Some Macroeconomic Effects of Tax Reform and Indexing

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The views expressed in this paper are those of the authors; no responsibility for them should be attributed to the Bank of Canada.
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INTRODUCTION

The period from 1971 to 1974 was one of fundamental change in the structure of the Canadian system of taxation. From the federal budget of June 18, 1971, which brought the long process of tax reform to a close, to the introduction of the personal income tax indexing scheme the variety and magnitude of the changes in taxation provide a particularly fruitful period for fiscal policy research.

In this paper we report upon and discuss the results of simulating a macroeconometric model that includes most of the tax reform measures and indexing. We propose to examine in some detail a number of questions arising from these changes. Generally, we are interested in the implications of tax reform and indexing for stabilization policy. More specifically, we explore two interesting questions relating to fiscal policy. First, "What has been the effect of the 1972 tax reforms and indexing on the yield and elasticity of the income tax system?" By comparing the estimated tax yield over the 1972-1978 period of the pre-reform system with that of the reformed system one can separate the effect of the changes in the structure from the effect on revenue of the current inflation under progressive taxation. In a like manner, the effect of indexing can be determined by comparing the estimated yield of an indexed with an unindexed tax system over the same period. The second question we ask is, "What has been the effect of tax reform and indexing on the built-in stability of the economy?" Here we look at a measure of built-in stability for the pre-tax reform system and
for the post-tax reform system with and without indexing. In addition we apply various expenditure shocks to the model both with and without the indexing scheme in operation. Related to this, we also administer to the model a direct price shock to ascertain the relative stabilizing properties of the various tax regimes under an imported or commodity inflation.

The model we use in our analysis is an updated version of RDX2, incorporating the revised tax system, which is well suited for this study. It is a policy model rich in structural detail, particularly in the government sector, facilitating the introduction, also in detail, of complex measures such as capital gains and indexing. In section 2 of this paper, we briefly describe the tax model in RDX2 and the changes that were made to include the tax reform measures and the indexing scheme. In section 3 we present and discuss the simulation results, and in section 4 our conclusions.
THE MODEL

The General Approach

Of the 488 equations in the current version of RDX2, 273 are used to explain the major revenues and expenditures of the federal and provincial-municipal governments, including identities that indicate the financing implications of changes in the net expenditure or asset accounts of the two levels of government. The 221 equations in the personal income tax model, with which we are primarily concerned in this paper, have been constructed to include, wherever possible, explicit policy parameters as well as an accurate representation of the underlying dynamic behaviour of the income tax system. Thus, in the three main tax collection equations, actual collections are regressed on a synthetic collections series (derived by multiplying a weighted average rate of tax by an appropriate base) and then adjusted to reflect payment or recording lags or both. Specification of the tax equations in this manner has two advantages. First, in constructing the synthetic tax series, we can take account of the essential complexities of the tax structure, and, as we demonstrate below, we can model with a fair degree of precision such comprehensive and complex changes in the tax structure as the tax reform measures and indexing. Second, by regressing actual taxes on our synthetic series, we should get a coefficient not significantly different from unity that provides a fairly simple and straightforward criterion of a successful specification. The adoption of this procedure also enables us to avoid the problems of multicollinearity present in attempts to estimate jointly separate coefficients for a tax rate and various components of the relevant base [3].
Tax Reform, Indexing, and the Equations

Initially we had to decide what tax reform measures were to be included in the tax model. All the measures not so included would eventually be reflected in the coefficients of the various equations, but until the relevant data become available and the equations are reestimated, the excluded measures would have to be the ones expected to have little effect on tax revenues or other economic variables. No new structure was required in order to incorporate major reforms into RDX2. Several variables were added to define new taxable income items and certain existing variables, such as average tax rates, were redefined on a new basis.

The following reform measures were incorporated into the RDX2 tax model:

(i) the new schedule of marginal tax rates and income brackets;
(ii) higher basic single, married, and old age exemptions;
(iii) the inclusion in assessed income of capital gains, unemployment insurance benefits, and employer-paid medicare premiums;
(iv) the allowance of certain child care and employment expenses, and unemployment insurance premiums as deductions from assessed income;
(v) a higher rate of dividend tax credit applied against grossed-up dividends earned from Canadian corporations.

Many changes in the corporate income tax were also included in the revisions, but these changes required only a reworking of
existing variables with no alteration in the form of the equation
and are not discussed here.

One of the major differences between the current tax model
and previous versions is that we endogenize tax rates as well as
the ratios that spread aggregate assessed wage income, nonwage
income, and the number of taxpayers across the fourteen income
groups and four income classes defined in the model. These
changes in the tax model, described in more detail below, provide
two major advantages: they allow us to index the personal income
tax precisely as the scheme is designed to operate and free us
from a dependence on tax data available only after a long lag.

In the flow chart on the next page we outline in a general
way the structure of the personal income tax model and show where
that model connects with the rest of RDX2.

There are six equations at the heart of the personal income
tax model: two equations for total tax collections (TPS, TPO),
one equation for provincial collections (IPYFM), an identity that
has federal collections (IFYF) as a residual, and two equations
defining personal tax accruals (IAW, TANW). All other equations
in the tax model are used to derive the income, exemption, and
tax rate variables that appear in the accruals equations.

At the first level of disaggregation we distinguish between
assessed wage income (YWAS) and assessed nonwage income (YNWAS),
since these types of income fluctuate differently, are taxed at
different effective rates, and are characterized by different
lags between the accrual of tax liability and the payment of tax.
This distinction between income types leads us to two basic tax
collection equations that reflect the two ways taxes are paid and
A STYLISED FLOW CHART OF THE PERSONAL INCOME TAX MODEL

From RDX2

Employment (NE) → Number of Taxpayers (NT) 9.7

National Accounts Income Variables

Assessed Wage Income (YWAS) 9.8

Assessed Nonwage Income (YNWAS) 9.9

Consumer Price Index (PCPI)

Average Group Exemptions (ZEXYG) Unindexed

Average Group Exemptions (ZEXYG) Indexed

Indexing Factor (RTI) 9.30

Displaced Lognormal Distribution

Quantiles

UQLI

UOMI

UQII

S9.1 - S9.9

Parameters

UDSI

UMEANI

USDMAI

S9.10 - S9.18

Spreading Ratios (Group & Class)

USRNW

S9.19 - S9.32

S9.61 - S9.63

USRW

S9.33 - S9.46

S9.64 - S9.66

USRNT

S9.47 - S9.60

S9.67 - S9.69

Average Class Exemptions (ZEXYC) Indexed

S9.112 - S9.135

Average Group Exemptions (ZEXYG) Indexed

S9.70 - S9.111

Marginal Tax Rate Schedule and Brackets (YBRACK) (RMARK)

Average Tax Rates (Group & Class)

(RTPYFB) (RTPYF)

(RTPYQ)

S9.138 - S9.191

Average Tax Accruals (TAW) 9.5

(TANW) 9.6

TAX COLLECTIONS (TPS) 9.1

TP0 9.2

(TPYPM) 9.3

To RDX2

Equation numbers for endogenous variables are listed below mnemonics.
collected. Because taxes on wage income are deducted at source (TPS), a one-month lag generally occurs between the accrual of tax and its collection. The tax on non-wage income (TPO) is usually paid in quarterly installments based on tax liabilities in the previous calendar year. As well as this difference in payment procedures, we must allow for refunds (generally involving wage income) and make-up payments made in the first and second quarters of the following year. Thus, we specify total tax collections as

\[
TPS = \alpha_0 (2/3 TAW + 1/3 TAW_{t-1}) + \alpha_1 Q_1 \sum_{i=1}^{4} TAW_{t-i} + \alpha_2 Q_2 \sum_{i=2}^{5} TAW_{t-i}
\]

\[
TPO = \beta_0 Q_1 \sum_{i=1}^{4} TANW_{t-i} + \beta_1 Q_2 \sum_{i=2}^{5} TANW_{t-i} + \beta_2 Q_3 \sum_{i=3}^{6} TANW_{t-i} + \beta_3 Q_4 \sum_{i=4}^{7} TANW_{t-i} + ECRPM
\]

where

TAW and TANW are accrued tax liabilities on wage and non-wage income, respectively, and ECRPM is an exogenous variable accounting for the increasing use of tax credits by provincial governments. The Q variables are quarterly dummies.

We expect that \(\alpha_0 + \alpha_1 + \alpha_2\) will not be significantly different from 1.0 and that \(\alpha_1\) and \(\alpha_2\) will both be negative to reflect refunds. Similarly, in the TPO equation we expect \(\beta_1\) to be greater than \(\beta_0\), \(\beta_2\), and \(\beta_3\) and the sum of the coefficients to be greater than 1.0, since the effects of income growth are not
explicitly allowed for but rather are reflected in the \( b_1 \) coefficient, which indicates larger make-up payments.

At this point, for the provincial tax collection (TPYPM) and accruals equations (TAW, TANW), a further level of disaggregation is introduced into the model. Both the wage and nonwage models are disaggregated according to four broad income classes, based on assessed income and ranging from $0-$3,000, $3,000-$5,000, $5,000-$10,000, and over $10,000. Each of the income classes is in turn disaggregated further into fourteen income groups - two groups each in income classes 1 and 2, and five groups in income classes 3 and 4. Disaggregation of the tax data by income level allows us to build progressivity into the tax structure reasonably and explicitly, so that in simulating alternative policies we can consider directly changes in the progressivity and composition of the tax rate structure. We use the group level of disaggregation only in the calculation of average tax rates. It can be switched-off, making the tax rate variables at the class level in effect exogenous, if such detail is not required. Class variables are weighted sums of their group components.

The two basic accrual identities for wage and nonwage income are:

\[
TAW = \sum_{i=1}^{4} \left( RTPYFiC \left[ YWASiC - (NTWiC) (ZEXYWiC) \right] \right)
+ \sum_{i=1}^{4} \left( RTPYFBiC \left[ YWASiC - (NTWiC) (ZEXYWiC) \right] (RTPYPXQ - RFAQ - RFAQ) \right)
+ \sum_{i=1}^{4} \left( RTPYQiC \left[ YWASiC - (NTWiC) (ZEXYWiC) \right] (RTPYPQ) \right)
\]
\[
I_{ANW} = \sum_{i=1}^{4} (RTPYFiC) \cdot (YNWASiyC - (NTNWic) \cdot (ZEXYNWic)) - 0.9 (RDC) \\
(\text{EYDIVA11}) (\text{YDIV11}) 
\]

\[
+ \sum_{i=1}^{4} (RTPYBiC) \cdot (YNWASiyC - (NTNWic) \cdot (ZEXYNWic)) - 0.9 (RDC) \\
(\text{EYDIViC}) (\text{EYDIVA11}) (\text{YDIV11}) (RTPYPXQ-RFAQ-RFAQ) 
\]

\[
+ \sum_{i=1}^{4} (RTPYQiC) \cdot (YNWASiyC - (NTNWic) \cdot (ZEXYNWic)) - 0.9 (RDC) \\
(\text{EYDIViC}) (\text{EYDIVA11}) (\text{YDIV11}) (RTPYPQ) 
\]

Tax accruals on wage income \((I_{ANW})\) in each income class (or group) are the sum of three terms, each the product of a tax rate and a tax base. The first term is the federal tax accrual, the product of a weighted federal rate \((RTPYFiC)\) and the tax base, total taxable wage income approximated by total assessed wage income \((YNWASiyC)\) less total exemptions and deductions (average exemptions and deductions times total tax returns filed by wage earners). This federal term represents total tax liabilities based on the assumption that the provinces levy taxes on the same base and at rates equal to the federal abatement (prior to 1972). Since they do not, the second and third terms represent the taxes incurred because of the imposition of provincial taxes (excluding Quebec taxes) at rates \((RTPYPXQ)\) greater than the federal abatement rates \((RFAQ, RFAQ)\), and because Quebec collects its own income tax at rates and in ways increasingly different than those prevailing the rest of the country. \(^5\)

The provincial tax rate is the weighted average rate of tax for each province except Quebec and the base is 'basic tax', calculated as the average federal 'basic tax' rate \((RTPYFiC)\) in
each income class times the income base as defined above. The Quebec term is similar to that for the other provinces, but, because Quebec does not index, we define an unindexed ‘basic tax’ rate (RTPYQiC) that is applied against the same income base. Up to the beginning of indexing, the RTPYBEiC and RTPYQiC are the same. The other rate in the Quebec term is the weighted average rate of tax in Quebec (RTPYPQ) and it is similar to the provincial rate excluding Quebec. Using these different tax rate variables we can model the increasing use by the federal government of tax changes - such as tax cuts and surtaxes - that do not affect provincial revenues, and vice versa.

