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Khalfaoui Rabeh, K and Boutahar Mohamed, B

GREQAM d'Aix-Marseille, IML université de la méditerranée

28 June 2011

A Time-Scale Analysis of Systematic Risk: Wavelet-Based Approach

R. Khalfaoui* and M. Boutahar†

Abstract

The paper studies the impact of different time-scales on the market risk of individual stock market returns and of a given portfolio in Paris Stock Market by applying the wavelet analysis. To investigate the scaling properties of stock market returns and the lead/lag relationship between them at different scales, wavelet variance and cross-correlations analyses are used. According to wavelet variance, stock returns exhibit long memory dynamics. The wavelet cross-correlation analysis shows that comovements between stock returns are stronger at higher scales (lower frequencies); scales corresponding to period of 4 months and longer, i.e. scales 7 and 8. The wavelet analysis of systematic risk shows that all individual assets and the diversified portfolio have a multi-scale behavior, which indicates that the systematic risk measured by Beta in the market model is not stable over time. The analysis of VaR at different time scales shows that risk is more concentrated at higher frequencies dynamics (lower time scales) of the data.

JEL Classification: C02; G12; G32

keywords: Wavelets, Systematic risk, Value-at-Risk

1 Introduction

There are several methods for analyzing financial time series, most of them used the time domain in econometric modeling. A natural concept in financial time series is the notion of multiscale features. That is, an observed time series may contain several structures, each occurring on a different time scale. Wavelet method was applied to separate the dynamics in a time series over a variety of different time horizons. Hence, wavelet analysis provides an efficient way to localize changes across time scales while maintaining the entropy conservation. This local property makes wavelets a suitable tool for analyzing economic and financial stochastic processes. Therefore, by decomposing a time series on different scales, one may expect to obtain a better understanding of the data generating process as well as dynamic market mechanisms behind the time series.

In recent years the interest for wavelet methods has increased in economics and finance. Ramsey and Zhang (1997) analyzed foreign exchange data using waveform dictionaries, Kim and In (2005) studied the relationship between stock markets and inflation using maximum overlap discrete wavelet transform estimator of the wavelet correlation. In and Kim (2006) examined the relationship between Australian stock and futures markets over various time horizons. Sharkasi et al. (2006) used wavelet transform to analyze the reaction of stock markets to crashes and events in emerging and mature markets, Kim and In (2007) studied the relationship between changes in stock prices and bond yields in the G7 countries. Durai and Bhaduri (2009) studied the relationship between stock prices, inflation and output using maximum overlap discrete wavelet transform.

In the area of finance, wavelet analysis appears useful, as different traders view the market with different time resolutions, for example hourly, daily, weekly or monthly. Markets consist of agents working in different time horizons. Therefore, the dynamics of interrelation between markets consist of scales that possibly behave differently. Different

*GREQAM laboratory, University of Mediterranean II, 2 rue de la charité Marseille 13002. FRANCE. e-mail: r.kalfaoui@gmail.com

†IML, University of Mediteranean II, Faculty of science of Luminy, 163 Luminy Avenue Marseille 13288. FRANCE. e-mail: boutahar@univmed.fr

types of traders analyze the multi-scale dynamics of time series. In fact, they analyzed the risk management at different time-horizon and tried to find the corresponding investment strategies.[1] Norsworthy et al. (2000) analyzed stocks from the US market and find that beta coefficients generally decrease as we move into higher scales. Some studies applied wavelet-based risk analysis for estimating Value-at-Risk of time series. Gençay et al. (2005) proposed a new method to estimate systematic risk (the Beta) using multiscale decomposition through wavelet filters. Their findings in US, UK and Germany markets provides a stronger relationship between portfolio return and risk as the scale increases. Fernandez (2006) used wavelet analysis to test multiscale CAPM using portfolio from emerging markets and find that beta coefficient changes with different time scales. Heni and Boutahar (2011) focused on modelling the conditional mean and conditional variance of exchange rates. They estimated the GARMA-FIGARCH model using the wavelet-based maximum likelihood estimator.

