# The long-run behavior of the S&P Composite Price Index and its risk premium

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***Abstract***

*We lay out here the basis for a long-term equity index model, with intent to extract the risk premium. This is done by first observing the behaviours of the S&P Composite price index, earnings and dividends over roughly 130 years of history, from 1871 to 1998, and then assessing whether they fit within an equilibrium and efficient-market framework. The notions of equilibrium and efficiency shall be defined and formalised here, as they relate to this work, using classical finance theory.*

*The conclusions derived so far are twofold. First, there is a transition in the market's behaviour at around 1945. It appears that prior to this, the dividend payment policy was, on aggregate, one of constant dividend yield. After this, the policy's focus seems to have shifted towards achieving market equilibrium and efficiency. Second, the backward-looking risk premium during the post-transition period is found, in theory, to be simply the negative percent rate of change in dividend yield. Moreover, under the special-case scenario where the equity price is discounted at a constant "infinite-horizon" discount rate, the forward-looking risk premium becomes identically the dividend yield.*

## 1. Introduction

The equity risk premium has firmly established itself as one of the most formidable and elusive puzzles in finance theory. To deal with this, numerous explanations have been proposed, each trying to describe how it must be measured, let alone how it should behave. With no unified approach in sight, it is, therefore, not surprising that experts have always found ways and reasons to argue about it.

Here we also try to tackle the question of the risk premium, but through a more elementary approach. We believe that only in such a way, where the very basic fundamentals are targeted, one could shed light on the controversies that underlie this puzzle.

We begin here by addressing the different time scales involved. This, in turn, should enable us to identify some of the possible ways for measuring the risk premium. For simplicity, we narrow down our attention on essentially the two asymptotic limits of time scale, namely the short and long run. While the former is on the order of months, the latter could span many years. What connects the two is most likely a transition regime, which, for the time being, will remain out of the scope of this work. This is because we

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need to understand the basics first before we delve into the complications.

In all, we anticipate that, since the short-run risk premium is inherently noisier than the long run, different methods of analysis are to be called for. A description of these follows next.

## 2. Possible Approaches to Extracting Market's Expectations and Risk Premiums

Having identified the two asymptotic cases of interest here, namely the short and long-term limits of time scale, we now proceed to discuss them. In advance, however, we should stress that in contrast to our objective here, which is to develop a long-term model, there is an extensive literature on extracting short-term risk measures. Although none of this will be covered here in any detail, a brief, qualitative explanation of the methodology will be provided.

### 2.1. Short-term Approach

The idea of using option prices to extract the market's near-term expectations of asset prices and implied measures of risk is not novel (Breeden and Litzenberger, 1978; Finucane, 1991; and many others). More lately, however, the notion has gained so much in popularity that even major investment firms and central banks have begun to commit much effort into studying it (Campa *et al*, 1998; Bahra, 1997, and references therein). These efforts have subsequently led to a variety of approaches to obtain expectations. Notwithstanding, it is still debated as to whether or not these expectations do contain any forecasting information.[4]

The connection between option prices and expectations stems from the belief that options have embedded in them the market's outlooks on near-term price movements. Insofar as work along these lines is concerned, derivatives-related research plays a fairly active role in it. The method of approach here, nonetheless, revolves around extracting "risk-neutral distributions" of markets' expectations using only the prices of calls and puts.

The overall advantage of using option prices to produce implied expectations and risk measures is that the methods involved are highly objective and process oriented, and, as such, they require no subjective inputs, whatsoever. With this in mind and noting that the aim of this paper is to develop a long-term model instead, we stop here and proceed with our own analysis.

### 2.2. Long-term Approach

As stated earlier, the long-run model we plan to develop here focuses on the other extremity of the time scale – that is, covering many years. This not only places more emphasis on trends than on short-term fluctuations, but it also deems the options methodology totally irrelevant since the long-term time scales involved far exceed the typical lifetime of any option.

To start, we need to formalise the notions of market equilibrium and efficiency, and explain how they fit into our proposed model. This will necessitate defining certain parameters, namely dividend yield, earnings yield, market's return, etc, after which we shall assess their behaviours. Clearly, therefore, we are relying here more on market fundamentals than on the fluctuations produced by "noise traders." It is very likely that the latter get washed out over the long run.

Let us begin by defining the realised dividend yield, $D(t+1)$, as

$$D(t+1) \equiv \frac{\delta(t+1)}{S(t)} \tag{1a}$$

[4] There is evidence that foreign exchange options exhibit some short-term forecasting capabilities (Campa and Chang, 1998).

where $\delta(t+1)$ is the dividend payment realised at the end of year $t$ [or the beginning of year $t+1$] and $S(t)$ the stock price at the beginning of year $t$. Where applicable, all other definitions will carry the similar notation that $t+1$ means either the end of year $t$ or the beginning of year $t+1$. In conjunction with the above, we also define the expected dividend yield, $D_f(t)$, as

$$D_f(t) \equiv \frac{\delta_f(t)}{S(t)} \tag{1b}$$

where $\delta_f(t)$ is the forecast of the dividend receivable at the end of year $t$.

Let us, in addition, express the earnings yield, $E(t+1)$, and the market's realised total rate of return, $R_M(t+1)$, as

$$E(t+1) \equiv \frac{e(t+1)}{S(t)} \tag{2}$$

and

$$R_M(t+1) = \frac{S(t+1) - S(t)}{S(t)} + D(t+1) \tag{3a}$$

where $e(t+1)$ is the firm's realised yearly earnings. Finally, we define the market's expected total rate of return, $R_E(t)$, as

$$R_E(t) = k_S(t) + D_f(t) \tag{3b}$$

where $k_S(t)$ is the time-$t$ expected capital gains, i.e.

$$k_S(t) \approx \frac{S_f(t) - S(t)}{S(t)} \tag{4}$$

and $S_f(t)$ the time-$t$ expectation of the price at the end of $t$ [or the beginning of $t+1$].

Note that we have avoided using the logarithmic form of the return and, instead, linearised the equations. Bearing in mind that the difference between the two is on the order of only a few percentage points, this helps to reduce complications in arriving at simple, closed-form solutions. What follows hereafter is the valuation method, along with the principles of efficiency and equilibrium, as they relate to this work.

### 2.2.a. Valuation

For valuation purposes, the widely accepted discounted-cash-flow methodology is enforced here. In its most elementary form, this assumes that the value of an asset is determined by the constant stream of cash that it can generate perpetually, all discounted at a constant discount rate. Although the form of the equation is simple, it happens to be quite specific in that it depends on who is receiving the cash. For instance, while the firm receives the cash in the form of asset-generated earnings, the investor collects it as dividends passed on to him by the firm.

Having said this and recognising that dividends are the prerogative of the firm, these payments must, therefore, be made to the investor by the firm in such a way that valuation becomes consistent, whether it is calculated by the firm based on earnings or the investor, based on dividends received. This, evidently, has direct implications on efficiency, which, along with equilibrium, will comprise the following section.

### 2.2.b. Efficiency and Equilibrium

To explain market efficiency and equilibrium and how they fit into this work, one needs to examine the mechanism that generates prices in an equity market. In a typical equity market, consisting of member firms and their investors, investors produce their own valuations given information passed on to them by firms. This leads to a buying and selling of shares, which, in turn, creates a market price that should coincide with the computed and, more ideally, the equilibrium price. In an efficient market, this price must be consistent whether it originates from the firm's valuation or the investor's, and, in either case, it should reflect the true value of the asset.

Normally, information made available by firms should include, among others, news and forecasts on earnings and dividends. In an efficient market, this information must be (a) fully reliable and 100% accurate, (b) fully available to and usable by all market participants and, (c) as mentioned earlier, it must lead to consistent valuations between the firms and investors. With these in mind, the basics that underlie such a market will be explained first, and followed then by a discussion on how the S&P Composite index has fared against it historically. Before this, however, the concepts of efficiency and equilibrium, as they apply to this work, must be quantified.

