixed prospect.

This requires the value of the loss aversion coefficient *λ*. Tversky and Kahneman (1992) conducted additional experiments in order to determine this value. The results, presented in Table 3.6 of the original publication, indicate that the mixed prospects are accepted if the profit is at least twice as great as the loss. An exact ratio of 2.07, as the mean value of *Θ* resulting from problems 1-6, is assumed for further calculations. The certainty equivalent of the loss outcome -*x* should thus correspond to the certainty equivalent of the gain outcome of 2.07*x*. This can be presented using (6.2):

$$-\lambda \, x \, D_l^{-1}(0.5) = 2.07 \, x \, D_g^{-1}(0.5) \qquad (6.4)$$

where D_l and D_g denote decision utility functions for losses and gains respectively. This leads to

$$\lambda = -1.99 \approx -2 \qquad (6.5)$$

It follows that a mixed prospect has a value of 0 for $CE_g = -2CE_l$. The certainty equivalent *CE* of a mixed prospect can thus be derived as:

$$CE = CE_g + 2CE_l \quad \text{if } CE_g \geq -2CE_l \qquad (6.6)$$

or:

$$CE = CE_g / 2 + CE_l \quad \text{if } CE_g < -2CE_l \qquad (6.7)$$

The approach proposed for evaluating mixed prospects easily explains the WTA-WTP disparity in risky choices. It has been stated in many experiments that both the 'willingness-to-accept' and 'willingness-to-pay' values differ both when riskless and risky options are considered. For example, subjects with a commodity to sell invariably demand a price substantially in excess of that which subjects in a position to purchase it are prepared to pay (Kahneman, Knetsch and Thaler (1990). The disparity between the two values has also been observed for risky options (Schmidt, Traub, 2009). The experimentally determined WTA/WTP ratio assumes a value in the neighborhood of 2 for a wide range of probabilities.

In this case, WTA may be considered the lottery certainty equivalent. The risk that the lottery may result in a zero outcome has to be taken into account. In this case, the investment will be negative and equal in value to the price paid (WTP). Following (6.6), both values should then satisfy the condition:

$$0 = \mathrm{WTA} + 2\,\mathrm{WTP} \tag{6.8}$$

Thus

$$\mathrm{WTA} / \mathrm{WTP} = -2 \tag{6.9}$$

which result is in accordance with the experimental data. The WTA-WTP disparity of risky options thus finds an interpretation in the phenomenon of loss aversion.

The question may be raised as to whether the proposed method of mixed prospect valuation negates the mental adaptation process described in this paper. This is because people should adapt to the minimum or maximum lottery outcome, according to the process. The answer is inconclusive at this moment. The level of 0, which separates gains and losses, may prevail over all other levels in the adaptation process. This leads, however, to the gain-loss separability violations mentioned earlier in this Point. A solution that satisfies the new evidence while taking the adaptation to those extreme lottery outcomes into account is certainly a future possibility.

6.4 Illustrative Example

Let us consider the following two lotteries to illustrate the valuation method:

A: \$100 with probability 1/3
\$50 with probability 1/3
-\$100 with probability 1/3

B: \$300 with probability 3/5
-\$200 with probability 2/5

The gain and loss parts of both prospects are considered separately. In those cases where they contain only one outcome, (6.2) applies. It follows that the certainty equivalent of the gain part of B is equal to $CEB_g = \$300\,D_p^{-1}\left(3/5\right) = \$139.7$, and the certainty equivalent for the loss part of B is equal to $CEB_l = -\$200\,D_n^{-1}\left(2/5\right) = -\$70.6$. The certainty equivalent of B is then determined to be \$139.7/2- \$70.6 = -\$0.75, according to (6.7).