Others studies are based on analyzing market risk by estimating Value at Risk at different time scales. Masih et al. (2010) analyzed stocks from emerging Gulf Cooperation Council (GCC) equity markets and found that VaR measured at different time scales suggests that risk tends to be concentrated more at the higher frequencies of the data.

This paper is organized as follows: In section 2, wavelet analysis is explained. In section 3, we provide wavelet Value-at-Risk methodology. In section 4, we defined the Wavelet-Market Model. Empirical results are discussed in section 5. An extension is given in section 6. We conclude in section 7.

2 Wavelets

2.1 The Maximal Overlap Discrete Wavelet Transform

An alternative wavelet transform for the discrete wavelet transform (DWT) [2] of a time series is the Maximal Overlap Discrete Wavelet Transform (MODWT). Unlike the classical DWT, the MODWT is a non-orthogonal transform. It has many advantages over the DWT such as non-dyadic length sample size, invariant translation (i.e. shifting the time series by an integer unit will shift the MODWT wavelet and scaling coefficients the same amount), provides increased resolution at coarser scales and produces more asymptotically efficient wavelet variance estimator than DWT (see Percival (1995)). The MODWT goes by several names in the statistical and engineering literature , such as, the "stationary DWT" (Nason and Silverman (1995)), "translation-invariant DWT" (Coifman and Donoho (1995)), and "time-invariant DWT" (Pesquet et al. (1996)).

Percival and Walden (2000) define the MODWT of a time series X_t, $t = 1, \ldots, N$ as follows: for an even positive inetegr L (L denotes the width of the initial filter), let $\{h_l;\ l = 0, \ldots, L-1\}$ and $\{g_l;\ l = 0, \ldots, L-1\}$ be the Daubechies wavelet and scaling filters, respectively. The MODWT wavelet and scaling coefficients are the solutions of multiresolution decomposition analysis (pyramid algorithm[3]) of Mallat (1989). Thus, we have

$$\tilde{\omega}_{jt} = \sum_{l=0}^{L_j-1} \tilde{h}_{jl} X_{t-l \ mod \ N},\ t = 0, 1, \ldots, N-1, \tag{2.1}$$

and

$$\tilde{v}_{jt} = \sum_{l=0}^{L_j-1} \tilde{g}_{jl} X_{t-l \ mod \ N},\ t = 0, 1, \ldots, N-1, \tag{2.2}$$

where $L_j \equiv (2^j - 1)(L-1) + 1$ is the length of the wavelet filter (see Gençay et al. (2002) for more detail) and where the MODWT wavelet and scaling filters $\tilde{h}_{jl}$ and $\tilde{g}_{jl}$ are calculated by rescaling the DWT filters coefficients, such that

[1] Investors work on many different time scales, and with wavelets we can separate these different time scales. Investors should take into account also their investment horizon when they make risk management and portfolio allocation decisions.

[2] See Percival and Walden (2000) for more details.

[3] A pyramid algorithm similar to that of DWT is utilized to compute the MODWT; see Percival and Guttorp (1994) and Percival and Mofjeld (1997)