Starting with the discounted-cash-flow principle of valuation, the firm, which expects to receive earnings [profits] from an asset, will value the asset according to the "earnings-discount model." This is given by

$$S(t) = \frac{e_f(t)}{R_F(t)} \tag{5}$$

where $S(t)$ is the current price of the asset at the beginning of year $t$, $e_f(t)$ is the beginning-of-the-year's generated forecast for the earnings at the end of year $t$, and $R_F(t)$ is the firm's discount rate at the beginning of year $t$, when valuation was done.[5]

For the purposes of this work, Equation 5 will be recast into:

$$R_F(t) = \frac{e_f(t)}{S(t)} \tag{6}$$

Note that under perfect foresight, where $e(t+1) = e_f(t)$, the earnings yield, as defined in Equation 2, becomes precisely the firm's discount rate in 6. It should further be stressed that the above assumes a constant stream of earnings, the implications of which will be discussed in more detail in Section 4.2.

The investor, on the other hand, will most likely value the asset according to some type of dividend-discount methodology. The simplest form for this is given by:

$$S(t) = \frac{\delta_f(t)}{R_I(t) - k_\delta(t)} \tag{7}$$

where $R_I(t)$ is the discount rate used by the investor, $\delta_f(t)$ is as defined in 1b and $k_\delta(t)$ is the expected dividend growth rate, which is expressible by

$$k_\delta(t) \equiv \ln\left[\frac{\delta_f(t)}{\delta(t)}\right] \approx \frac{\delta_f(t) - \delta(t)}{\delta(t)} \tag{8}$$

with $\delta(t)$ being the dividend just realised. Substituting Equation 8 into 7 yields after some rearrangement

$$R_I(t) = k_\delta(t) + D_f(t) \tag{9}$$

Equations 3b, 6 and 9 allow us now to formalise and unite the notions of market efficiency and equilibrium. This is done through the pair of propositions below.

***Proposition 1*** **–** *Market efficiency implies that the earnings-discount-based valuation produced by the firm equals the dividend-discount-based valuation computed by the investor.*

[5] It has been argued whether earnings or dividends should be used for valuation (Shiller, 1981; and references therein). It can, never the less, be shown that, taking reinvestment into account, both lead to identical results. This is essentially a consequence of Gordon's growth model, which we shall allude to again later.

***Proposition 2*** *– Market equilibrium implies that the discount rates, $R_I(t)$ and $R_F(t)$, as implemented in Equations 6 and 9, should be equal to each other, as well as to the market's expected total rate of return, $R_E(t)$, as defined in Equation 3b.*[6]

Moreover, being the prerogative of the firm, the dividend payments could be adjusted to fit within the framework of Propositions 1 and 2. We illustrate this in Figure 1.

Figure 1 may be put into clearer perspective by substituting Equations 1b and 8 into 9 and equating the outcome to 6. This leads to

$$\frac{\delta_f(t)}{\delta(t)} = \frac{1 + e_f(t)/S(t)}{1 + \delta(t)/S(t)} \tag{10}$$

which is simply the dividend payment policy that brings in line the valuation of the firm with that of the investor. This basically re-enforces the notions stated in Proposition 1 and part of Proposition 2.[7]

The rationale behind Figure 1, and subsequently Equation 10, is that, acting on economic outlooks, the firm, at the beginning of year $t$, generates an earnings forecast for the end of year $t$. This is simply $e_f(t)$, which is produced through some "black-box" process that is not of concern here. Given this, along with concurrent values of $S(t)$ and $\delta(t)$, the firm could, thus, turn out a forecast for the end-of-the-year's dividend payment, $\delta_f(t)$, which guarantees to satisfy the necessary criterion $R_I = R_F$, leading to Equation 10. This not only relieves the firm from having to make ad-hoc decisions on dividends, but also warrants that the investor receive accurate valuation-related information from the firm via the dividend-payment process.

Next, we extend Figure 1 to include the whole of the second proposition as well. This ensures equilibrium by bringing the market's expected total rate of return, $R_E(t)$, in harmony with the discount rates. This process, which results in "fair" valuation, is illustrated in Figure 2.

Figure 2 essentially depicts how a "fair" expected market rate-of-return forecast could be recovered by the investor via a procedure that ensures both efficiency and equilibrium. To demonstrate, let us begin where we left off.

Recall that the economic outlook at the beginning of year $t$ leads to the firm's earnings forecast, $e_f(t)$, for the end of year $t$. This, in turn, allows a forecast for the dividend payment, $\delta_f(t)$, for the end of year $t$ [via Equation 10]. Finally, applying Proposition 2, which brings in the equality $R_I(t) = R_E(t)$, yields

$$\frac{S_f(t)}{S(t)} = \frac{\delta_f(t)}{\delta(t)} \tag{11}$$

on equating Equations 3b with 9 and incorporating 4 and 8 as well. Simply

[6] Note that Proposition 1, along with the equality $R_I(t) = R_F(t)$ stated in Proposition 2, readily lead to Gordon's Growth model, whereby dividend growth rate is related to the rates of discount and re-investment. It further follows that by including the expected total market return, $R_E(t)$, as well, one could extend the above into the equilibrium framework of Proposition 2. This is simply equating the discount rates with the required or expected rate of return [see, for instance, Fabozzi (1999) or Ross *et al* (1998) for related discussions].

[7] At first glance, the constraint set here on dividends appears to violate one of Miller-Modigliani's propositions, namely, dividend payments can be made arbitrarily since they should have no impact on equity price. Based on this, dividends withheld today could be paid later. In the long run, however, aggregating this short-term, seemingly ad-hoc behaviour over time must ultimately lead to a pattern, which is what this work is concerned with.

stated, Equation 11 enables one to extract a forecast for next year's price, $S_f(t)$, which satisfies both propositions, 1 and 2.

Stemming from a highly idealised scenario, the price forecast, $S_f(t)$, should, in reality, be different from the actual, $S(t+1)$, realised a year later. The difference between the two, which presumably is an uncorrelated error, will be shown afterwards to lead to the risk premium. At this stage, though, we need to discuss the implications of a constant dividend-yield policy on equilibrium and efficiency.

### 2.2.c. The Effects of a Constant-Dividend Yield Policy

We have thus far described a methodology by which both, market efficiency and equilibrium, could be accomplished simultaneously. The process, which is displayed schematically in Figure 2, as well as quantitatively in Equations 10 and 11, suggests that, in causal terms, the earnings forecast precedes the dividend payment decision, which leads ultimately to a "fair" forecast of the price or expected return. This process, thereby, enables the firm to use dividends as an instrument to convey information on expected earnings to the investor. It is, therefore, essential here that dividends be not only linked to, but also made to follow the earnings forecast. More important as well, dividend payments should be set free of all artifical constraints, as these tend to violate the notion of efficiency. This is clarified through the example below.

Consider, for instance, that a constant dividend-yield policy is in place. Based on the definition in Equation 1b, this may be written as

$$\frac{\delta_f(t)}{S(t)} = \lambda_\delta = \textit{constant in time} \qquad (12)$$

which describes a dividend forecast, $\delta_f(t)$, that is constrained to be directly proportional to the price one year prior, $S(t)$. Purely because there is no useful information [i.e. on earnings forecast] embedded in the dividend payment expressed in 12, a constant dividend-yield policy will, thus, not allow information to flow efficiently from the firm to the investor. As a result, this constraint, along with any other artifical ones that may be imposed on dividends, will force both the firm and the investor to arrive at different conclusions regarding the net asset value. This, consequently, leads to the next proposition, which is:

***Proposition 3*** – *Unless it is identically equal to zero, a constant-dividend yield policy cannot be sustained in a market that is already, or is striving to become, efficient.*

## 3. An Examination of the Historical S&P Composite Index

We rely here on long-term historical S&P Composite data to assess the behaviours of some of the variables discussed above. The details of how the earnings and dividends, dating back approximately 130 years, were collated will not be discussed. Instead, the interested reader is referred to the web site containing the data[8].

Altogether, we are mainly interested in how the three rates, $R_E$, $R_F$ and $R_I$, compare against each other. Recall that if $R_F = R_I$ only, then Proposition 1, as well as part of Proposition 2, will be satisfied. Aside from conforming to Gordon's Growth model [see Footnote 6], this also indicates that the firm is relaying information efficiently to the investor via dividends.

Moreover, if all three were equal to each other, Propositions 1 and 2 will be obeyed simultaneously. This suggests

[8] http://www.econ.yale.edu/~shiller/chapt26.html.

that, in addition to the efficient flow of information from the firm to the investor, equilibrium with market prices is also established. Of course, other relationships will emerge as we provide a wider picture of these behaviours.