The nonwage accruals identity (TANW) is similar to TAW except that the income base is taxable nonwage income \([YNWASiC-NINWiC(ZEXYNWiC)]\) with allowance made for the dividend tax credit. The credit is defined as \((RDCEYDIVA11)(YDIV11)\) where \(RDC\) is the rate of dividend tax credit, \(YDIV11\) is dividends paid to Canadians by Canadian corporations, and \(EYDIVA11\) is the proportion of \(YDIV11\) assessed for tax purposes. Since the basic tax and the basic tax rate are net of the dividend tax credit, the tax base for the provincial terms excludes the dividend credit.

These equations and the equation for provincial tax collections\(^6\) provide the backbone of the personal income tax model. The rest of the model contains the equations used to define reasonable series for the independent variables in the two accrual identities. It is at this point that tax reform, indexing, and any other changes in the tax system enter the model.
The Tax Base

The tax base in the model is a function of assessed wage income, assessed nonwage income, the number of tax returns, and average exemptions and deductions. We generate total wage and nonwage assessed income (YWAS, YNWAS) and total tax returns filed (NT) by what we refer to as behavioural identities. Since these data are obtained from annual taxation statistics [12], we regress the annual series on appropriate national accounts and other model variables and then use the estimated coefficients along with quarterly values of the independent variables to generate quarterly series for the dependent variables. The equations for these total variables are

\[
NT = -2.6198 + 1.2398 (NE + NCAPR)
\]

\[
YWAS = 0.80472 \times YWNA + 0.00659 \times YWNA (QTSTEP)
\]

\[
+ (QTXREM) (YWSLIMED + GTPUIBF) + (QITA) (GTPFAF)
\]

\[
YNWAS = 0.37401 \times YNWNA + 0.00331 \times YNWNA (QTSTEP)
\]

The NT equation relates the total number of tax returns filed to an approximation of the number of people obliged to pay tax, comprised primarily of those in the workforce earning income (NE) or receiving taxable transfer payments such as old age pensions (NCAPR). We suspect that the structure of this equation is not as well specified as it should be, but the variable is not critical enough to the functioning of the model to warrant an overly complicated specification.

The variables YWNA and YNWNA are the national accounts counterparts of YWAS and YNWAS, respectively. There are a number
cf differences between income as defined for tax purposes and income as it appears in the national accounts [13]. The independent variables have been constructed to approximate closely the tax definition. For assessed wage income the counterpart is national accounts wages and salaries plus military pay and allowances, excluding supplementary labour income not assessed for tax purposes. For assessed nonwage income the counterpart is the remainder of personal income net of all items we treat separately such as dividends. A time trend variable (QSTEP) is included to capture the growth in assessed income as a proportion of national accounts income.

The first of our adjustments for tax reform enters at this point. A term has been added to the YWAS equation to include as taxable income unemployment insurance benefits (GTPUIBF) and employer-paid medicare premiums (YWSIMED). Since these items were not taxable prior to 1972, they are multiplied by a dummy variable (QTXRFM) defined as zero prior to 1972 and 1 thereafter. (The remaining term in the YWAS equation picks up the taxation of family allowances payments (GTPFAF) beginning in 1974.)

We have assumed that tax reform did not affect in a major way the NT relationship. There may possibly have been a shift in this relationship, depending upon whether, on balance, the various reforms have added people to or removed them from the tax rolls. The trend has been for NT to grow as a proportion of NE+NOAPR and we think that if tax reform has affected the relationship in any way it has merely accelerated this trend rather than shifted it.
The most significant change in the tax system introduced by tax reform is the inclusion in the tax base of realized capital gains. But, before describing how this measure has been handled in the model, we must briefly outline the procedure used to disaggregate the aggregate income and tax return variables into fourteen income groups and four income classes.

The Spreading Ratios

Given the quarterly series for YWAS, YNWAS and NT, we disaggregate by income group and income class using spreading ratios. The spreading ratio for the jth income group (or ith income class) is the proportion of total assessed wage or nonwage income or taxpayers in the jth group; i.e. for assessed wage income USRWj = YWASj/YWAS; for assessed nonwage income USRNWj = YNWASj/YNWAS; and for the number of taxpayers USRNTj = NTj/NT (j = 1,14), and where

\[
\begin{align*}
14 & \quad 14 & \quad 14 \\
\sum_{j=1}^{14} USRWj & = \sum_{j=1}^{14} USRNWj & = \sum_{j=1}^{14} USRNTj = 1
\end{align*}
\]

The four class-spreading ratios for each of the aggregate variables are derived by summing the group ratios in each class.

Until the present version of the model was constructed these spreading ratios based on taxation data were treated as exogenous. This proved to be unsatisfactory, since the values of these spreading ratios are highly dependent on the level of assessed income per taxpayer. It is a deficiency we find particularly serious in forecasting and simulation exercises where the level of assessed income per taxpayer is likely to
change substantially. Without a specific model of how the income distribution behind the spreading ratios changes over time, one cannot project income tax revenues accurately. We chose as a solution to this problem the estimation of a distribution across income groups for each spreading ratio and the explanation of the parameters that define the distribution in terms of other RDX2 variables.

There are a multitude of distributions from which to choose. In studies of income distribution, however, the lognormal distribution has received the most attention and is preferred to the normal distribution because it exhibits the positive skewness characteristic of the frequency distribution of income. Both are two-parameter distributions. More recently a case has been put forward by C.E. Metcalfe [20] for the superiority of another member of this same family, the displaced lognormal distribution. This distribution is sufficiently flexible to allow for non-symmetric shifts in income distribution, and the additional parameter gives a tighter empirical fit.

A variable has a displaced lognormal distribution if the logarithm of the variable plus some constant is normally distributed. The three parameters are the displacement (UDIS), the mean (UMEAN), and the standard deviation (USIGMA) of the transformed variable. There are various ways of estimating these parameters. A technique suggested by Aitchison and Brown [1] for use with grouped data is the method of quantiles. They advise the choice of three symmetric quantiles under which lie 10 percent, 50 percent and 90 percent of the cumulative frequency. If these quantiles are called UQL, UCM, and UCU, the three
parameters of the distribution can be determined from the following formulas:

\[ \text{UDIS} = \left( \frac{\text{UQM}}{\left( \frac{(\text{UQL} + \text{UQU})}{(\text{UQM}^2)} \right) - 1} \right) \left( \frac{2-\text{UQL}/\text{UQM} - \text{UQU}/\text{UQM}}{} \right) \]

\[ \text{UMEAN} = \ln(\text{UQM} + \text{UDIS}) \]

\[ \text{USIGMA} = \ln\left( \frac{\text{UQU} + \text{UDIS}}{\text{UQM} + \text{UDIS}} \right) / 1.2815 \]

The constant 1.2815 is the number of standard deviation units within which, on either side of the mean, lie 80 percent of the observations.

The parameter of displacement (UDIS) provides the displaced lognormal distribution with an added degree of flexibility over the normal and lognormal. Positive values of UDIS enable the displaced lognormal to fit an empirical distribution intermediate between a lognormal and a normal, that is a distribution with some degree of positive skewness. Negative values of UDIS enable the displaced lognormal to fit an empirical distribution that is even more positively skewed than the lognormal.

The displaced lognormal was fitted to disaggregated taxation statistics for the distribution across income groups for NT, YWAS, and YNWAS. Nineteen income groups were used. Regressions of the actual frequencies in these groups on those predicted by the displaced lognormal for all three distributions yielded coefficients of determination in the neighborhood of .99. Thus we considered the displaced lognormal to be a satisfactory representation of the empirical distribution. All three of these distributions exhibited the positive skewness of the displaced lognormal, with the distribution of NT being the least positively
skewed and that for YNWAS being the most, primarily because of
the heavy concentration of pension income in the low income
groups.

Having found a satisfactory distribution, we made the
spreading ratios endogenous by explaining the parameters of the
distribution in terms of other economic variables. We estimated
the quantiles and used the above formulas to derive the
parameters of the distribution instead of estimating the
parameters directly, since the economic interpretation of the
quantiles is more clear cut. Changes in the lower quantile
represent changes in the bottom of the income distribution,
changes in the median quantile represent changes in the central
tendency, and changes in the upper quantile represent changes in
the upper tail of the income distribution.

The logarithm of each of the nine quantile variables (three
each for the NT, YWAS, and YNWAS distributions) is estimated as a
linear function of assessed income per taxpayer, seasonally
adjusted at annual rates and a constant. For example, the
equation for the lower quantile of the distribution of NT across
the fourteen income groups is,

\[ \ln(UQLNT) = 5.7223 + .13097 \ln\left( \frac{(YWAS+YNWAS)}{(NT)(ESAYAS)} \right) \]

where

ESAYAS is the seasonal adjustment factor.

This specification maintains a constant elasticity of the
quantiles with respect to assessed income per taxpayer. The
estimation results show the elasticity for the lower quantile to
be the smallest and for the upper the highest. Thus, the
distributions tend to spread out over time, as one would expect.
With the quantile variables, the parameters of the distributions
can be obtained from the formulas described above. The spreading
ratios are then easily derived by simple integration over the
distributions as defined by these parameters.

The spreading ratios times the respective aggregate give the
group or class variable, ie for NI, NTj = USRNTj(NI). The class
variable is obtained by aggregating over groups in the class
NTiC = Σ NTj and similarly for YNWAS and YWAS. Finally, we must
disaggregate each NTiC into returns filed by taxpayers who are
having deductions made at source (NTWiC) and taxpayers who are
self-assessed (NTNWiC). We proceed by assuming that in each
income class, mean assessed wage income is equal to mean assessed
nonwage income. This assumption is obviously not valid for all
classes, if indeed for any, since a taxpayer can be a wage earner
as well as a nonwage earner. Nonetheless it yields results that
appear to be fruitful and for want of a better assumption, we
proceed by writing:

YWASiC/NTWiC = YNWASiC/NTNWiC.

Solving for NTWiC and substituting the identity
NTiC = NTWiC + NTNWiC,
we obtain

NTWiC = [ YWASiC/(YNWASiC+YWASiC) ]NTiC.

Given the NTWiC, we define the NTNWiC from the NTiC identities.
Capital Gains Taxation

The most significant change in the tax system to be introduced by tax reform is the addition of certain capital gains income into the tax base. A capital gains variable (YKGP) is defined in RDX2 [18], so this source of income is easily brought into the tax sector. To do this we redefine YKGP as annual accrued capital gains and losses (YKGPA), cumulate net accruals to yield a stock of unrealized accruals, and then assume that some proportion of this stock is realized (YKGPR) each year. Half these realized gains (losses) are then added to (subtracted from) assessed income.

Accrued capital gains in RDX2 are derived from two sources: pure capital gains from the increase in the market value of the resident-owned business capital stock (ie the change in the market value less the value of new investment), and capital gains resulting from retained earnings.8

Accrued capital gains as calculated above exclude gains on real estate, other than principal residences, that are subject to the tax. Kul Bhatia [5] estimates that these accrued gains would be approximately $119.1 million in 1972. This means realized capital gains would be approximately $12 million higher in 1972 than the amount obtained with our calculations. At this rate it would take six years before the understatement would reach $60 million. The omission of this type of capital gain therefore is not likely to be important.

Realized capital gains are taxable, and we define YKGPR as a proportion of the stock of unrealized accrued gains. The stock is

\[ AYKGPA = AYKGPA_{t-1} + YKGPA - YKGPR \]
where $AYKGPA = 0$ beginning, in 1972

Realizations are

$$YKGPR = 0.018(AYKGPA_{t-1} + YKGPA)$$

The proportion of accrued gains realized is assumed to be 1.8 percent per quarter. The proportion can vary depending on economic conditions and, in fact, in the United States it is closely related to the volume of trading in the stock market. But since the Canadian experience with a capital gains tax is too short to provide a precise estimate of this proportion, we decided to fall back on the U.S. experience. The actual proportion in the United States is approximately 5 percent, and to this we have added a further 2.3 percent to allow for the fact that in Canada, in contrast to the United States, capital gains are deemed to be realized at death.  Consequently, our final estimate is 7.3 percent per year or 1.8 percent per quarter.

Since only half of realized capital gains are taxable, fifty percent of $YKGPR$ is added to assessed nonwage income and spread among the income classes using exogenous dividend spreading ratios ($EYDIViC$).