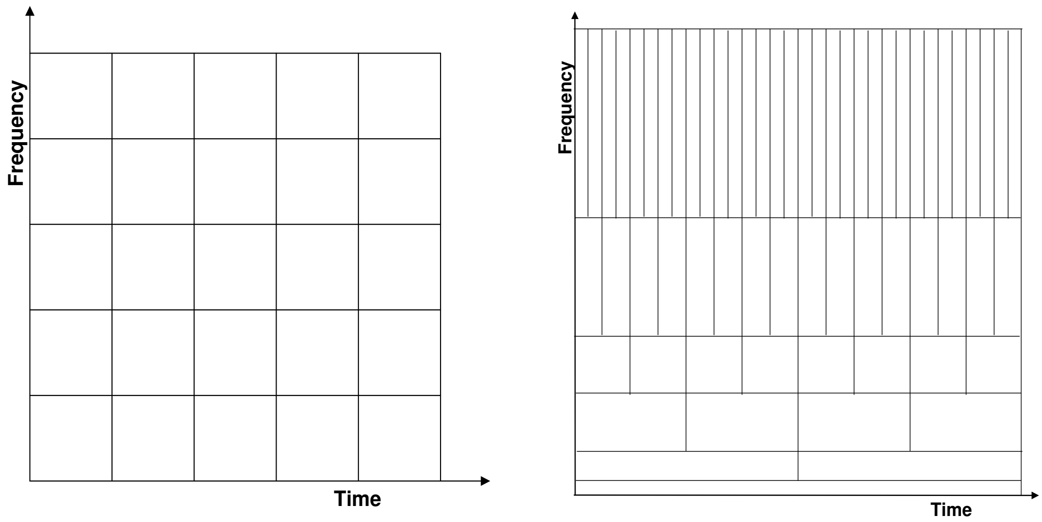

Figure 2.1 The short-time Fourier transform (left panel) and the wavelet transform (right panel) partitioning of the time–frequency plane.

$\tilde{h}_{jl} = h_{jl}/2^{j/2}$ and $\tilde{g}_{jl} = g_{jl}/2^{j/2}$ and circularly shifting by unit intervals for all levels of the transform. The MODWT filters satisfies the following three properties:

$$\sum_{l=0}^{L-1} \tilde{h}_l = 0, \sum_{l=0}^{L-1} \tilde{g}_l = 1, \tag{2.3}$$

$$\sum_{l=0}^{L-1} \tilde{h}_{jl}^2 = \sum_{l=0}^{L-1} \tilde{g}_{jl}^2 = \frac{1}{2^j}, \tag{2.4}$$

$$\sum_{l=-\infty}^{\infty} \tilde{h}_l \tilde{h}_{l+2n} = \sum_{l=-\infty}^{\infty} \tilde{g}_l \tilde{g}_{l+2n} = 0. \tag{2.5}$$

Figure. 2.1 shows the time-frequency resolution properties of the Gabor transform or short-time (time-variable) Fourier transform and the wavelet transform. The gabor transform (left panel) has constant resolution at all times and frequencies and the wavelet transform provides good frequency resolution (and poor time resolution) at low frequencies and good time resolution (and poor frequency resolution) at high frequencies.[4]

2.2 Wavelet variance, covariance and correlation

2.2.1 Analysis of variance

Because wavelet transform can break down the original time series into components of different scales, it provides a powerful tool to detect the pattern of variations in observed data. In particular, it is interesting to calculate the wavelet variance on scale-by-scale basis. Percival (1995) and Percival and Mofjeld (1997) proved that the variance of a given time series is captured by the variance of the MODWT coefficients. Hence, the total variance of a time series can be partitioned using the MODWT as

$$\|X\|^2 = \sum_{j=1}^{J} \left\|\tilde{\omega}_j\right\|^2 + \|\tilde{v}_J\|^2. \tag{2.6}$$

[4] Both Gabor and wavelet transform are based on Fourier transform. The Gabor transform uses constant window-function which cannot face the problem when signals have very frequency components with short time spans, and low frequency components with long time spans. The wavelet transform uses varying window-function. Indeed, it allows the use of long time intervals where we want more precise low-frequency information, and shorter regions where we want high-frequency information.