Prior to doing so, however, we need to enforce the assumption of "perfect foresight," which requires that $\delta_f(t) = \delta(t+1)$, $e_f(t) = e(t+1)$ and $S_f(\mathrm{t}) = S(t+1)$. In other words, we shall derive and compare implied rates of discount and return. This assumption, which also renders Equations 3a and 3b identical, is not only necessary because it is the only means for producing the charts that follow next, but also valid since if the model were to fail here, then it should fail even more severely under alternative assumptions.

Figures 3-5, respectively, display $R_F(t)$ and $R_I(t)$, $R_M(t+1)$ and $R_I(t)$ and, finally, $R_F(t)$ and $R_M(t+1)$, all against time. Note that because of the perfect-foresight assumption, $R_M(t+1)$ is used instead of $R_E(t)$. These parameters have been computed using the definitions in Equations 3a, 6 and 9.

Based on Figures 3-8, we arrive at the following conclusions:

(a) $R_I$'s volatility closely matches $R_M$'s between 1871 to 1940 [see Figures 4 and 6]. Following a transition period of roughly 5-10 years between 1940 and 1950, the behaviour shifted such that $R_I$ began to follow $R_F$ instead [see Figures 3 and 7]. This reveals that prior to 1940, the investor's uncertainties regarding his discount rate went, more or less, with uncertainties on market's rate of return, whereas, after 1950, they coincided more with the firm's discount rate. For convenience, these behaviours, which are evident in Figures 3-5, are shown expanded in Figures 6 and 7. Note that no adjustable or fitting parameters have been incorporated in any of the figures.

(b) Figure 5 demonstrates that nowhere in time does $R_F$'s volatility match that of $R_M$'s. The latter has always been significantly more volatile than the former.

(c) The logarithms of price, earnings and dividends seem to move roughly in parallel with one another, as Figure 8 indicates. This suggests that the growth rates of the three parameters are tied closely to one another.

(d) The above-mentioned transition, which occurred between 1940-1950, seems to also prevail in Figure 8, where a marked shift in the magnitude of growth [slope of the logarithmic curves] is shown to occur in all cases. This transition, which connects the pre- and post-WWII US markets, has been documented (Siegel, 1998).

In addition, we display in Figure 9 $k_\delta(t)$ and $k_S(t\text{-}1)$, as defined in Equations 4 and 8, respectively, against time from 1871 to 1998. The pre- and post-transition periods are again expanded and depicted separately in Figures 10 and 11, respectively. This is done in order to determine whether or not the S&P market has ever pursued a constant dividend-yield policy. Such a policy would cause Equation 12 to be followed closely, meaning that $k_\delta(t)$ and $k_S(t\text{-}1)$ should be equal to, as well as highly correlated with, each other.

It is, in fact, evident in Figure 10 that during 1871-1940, $k_\delta(t)$ and $k_S(t\text{-}1)$ were not only highly correlated, but also close to one another. This observation, however, does not seem to hold for the post-transition period, which is illustrated in Figure 11.

We scrutinise this further by examining the coefficients generated by either of the following regressions:

$$k_\delta(t) = \alpha_0 + \alpha_1 k_S(t-1) \qquad (13a)$$

or

$$\ln[\delta(t)] = \beta_0 + \beta_1 \ln[S(t-1)] \qquad (13b)$$

both originating from Equation 12, which is the constant-dividend yield policy. For this to hold, we need to have $\alpha_0 = 0$, $\beta_0 = constant$, and both $\alpha_1$ and $\beta_1$ equal to unity. Interestingly, the results of the above, which are displayed in Table 1 [coupled with the fact that the pre- and post-transition correlations between $k_\delta(t)$ and $k_S(t\text{-}1)$ are calculated to be 0.77 and 0.32, respectively], do indeed portray a market that, prior to 1940, appears to have followed a policy of constant dividend yield. Having concluded this, we now turn to the post-transition period shown in Figure 7 to determine the type of dividend-payment policy in place then.

Recall that Equation 10 delineates the dividend payment scheme that leads to market efficiency. Assuming perfect foresight again, we write this in regression form as:

$$\frac{\delta(t+1)}{\delta(t)} = \gamma_0 + \gamma_1 \left[ \frac{1 + e(t+1)/S(t)}{1 + \delta(t)/S(t)} \right] \qquad (14)$$

whereby, if Equation 10 were to hold, the coefficients $\gamma_0$ and $\gamma_1$ must equal zero and one, respectively.

These coefficients, along with their t-statistics, are presented in Table 2. We conclude here that the post-transition market, in contrast, appears to exhibit a tendency towards heading in the direction of efficiency and equilibrium. Moreover, the relatively high correlation of 0.53 between the two sides of Equation 14 helps to confirm this over the post-transition period. As we shall demonstrate next, these conclusions are important because they could provide a theoretical basis for the equity risk premium.

## 4. The Equity Risk Premium

One of the major hurdles to understanding the equity risk premium is the question of how it must be defined. Even though it is widely agreed that the risk premium should be the difference between the market's rate of return and the risk-free rate, among the important questions is what risk-free rate must one use? Furthermore, how does one attain a measure of the forward-looking risk premium? Recognising that these tend to be quite subjective, it is then no wonder why we encounter so much difficulty in trying to obtain the risk premium.

### 4.1. Defining the Equity Risk Premium

To circumvent the above-mentioned problems, we will try here to derive a relationship for the risk premium in a particular, but plausible, way, and will adhere to it firmly throughout the rest of this paper. Consider first the valuation of a stock index futures contract, which is written at time $t$ and deliverable a year later, at $t$+1. Letting the value of this contract be $F(t)$, then according to first principles (Fabozzi, 1999),

$$F(t) = S(t)[1 + r_f(t)] - \delta_f(t) \qquad (15)$$

where we have defined $r_f(t)$ to be the relevant risk-free rate and neglected all transaction costs. Combining the above with Equations 3a and 4 and expressing the backward-looking risk premium, $\hat{r}_p(t)$, as

$$\hat{r}_p(t) \equiv R_M(t+1) - r_f(t) \qquad (16)$$

we obtain

$$\hat{r}_p(t) = \frac{S(t+1)}{S(t)} - \frac{F(t)}{S(t)} + \frac{\delta(t+1)}{S(t)} - \frac{\delta_f(t)}{S(t)} \qquad (17)$$

Note that the above is backward looking because it utilises realised values. The forward-looking risk premium shall be discussed shortly.

Let us now compare the ratios $\delta(t+1)/S(t)$ and $\delta_f(t)/S(t)$ over the pre- and post-transition periods. Recall that these are the realised and forecast dividend yields - i.e. $D(t+1)$ and $D_f(t)$ - respectively, the latter coming from Equation 10, using the perfect-foresight assumption that $e_f(t) = e(t+1)$. The two are compared against each other in Figure 12, with the transition period shaded.

It is clear here that after 1950, where efficiency and equilibrium were the market's objectives, the two dividend yields, namely the realised and forecast, have been in remarkably good agreement. Prior to this, however, when the constant-dividend yield policy was in place, the relation appears to break down.

With this in mind, therefore, our definition for the backward-looking risk premium in 17 reduces to

$$\hat{r}_p(t) \approx \frac{S(t+1)}{S(t)} - \frac{F(t)}{S(t)} \qquad (18)$$

which should be valid post 1950 when $\delta(t+1)/S(t) \approx \delta_f(t)/S(t)$. Moreover, in a perfect market one would expect the futures price, $F(t)$, to exactly reflect the fair asset price, $S_{fair}(t+1)$, a year later. This, subsequently, yields

$$\hat{r}_p(t) = \frac{S(t+1)}{S(t)} - \frac{S_{fair}(t+1)}{S(t)} \qquad (19)$$

Equation 19, therefore, indicates that the risk premium can be written as the difference between the realised and the "fair" rates of return. This, in turn, poses the question of what should the market's fair rate of return be? The answer to this is proposed as follows:

***Proposition 4*** *– The market's "fair" rate of return is the one that satisfies both, efficiency and equilibrium.*

With the above in place, it is now possible to derive an expression for the backward-looking risk premium. Quantitatively, Proposition 4 takes us back to Equation 11, which may be re-expressed as:

$$\frac{S_{fair}(t+1)}{S(t)} = \frac{S_f(t)}{S(t)} = \frac{\delta_f(t)}{\delta(t)} \qquad (20)$$

where $S_{fair}(t+1)$ has been set equal to $S_f(t)$. Recall that, just as Equation 11 is meant to satisfy both efficiency and equilibrium, the above should do so as well. Now, inserting Equation 20 into 19 yields

$$\hat{r}_p(t) = \frac{S(t+1)}{S(t)} - \frac{\delta_f(t)}{\delta(t)} \qquad (21)$$

which leads to

$$\hat{r}_p(t) \approx -\frac{d\ln[D(t+1)]}{dt} \qquad (22)$$

after differentiating either Equation 1a or 1b with respect to time, letting $\delta(t+1) \approx \delta_f(t)$ [based on Figure 12c] and implementing 21. We need to stress here that the above incorporates the simplifying assumption that the long-term rate of growth in price, $k_S(t)$, is constant [refer to the next section], which, given Figure 8, appears to be fairly reasonable post 1950.