The final change in the income base due to tax reform is the treatment of dividends and the dividend tax credit. On the one hand, the dividend tax credit has gone up from 20 percent to 26.7 percent, and on the other, tax is now levied on dividends after they have been grossed up by one third. The dividend tax credit is already included in personal income tax accruals on nonwage income ($TANW$) so it is only necessary to spread grossed-up dividends among the nonwage income classes. To do this we use
the exogenous dividend-spreading ratios, being the proportion of dividends assessed in each of the four income classes.

For assessed nonwage income in each income class, therefore, we have

\[ YNWAS_iC = USRN\text{W}_{iC}(YNWAS) + [1 + 0.3(CTXRFM)(EYDIV_iC)(EYDIV_{11})(YDIV_{11}) + (CTXRFM)(EYDIV_iC)(0.5 \ YKGFR) \]

Two items affected by tax reform and particularly by indexing remain to be considered: exemptions and deductions, and tax rates.

Exemptions and Deductions

Average annual levels of total exemptions and deductions (ZEXYGj) and personal exemptions (ZEXPERj) in each income group are exogenous to the model and are derived from the taxation statistics. Quarterly indexed average exemptions and deductions by class are endogenous, however, because the quarterly pattern of the annual group variables is determined by the quarterly pattern of YWAS and YNWAS. To do this we assume that average exemptions and deductions for wage income are equal to those for nonwage income in each income group. As well, the indexing factor (RTI), derived from the consumer price index (PCPI), is endogenous and average exemptions by class are weighted sums of the group variables with the USRN\text{Tj serving as weights.

Exemptions and deductions have been changed not only by the tax reform measures but also by the May 8, 1972, February 19,
1973 and the November 18, 1974 federal budgets. Tax reform and the May 8 budget changes for 1972 included: (i) an increase in the basic personal exemption for single taxpayers from $1,000 to $1,500, (ii) an increase in the married exemption from $2,000 to $2,850, (iii) an increase in the old age exemption from $500 to $1,000, (iv) an employment expense deduction of 3 percent of employment income to a maximum of $150, and (v) the allowance of unemployment insurance contributions as a deduction from assessed income. In 1973 the exemption for single taxpayers was increased by a further $100, and for married taxpayers by a further $150. In 1974, up to $1,000 of interest income became exempt and in 1975 this was extended to include dividend income. Also in 1975 an additional $1,000 exemption for pension income was allowed.

An appreciation of the size of the increase in average exemptions and deductions produced by these measures can be obtained from an examination of the following table.

<table>
<thead>
<tr>
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</thead>
<tbody>
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<td>$0-3,000</td>
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<td>90</td>
<td>236</td>
<td>172</td>
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<td>$3,000-5,000</td>
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<td>135</td>
<td>647</td>
<td>318</td>
</tr>
</tbody>
</table>

* 1971 taxation statistics weights [12].
** In the $2,000 class only 70 percent of the increase is included because of the underutilization of exemptions.

It is evident in Table 1 that the effect of changes in 1972 introduced as a result of tax reform are much greater than
changes in any subsequent year. The cumulative effect of the 1973-1975 changes, however, is about equal to 1972 tax reform.

The indexing of exemptions is straightforward. Total average group exemptions and deductions are increased at the beginning of each year by the amount of the increase in personal exemptions (ZEXPERj) due to indexing.

\[ ZEXYGj = ZEXYGj + (.86 \text{ RTI}-1)(ZEXPERj) \]

where

.86 is an estimate of the proportion of personal exemptions indexed, given that Quebec does not index. The indexing factor (RTI) is discussed below.

Tax Rates

We now consider in detail the calculation of the various tax rates used in the model. The attention given to the appropriate and explicit specification of tax rates reflects to a large degree the important role they have in fiscal policy. The need for more than one tax rate arises because both the federal and provincial governments tax personal income and because both levels of government use the income tax, and particularly income tax rates, as instruments of fiscal policy. For the federal income tax there are the average federal tax rates - one for each income group defined as the tax payable on the mean taxable income in the jth income group as a ratio to the mean taxable income in that group. The tax payable is obtained directly by applying the appropriate tax schedule (defined by its marginal
rates ($RMARF_k$) and the income brackets ($YERACK_k$), which is exogenous to the model, and by utilizing all the group income, exemption, and tax-return data calculated in other equations. Thus

\[
RTFYF_j = \left( \sum_{k=1}^{n-1} \left[ (RMARF_k) (YERACK+1-YERACK) (RT1) \right] + (RMARFn) [YTAj-(YERACn) (RT1)] / YTAj \right)
\]

where

$YTAj$ is the appropriate tax base.

In the above case, the tax base is average assessed wage and nonwage income less average exemptions and deductions in the $j$th group.

\[
YTAj = \left[ (USRWj) (4YWAS) / ((USRNTj) (NT) (EASYW)) + (USRNWj) (4YNWAS) / ((USRNTj) (NT) (EASYNW)) \right] - ZEXYGj
\]

(All rates are annual, hence the quarterly values must be seasonally adjusted ($EASYW$, $EASYNW$) and put at annual rates ($4YWAS$, $4YNWAS$).

The other major rate of tax we use is the basic rate ($RTFYFBj$). The basic rate is required because it defines the base for provincial income taxes. It is calculated in the same way as $RTFYFj$ except that the numerous federal tax cuts, surtaxes, etc. are not included in the calculation. This is the only difference between $RTFYFj$ and $RTFYFBj$. All the federal tax measures in recent budgets,\textsuperscript{11} designed so as not to affect
provincial revenues, can and have been explicitly worked into the tax model.

The other tax rates, in conjunction with the basic tax rate, are provincial rates. Prior to 1972, under a series of agreements, the federal government partially withdrew from the income tax field to allow the provincial governments to levy their own income taxes without significantly increasing the total tax burden. The federal withdrawal is reflected in the tax abatement rates (\( RFXQ \), \( RFAQ \)) and in provincial taxation by the weighted average rate of provincial tax (\( RFTPXPQ \), \( RTPXPQ \)). There are two abatement rates and two weighted provincial rates owing to the different treatment accorded Quebec (higher abatements for Quebec because it has opted-out of several federal-provincial shared-cost programs and the fact that Quebec collects its own income tax). We assume that the Quebec tax schedule and tax base are approximated by a single rate (\( RTPYPQ \)) and by the tax base that is used for other provinces. Initially, with the introduction of tax reform in 1972, Quebec attempted to harmonize its tax system with that of the rest of Canada. But following this initial attempt, Quebec has gone its own way - the most important difference now between it and the other provinces is its refusal to index [22]. We have therefore made changes in our model, such as the introduction of non-indexed exemptions and a non-indexed basic tax rate (\( RTPYQ_iC \)), to reflect these differences. If this trend continues (Ontario has indicated it will collect its own income tax after the tax collection agreement expires in 1977) it may be necessary to have eleven tax models, but in the meantime we think the present structure
captures the essential differences between the Quebec system and the federal system adequately.

How has the tax model been affected by tax reform and indexing? The structure of tax rates was changed significantly by tax reform, and indexing has changed average rates. In Table 2 we compare the pre- and post-reform tax schedules. The most striking differences between them are the higher marginal tax rates on taxable income up to $15,000 and the higher average tax rates on taxable income up to $125,000 under the new system. Looking at these rate changes and the higher exemptions one could easily conclude that the income elasticity of the new system would be higher than that of the former system. But, as we demonstrate later, this does not prove to be the case.

In our model, the \( RMARF_k \) and \( YERACK \) variables come from the tax rate schedule. The model has been designed so that different schedules with varying degrees of progressivity can be easily introduced by changing the values of \( RMARF_k \) and \( YERACK \).

Under the new system provincial rates are higher, since they now apply to a new lower tax base not inflated by abatements. Furthermore, the abatement rate to all provinces excluding Quebec \( (RFAQ) \) must be set equal to zero, and the abatement rate to Quebec \( (RFAQ) \) must be substantially reduced.\(^{13}\) The weighted average provincial tax rates calculated for the new tax system are as shown in Table 3. The rate for all provinces excluding Quebec \( (RTFYFXQ) \) embodies, from 1973 on, the expiration of the 3 percent Ontario tax cut; the increase in the Saskatchewan rate from 37 to 40 percent; and, from 1974 on, the increase in the
Table 2

COMPARISON OF PERSONAL INCOME TAX RATES
BEFORE AND AFTER 1972 TAX REFORM

<table>
<thead>
<tr>
<th>Taxable Income Bracket</th>
<th>Before Reform*</th>
<th></th>
<th>After Reform**</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
<td>Marginal</td>
<td>Average</td>
<td>Marginal</td>
</tr>
<tr>
<td>0 - 500</td>
<td>0</td>
<td>14.80</td>
<td>0</td>
<td>22.20</td>
</tr>
<tr>
<td>500 - 909</td>
<td>14.80</td>
<td>17.00</td>
<td>22.79</td>
<td>23.5</td>
</tr>
<tr>
<td>909 - 1,000</td>
<td>15.00</td>
<td>20.00</td>
<td>22.85</td>
<td>24.8</td>
</tr>
<tr>
<td>1,000 - 1,643</td>
<td>16.96</td>
<td>20.42</td>
<td>23.61</td>
<td>24.8</td>
</tr>
<tr>
<td>1,643 - 2,000</td>
<td>17.58</td>
<td>23.51</td>
<td>23.83</td>
<td>26.1</td>
</tr>
<tr>
<td>2,000 - 3,000</td>
<td>19.55</td>
<td>25.57</td>
<td>24.58</td>
<td>27.4</td>
</tr>
<tr>
<td>3,000 - 4,000</td>
<td>21.06</td>
<td>28.66</td>
<td>25.29</td>
<td>27.4</td>
</tr>
<tr>
<td>4,000 - 5,000</td>
<td>22.58</td>
<td>28.66</td>
<td>25.71</td>
<td>30.0</td>
</tr>
<tr>
<td>5,000 - 6,000</td>
<td>23.59</td>
<td>26.78</td>
<td>26.43</td>
<td>30.0</td>
</tr>
<tr>
<td>6,000 - 7,000</td>
<td>24.05</td>
<td>26.78</td>
<td>26.94</td>
<td>32.6</td>
</tr>
<tr>
<td>7,000 - 8,000</td>
<td>24.39</td>
<td>30.90</td>
<td>27.64</td>
<td>32.6</td>
</tr>
<tr>
<td>8,000 - 9,000</td>
<td>25.11</td>
<td>30.90</td>
<td>28.19</td>
<td>35.2</td>
</tr>
<tr>
<td>9,000 - 10,000</td>
<td>25.69</td>
<td>36.05</td>
<td>28.90</td>
<td>35.2</td>
</tr>
<tr>
<td>10,000 - 11,000</td>
<td>26.63</td>
<td>36.05</td>
<td>29.47</td>
<td>40.5</td>
</tr>
<tr>
<td>11,000 - 12,000</td>
<td>27.42</td>
<td>41.20</td>
<td>30.39</td>
<td>40.5</td>
</tr>
<tr>
<td>12,000 - 14,000</td>
<td>29.39</td>
<td>41.20</td>
<td>31.83</td>
<td>45.7</td>
</tr>
<tr>
<td>14,000 - 15,000</td>
<td>30.17</td>
<td>46.35</td>
<td>32.76</td>
<td>45.7</td>
</tr>
<tr>
<td>15,000 - 24,000</td>
<td>36.24</td>
<td>46.35</td>
<td>37.61</td>
<td>50.9</td>
</tr>
<tr>
<td>24,000 - 25,000</td>
<td>36.64</td>
<td>51.50</td>
<td>38.14</td>
<td>50.9</td>
</tr>
<tr>
<td>25,000 - 39,000</td>
<td>41.98</td>
<td>51.50</td>
<td>42.72</td>
<td>56.1</td>
</tr>
<tr>
<td>39,000 - 40,000</td>
<td>42.22</td>
<td>56.65</td>
<td>43.06</td>
<td>56.1</td>
</tr>
<tr>
<td>40,000 - 60,000</td>
<td>47.03</td>
<td>61.80</td>
<td>47.40</td>
<td>61.3</td>
</tr>
<tr>
<td>60,000 - 90,000</td>
<td>51.95</td>
<td>66.95</td>
<td>52.04</td>
<td>61.3</td>
</tr>
<tr>
<td>90,000 - 125,000</td>
<td>56.95</td>
<td>72.10</td>
<td>54.63</td>
<td>61.3</td>
</tr>
<tr>
<td>125,000 - 225,000</td>
<td>63.68</td>
<td>77.25</td>
<td>57.59</td>
<td>61.3</td>
</tr>
<tr>
<td>225,000 - 400,000</td>
<td>69.62</td>
<td>82.40</td>
<td>59.70</td>
<td>61.3</td>
</tr>
</tbody>
</table>

* Combined federal and provincial taxes include the old age security tax, the social development tax, and the 3 percent surtax, and are after deducting the 20 percent reduction (maximum $20).