where $\left\|\tilde{\omega}_j\right\|^2$ is the detail variance (variance of X due to changes at scales λ_j) and $\|\tilde{v}_J\|^2$ is the smooth variance (variance due to changes at scales λ_J). The wavelet-variance analysis consists of partitioning the variance of a time series X_t into pieces that are associated to different time scales. It tells us what scales are important contributors to the overall variability of a series (see Percival and Walden (2000) and Gençay et al. (2002)). The wavelet variance of the wavelet coefficients $\tilde{\omega}_{jt}$ at scale λ_j is defined as

$$\tilde{\sigma}_X^2(\lambda_j) = \frac{1}{2\lambda_j} Var(\tilde{\omega}_{jt}), \tag{2.7}$$

where $\tilde{\omega}_{jt}$ is defined in equation (2.1) and scale λ_j is associated with frequency interval $\left[1/2^{j+1}, 1/2^j\right]$.
The total variance of X can be decomposed as

$$\sum_{j=1}^{J} \tilde{\sigma}_X^2(\lambda_j) = Var(X_t). \tag{2.8}$$

Mondal and Percival (1995) defined an unbiased MODWT estimator of $\tilde{\sigma}_X^2(\lambda_j)$ as follows

$$\hat{\sigma}_X^2(\lambda_j) = \frac{1}{M_j} \sum_{t=L_j}^{N} \tilde{\omega}_{jt}^2, \tag{2.9}$$

where $M_j = N - L_j + 1$ is the number of maximum overlap coefficients and $L_j = (2^j - 1)(L-1) + 1$ is the length of the wavelet filter.
Given the usefulness of the wavelet variance for univariate time series, the following section investigates the *wavelet covariance* and *wavelet correlation* for bivariate time series.

2.2.2 Analysis of covariance

To determine the relationship between two time series on a scale-by-scale basis the notion of wavelet covariance has to be used. Whitcher et al. (2000a) has been introduced the definition of wavelet covariance and wavelet correlation between two processes. Let X_t and Y_t be two stationary discrete time series, and let $\tilde{\omega}_{X,jt}$ and $\tilde{\omega}_{Y,jt}$ be the scale λ_j wavelet coefficients computed from applying MODWT to each time seris X_t and Y_t, respectively. The wavelet covariance of (X_t, Y_t) for scale λ_j is defined as

$$\gamma_{XY}(\lambda_j) = \frac{1}{2\lambda_j} Cov\left(\tilde{\omega}_{X,jt}, \tilde{\omega}_{Y,jt}\right). \tag{2.10}$$

An unbiased estimator of the wavelet covariance based upon the MODWT is given by

$$\hat{\gamma}_{XY}(\lambda_j) = \frac{1}{M_j} \sum_{t=L_j}^{N} \tilde{\omega}_{X,jt} \tilde{\omega}_{Y,jt} \tag{2.11}$$

By introducing an integer τ between $\tilde{\omega}_{X,jt}$ and $\tilde{\omega}_{Y,jt}$, Whitcher et al. (2000a) defined the *wavelet cross-covariance* of (X_t, Y_t) for a scale λ_j and lag τ as

$$\gamma_{XY,\tau}(\lambda_j) = \frac{1}{2\lambda_j} Cov\left(\tilde{\omega}_{X,jt}, \tilde{\omega}_{Y,j,t+\tau}\right). \tag{2.12}$$

Following Gençay et al. (2002), the MODWT estimator of the wavelet cross-covariance is biased.