It is interesting to note from Equation 22 that, based on what we have presented so far, the backward-looking risk premium is the negative percent rate of change in the dividend yield. This result can be extended even further upon examining the market's behaviour under a constant discount rate.

### 4.2. The Market under a Constant Discount Rate

How a market that is efficient and in equilibrium should behave under constant discount rate is another issue that needs to be ironed out. It should, never the less, be pointed out that this question, which involves also the notion of excess volatility, instigated some debates in the 1980s (Shiller, 1981; Grossman and Shiller, 1981; Kleidon, 1986; among others). With respect to our work, however, we need not to worry about excess volatility because it is irrelevant.

Let us, instead, approach the question from the perspective of the investor [or firm], who wishes to forecast in the "infinite horizon," where discount rates are indeed set to be constant. Lacking the ability to see infinitely with precision, as we all do, the investor will, most likely, also foresee all relevant market variables, particularly the rates of discount, return and growth [of dividends, earnings, prices, etc.] as constants too. This, of course, is consistent with the classical present-value equations presented in Equations 5 and 7.

The outcome of this idealisation is as follows. A market that is efficient, in equilibrium and pursues a constant discount rate will simply follow:

$$R_F = R_I = R_E = R^* \qquad (23)$$

where $R^*$ denotes that constant. Note that this is simply Proposition 2 extended. Moreover, at the infinite horizon, all expected rates of growth will be foreseen as constant in time, i.e.

$$k_S \equiv \frac{S_f(t) - S(t)}{S(t)} = constant \qquad (24a)$$

$$k_e \equiv \frac{e_f(t) - e(t)}{e(t)} = constant \qquad (24b)$$

and

$$k_\delta \equiv \frac{\delta_f(t) - \delta(t)}{\delta(t)} = constant \qquad (24c)$$

where $k_S$, $k_e$ and $k_\delta$, respectively, are the expected rates of growth in price, earnings and dividends, as defined.

Returning now to Equations 3 and 9, we find that under such a scenario, the dividend yield must be constant as well. This conclusion, however, becomes rational under Proposition 3 if and only if the dividend yield were identically equal to zero. This, therefore, leads to our last proposition, which is:

***Proposition 5 –*** *In a market that is in pursuit of efficiency and equilibrium, the investor bases his long-term, infinite-horizon, forward-looking asset-pricing decision on a <u>constant</u> discount rate and <u>zero</u> dividend yield.*

Proposition 5, therefore, suggests that the investor's one-year-ahead price forecast, which we shall denote here by $\breve{S}_{investor}(t+1)$, is calculated on the basis of a zero dividend yield. This, of course, should still be consistent with his discount rate, $R_I(t)$.

On the other hand, fair valuation comes from Equation 20, which, when linked with Equation 9 and Proposition 5, yields:

$$\left[\frac{\breve{S}_{investor}(t+1)}{S(t)} - 1\right] = \left[\frac{S_{fair}(t+1)}{S(t)} - 1\right] + D_f(t) \qquad (25)$$

It follows, therefore, that upon combining Equations 25 and 19, the <u>forward-looking</u> risk premium, $\breve{r}_p(t)$, becomes

$$\breve{r}_p(t) = D_f(t) \qquad (26)$$

which is simply the expected dividend yield itself.

## 5. The Dividend Yield and the Forward-looking Risk Premium in a Market approaching Efficiency and Equilibrium

We have just demonstrated that the equity risk premium in a market that is approaching efficiency and equilibrium, and in which the investor bases his forecasts on infinite-horizon perceptions, is the expected dividend yield itself. This, coupled with Equation 22, gives the differential equation

$$\frac{d\ln[D_f(t)]}{dt} + D_f(t) = 0 \tag{27}$$

which has the solution:

$$\frac{1}{D_f(t)} = \frac{1}{D_f(0)} + t \tag{28}$$

where $D_f(0)$ is the initial condition. In arriving at 27, we have let $D_f(t) = D(t+1)$, thus imposing a continuity between the forward and backward-looking risk premiums. This essentially rationalises the market's behaviour at any point in time.

Conditional on infinite-horizon-based projections, therefore, Equations 22 and 26 should apply to the historical S&P Composite index post transition, following 1950. Moreover, Equation 28 suggests that, subject to the assumptions stated, the dividend yield, and hence the risk premium, should decay in time continuously and asymptotically approach zero.

Conversely, prior to 1940, when the constant-dividend yield policy was in place, the risk premium measures provided by Equations 22 and 26 should not hold. The reason is that these equations have been derived strictly to fulfil our five propositions, which apply more suitably to after 1950.

We now assess the validity of Equation 28 by comparing it with the inverted historical dividend yield data, i.e. $1/D(t)$, plotted against time. Based on the above, before 1940, which is the constant-dividend-yield era, $1/D(t)$ should be constant. In contrast, $1/D(t)$ must rise linearly in time post transition, as the model suggests.

Figure 13 displays $1/D(t)$, both theoretical and historical, against time. We observe here that the constant-dividend yield model holds rather tightly prior to 1940. Post 1950, however, the fit between model and data appears to deteriorate. This is possibly due to a violation of the equality between the backward and forward-looking risk premiums, which is a requirement of our model [refer to the last paragraph of this section]. Never the less, putting all these together and considering that no adjustable parameters have been implemented anywhere, comparison between model and data, in terms of orders of magnitude and trends, is perhaps reasonable given the relative simplicity and straightforwardness of the underlying model.

Alternatively, let us concentrate on the post-transition period, which began in 1950. Recalling that this is when the market focused on achieving efficiency and equilibrium, the theoretical infinite-horizon-based risk premium was, therefore, the expected dividend yield, as specified by Equation 26. The dynamics of this, which are described by Equation 28, are displayed in Figure 14. For comparison, the dividend yield is also included. Evidently, the theoretical risk premium is seen to decline slowly in time, asymptotically approaching zero. This limit signifies a highly idealised scenario, whereby all five propositions are satisfied in unison.

Finally, we refer to typical numbers generated by Equations 22 and 26. Note that the former is the backward-looking risk premium, defined by the negative rate of change of the dividend yield, and the latter the forward-looking described by the dividend yield itself. Based on recent data - i.e. of the last 1-2 years – these turn out to be about 20% and 2%, respectively. Very clearly, the two numbers are in conflict with one another. This is interesting because recent literature also provides similarly incompatible figures (Cornell, 1999). In fact, this discrepancy between the forward and backward-looking risk premiums might be the cause for the poor post-transition agreement between model and data observed in Figures 13 and 14. This, of course, is highly speculative and, like anything else, it remains open to criticisms and debate.

## 6. General Remarks

We need to emphasise now that there are basically three important issues at work here and mixing them up appears to be a major source of confusion. The issues are (a) market efficiency, (b) market equilibrium and (c) steady-state equilibrium. The implications of a and b are covered by Propositions 1-4, while those of c are described by Proposition 5. These are completely separate issues and, hence, they must be treated as such.

Moreover, we have so far tried to prove the following. If we were in a market that was efficient and in equilibrium [this does not necessarily imply stationarity or steady state], then Gordon's growth model is generally applicable. However, if the market were at steady state as well [i.e. where all growth and discount rates are constant in time], then we have the case where all discount rates are equal to the rates of growth and return. This should happen in the "long run", or, more formally, as time approaches infinity.