** Combined federal tax and 30.5 percent provincial tax. The initial federal rate is 17 percent as in 1972.
Newfoundland rate from 36 to 40 percent. None of the announced changes for 1975 has been included in these calculations.

Table 3

<table>
<thead>
<tr>
<th></th>
<th>RTPYPQ</th>
<th>RTPYPXQ</th>
<th>RFAQ</th>
<th>RFAXQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1972</td>
<td>15.544</td>
<td>23.439</td>
<td>6.432</td>
<td>0.0</td>
</tr>
<tr>
<td>1973</td>
<td>15.544</td>
<td>23.905</td>
<td>6.432</td>
<td>0.0</td>
</tr>
<tr>
<td>1974</td>
<td>15.544</td>
<td>23.949</td>
<td>6.432</td>
<td>0.0</td>
</tr>
<tr>
<td>1975</td>
<td>15.544</td>
<td>23.949</td>
<td>6.432</td>
<td>0.0</td>
</tr>
</tbody>
</table>

The last major item to consider is the indexing of tax brackets, beginning in 1974. Indexing, as introduced into the Canadian personal income tax system [10], [2], is an attempt to prevent average tax rates from increasing as a result of inflation. The major element in this increase in effective tax rates is the movement of income into higher tax rate brackets and hence its taxation at a higher marginal rate. If the increase in income that puts a taxpayer into a higher tax bracket has been less than the rate of inflation, real disposable income declines. This inflation-induced tax increase is removed by the simple expedient of increasing the width of the taxable income brackets by an indexing factor.

In RCX2 the indexing factor (RTI) is defined as being equal to the increase in the consumer price index (PCPI) during the year ending in the third quarter of the preceding year over the year ending in the third quarter of 1972.14

\[
RTI = Q1\text{ FACTOR}_{t-2} + Q2\text{ FACTOR}_{t-3} + Q3\text{ FACTOR}_{t-4} + Q4\text{ FACTOR}_{t-5} + 1 - QINDEX
\]
where

\[
\text{FACTOR} = Q\text{INDEX}^{.25} \sum_{i=0}^{3} \frac{\text{PCPI}_{t-i}}{1.38025}
\]

The variable FACTOR gives the increase in the consumer price index from the average level of the index over the quarters ending the 3rd quarter of 1972 (1.38025). It is zero whenever indexing is not required. This is accomplished through the use of the dummy variable QINDEX (equal to 1 when indexing is in effect and zero otherwise). The Q variables are quarterly dummies ensuring that the same RTI applies in each quarter of a calendar year. RTI is applied at the appropriate places in the model to increase exemptions and deductions (the ZEXYGTj equations) and to increase the width of the tax brackets (XBRAcK in the RTPYFj and RTPYFEj equations).

The Performance of the Model

Before proceeding with the comparative analysis of the old and new tax systems, we present an evaluation of the predictive performance of the new tax model in order to establish that it is a suitable tool for such analysis. In Table 4 a comparison is provided between the intra-sample root mean square error as a percentage of the mean for the four key tax sector variables. The new tax sector performs almost as well as the old. This result is remarkable because the old tax sector was heavily dependent on exogenous inputs from the taxation statistics [12], whereas in the new tax model all these variables are endogenous. Thus, the advantage for forecasting in eliminating reliance on the taxation statistics (only available with a two-year time lag)
has been gained at a minimal cost in terms of the intra-sample tracking performance.

Table 4

THE ROOT-MEAN-SQUARE ERROR AS A PERCENTAGE OF THE MEAN FOR THE PERSONAL INCOME TAX SECTOR 1961-70

<table>
<thead>
<tr>
<th>Variables</th>
<th>Old</th>
<th>New</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPS</td>
<td>6.40</td>
<td>6.46</td>
</tr>
<tr>
<td>TPO</td>
<td>7.12</td>
<td>9.61</td>
</tr>
<tr>
<td>TPYPM</td>
<td>9.05</td>
<td>9.13</td>
</tr>
<tr>
<td>TPYF</td>
<td>4.83</td>
<td>5.15</td>
</tr>
</tbody>
</table>

* Documented in [17].

Since personal income tax data are now available for the first three years of the new tax system, including the first year of indexing, it is also possible to test the ability of the model to predict revenues under the new system. A satisfactory score on this test serves as evidence that the many modifications to the model made on account of tax reform and indexing are an accurate portrayal of the new system and that the coefficients of the model estimated under the old tax regime have not changed significantly. As we show in Table 5, the model adequately forecasts tax revenue during the 1972-1974 period.
Table 5

FORECASTING RESULTS FOR THE PERSONAL INCOME TAX SECTOR UNDER REFORMED TAX SYSTEM
1972-1974
(Millions of dollars)

<table>
<thead>
<tr>
<th>Year</th>
<th>Federal tax</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual</td>
<td>1972</td>
<td>1973</td>
<td>1974</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Predicted</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Error</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>7,914</td>
<td>9,256</td>
<td>11,105</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>8,225</td>
<td>8,870</td>
<td>10,657</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-319</td>
<td>+386</td>
<td>+448</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Province</th>
<th>Federal tax</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual</td>
<td>1972</td>
<td>1973</td>
<td>1974</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Predicted</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Error</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3,496</td>
<td>4,052</td>
<td>5,036</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3,482</td>
<td>3,948</td>
<td>4,815</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>+12</td>
<td>+104</td>
<td>+221</td>
<td></td>
</tr>
</tbody>
</table>
RESULTS

The first question relating to fiscal policy we take up is, "What is the effect of tax reform and indexing on the yield and elasticity of the income tax system?" In order to provide an answer to the effect of the 1972 tax reform we solved the tax model five times over the 1972-1978 period using projected values for all variables not endogenous to the tax sector. Each of the five solutions involves a different tax regime. They are:

(1) the 1971 pre-tax reform system, which we call the old system,
(2) the old system with new definitions of income,
(3) the old system with new exemptions,
(4) the old system with new rates including the 3 percent tax cut, and
(5) the complete 1972 reformed system, which we call the new system.

The effect of these reforms on federal and provincial income tax revenues is obtained by comparing the revenue raised in the last four solutions of the tax model with that raised in the first solution. Our results are furnished in Table 6.

The most striking feature of these comparisons is the little difference the 1972 tax reform has made in total personal income tax revenue. Over the whole period the new system generates slightly less revenue than the old. This seems to indicate that inflation and not tax reform is behind the recent rapid growth of personal income tax revenue.

Table 6 is also interesting in that it shows how the large changes in revenue caused by changes in the definition of income,
exemptions, and tax rates offset each other. The revenue lost because of higher exemptions is recouped by the new tax rates and income definitions. At first changes in tax rates are the most important factor in making up the revenue loss, but by the end of the period changes in the definition of income, the most significant of which is the maturing capital gains taxation, have become almost as important.

The effect of tax reform on the elasticity of the tax system, based on the same five solutions of the tax model, is set out in Table 7. Our analysis shows that the elasticity of the personal income tax has not been changed by tax reform. The higher elasticity imparted to the system through increased tax exemptions is offset by the lower elasticity due to the new rate structure.

We next examine the effect of indexing on the yield and elasticity of the tax system. To do this we solve the tax model once again to obtain revenue estimates for the indexed new system. In Table 8 we show the revenue loss during 1974 and 1975 due to indexing the 1972 tax system. Over the 1976-1978 simulation period the average reduction in revenues from a 1 percent indexation of total personal income tax grows from $103 million to $137 million.

The effect of indexing on the elasticity of the income tax is shown in Table 9. It would appear that an indexed tax system has an elasticity just under four-tenths less than that of an unindexed system. Indexing brings the elasticity down to a level not that much greater than unity. Under an indexed system only real growth in income per capita is taxed at progressively higher
Table 6

THE EFFECT OF TAX REFORM ON PERSONAL INCOME TAX REVENUES
(Millions of dollars)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue from changes in:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The definition of income</td>
<td>680</td>
<td>969</td>
<td>1,091</td>
<td>1,394</td>
<td>1,695</td>
<td>2,137</td>
<td>2,333</td>
</tr>
<tr>
<td>Tax exemptions</td>
<td>-1,662</td>
<td>-2,354</td>
<td>-2,725</td>
<td>-3,126</td>
<td>-3,560</td>
<td>-4,040</td>
<td>-4,614</td>
</tr>
<tr>
<td>Tax rates</td>
<td>1,295</td>
<td>1,627</td>
<td>1,681</td>
<td>1,815</td>
<td>1,951</td>
<td>2,232</td>
<td>2,550</td>
</tr>
<tr>
<td>Total*</td>
<td>18</td>
<td>-6</td>
<td>-199</td>
<td>-190</td>
<td>-186</td>
<td>5</td>
<td>-87</td>
</tr>
</tbody>
</table>

* The total effect of the three types of tax changes taken together is not equal to the sum of the effects of the changes taken separately because of interactions between definitions, exemptions and rates. In addition in 1972, the total effect is subject to a timing adjustment for the quarterly installment payment of taxes on nonwage income.

Table 7

EFFECT OF TAX REFORM ON THE ELASTICITY OF THE PERSONAL INCOME TAX WITH RESPECT TO NOMINAL GNE

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax system</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Old</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Old: new income</td>
<td>1.29</td>
<td>1.30</td>
<td>1.70</td>
<td>1.44</td>
<td>1.58</td>
<td>1.46</td>
<td>1.45</td>
</tr>
<tr>
<td>Old: new exemptions**</td>
<td>1.38</td>
<td>1.26</td>
<td>1.73</td>
<td>1.45</td>
<td>1.61</td>
<td>1.41</td>
<td>1.46</td>
</tr>
<tr>
<td>Old: new rates</td>
<td>1.02</td>
<td>1.38</td>
<td>1.80</td>
<td>1.53</td>
<td>1.67</td>
<td>1.53</td>
<td>1.48</td>
</tr>
<tr>
<td>New***</td>
<td>1.34</td>
<td>1.18</td>
<td>1.60</td>
<td>1.37</td>
<td>1.55</td>
<td>1.43</td>
<td>1.40</td>
</tr>
</tbody>
</table>

* Elasticity is defined to be the percentage change in tax divided by the percentage change in gross national expenditure.

** The old system with new exemptions exhibits a low elasticity in 1973 because the tax on nonwage income is still declining to a new lower level. It is only in 1974 after this level is reached that the elasticity picks up.

*** The timing adjustment referred to in Table 6 is applied to the new system before elasticities are calculated.
Table 8
THE REVENUE LOSS FROM INDEXING
(Millions of dollars)

<table>
<thead>
<tr>
<th></th>
<th>1974</th>
<th>1975</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total tax</td>
<td>447</td>
<td>1,487</td>
</tr>
<tr>
<td>Indexing factor</td>
<td>6.6</td>
<td>17.4</td>
</tr>
</tbody>
</table>

* The values are obtained using the 1972 reformed tax system. It does not include the effect of tax changes in the February 1973 and November 1974 budgets. These changes increase the revenue loss due to indexing in 1974 and 1975.

Table 9
THE EFFECT OF INDEXING ON THE ELASTICITY OF THE PERSONAL INCOME TAX WITH RESPECT TO NOMINAL GNE

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>New without indexing</td>
<td>1.22</td>
<td>1.72</td>
<td>1.46</td>
<td>1.65</td>
<td>1.44</td>
<td>1.48</td>
</tr>
<tr>
<td>New with indexing</td>
<td>1.03</td>
<td>1.27</td>
<td>.95</td>
<td>1.19</td>
<td>1.08</td>
<td>1.10</td>
</tr>
<tr>
<td>Effect of indexing</td>
<td>.19</td>
<td>.45</td>
<td>.51</td>
<td>.46</td>
<td>.36</td>
<td>.38</td>
</tr>
<tr>
<td>Avg.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
rates. Any growth coming from inflation or a larger employed labour force is taxed at roughly proportionate rates. This development has significant implications for the growth of the government sector since the personal income tax has been the most rapidly expanding of the revenue sources financing that growth.

The second question relating to fiscal policy we take up is, "What is the effect of tax reform and indexing on the built-in stability of the economy?" One measure of the effect of these tax changes on built-in stability is called built-in flexibility. As defined by Pechman [21] p 392 built-in flexibility is the absolute increase in tax liability for every dollar increase in the base. Pechman used a modified version of personal income as his base whereas we use current-dollar gross national expenditure. The rationale underlying this concept is that a tax system is more effective in reducing fluctuations in aggregate demand the more money it withdraws from the economy in a boom and the less in a downturn. Thus, a higher value of built-in flexibility is thought to imply a more stable system.