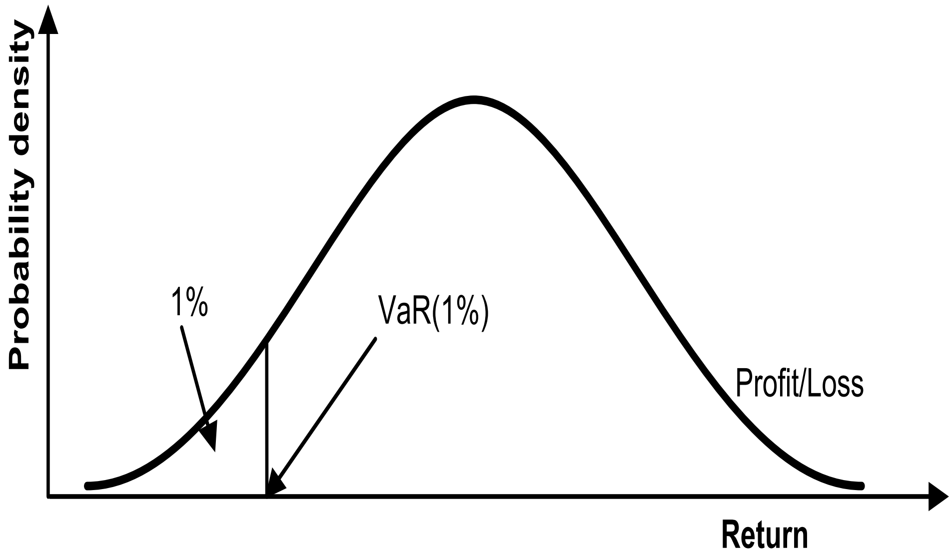

Figure 3.1 Value-at-Risk quantification using the normal probability density function

The MODWT wavelet cross-correlation coefficients for a scale λ_j and lag τ are simply obtained by using the wavelet cross-covariance $\gamma_{XY,\tau}(\lambda_j)$ and the standard deviations $\tilde{\sigma}_X^2(\lambda_j)$ and $\tilde{\sigma}_Y^2(\lambda_j)$:

$$\rho_{XY,\tau}(\lambda_j) = \frac{\gamma_{XY,\tau}(\lambda_j)}{\tilde{\sigma}_X(\lambda_j)\tilde{\sigma}_Y(\lambda_j)} \tag{2.13}$$

where $\tilde{\sigma}_X^2(\lambda_j)$ and $\tilde{\sigma}_Y^2(\lambda_j)$ are the MODWT wavelet variance defined in equation (2.7). The wavelet cross-correlation is used to determine lead/lag relationships on a scale by scale between two time series.

3 Value at Risk: VaR

Measure of risk management is the interest of traders in financial markets. There are many types of measures of risk, such that volatility, semi-variance or downside risk and expected shortfall. One of the most important risk measures in finance is the VaR, which measures the maximum trading loss that a bank can face over a given horizon (usually one day) and under a specified significance level (popular significance levels usually are 99% and 95%).[5] Consider p_t, the value of a given portfolio P at a particular time t (for example, day t). Let r_t be the return of this portfolio during the period $(t-1,t)$. The VaR is interpreted as the maximum loss of the portfolio not exceeded with a given probability over the period $(t-1,t)$. Mathematically, VaR is defined at the period time Δt and for significance level $\alpha\%$ as

$$P(r_t \leq VaR(\alpha)) = \alpha, \tag{3.1}$$

From the equation (3.1) and Figure 3.1 we observed that VaR estimates the statistically significant losses at the distribution tails.

Several methods for VaR estimation are discussed in the literature.[6] These methods are grouped in three categories: non-parametric methods (Historical Simulation, Weighted Historical Simulation, Filtered Historical Simulation, ...), semi-parametric methods (extreme value theory, ...) and parametric methods (ARCH, univariate GARCH, multivariate GARCH, RiskMetrics). When looking at the time-varying volatility models for VaR estimation, one can see that these methods look at the historical data of the time-horizon chosen. Therefore, a tool is needed which enables the analyst

[5] For more detailed discusion about Value at Risk see, Jorion (1996).

[6] See, Jorion (1996)

to decompose the signal into all of its components, separating higher frequent behavior from the lower frequent one, in order to analyze which of these components produce relevant information. The wavelet VaR fills this gap.