This, therefore, identifies two distinct scenarios, namely the unsteady and steady-state, with the disparity between the two arising from time-dependent variations in growth and discount rates. Since this difference is embodied solely in the dividend yield, as we have argued here, then the dividend yield should represent the risk premium that a "far-sighted" investor will require for the risk implied by assuming steady state conditions in his valuation. A steady-state market, in fact, represents the ideal situation where no volatility and, hence, no risk is involved.

An important question now is whether the market is currently efficient and in equilibrium, so that one could utilise the dividend yield as a measure of the risk premium. The answer is certainly NO. This could be observed in Figures 3-5, whereby the discount rates and the market's return are shown to be quite different from each other. Never the less, these figures do indicate that the market is closer to being efficient and in equilibrium post 1950 than pre 1940. Pre 1940 is a completely different story, altogether.

Consequently, what is the long-term, forward-looking risk premium appropriate to current market conditions? The model proposed here is not developed far enough to tell us. However, it does suggest that a suitable, theoretical estimate for it is the dividend yield. As for practical applications, perhaps in a multistage dividend-discount model, the dividend yield could be incorporated as an estimate for the risk premium in the final stage, where all growth and discount parameters are assumed constant.

## 7. Conclusions

The behaviour of the S&P market over the past 130 years was the subject of our investigation. Based on five propositions, one being that market efficiency may be achieved via dividend-related information conveyed to the investor by the firm, we arrive at several conclusions, some of which are outlined below.

(a) The overall behaviour of the market between 1871-1998 can be broken down into two stages – pre 1940 and post 1950, with the period in between being transitional.

(b) The pre-transition period was dictated by a policy of constant dividend yield.

(c) The post-transition period is characterised by a falling dividend yield. This is consistent with a market that is pursuing efficiency and equilibrium. In such a case, a constant dividend yield, unless it is identically equal to zero, cannot be sustained. Therefore, if not already zero, the dividend yield should slowly, but steadily, decline in time and asymptotically approach zero.

(d) Also, within such a market, the forward-looking risk premium could be shown to be equal to the dividend yield itself. The declining dividend yield, therefore, translates into a falling risk premium, which also asymptotically approaches zero. This is the limit where the market becomes perfect in every aspect – that is, 100% efficient and in equilibrium and has all rates of growth and interest constant. This, of course, describes the ideal situation where all five propositions are satisfied at the same time.

We finally conclude here by pointing out that from 1950 onward, the S&P market has, according to our model, been actively in pursuit of efficiency and equilibrium. Although this perfect state has not been fully realised yet, there are indications, such as a decaying [forward-looking] risk premium, that the market is headed strongly in that direction.

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| Coefficient | Time period | |
|---|---|---|
| | 1871-1940 | 1950-1998 |
| $\alpha_0$ | -2.7e-4 (-0.18) | 0.015 (0.40) |
| $\alpha_1$ | 0.94 (9.77) | 1.37 (2.30) |
| $\beta_0$ | -2.91 (-29.37) | -2.30 (-13.11) |
| $\beta_1$ | 0.98 (20.13) | 0.79 (21.95) |

Table 1 – Statistical tests on the coefficients of the regressions in Equations 13 a and b. The numbers in the parentheses are the t-statistics.

| Coefficient | Time period | |
|---|---|---|
| | 1871-1940 | 1950-1998 |
| $\gamma_0$ | -3.46 (-6.00) | -0.18 (-0.61) |
| $\gamma_1$ | 4.39 (7.78) | 1.18 (4.21) |

Table 2 – Statistical tests on the coefficients of the regression in Equation 14. The numbers in the parentheses are the t-statistics.

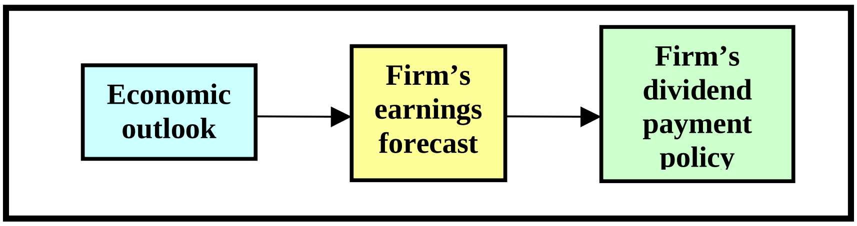


Figure 1 – Schematic diagram of the process leading to the firm's dividend payment policy.

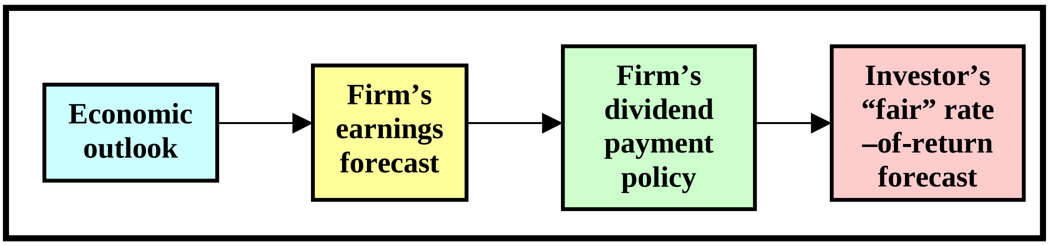


Figure 2 – Extension of Figure 1, leading to the process of "fair" rate-of-return forecast in the context of equilibrium and efficiency. This fulfils both propositions, 1 and 2.

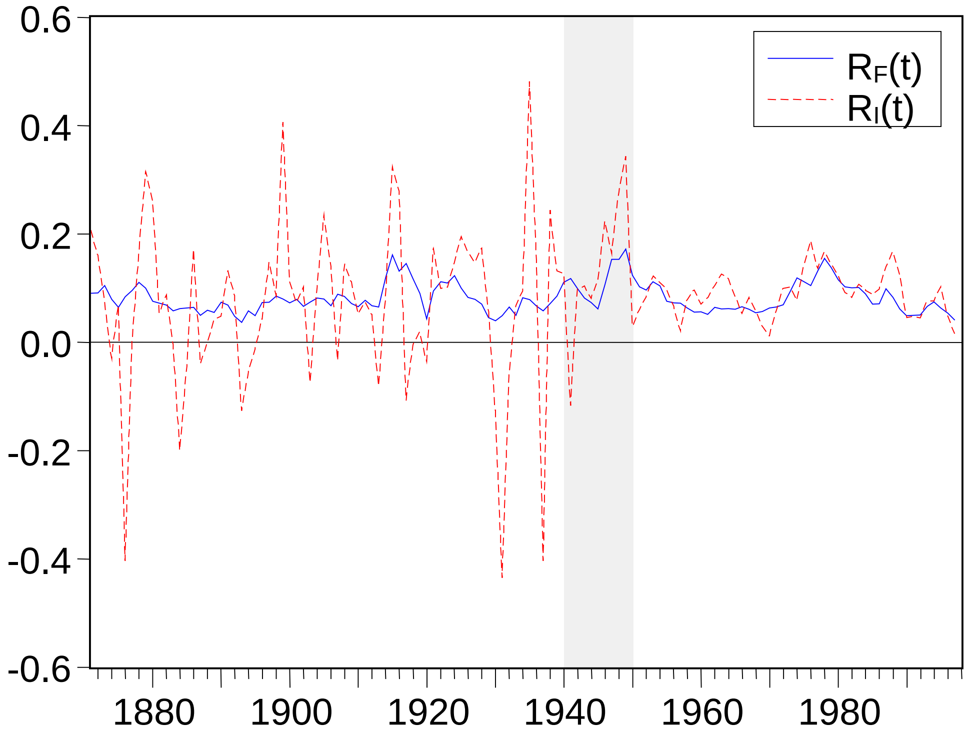


Figure 3 – The implied discount rates of the firm, $R_F$, and investor, $R_I$, plotted together as functions of time. Shaded region signifies transition between 1940-1950.