The built-in flexibility of the new tax system and that of the old as displayed in Table 10 do not differ greatly except in 1974 when this flexibility is higher for the old system and in 1977 when it is higher for the new. On the basis of this measure, neither one of the systems seems to possess a greater amount of built-in stability than the other. Thus, we conclude that tax reform has not affected the stability of the economic system. However, the built-in flexibility of the new system with indexing is over thirty percent less than that of either the old system or the new system.
without indexing. Thus there would seem to be a prima facie case to be made for the contention that indexing has reduced the built-in stability of the economy.

Built-in flexibility tells us only part of the story however. On the one hand, if the built-in flexibility of two tax systems is the same, then the dynamic multipliers of any macro-econometric model incorporating these two tax systems will also be the same. The performance of the model under shock will be invariant with respect to the tax system. Thus the built-in stability of the two tax systems will be identical. On the other hand, if the built-in flexibility of the two tax systems is different, the dynamic multipliers will also be different. In this case the best way to decide which tax system possesses the greater amount of built-in stability is to apply the same representative shocks to a macro-econometric model embodying the two tax systems and to examine the results. This is exactly what we do to determine the effect of indexation on built-in stability.

With this end in mind, we established a control solution with the full model RDX2 over the 1973-1978 period. An indexed response to a shock is obtained by having the indexing factor calculated endogenously and an unindexed response is obtained by setting the indexing factor at its control value exogenously.

The first shock we administer is a sustained $100 million per quarter increase in real government current nonwage expenditure starting in the first quarter of 1973. The responses of an indexed and unindexed system to this shock, as portrayed in Table 11, are different. As one would expect, the indexed system
Table 10
THE BUILT-IN FLEXIBILITY* OF THE TOTAL PERSONAL INCOME TAX UNDER VARIOUS TAX SYSTEMS

<table>
<thead>
<tr>
<th></th>
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<th></th>
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<th></th>
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<tr>
<td>Old</td>
<td>.146</td>
<td>.152</td>
<td>.208</td>
<td>.190</td>
<td>.220</td>
<td>.217</td>
</tr>
<tr>
<td>New without indexing</td>
<td>.144</td>
<td>.143</td>
<td>.208</td>
<td>.191</td>
<td>.228</td>
<td>.214</td>
</tr>
<tr>
<td>New with indexing</td>
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<td>.121</td>
<td>.149</td>
<td>.115</td>
<td>.143</td>
<td>.133</td>
</tr>
</tbody>
</table>

* Defined to be the marginal tax rate with respect to GNE.

Table 11
THE EFFECT OF AN INCREASE IN GOVERNMENT EXPENDITURE WITH AND WITHOUT INDEXING*
(Millions of dollars)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td>405</td>
<td>600</td>
<td>736</td>
<td>828</td>
<td>846</td>
<td>953</td>
</tr>
<tr>
<td>With indexing</td>
<td>405</td>
<td>602</td>
<td>743</td>
<td>844</td>
<td>881</td>
<td>1027</td>
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<td>Consumer price index(%)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Without indexing</td>
<td>.09</td>
<td>.31</td>
<td>.62</td>
<td>.98</td>
<td>1.35</td>
<td>1.70</td>
</tr>
<tr>
<td>With indexing</td>
<td>.09</td>
<td>.31</td>
<td>.63</td>
<td>.99</td>
<td>1.37</td>
<td>1.74</td>
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<tr>
<td>Personal income tax</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without indexing</td>
<td>32</td>
<td>117</td>
<td>260</td>
<td>395</td>
<td>582</td>
<td>890</td>
</tr>
<tr>
<td>With indexing</td>
<td>32</td>
<td>109</td>
<td>229</td>
<td>346</td>
<td>471</td>
<td>661</td>
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</tbody>
</table>

* An increase of $100 million per quarter in real government current nonwage expenditure.
has the higher multiplier. Income tax revenues start to diverge in the second year of the shock and by the sixth year revenues are $229 million lower for the indexed system than for the unindexed. This difference in the response of tax revenues to the shock causes the dynamic government expenditure multiplier as portrayed in Figure 2 to be .24 higher for the indexed system. If we had run the shock out longer, the difference would have been even greater. Thus, one can probably conclude in this case that indexing has reduced built-in stability.

In a recent paper on the economic effects of indexing John Bossons and T.A. Wilson [7] p 196 argue that built-in stabilizers are important only as a dampener for transient shocks and that persistent shocks can be adequately dealt with only through appropriate discretionary changes in policy. They acknowledge that an indexed system tends to exaggerate a persistent expansionary shock, as we show for the government expenditure increase discussed above. However, they contend that the differences between an indexed and unindexed system take so long to emerge that these differences, as well as part of the shock itself, could be offset by discretionary policy changes. As a result in their analysis they focus on a transient shock. They applied a shock to the University of Toronto Quarterly Forecasting Model (QFM) of a $500 million real increase in exports during 1965. Their results, which are included in Table 12, they interpret as "demonstrating that the important short-run response of the tax system to an expansionary shock would not be weakened [by indexing] and that the problem of the perverse timing of fiscal adjustments arising from the lags in the price
Figure 2

THE DYNAMIC MULTIPLIER FOR A $100 MILLION INCREASE IN REAL GOVERNMENT EXPENDITURE WITH AND WITHOUT INDEXING
Table 12

COMPARISON OF THE RESULTS FROM QFM
AND RDX2 FOR AN ON-AND-OFF SHOCK** WITH
AND WITHOUT INDEXING

(Shock minus control as a percentage of the control)

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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</thead>
<tbody>
<tr>
<td><strong>Real GNE</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Export Shock</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>QFM without indexing</td>
<td>1.33</td>
<td>.75</td>
<td>.28</td>
<td>-.07</td>
<td>-.26</td>
<td>n.a.</td>
</tr>
<tr>
<td>QFM with indexing</td>
<td>1.33</td>
<td>.75</td>
<td>.32</td>
<td>.06</td>
<td>-.06</td>
<td>n.a.</td>
</tr>
<tr>
<td>RDX2 without indexing</td>
<td>.94</td>
<td>.29</td>
<td>.06</td>
<td>-.23</td>
<td>-.47</td>
<td>-.54</td>
</tr>
<tr>
<td>RDX2 with indexing</td>
<td>.94</td>
<td>.29</td>
<td>.07</td>
<td>-.22</td>
<td>-.47</td>
<td>-.54</td>
</tr>
<tr>
<td><strong>Government Shock</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RDX2 without indexing</td>
<td>1.06</td>
<td>.49</td>
<td>.38</td>
<td>.08</td>
<td>-.16</td>
<td>-.22</td>
</tr>
<tr>
<td>RDX2 with indexing</td>
<td>1.06</td>
<td>.49</td>
<td>.40</td>
<td>.10</td>
<td>-.12</td>
<td>-.16</td>
</tr>
</tbody>
</table>

| **GNE Price** |      |      |      |      |      |      |
| Export Shock |      |      |      |      |      |      |
| QFM without indexing | .01  | .57  | 1.39 | 1.73 | 1.91 | n.a. |
| QFM with indexing | .01  | .57  | 1.39 | 1.77 | 2.00 | n.a. |
| RDX2 without indexing | .12  | .29  | .40  | .27  | .02  | -.29 |
| RDX2 with indexing | .12  | .29  | .40  | .28  | .02  | -.28 |
| **Government Shock** |      |      |      |      |      |      |
| RDX2 without indexing | .21  | .36  | .61  | .68  | .62  | .47  |
| RDX2 with indexing | .21  | .36  | .61  | .69  | .64  | .51  |

* Results are those reported in [7] p 195.

** The QFM export shock occurs in 1965 and is $500 million or 1 percent of real GNE. The RDX2 export and government shock occur in 1973 and are $800 million, which is also 1 percent of real GNE.
level response would be mitigated (although not eliminated)[by indexing]."

We ran similar experiments to those of Bossons and Wilson on RDX2 to see if their results are model-specific or are more generally applicable. Our shock is a $800 million increase in real exports in 1973 that bears the same relationship to real gross national expenditure in 1973 as did the $500 million shock in 1965. Our results bear out the first of the Bossons and Wilson conclusions that the short-run response is not weakened since the responses of the unindexed and indexed system to the shock are identical. Our results, however, do not support their second conclusion concerning the perverse timing of fiscal adjustments because the output and price responses remain the same in the longer run. One reason our results differ from those of Bossons and Wilson is that RDX2 has an endogenous exchange rate whereas QFM has an exogenous rate. In RDX2 an increase in exports causes the Canadian dollar to appreciate relative to its control value and puts downward pressure on domestic prices. The upward pressure on prices because of increased aggregate demand is, thus, blunted by the appreciation of the dollar. Therefore the endogenous indexing factor used in adjusting taxes shows little increase. As a result the difference in taxes between the unindexed and the indexed system, which is the source of the differences in response to the shock, is minor.

To avoid in part the problems arising from the dissimilar treatment of the exchange rate in the two models, we ran a $800 million increase in real government expenditure, which does not have such a direct effect on the exchange rate as the earlier
export shock. These results, also reported in Table 12, weakly support the second conclusion of Bossons and Wilson. The difference in the degree of support given to their second conclusion by their results and by our results stems from the difference in the size of the price response of the two models to the shock. The response of prices in QFM appears to be unrealistically high relative to the size and duration of the shock. In RDX2, because of the small change in prices, the difference in response between the unindexed and the indexed system is again so slight as to be negligible. Consequently, we think that Bossons and Wilson [7] p 196 have placed too much emphasis on a difference in response, which could turn out to be insignificant when subjected to further analysis, by drawing their overall conclusion that, on balance, indexing "would ... increase the stabilizing properties of the tax system with respect to its response to purely transient shocks."

Having examined the implications of several types of real expenditure shocks to the economic system as schematically represented by RDX2, we move on to the unexplored realm of the price shock. During the last few years the economy has been subjected to one price shock after another, the most important of which were the explosions in the prices of food and crude petroleum. Price flare-ups like these erode the real personal disposable income available for consumption because they transfer income from consumers to producers. They thus tend to have a deflationary impact on the economy.

With food and oil in mind, we increased the price deflator for non-durables and semi-durables by 4 percent. The results of
these simulations are set out in Table 13. Because of the immediate effect on price of this shock the results start to diverge as early as the second year. By the end of the period, personal income tax revenue is $680 million lower with indexing. Consequently the reduction in real gross national expenditure with the indexed system is only 85 percent of that with the unindexed system. The mitigation of the fall in real GNE is obtained with only a moderate accentuation of the increase in the consumer price index. On the basis of these results, we conclude that, in the case of an exogenous price shock unaccompanied by changes in the nominal income available for consumption, indexing enhances the built-in stability of the system by cushioning the fluctuations in real disposable income.

Table 13

THE EFFECT OF A PRICE SHOCK WITH AND WITHOUT INDEXING*
(Millions of dollars)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GNE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without indexing</td>
<td>-429</td>
<td>-474</td>
<td>-897</td>
<td>-1,134</td>
<td>-1,799</td>
<td>-2,335</td>
</tr>
<tr>
<td>With indexing</td>
<td>-429</td>
<td>-443</td>
<td>-807</td>
<td>-974</td>
<td>-1,551</td>
<td>-1,981</td>
</tr>
<tr>
<td>Consumer price index(%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without indexing</td>
<td>2.20</td>
<td>2.94</td>
<td>3.54</td>
<td>3.92</td>
<td>3.95</td>
<td>3.68</td>
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<tr>
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<td>2.94</td>
<td>3.57</td>
<td>4.00</td>
<td>4.13</td>
<td>4.01</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without indexing</td>
<td>149</td>
<td>424</td>
<td>623</td>
<td>787</td>
<td>800</td>
<td>790</td>
</tr>
<tr>
<td>With indexing</td>
<td>149</td>
<td>275</td>
<td>288</td>
<td>325</td>
<td>185</td>
<td>110</td>
</tr>
</tbody>
</table>

* The price shock is a 4 percent increase in the price of non-durables and semi-durables.
In our analysis of the effect of indexing on built-in stability, we have presented the responses of unindexed and indexed systems to four shocks. Indexing turned out to be destabilizing for a persistent volume shock, stabilizing for a persistent price shock, and neutral for the two transient volume shocks. Thus, the answer to the question of the effect of indexing on built-in stability would seem to depend on a subjective assessment of the future frequency of these various types of shocks.
CONCLUSIONS

Our simulations with the tax model have lead us to draw five conclusions concerning the macroeconomic effect of tax reform and indexing. They are:

(1) Contrary to the views expressed in the financial press, tax reform has not raised significantly more money.