4 The Wavelet-market model

The market price is influenced by different market participants, such as intraday traders, daily traders, short term traders and long term traders. These participants have different trading strategies over different investment time-horizons. Therefore, market prices are formed by the influence of financial participants which caracterized by different time-frequencies. Thus, the market risk has a multi-scale structure. Most of the traders used a constant risk over a period (day, ...). In this section we introduced a market model at different scales: The Wavelet-market model. After decomposing a return series into j crystals $j = 1,\dots,J$ (details and smooths), The decomposition is based on MODWT using Mallat's algorithm

$$r_t = D_1(t) + \dots + D_{J-1}(t) + D_J(t) + S_J(t). \tag{4.1}$$

where $S_J(t)$ and $D_j(t), j = 1,\dots,J$ are defined as follows

$$S_J(t) = \sum_k s_{Jk}\phi_{Jk}(t), \tag{4.2}$$

$$D_j(t) = \sum_k d_{jk}\psi_{jk}(t),\ j = 1,\dots,J. \tag{4.3}$$

where $\phi_{jk}(t)$ and $\psi_{jk}(t)$ are the father and mother wavelet that are given by the following two equations.[7] j and k are the number of scale crystals (intervals or frequencies) and the number of coefficients in each component.

$$\phi_{jk}(t) = 2^{-j/2}\phi\left(\frac{t-2^jk}{2^j}\right)\ for j = 1,\dots,J \tag{4.4}$$

$$\psi_{jk}(t) = 2^{-j/2}\psi\left(\frac{t-2^jk}{2^j}\right)\ for j = 1,\dots,J \tag{4.5}$$

To define the Wavelet-market model we run an Ordinary Least Square regression of each stock crystals on each crystals of the market portfolio r_m:

$$r_{it}(\lambda_j) = \alpha_i(\lambda_j) + \beta_i(\lambda_j)r_{mt}(\lambda_j) + \varepsilon_{it}(\lambda_j),\ j = 1,\dots,J. \tag{4.6}$$

where $r_{it}(\lambda_j)$ and $r_{mt}(\lambda_j)$ are the details at scale (λ_j) of the asset i and the market return. In the Wavelet-market model (equation 4.6), the wavelet beta estimator for asset i, at scale λ_j, is defined as

$$\hat{\beta}_i(\lambda_j) = \frac{\hat{\gamma}_{r_i r_m}(\lambda_j)}{\hat{\sigma}^2_{r_m}(\lambda_j)},\ j = 1,\dots,J. \tag{4.7}$$

where $\hat{\gamma}_{r_i r_m}(\lambda_j)$ and $\hat{\sigma}^2_{r_m}(\lambda_j)$ are defined in (2.11) and (2.9). We define also, the wavelet R^2 coefficient for asset i, at scale λ_j, as

$$R^2_i(\lambda_j) = \hat{\beta}^2_i(\lambda_j)\frac{\hat{\sigma}^2_{r_m}(\lambda_j)}{\hat{\sigma}^2_{r_i}(\lambda_j)},\ j = 1,\dots,J. \tag{4.8}$$

[7] See Daubechies (1992) for more details.

From the Wavelet-market model, the market risk of a given asset i, at scale (λ_j), is decomposed to systematic risk and unsystematic risk (independent of the market), thus we write

$$\sigma_i^2(\lambda_j) = \beta_i^2(\lambda_j)\sigma_m^2(\lambda_j) + \sigma_{\varepsilon_i}^2(\lambda_j),\ i = 1,\ldots,N\ j = 1,\ldots,J. \tag{4.9}$$

where $\beta_i^2(\lambda_j)\sigma_m^2(\lambda_j)$ and $\sigma_{\varepsilon_i}^2(\lambda_j)$ are the systematic and unsystematic risks, at scale λ_j, respectively.
For a given portfolio P of N assets, the wavelet variance-covariance matrix of the N asset returns, at scale λ_j is defined as follows

$$\Sigma_P(\lambda_j) = \beta(\lambda_j)\beta^{'}(\lambda_j)\sigma_m^2(\lambda_j) + \Sigma_\varepsilon(\lambda_j),\ j = 1,\ldots,J. \tag{4.10}$$

where $\beta(\lambda_j) = \begin{pmatrix} \beta_1(\lambda_j) \\ \beta_2(\lambda_j) \\ \vdots \\ \beta_N(\lambda_j) \end{pmatrix}$ and $\Sigma_\varepsilon(\lambda_j) = \begin{pmatrix} \sigma_{\varepsilon_1}^2(\lambda_j) & 0 & \cdots & 0 \\ 0 & \sigma_{\varepsilon_2}^2(\lambda_j) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{\varepsilon_N}^2(\lambda_j) \end{pmatrix}$, $j = 1,\ldots,J.$