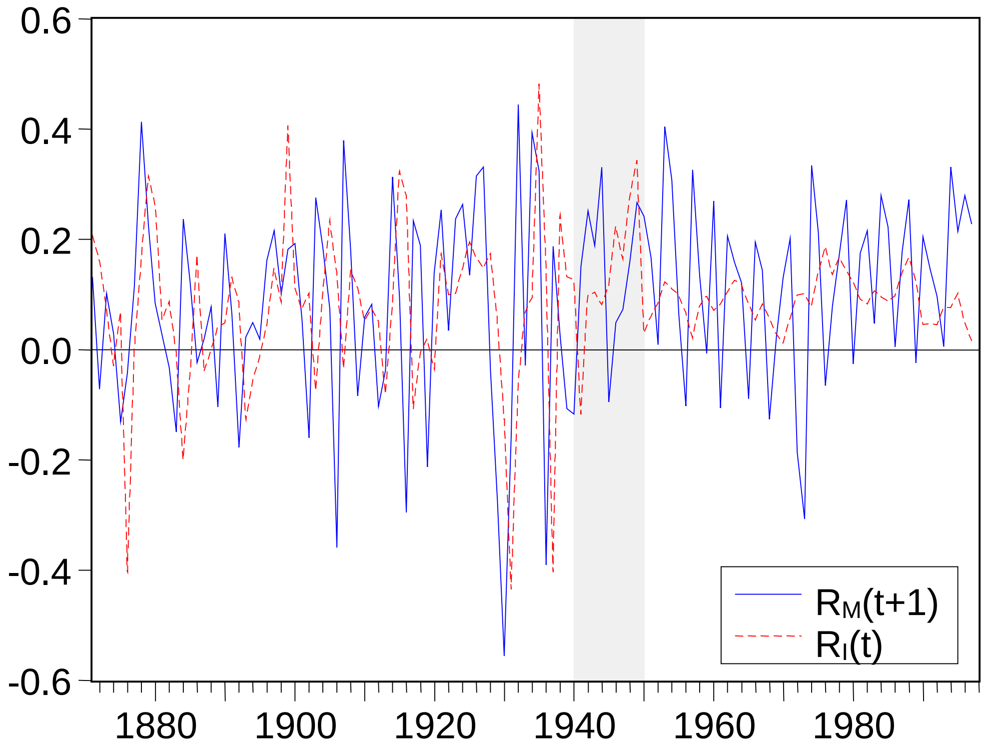


Figure 4 – The implied discount rate of the firm, $R_F$, and total rate of return of the market, $R_M$, plotted together as functions of time. Shaded region signifies transition between 1940-1950.

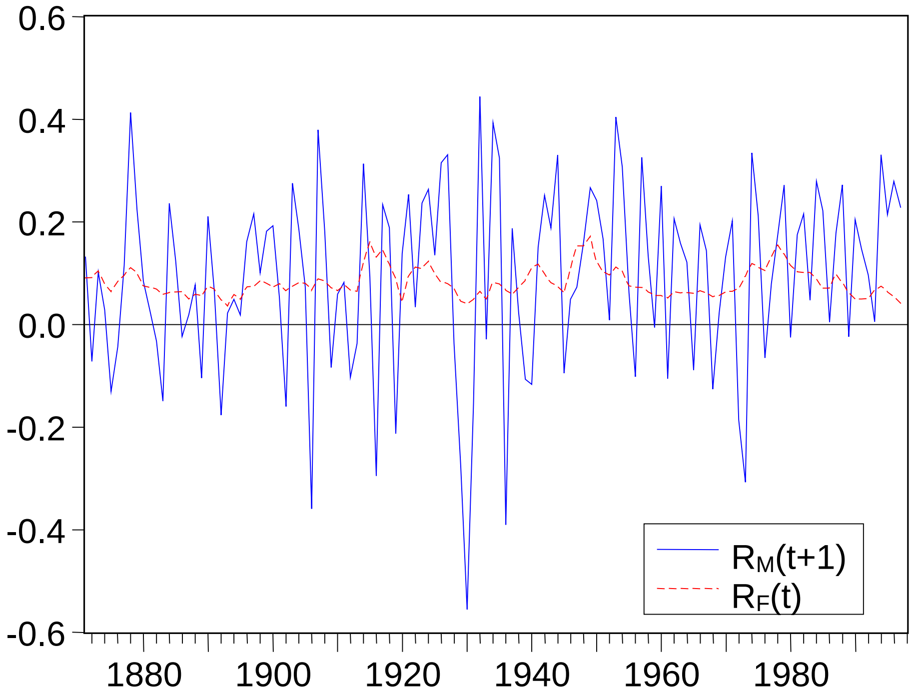


Figure 5 – The implied discount rate of the investor, $R_I$, and realised rate of return of the market, $R_M$, plotted together as functions of time.

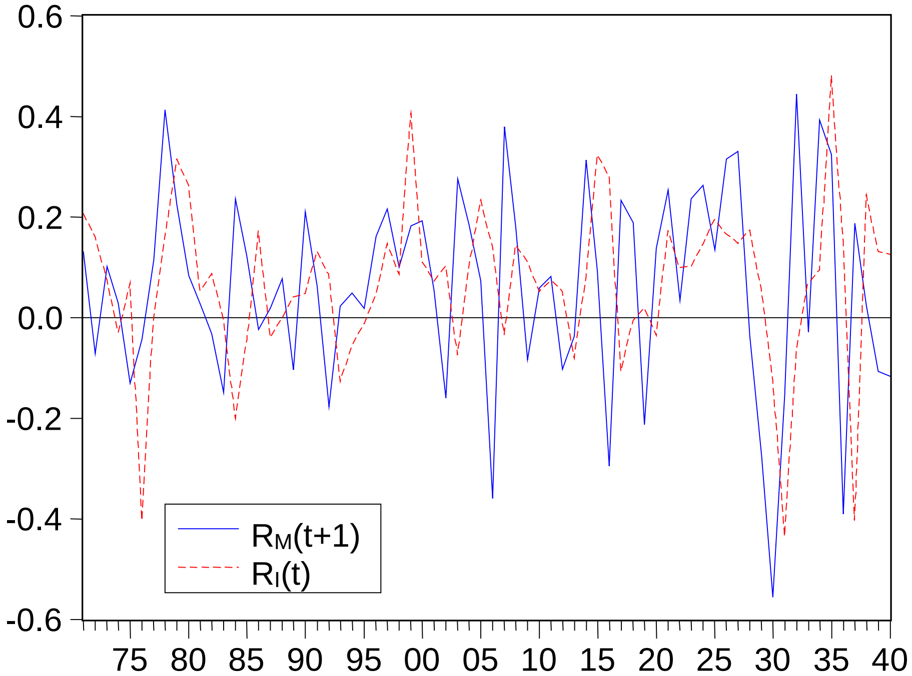


Figure 6 – The implied discount rate of the investor, $R_I$, and realised rate of return of the market, $R_M$, plotted together over the pre-transition period 1871-1940.

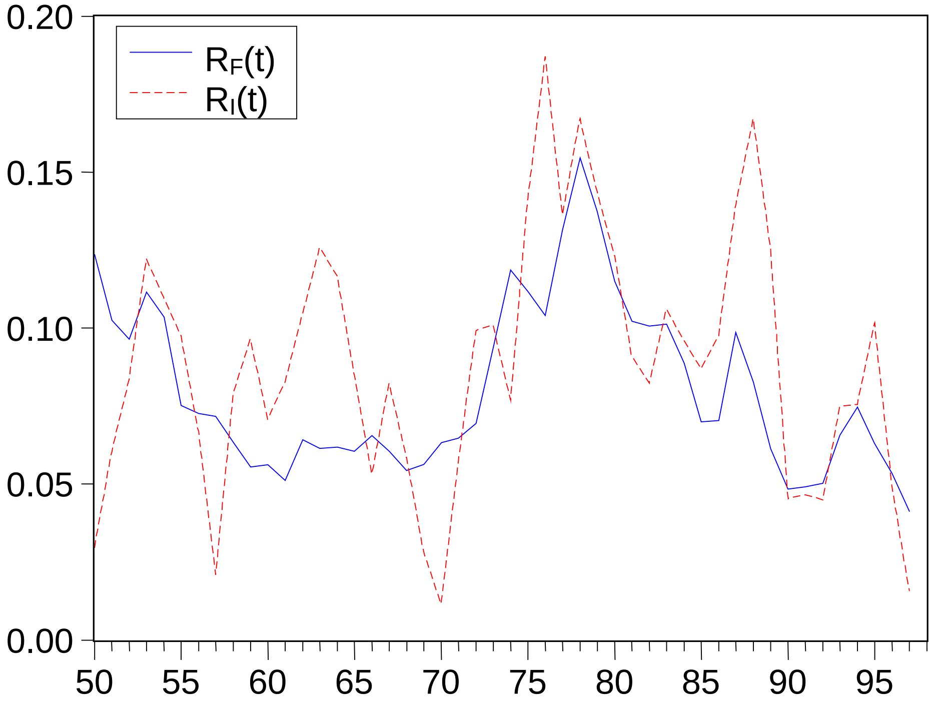


Figure 7 – The implied discount rates of the firm, $R_F$, and investor, $R_I$, plotted together over the post-transition period 1950-1998.