It further follows that the certainty equivalent of the loss part of A equals

$CEA_l = -\$100\, D_n^{-1}(1/3) = -\$30.8$. The gain part of A, however, has more than 1 outcome, so its equivalent probability has first to be calculated using (6.3):

$$p_{eq} = \frac{1}{3} D\left(\frac{100}{100}\right) + \frac{1}{3} D\left(\frac{50}{100}\right) + \frac{1}{3} D\left(\frac{0}{100}\right) = \frac{1}{3} + \frac{1}{3} D\left(\frac{1}{2}\right) = 0.551 \quad (6.10)$$

The certainty equivalent thus equals $CEA_g = \$100\, D_p^{-1}(0.551) = \$43.6$. It follows that the certainty equivalent of A equals \$43.6 / 2 – \$30.8 = -\$9.0. It turns out that B is better than A by \$8.25.

A different result is achieved using Cumulative Prospect Theory. The value of prospect A is -20.4, whereas the value of prospect B is -21.6. This means that both prospects are of similar negative values with a small preference for A[8]. A clear disadvantage of this approach is that the prospect values are expressed in meaningless units.

7 Including Logarithmic Perception of Financial Stimuli

The decision utility function was derived in Point 3 using linearly assessed outcomes. The function derived this way works well in most cases. An important objection raised by one reviewer was that the proposed model fails to interpret the following case. Let us suppose that $P_{min} = 0$ and $p = 0.5$. The model implies that CE/P is constant according to (6.2). This seems unrealistic, because this ratio should decrease as P becomes very large. For instance, somebody may well be indifferent between a certain \$40 and a 50% chance of winning \$100, but will definitely prefer a certain \$40 million to a 50% chance of winning \$100 million.

This objection is justified. Therefore, a nonlinear perception utility function should be implemented, especially for wider outcome ranges, as in the case considered. Such a function leads to a different evaluation of the relative outcome, than presented by (3.9):

$$r = \frac{u(CE) - u(P_{\min})}{u(P_{\max}) - u(P_{\min})} \quad (7.1)$$

where u denotes the perception utility. “Perception utility” is a psychophysical function describing how stimuli, e.g. monetary outcomes, are perceived. Per contra, “decision utility” describes how these perceived and framed outcomes are factored into risky decisions. Perception utility,

[8] The example considered was suggested by a reviewer. In order to compare the predictive power of the two models, a quick test was conducted among the staff of Acnet, a small telecommunications company in Warsaw, Poland. 19 people of varying sex, age, and educational level responded. Prospect A was chosen by 5 people (26.3%) and prospect B by 14 people (73.7%). The decision utility model therefore made a more accurate prediction in this case.

framing, and decision utility are thus three layers of a multi-layer decision making model. This multi-layer approach is in agreement with models of psychophysical judgment (Birnbaum, 1974).

In principle, perception utility represents the same concept as the Prospect Theory value function. The two functions, however, differ in detail in almost every respect (Kontek, 2011). First of all, the perception utility function is assumed to be logarithmic as the power function used by Cumulative Prospect Theory surprisingly fails to explain the case considered. This means that increasing prospect size does not alter preference, according to Cumulative Prospect Theory. This property of the theory is not accidental; Tversky and Kahneman (1992) refer to it as "preference homogeneity".

The inability of the power function to explain this particular case might account for the other known Cumulative Prospect Theory failures. Neilson and Stowe (2002) demonstrate that Cumulative Prospect Theory cannot simultaneously explain participation in lotteries and the original Allais paradox. Blavatskyy (2005) similarly demonstrated that Cumulative Prospect Theory, with its power value function, could not explain the St. Petersburg Paradox. This preference change can only be explained by assuming a value function of decreasing elasticity, whereas the power function is of constant elasticity (Scholten & Read, 2010). They propose using the logarithmic function $v(x) = 1/a \ln(1 + a\,x)$ in order to explain similar observations.