(2) Tax reform has not materially increased the elasticity of the tax system.

(3) Indexing has reduced the elasticity of the personal income tax by four-tenths to within shooting distance of unity.

(4) Tax reform has not had an important effect on built-in stability.

(5) Indexing has reduced built-in stability with respect to certain types of shocks, and enhanced it with respect to others.

We hope that these conclusions demonstrate the utility of what is unique in our approach, that is the incorporation of a disaggregated tax model into a macroeconometric model such as RDX2.
FOOTNOTES

1. The original version of RDX2 is documented in [18]. In this study we use the 'Green Book' version of RDX2 [17].

2. The latest version of RDX2 is referred to as the Red Book [4]. This version documents the new tax sector.

3. Our approach to constructing the tax model is described in detail in [16] and [14]. The present version of the government sector of RDX2 differs somewhat from [16] but the approach remains the same. The tax model is now in two parts. The basic structure is composed of twenty-nine equations. A supplemental set of 192 equations is used to endogenize a set of variables that are treated as exogenous in the basic model if the supplemental section is switched off. This arrangement of the model allows for a high degree of precision when required in studies involving fiscal policy, and at the same time simplifies the model at a minimum cost when such precision is not required.

4. The best synopsis of the tax reform package compared to the old system and with suggested amendments from the numerous reports preceeding the legislation is the booklet Summary of 1971 Tax Reform Legislation [11].

5. Until the introduction of indexing, the Quebec tax system did not differ significantly from the federal system and could be treated in our model as being similar to that of
the other provinces. However, recent changes make this impossible and we have separated Quebec from the other provinces. This has required a significant increase in the size of our model. Of the 221 equations in the personal income tax model, forty are needed for the Quebec system at the level of disaggregation we have defined.

6. This equation is not shown here since it is used only to divide total personal income taxes (TPS+TPO) into federal and provincial shares.

7. For a discussion of the possible functions and the techniques employed in fitting them to frequency distributions see Kendall and Stuart [19].


11. For instance, the 3 percent cut in the October 14, 1971 budget; the 5 percent cut with a $100 minimum and a $500
maximum in the February 19, 1973 budget; the increase in the minimum cut to $150 for 1974 in the November 8, 1974 budget; and the 8 percent cut with a minimum of $200 and a maximum of $750 also in the November 8, 1974 budget.

12. For a description of the Quebec system see [23].

13. Abatements of 24 percent are still provided to Quebec because of the opting-out arrangement.


15. The revenue loss due to indexing reported here is the result of a straightforward calculation involving increasing both exemptions and rate brackets by the indexing factor. The larger question of the net revenue loss compared to other methods of offsetting the effects of inflation is much more complicated.
LIST OF VARIABLES

Variables endogenous to the tax sector of RDX2 [17] are followed by equation numbers. The other endogenous variables are followed by X. Variables exogenous to RDX2 are followed by E. Variables not annotated are used only to simplify presentation.

J is an operator. The J is always followed either by a numeral or by W. The numeral refers to the number of quarters, including the current quarter, involved in the operation. The following operations are defined in a case where J is followed by some numeral.

J3A is a three-quarter unweighted moving average starting in the current quarter.

J3L is a three-quarter lag.

J3S is a three-quarter moving sum.

The JW operator is a weighted moving average (or an unweighted moving average not starting in the current quarter). The weights are listed under the equation in which the operator is used. The variable to which the J operator applies is shown in parenthesis immediately behind the operator.
<table>
<thead>
<tr>
<th>Mnemonic</th>
<th>Description</th>
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<tbody>
<tr>
<td>AYKG1A</td>
<td>Stock of accrued unrealized capital gains.</td>
</tr>
<tr>
<td>ECRPM</td>
<td>Provincial tax credits.</td>
</tr>
<tr>
<td>EASYAS</td>
<td>Seasonal adjustment factor for assessed income per taxpayer.</td>
</tr>
<tr>
<td>EASYANW</td>
<td>Seasonal adjustment factor for assessed nonwage income per nonwage taxpayer.</td>
</tr>
<tr>
<td>EASYW</td>
<td>Seasonal adjustment factor for assessed wage income per wage taxpayer.</td>
</tr>
<tr>
<td>EYDIVA11</td>
<td>Ratio of assessed dividend income to total dividends paid to Canadians by Canadian corporations.</td>
</tr>
<tr>
<td>EYDIVIC</td>
<td>Proportion of assessed dividend income earned by taxpayers with assessed nonwage income in class i (i = 1, 3).</td>
</tr>
<tr>
<td>FIYC12</td>
<td>Canadian corporate retained earnings accruing to U.S. shareholders.</td>
</tr>
<tr>
<td>FIYC13</td>
<td>Canadian corporate retained earnings accruing to shareholders of countries other than the United States.</td>
</tr>
<tr>
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<td>Military Pay and Allowances.</td>
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<td>GTPFAF</td>
<td>Family Allowances.</td>
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<td>GTPUIBF</td>
<td>Unemployment insurance benefits.</td>
</tr>
<tr>
<td>GWPASPM</td>
<td>Provincial and municipal wage supplements paid to employees in public administration.</td>
</tr>
<tr>
<td>GWSF</td>
<td>Federal wage supplements paid to employees in public administration and defence.</td>
</tr>
<tr>
<td>GWSSM</td>
<td>Municipal wage supplements paid to employees in elementary and secondary schools.</td>
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<td>IME</td>
<td>Business investment in machinery and equipment.</td>
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<td>INRC</td>
<td>Business investment in non-residential construction.</td>
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<td>Code</td>
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quarter of 1950, 2.0 in each quarter of 1951, etc.

QTXRFM E Variable to reflect tax reform, equals 1.0 from 1Q72 on.

RDC E Rate of dividend tax credit.

RFAQ E Rate of federal personal income tax abatement to Quebec.

RFAXQ E Weighted average rate of federal personal income tax abatement to all provinces except Quebec.

RMARFK E Marginal rate of federal income tax in income bracket \( k (k = 1, 17 \text{ before 1972}) (k = 1, 13 \text{ after 1972}).

RMARFfk E Marginal rate of basic tax in income bracket \( k (k = 1, 17 \text{ before 1972}). (k = 1, 13 \text{ after 1972}).

RTI 9.30 Indexing Factor.

RTPYFiC S9.180-S9.183 Weighted average rate of federal income tax payable by taxpayers in income class \( i (i = 1, 4).

RTPYFj S9.138-S9.151 Average rate of federal income tax payable by taxpayers in income group \( j (j = 1, 14).

RTPYFBIC S9.184-S9.187 Weighted average rate of basic tax payable by taxpayers in income class \( i (i = 1, 4).

RTPYFBj S9.152-S9.165 Average rate of basic tax payable by taxpayers in income group \( j (j = 1, 14).

RTPYPQ E Weighted average rate of provincial income tax payable by Quebec residents.

RTPYPXQ E Weighted average rate of provincial income tax payable by residents of all provinces except Quebec.

RTPYQiC S9.183-S9.191 Weighted average rate of non-indexed basic tax payable by taxpayers in income class \( i (i = 1, 4).

RTPYQj S9.166-S9.179 Average rate of non-indexed basic tax payable by taxpayers in income group \( j (j = 1, 14).
<table>
<thead>
<tr>
<th>Code</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RVB12</td>
<td>X</td>
<td>Return to U.S. residents from Canadian business assets (percentage of total return).</td>
</tr>
<tr>
<td>RVB13</td>
<td>X</td>
<td>Return to residents of other countries from Canadian business assets (percentage of total return).</td>
</tr>
<tr>
<td>TANW</td>
<td>9.6</td>
<td>Personal income tax accruals on nonwage income.</td>
</tr>
<tr>
<td>TAW</td>
<td>9.5</td>
<td>Personal income tax accruals on wage income.</td>
</tr>
<tr>
<td>TPO</td>
<td>9.2</td>
<td>Personal income tax collections not withheld at source.</td>
</tr>
<tr>
<td>TPS</td>
<td>9.1</td>
<td>Personal income tax collections withheld at source.</td>
</tr>
<tr>
<td>TPYF</td>
<td>9.4</td>
<td>Federal personal income tax collections.</td>
</tr>
<tr>
<td>TPYPM</td>
<td>9.3</td>
<td>Provincial personal income tax collections.</td>
</tr>
<tr>
<td>UDISi</td>
<td>S9.10-S9.12</td>
<td>Displacement factor for displaced lognormal distribution for i = NT, YWAS, YNWAS.</td>
</tr>
<tr>
<td>UMEANi</td>
<td>S9.13-S9.15</td>
<td>Mean for displaced lognormal distribution for i = NT, YWAS, YNWAS.</td>
</tr>
<tr>
<td>UQiNT</td>
<td>S9.1-S9.3</td>
<td>Level of assessed income below which lie 10 percent, 50 percent, and 90 percent of the number of taxpayers for i = L, M, U, respectively.</td>
</tr>
<tr>
<td>UQiYNW</td>
<td>S9.7-S9.9</td>
<td>Level of assessed income below which lie 10 percent, 50 percent, and 90 percent of assessed nonwage income for i = L, M, U, respectively.</td>
</tr>
<tr>
<td>UQiYW</td>
<td>S9.4-S9.6</td>
<td>Level of assessed income below which lie 10 percent, 50 percent, and 90 percent of assessed wage income for i = L, M, U, respectively.</td>
</tr>
<tr>
<td>USIGMAi</td>
<td>S9.16-S9.18</td>
<td>Standard deviation for displaced lognormal distribution for i = NT, YWAS, YNWAS.</td>
</tr>
<tr>
<td>USRNTiC</td>
<td>S9.61-S9.63</td>
<td>Proportion of tax returns filed in income class i(i = 1, 3).</td>
</tr>
<tr>
<td>USRNTj</td>
<td>S9.19-S9.32</td>
<td>Proportion of tax returns filed in</td>
</tr>
</tbody>
</table>
income group \( j (j = 1, 14) \).

**USRNWic** S9.67-S9.69 Proportion of assessed nonwage income in income class \( i (i = 1, 3) \).

**USRNWj** S9.47-S9.60 Proportion of assessed nonwage income in income group \( j (j = 1, 14) \).

**USRWic** S9.54-S9.66 Proportion of assessed wage income in income class \( i (i = 1, 3) \).

**USRWj** S9.33-S9.46 Proportion of assessed wage income in income group \( j (j = 1, 14) \).

**WZEXNW** S9.136 Quarterly spreading ratio for exemptions against nonwage income.

**WZEXW** S9.137 Quarterly spreading ratio for exemptions against wage income.

**YASP** Average assessed income, seasonally adjusted at annual rates.

**YBRACK** E Lower income level in taxable income bracket \( k (k = 1, 18 \text{ before } 1972) \) \( (k = 1, 14 \text{ after } 1972) \).

**YDIV11** X Dividends paid to Canadian residents by Canadian corporations.

**YGj** E Upper income level of group \( j (j = 1, 14) \).

**YIVA** X Inventory valuation adjustment.

**YKGPA** 8.13 Accrued capital gains.

**YKGPR** 8.15 Realizations of capital gains.

**YNWAS** 9.9 Assessed nonwage income (excluding dividends and capital gains).

**YNWASIC** 9.18-9.21 Assessed nonwage income (including dividends and capital gains) in income class \( i (i = 1, 4) \).