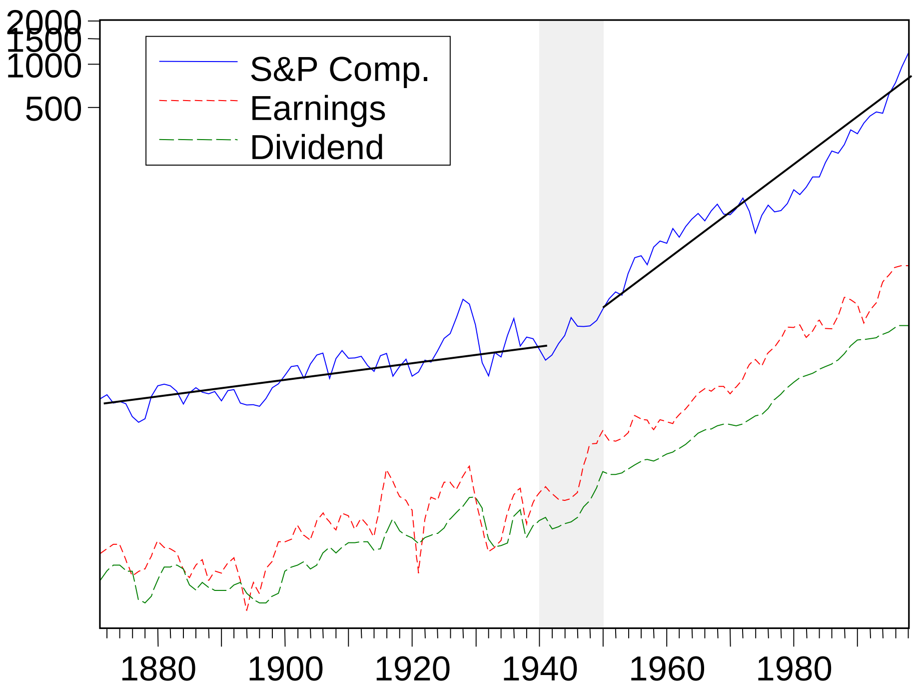


Figure 8 – S&P Composite index, earnings and dividends, all plotted logarithmically against time. Straight lines through S&P index elucidate the variation in trend across the transition. Shaded region signifies the transition period between 1940-1950.

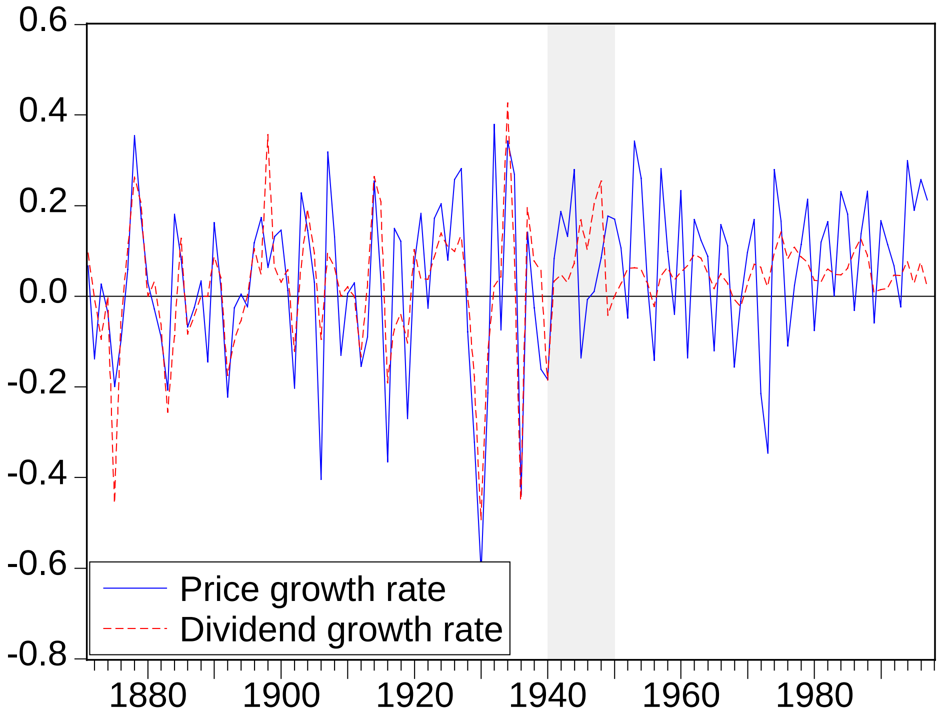


Figure 9 – Realised price growth rate, $k_S(t\text{-}1)$, and dividend growth rate, $k_\delta(t)$, both plotted against time. Shaded region signifies the transition period between 1940-1950.

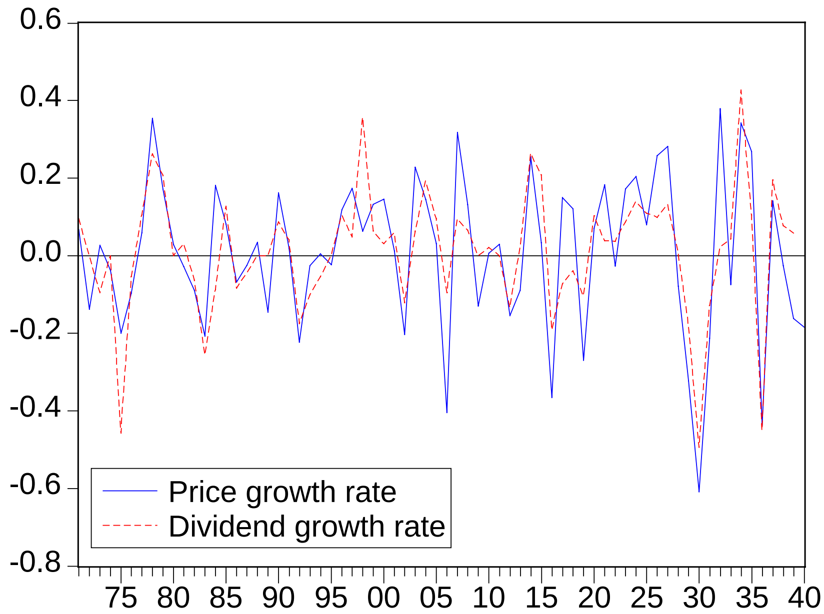


Figure 10 – Realised price growth rate, $k_S(t\text{-}1)$, and dividend growth rate, $k_\delta(t)$, both plotted against time over the pre-transition period between 1871-1940.

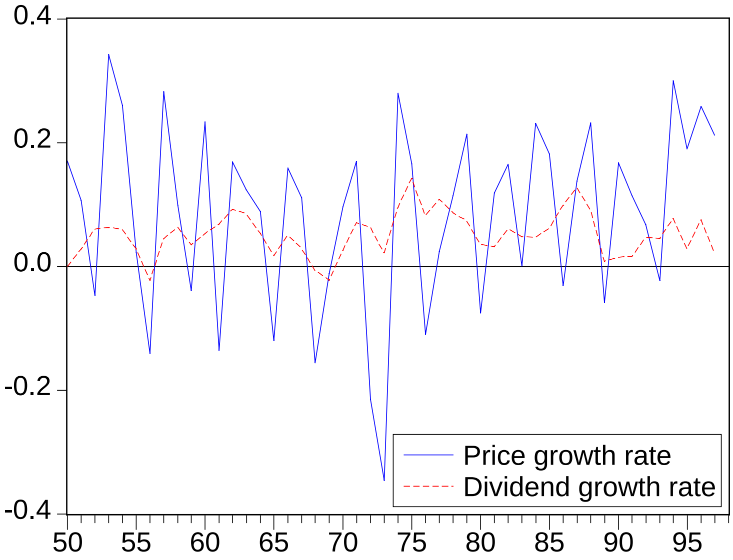


Figure 11 – Realised price growth rate, $k_S(t\text{-}1)$, and dividend growth rate, $k_\delta(t)$, both plotted against time over the post-transition period between 1950-1998.