Using a logarithmic function to describe perception utility explains the case considered on the basis of the decision utility model. Please note that the relatively perceived outcome r is greater in the case of the lottery with greater outcomes:

$$\frac{\ln(1 + a\,40)}{\ln(1 + a\,100)} < \frac{\ln(1 + a\,40{,}000{,}000)}{\ln(1 + a\,100{,}000{,}000)} \tag{7.2}$$

This shifts considerations more to the right part of the decision utility function, which represents greater risk aversion, and explains the preference for a sure payment in the case of a million dollar lottery. Such an explanation would not be possible with linearly assessed outcomes, in which case the relative outcomes are the same:

$$\frac{40}{100} = \frac{40{,}000{,}000}{100{,}000{,}000} \tag{7.3}$$

or with perception utility described using a power function, in which case the ratio CE/P remains constant whatever the power coefficient value:

$$\frac{40^{\alpha}}{100^{\alpha}} = \frac{40,000,000^{\alpha}}{100,000,000^{\alpha}} \tag{7.4}$$

The latter case shows why Cumulative Prospect Theory with its power value function fails to interpret lotteries with increasing stakes.

The logarithmic shape of the perception utility function is not the only difference with the Prospect Theory value function. More importantly, Kontek (2011) puts forward a hypothesis that perception utility is generally logarithmic in shape both for gains and losses and only happens to be convex for losses when gains are not present in the problem context. This explains the gain-loss separability violations discussed in Point 6.3, but is in sharp contrast with the Prospect Theory premise that the value function is concave for gains and convex for losses. If true, this hypothesis would mean a complete departure form the Prospect Theory model both on the perception and decision making level.

8 Summary

This article presents an alternative interpretation of the experimental data published by Kahneman and Tversky in their 1992 paper "Advances in Prospect Theory". Mental transformations, crucial to deriving the results, were discussed in the introduction. Later, the solution was derived without using the probability weighting function. The obtained function has a double S-type shape that strongly resembles the utility curve specified by the Markowitz hypothesis (1952). The presented decision utility function shows that risk seeking appears for relative outcomes below the aspiration level. On the other hand, risk aversion is present for relative outcomes above the aspiration level. This pattern is reversed for losses. The explanation of risk attitudes given by the convex-concave-convex-concave shape of the decision utility function substitutes the fourfold pattern introduced by CPT.

The decision utility function is only the upper layer of a multi-layer decision making model. The bottom layer is perception utility, which is a psychophysical function describing how stimuli, e.g. monetary outcomes, are perceived. It should be described using a logarithmic function, rather than a power function as assumed by Cumulative Prospect Theory. This would enable the change in preference observed as prospect size increases to be explained. In many cases, however, the outcome range is narrow. Perception utility, then, plays no, or only a very limited, role in decision making, and the logarithmic function can be approximated by a linear function. This is the case with the original Tversky and Kahneman data (1992) which allowed the decision utility function to be derived in this paper using linearly assessed outcomes.

The middle layer of the model is framing. This is the combination of the mental adaptation and prospect scaling processes described in detail in this paper. The framing operation plays a crucial role in the model presented. It is framing that leads people to consider changes in wealth in relative, rather than absolute terms (see also Kontek, 2009).

The decision utility derived in this paper thus describes how these perceived and framed outcomes are treated in risky decisions. This function is expressed in terms of probability and follows the Expected Utility Theory method of assessing multi-outcome lotteries. This multi-layer decision-making model is in agreement with generally accepted models of psychophysical judgment. As a result, lotteries are evaluated in terms of money rather than hypothetical utils.

As the solution presented here is based on well known and well documented mental transformations, the results provide a basis for negating Prospect Theory as a theory that correctly describes how decisions are made under conditions of risk. One of the main postulates of Prospect Theory (and at the same time one of the main sources of its problems) is probability weighting. As argued, it is an artificial concept. Moreover, it makes this theory difficult to apply to more complex applications (for instance multi-outcome lotteries). Coming back to the classical notion of probability should make it possible to adapt it to model real world conditions.

One issue requires further clarification. Recently evidenced gain-loss separability violations have called the approach of evaluating mixed prospects into question. Tversky and Kahneman's data, obtained separately for gain and loss prospects, were used in this paper to derive the decision utility function. Therefore no decision utility function can be presented for mixed prospects without any further data sets or new experiments. The valuation of mixed prospects therefore seems to be one of the most important subjects for future research.

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