**YNWASj** Assessed nonwage income (excluding dividends and capital gains) in income group \( j (j = 1, 14) \).

**YNWNA** National accounts proxy for nonwage income.

**YP** X Personal income.

**YRENT** X Imputed rent on owner-occupied dwellings.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>YTAj</td>
<td>Average taxable income in income group j, seasonally adjusted at annual rates (j = 1, 14).</td>
</tr>
<tr>
<td>YW</td>
<td>Labour income.</td>
</tr>
<tr>
<td>YWAS</td>
<td>Assessed wage income.</td>
</tr>
<tr>
<td>YWASiC</td>
<td>Assessed wage income in income class i(i = 1, 4).</td>
</tr>
<tr>
<td>YWASj</td>
<td>Assessed wage income in income group j(j = 1, 14).</td>
</tr>
<tr>
<td>YWNA</td>
<td>National accounts proxy for wage income.</td>
</tr>
<tr>
<td>YWSLMED</td>
<td>E Medicare premiums paid by employer on behalf of employee.</td>
</tr>
<tr>
<td>YWSLP</td>
<td>E Supplementary labour income, private sector.</td>
</tr>
<tr>
<td>ZEXINWiC</td>
<td>S9.120-S9.123 Average indexed exemption and deduction for taxpayer earning nonwage income in income class i(i = 1, 4).</td>
</tr>
<tr>
<td>ZEXIWIC</td>
<td>S9.124-S9.127 Average indexed exemption and deduction for taxpayer earning wage income in income class i(i = 1, 4).</td>
</tr>
<tr>
<td>ZEXPERj</td>
<td>E Average personal exemption in income group j(j = 1, 14).</td>
</tr>
<tr>
<td>ZEXQNWIC</td>
<td>S9.128-S9.131 Average non-indexed exemption and deduction for taxpayer earning nonwage income in income class i(i = 1, 4).</td>
</tr>
<tr>
<td>ZEXQWIC</td>
<td>S9.132-S9.135 Average non-indexed exemption and deduction for taxpayer earning wage income in income class i(i = 1, 4).</td>
</tr>
<tr>
<td>ZEXYG1j</td>
<td>S9.84-S9.97 Average indexed exemption and deduction in income group j(j = 1, 14).</td>
</tr>
<tr>
<td>ZEXYGj</td>
<td>E Average non-indexed exemption and deduction in income group j(j = 1, 14).</td>
</tr>
<tr>
<td>ZEXYGQj</td>
<td>S9.98-S9.111 Average non-indexed exemption and deduction used in calculating Quebec tax rate in income group j(j = 1, 14).</td>
</tr>
<tr>
<td>ZEXYGtj</td>
<td>S9.70-S9.83 Average exemption and deduction used in calculating combined federal and</td>
</tr>
</tbody>
</table>
provincial tax in income group $j(j = 1, 14)$.

**ZEXYNWiC S9.112-S9.115**

Average exemption and deduction for taxpayer earning nonwage income in income class $i(i = 1, 4)$ used in calculating combined federal and provincial tax.

**ZEXYWiC S9.116-S9.119**

Average exemption and deduction for taxpayer earning wage income in income class $i(i = 1, 4)$ used in calculating combined federal and provincial tax.
MODEL EQUATIONS

SECTOR 9

9.1 TPS  Personal income tax collections withheld at source
        (X=179)

        1Q62-4Q70  OLS

        TPS = 1.1861 J0(TAW) - .01421 [ Q1(J1L[J4S(TAW)]] ]
        (83.55)  (1.92)
        - .11269 [ Q2(J2L[J4S(TAW)]] ]
        (15.03)

        z  J0(TAW)
        0  .667
        -1  .333
        Sum W = 1.000

        see = 58.52  RB2 = .986  cov = 6.46%  dw = 2.65

9.2 TPO  Personal income tax collections not withheld at source

        1Q62-4Q70  OLS

        TPO = .32665 [ Q1(J1L[J4S(TANW)]] ]
        (21.29)
        + .99243 [ Q2(J2L[J4S(TANW)]] ]
        (64.70)
        + .35261 [ Q3(J3L[J4S(TANW)]] ]
        (22.99)
        + .27625 [ Q4(J4L[J4S(TANW)]] ] - ECRFM
        (18.01)

        see = 26.38  RB2 = .982  cov = 9.99%  dw = .87
9.3 TPYPM Provincial personal income tax collections
(X=181)

2Q62-4Q70 OLS

\[
TPYPM = 1.1690 \left[ J1W \left( \sum_{i=1}^{4} (.01 \times RTPYPM_i) \right) \right] \left( \frac{.9 \times RTPYFB_iC}{YWASiC+YNWASiC} \right) \left( NTWiC \right) (ZEXWiC)
\]

\[
- (NTNW\text{W}iC) (ZE\text{}\text{}XNW\text{}iC) \right] - .9 (EYDIVA11)
\]

\[
(YDIV11) (.01 RDC) \right] + J2W\left[ \left( \sum_{i=1}^{4} (.01 \times RTPYQ_iC) \right) \right] \left( \frac{.9 \times RTPYQ_iC}{YWASiC+YNWASiC} \right) \left( NTWiC \right) (ZE\text{}\text{}X\text{}W\text{}iC) \left( NTNW\text{W}iC \right) (ZE\text{}\text{}X\text{}QNW\text{}iC) \right] \}
\]

\[
- (.01 \times RTPYQ) [ (EYDIV1C) \left( RTPYQ1C \right) \left( RTPYFB1C \right) \left( RTPYQ2C \right) \left( RTPYFB2C \right) + (EYDIV3C)
\]

\[
\left( RTPYQ3C \right) \left( RTPYFB3C \right) + (1-\text{}EYDIV1C - EYDIV2C-\text{}EYDIV3C) \left( RTPYQ4C \right) \left( RTPYFB4C \right) ]
\]

\[
.9 (EYDIVA11) \times YDIV11 (.01 RDC) \right] \right] - ECRPM
\]

<table>
<thead>
<tr>
<th>t</th>
<th>J1W</th>
<th>J2W</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.333</td>
<td>.667</td>
</tr>
<tr>
<td>1</td>
<td>.667</td>
<td>.333</td>
</tr>
</tbody>
</table>

see = 25.59  RB2 = .982  cov = 8.58%  dw = 2.43
Technical Relationships

9.4 TPYF Federal personal income tax collections
(X=180)

TPYF = TPS + TPO - TPYPM

9.5 TAW Personal income tax accruals on wage
(X=165) income

\[ TAW = \sum_{i=1}^{4} \left[ (0.01 \text{ RTPYFBiC}) \cdot 0.01(\text{RTPYFQ} - \text{RFAX} - \text{RFAQ}) \\
+ (0.01 \text{ RTPYQiC}) \cdot 0.01 \text{ RTPYPO} + 0.01(\text{RTPYFiC}) \\
\right] (\text{YWASiC} - (\text{NTWiC})(\text{ZEXYWiC})) \]

9.6 TANW Personal income tax accruals on nonwage
(X=164) income

\[ TANW = \sum_{i=1}^{4} \left[ (0.01 \text{ RTPYFBiC}) \cdot 0.01(\text{RTPYFQ} - \text{RFAX} - \text{RFAQ}) \\
+ (0.01 \text{ RTPYQiC}) \cdot 0.01 \text{ RTPYPO} + 0.01(\text{RTPYFiC}) \\
\right] (\text{YNWASiC} - (\text{NTNWiC})(\text{ZEXYNWiC})) \\
- 0.9 (0.01 \text{ RDC}) (\text{EYDIVA11})(\text{YDIV11}) \\
\left[ 1+0.01(\text{RTPYFQ} - \text{RFAX} - \text{RFAQ}) \\
3 \left( \sum_{i=1}^{3} \frac{\text{EYDIViC}}{(\text{RTPYFiC})} \right) \\
+ (1-\left( \sum_{i=1}^{3} \frac{\text{EYDIViC}}{(\text{RTPYF4iC})} \right) \right) \\
\right] (0.01 \text{ RTPYPO}) \]
9.7 \( NT \quad \text{Tax returns filed} \quad (X=114) \)

\[ NT = 1.2398(NE+NOAPR) - 2.6198 \]

9.8 \( YWAS \quad \text{Assessed wage income} \quad (X=236) \)

\[ YWAS = 0.80472(YW+GMPF-YWSLP-GWSF-GWPASPM-GWSSM) \]
\[ + \ 0.00659(YW+GMPF-YWSLP-GWSF-GWPASPM-GWSSM) \times QTSTEP \]
\[ + (QTXRFM)(YWSLMED+GTPUIBF) + (QTFA)(GTPFAF) \]

9.9 \( YNWAS \quad \text{Assessed nonwage income (excluding dividends)} \quad (X=230) \)

\[ YNWAS = 0.37401(YP-GMPF-YW-YRENT-YDIV11-GTPUIBF-GTPFAF) \]
\[ + \ 0.00331(YP-GMPF-YW-YRENT-YDIV11-GTPUIBF-GTPFAF) \times QTSTEP \]

9.10 \( NT1C \quad \text{Tax returns filed, income class 1} \quad (X=115) \)

\[ NT1C = \text{(USRNT1C)}(NT) \]

9.11 \( NT2C \quad \text{Tax returns filed, income class 2} \quad (X=116) \)

\[ NT2C = \text{(USRNT2C)}(NT) \]
9.12 NT3C          Tax returns filed, income class 3
               (X=117)

NT3C = (USRNT3C) (NT)

9.13 NT4C          Tax returns filed, income class 4
               (X=118)

NT4C = NT - (NT1C+NT2C+NT3C)

9.14 YWAS1C       Assessed wage income, income class 1
               (X=237)

YWAS1C = (USRW1C) (YWAS)

9.15 YWAS2C       Assessed wage income, income class 2
               (X=238)

YWAS2C = (USRW2C) (YWAS)

9.16 YWAS3C       Assessed wage income, income class 3
               (X=239)

YWAS3C = (USRW3C) (YWAS)

9.17 YWAS4C       Assessed wage income, income class 4
               (X=240)

YWAS4C = YWAS - (YWAS1C+YWAS2C+YWAS3C)
9.18 \( YNWAS1C \) Assessed nonwage income (including dividends), income class 1

\[
YNWAS1C = (USRNW1C) (YNWAS) + (1+.333 \text{ QTXRFM}) (EYDIV1C) (EYDIVA11) (YDIV11) + (\text{QTXRFM})(EYDIV1C) (.5 \ YKGPR)
\]

9.19 \( YNWAS2C \) Assessed nonwage income (including dividends), income class 2

\[
YNWAS2C = (USRNW2C) (YNWAS) + (1+.333 \text{ QTXRFM}) (EYDIV2C) (EYDIVA11) (YDIV11) + (\text{QTXRFM})(EYDIV2C) (.5 \ YKGPR)
\]

9.20 \( YNWAS3C \) Assessed nonwage income (including dividends), income class 3

\[
YNWAS3C = (USRNW3C) (YNWAS) + (1+.333 \text{ QTXRFM}) (EYDIV3C) (EYDIVA11) (YDIV11) + (\text{QTXRFM})(EYDIV3C) (.5 \ YKGPR)
\]

9.21 \( YNWAS4C \) Assessed nonwage income (including dividends), income class 4

\[
YNWAS4C = YNWAS + (1+.333 \text{ QTXRFM}) (EYDIVA11) (YDIV11) + (\text{QTXRFM})(.5 \ YKGPR) - (YNWAS1C+YNWAS2C+YNWAS3C)
\]
9.22 NTW1C  Wage earners tax returns filed, income class 1
(X=119)

NTW1C = \left[ \frac{YWAS1C}{(YWAS1C+YNWAS1C)} \right] (NT1C)

9.23 NTW2C  Wage earners tax returns filed, income class 2
(X=120)

NTW2C = \left[ \frac{YWAS2C}{(YWAS2C+YNWAS2C)} \right] (NT2C)

9.24 NTW3C  Wage earners tax returns filed, income class 3
(X=121)

NTW3C = \left[ \frac{YWAS3C}{(YWAS3C+YNWAS3C)} \right] (NT3C)

9.25 NTW4C  Wage earners tax returns filed, income class 4
(X=122)

NTW4C = \left[ \frac{YWAS4C}{(YWAS4C+YNWAS4C)} \right] (NT4C)

9.26 NTNW1C  Nonwage earners tax returns filed, income class 1
(X=123)

NTNW1C = NT1C - NTW1C

9.27 NTNW2C  Nonwage earners tax returns filed, income class 2
(X=124)

NTNW2C = NT2C - NTW2C
9.28 \text{NTNW3C} \quad \text{Nonwage earners tax returns filed, income class 3} \quad (X=125) \\
\text{NTNW3C} = \text{NT3C} - \text{NTW3C}

9.29 \text{NTNW4C} \quad \text{Nonwage earners tax returns filed, income class 4} \quad (X=126) \\
\text{NTNW4C} = \text{NT4C} - \text{NTW4C}

9.30 \text{RTI} \quad \text{Indexing factor} \quad (X=295) \\
\text{RTI} = \text{QINDEX} \left[ (Q1) (J2L (J4A (PCPI)) / 1.38025) \right. \\
\left. + (Q2) (J3L (J4A (PCPI)) / 1.38025) \right. \\
\left. + (Q3) (J4L (J4A (PCPI)) / 1.38025) \right. \\
\left. + (Q4) (J5L (J4A (PCPI)) / 1.38025) \right] \\
+ 1 - \text{QINDEX}

where

1.38025 is the average of the consumer index in the twelve months ending September 31, 1973 which is the base period for indexing of personal income tax.
SECTOR 9 - SUPPLEMENTAL

Endogenization of Spreading Ratios, Exemptions and Tax Rates

I. SPREADING RATIOS

1) Quantiles (L = lower, M = middle, U = upper)