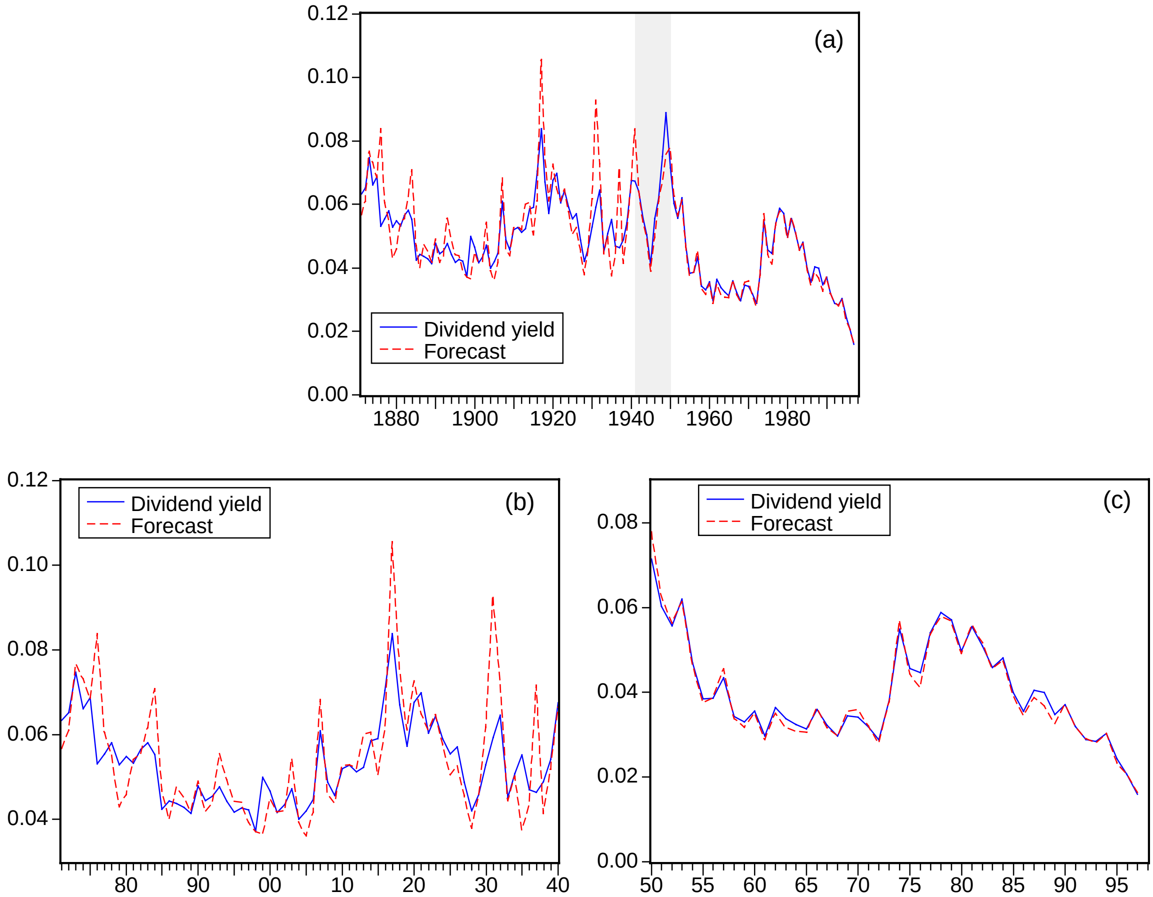


Figure 12 – Realised and forecast dividend yields as functions of time. The forecast is based on Equation 10 using the perfect-foresight assumption that $e_f(t) = e(t+1)$. Figure 12a compares the behaviours over the full range of historical data (1871-1998), with the transition period shaded. Figures 12b and 12c, respectively, show expanded views of the pre- and post-transition periods.

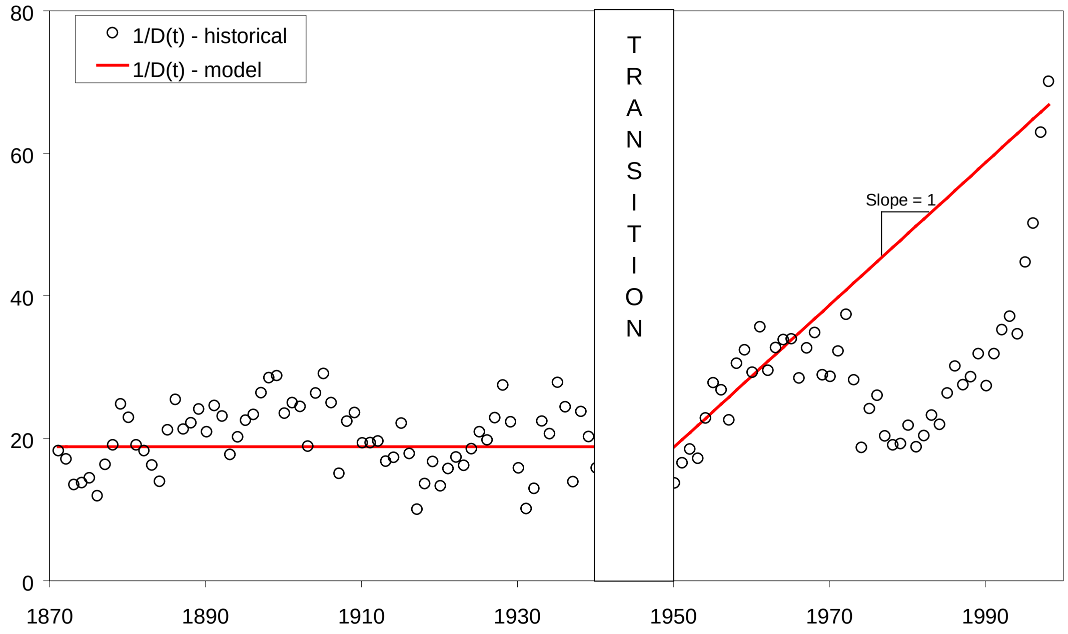


Figure 13 – Comparison of the inverted dividend yield, $1/D(t)$, between model and historical data. The lines arise from the model, whereby prior to 1940, a constant dividend yield was in effect, and after 1950, the dividend yield followed a behaviour [see Equation 28] consistent with that of a market in pursuit of efficiency and equilibrium.

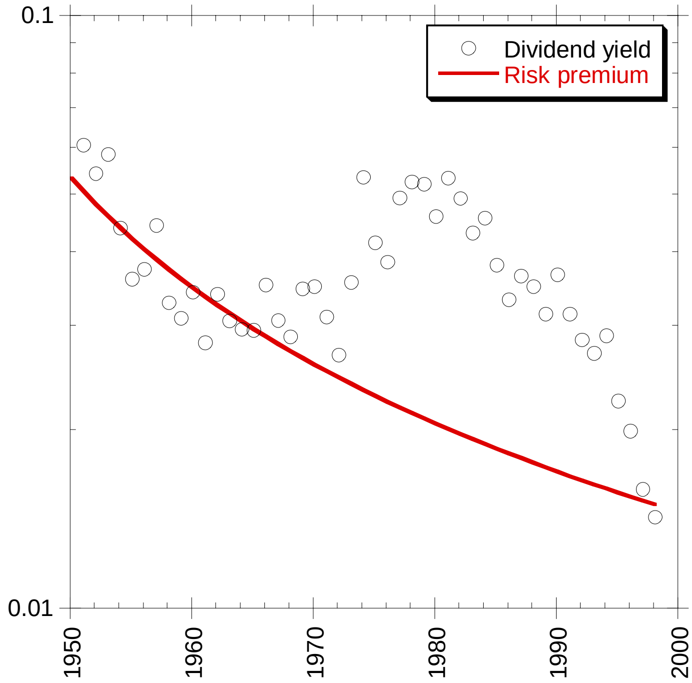


Figure 14 – The long-term, forward-looking risk premium based on Equations 26 and 28, illustrating its theoretical behaviour from the perspective of the investor's infinite-horizon perceptions. Note the slow decay to zero, as the market asymptotically approaches efficiency and equilibrium. For comparison, the dividend yield is included as well.