S9.1 \[ \ln(UQLNT) = 5.7223 + 0.13097 \ln(YASP) \]
\[ (PG = 1) \quad (50.93) \quad (9.70) \]
\[ \text{see} = 0.0175 \quad \text{RB2} = 0.64 \quad \text{cov} = 0.26 \% \quad \text{dw} = 0.11 \]

S9.2 \[ \ln(UQMNT) = 0.87804 + 0.87683 \ln(YASP) \]
\[ (PG = 2) \quad (16.01) \quad (133.06) \]
\[ \text{see} = 0.0085 \quad \text{RB2} = 0.997 \quad \text{cov} = 0.10 \% \quad \text{dw} = 0.98 \]

S9.3 \[ \ln(UQUNT) = -1.0801 + 1.2032 \ln(YASP) \]
\[ (PG = 3) \quad (6.91) \quad (64.10) \]
\[ \text{see} = 0.0243 \quad \text{RB2} = 0.987 \quad \text{cov} = 0.27 \% \quad \text{dw} = 0.26 \]

S9.4 \[ \ln(UQLYW) = -0.79136 + 1.0319 \ln(YASP) \]
\[ (PG = 4) \quad (8.95) \quad (97.16) \]
\[ \text{see} = 0.0137 \quad \text{RB2} = 0.994 \quad \text{cov} = 0.18 \% \quad \text{dw} = 0.51 \]
\[ \ln(UQMYW) = -1.5050 + 1.2124 \ln(YASP) \]
\[ (PG = 5) \quad (13.98) \quad (93.75) \]

\[ \text{see} = 0.0167 \quad \text{RB2} = 0.994 \quad \text{cov} = 0.20\% \quad \text{dw} = 0.47 \]

\[ \ln(UQUYW) = -1.4726 + 1.2967 \ln(YASP) \]
\[ (PG = 6) \quad (13.93) \quad (102.09) \]

\[ \text{see} = 0.0164 \quad \text{RB2} = 0.995 \quad \text{cov} = 0.18\% \quad \text{dw} = 0.56 \]

\[ \ln(UQLYNW) = 4.4740 + 0.39482 \ln(YASP) \]
\[ (PG = 7) \quad (32.58) \quad (23.93) \]

\[ \text{see} = 0.0213 \quad \text{RB2} = 0.915 \quad \text{cov} = 0.28\% \quad \text{dw} = 0.07 \]

\[ \ln(UQMYNW) = 0.70303 + 0.98201 \ln(YASP) \]
\[ (PG = 8) \quad (6.47) \quad (75.15) \]

\[ \text{see} = 0.0169 \quad \text{RB2} = 0.991 \quad \text{cov} = 0.19\% \quad \text{dw} = 0.36 \]

\[ \ln(UQUYNW) = 1.2303 + 1.1051 \ln(YASP) \]
\[ (PG = 9) \quad (10.77) \quad (80.49) \]

\[ \text{see} = 0.0178 \quad \text{RB2} = 0.992 \quad \text{cov} = 0.17\% \quad \text{dw} = 0.51 \]

where

\[ YASP = 4(YWAS+YNWAS)/(NT)(ESAYAS) \]
2) Distribution Parameters

S9.10-S9.12  Displacement Factor

\[ UDIS_i = UQMI \left[ \frac{(UQLi)(UQUI)/(UQMi)^2 - 1}{2 - \frac{(UQLi)/(UQMi) - (UQUI)/(UQMi)}{2}} \right] \]

S9.13-S9.15  Mean

\[ UMEAN_i = \ln(UQMi + UDIS_i) \]

S9.16-S9.18  Standard Deviation

\[ USIGMA_i = \ln \left( \frac{UQUI + UDIS_i}{UQMi + UDIS_i} \right) / 1.2815 \]

where

i = NT, YWAS, YNWAS
3) Ratios

a) Group

\[ S9.19-S9.32 \quad \text{Tax Returns Filed} \]

\[ \text{USRNT}_j = \frac{1}{2} \int \left[ e^{-\left( \frac{(X-\text{UMEANNT})/\text{USIGNANT})^2}{2} \right)} \right] \ln (Y_Gj+\text{UDISNT}) dx \]

\[ \left( j = 1, 14 \right) \]

\[ S9.33-S9.46 \quad \text{Assessed Wage Income} \]

\[ \text{USRW}_j = \frac{1}{2} \int \left[ e^{-\left( \frac{(X-\text{UMEANYWAS})/\text{USIGMAYWAS})^2}{2} \right)} \right] \ln (Y_Gj+\text{UDISYNWAS}) dx \]

\[ \left( j = 1, 14 \right) \]
USRNWj = \frac{1}{2} \int \frac{\ln(Yj+UDISYNWAS) - \left[ \frac{((X-UMEANYNWAS)/USIGMAYNWAS)^2}{2} \right]}{e} dx

\ln(Yj-1+UDISYNWAS)

(j = 1, 14)
b) Class

$S9.61-S9.63$ Tax Returns Filed

$\text{USRNTiC} = \sum_{m} \text{USRNTj}$

$(E = 110 + i)$

$(i = 1, 3)$

$S9.64-S9.66$ Assessed Wage Income

$\text{USRWiC} = \sum_{m} \text{USRWj}$

$(E = 150 + i)$

$(i = 1, 3)$

$S9.67-S9.69$ Assessed Nonwage Income

$\text{USRNWiC} = \sum_{m} \text{USRNWj}$

$(E = 116 + i)$

$(i = 1, 3)$

where

$m$ includes all the groups in class $i$
II. EXEMPTIONS

1) Group

S9.70-S9.83 Total

\[ Z_{\text{EXYG}Tj} = Z_{\text{EXYG}j} + (.8558RTI-1)(Z_{\text{EXPER}j}) \]

\((j = 1, 14)\)

where

.8558 is an estimate of the proportion of personal exemptions (ZEXPER) indexed, given that Quebec has chosen not to index the personal income tax.

S9.84-S9.97 Indexing Provinces

\[ Z_{\text{EXYG}Ij} = Z_{\text{EXYG}j} + (RTI-1)(Z_{\text{EXPER}j}) \]

\((j = 1, 14)\)

S9.98-S9.111 Quebec

\[ Z_{\text{EXYG}Qj} = Z_{\text{EXYG}j} \]

\((j = 1, 14)\)
2) Class

S9.112-S9.115 Total Nonwage

\[ ZEXYNWiC = \left( \text{WZEXNW} \right) \sum_m \left( \frac{\text{USRNT}_j}{\sum \text{USRNT}_j} \right) (ZEXYG\text{T}_j) \]

\( (E = 136 + i) \)
\( (i = 1, 4) \)

S9.116-S9.119 Total Wage

\[ ZEXYW\text{iC} = \left( \text{WZEXW} \right) \sum_m \left( \frac{\text{USRNT}_j}{\sum \text{USRNT}_j} \right) (ZEXYG\text{T}_j) \]

\( (E = 119 + i) \)
\( (i = 1, 4) \)

S9.120-S9.123 Indexing Provinces Nonwage

\[ ZEXINWiC = \left( \text{WZEXNW} \right) \sum_m \left( \frac{\text{USRNT}_j}{\sum \text{USRNT}_j} \right) (ZEXYG\text{I}_j) \]

\( (PG = 13 + i) \)
\( (i = 1, 4) \)
S9.124-S9.127       Indexing Provinces Wage

\[ ZEXIWic = (WZEXW) \sum \left( \frac{USRNTj}{m} \right) \left( \sum \frac{USRNTj}{m} \right) \left( ZEXYGij \right) \]

\( (PG = 17 + i) \)
\( (i = 1, 4) \)

S9.128-S9.131       Quebec Nonwage

\[ ZEXQNWic = (WZEXNW) \sum \left( \frac{USRNTj}{m} \right) \left( \sum \frac{USRNTj}{m} \right) \left( ZEXYGQj \right) \]

\( (PG = 21 + i) \)
\( (i = 1, 4) \)

S9.132-S9.135       Quebec Wage

\[ ZEXQWiC = (WZEXW) \sum \left( \frac{USRNTj}{m} \right) \left( \sum \frac{USRNTj}{m} \right) \left( ZEXYGQj \right) \]

\( (PG = 25 + i) \)
\( (i = 1, 4) \)

where

\( m \) includes all the groups in class \( i \)
3) Quarterly Spreading Ratios

$S9.136 \quad \text{Nonwage}$

$WZEXNW = (Q1) \ (J4L(YNWAS)/J1L(J4S(YNWAS)))$
$\quad + (Q2) \ (J4L(YNWAS)/J2L(J4S(YNWAS)))$
$\quad + (Q3) \ (J4L(YNWAS)/J3L(J4S(YNWAS)))$
$\quad + (Q4) \ (J4L(YNWAS)/J4L(J4S(YNWAS)))$

$S9.137$

$WZEXW = (Q1) \ (J4L(YWAS)/J1L(J4S(YWAS)))$
$\quad + (Q2) \ (J4L(YWAS)/J2L(J4S(YWAS)))$
$\quad + (Q3) \ (J4L(YWAS)/J3L(J4S(YWAS)))$
$\quad + (Q4) \ (J4L(YWAS)/J4L(J4S(YWAS)))$
III. TAX RATES

1) Group

S9.138-S9.151 Federal Rate

$$\text{RTPYF}_j = \left( \sum_{k=1}^{n-1} \left[ (RMARFk)(YBRACK+1-YBRACK)(RTI) \right] \right) + \frac{(RMARFn)[YTAj - (YBRACn)(RTI)]}{YTAj}$$

$$j = 1, 14$$

S9.152-S9.165 Basic Federal Rate

$$\text{RTPYFB}_j = \left( \sum_{k=1}^{n-1} \left[ (RMARFBk)(YBRACK+1-YBRACK)(RTI) \right] \right) + \frac{(RMARFBn)[YTAj - (YBRACn)(RTI)]}{YTAj}$$

$$j = 1, 14$$

S9.166-S9.179 Quebec Rate

$$\text{RTPYQ}_j = \left[ \sum_{k=1}^{n-1} \left[ (RMARFBk)(YBRACK+1-YBRACK) \right] \right] + \frac{(RMARFBn)(YTAj-YBRACn)}{YTAj}$$

$$j = 1, 14$$
where

\[ YTA_j = \frac{(USRw) (4YWAS)}{(USRNTj) (NT) (ESAYW)} \]

\[ + \frac{(USRNWj) (4YNWAS)}{(USRNTj) (NT) (ESAYNW)} \]

\[ - ZEXYGij \]

the i in ZEXYGij refers to T, I, and Q; n is the marginal tax rate bracket for group j
2) Class

**S9.180-S9.183** Federal Rate

\[ RTPYFiC = \sum_m \left( \frac{YASj}{\sum_m YASj} \right) (RTPYFj) \]

\((E = 125 + i)\)

\((i = 1, 4)\)

**S9.184-S9.187** Basic Federal Rate

\[ RTPYFBiC = \sum_m \left( \frac{YASj}{\sum_m YASj} \right) (RTPYFBj) \]

\((E = 129 + i)\)

\((i = 1, 4)\)

**S9.188-S9.191** Quebec Rate

\[ RTPYQiC = \sum_m \left( \frac{YASj}{\sum_m YASj} \right) (RTPYQj) \]

\((PG = 9 + i)\)

\((i = 1, 4)\)

where

\(m\) includes all the groups in class \(i\)

\(YASj = (YWASj + YNWASj)\)
SECTOR 8

Capital Gains

8.13 YKGPA  Accrued capital gains
       (X=296)

\[
YKGPA = (\text{QTXRFM}) \cdot 0.775 [J1D[(VKB)(1-.01 RVB12-.01 RVB13)] \\
- (1-.01 RVB12-.01 RVB13)[J1D[PKIB]KIB] \\
+ YIVA + (\text{PIME}) (\text{IME-.05 J1L[KME]} + (\text{PINRC}) \\
\quad (\text{INRC-.01 J1L[KNRC]}))] \\
+ (1 - [\text{LPCV12}/((1-.01 RVB13)(VKB))]) \\
\quad (\text{YCR-FIYCRE12-FIYCRE13}))
\]

8.14 AYKGPA  Stock of accrued unrealized capital gains
       (X=297)

\[
AYKGPA = J1L(AYKGPA) + YKGPA - YKGPR
\]

8.15 YKGPR  Realizations of capital gains
       (X=298)

\[
YKGPR = 0.018[J1L(AYKGPA) + YKGPA]
\]

where:

0.018 is an estimate of proportion of accrued gains that would be realized based on U.S. experience adjusted for effect of deemed realization at death in Canada.
REFERENCES