Assume that $E(r_P) = 0$, the Wavelet-VaR of the portfolio P is given by

$$VaR_{\lambda_j}(\alpha) = F^{-1}(\alpha)\sqrt{\kappa'\Sigma_P(\lambda_j)\kappa},\ j = 1,\ldots,J. \tag{4.11}$$

where κ is a $N\times 1$ vector of portfolio weights and $F^{-1}(\alpha)$ is the inverse of the cumulative normal distribution function. For an equally weighted portfolio, such that $\kappa_i = 1/N\ \forall i$, the Wavelet-VaR is

$$VaR_{\lambda_j}(\alpha) = F^{-1}(\alpha)\left\{\sigma_m^2(\lambda_j)\left(\sum_{i=1}^N \beta_i(\lambda_j)/N\right)^2 + \frac{1}{N^2}\sum_{i=1}^N \sigma_{\varepsilon_i}^2(\lambda_j)\right\}^{1/2},\ j = 1,\ldots,J. \tag{4.12}$$

For a well-diversified portfolio, i.e. N is large, the Wavelet-VaR is approximately calculated by the systematic risk, such that we have

$$VaR_{\lambda_j}(\alpha) \approx \frac{1}{N}F^{-1}(\alpha)\sigma_m(\lambda_j)\left|\sum_{i=1}^N \beta_i(\lambda_j)\right|,\ j = 1,\ldots,J. \tag{4.13}$$

5 Data and empirical results

In this section, our analysis is based on stock market price of ten assets from Paris Stock Market. We decompose the returns series into their time-scale components using the MODWT analysis based on the Daubechies least asymmetric (LA) wavelet filter of length $L = 8$.[8] For any given time series, the level j wavelet coefficients are associated with changes at scale $\lambda_j = 2^{j-1}$. Hence, the scale 2^{j-1} corresponds to frequencies in the interval $f \in \left[1/2^{j+1}, 1/2^j\right]$, the wavelet coefficient (detail) $\tilde{\omega}_1$ associated with changes on the scale λ_1 captures frequencies $f \in [0.25, 0.5]$, thus is associated to 2-4 days periods, similarly $\tilde{\omega}_2$ contains frequencies $f \in [1/8, 1/4]$, is associated with a period length of 4 to 8 days. While scales λ_3 to λ_8 are associated to 8-16, 16-32, 32-64, 64-128, 128-256 and 256-512 day periods, respectively.

[8] It has been shown that LA(8) filter gives the best performence for the wavelet time series decomposition. This wavelet filter has been widely used and applied in a wide variety of data types. Following Gençay et al. (2002), the choice of filter length depends on three aspects: length of data, complexity of spectral density function, and the underlying shape of features in the data.

5.1 Data

The data employed are daily closing stock market price for ten assets of many different sectors that are listed in France SBF 120 stock market Index, obtained from DataStream. The sample period runs from December 29, 2000 to September 06, 2010 (2529 observations). Table 5.1 shows additional information about the data used. For each stock i, we collect daily returns series, defined as returns of daily closing price p_{it}: $r_{it} = ln(p_{it}) - ln(p_{i,t-1})$. Figure 5.1 depicts Level price series for the data and Figure 5.2 shows growth in the return series for all the stock prices.

Table 5.1

Definition of variables used in the study

Variable name	Abbreviation	Sector name
Panel A: Firms		
Air France	AF	Airlines
Alcatel-Lucent	ALU	Telecommunications Equipement
BNP Paribas	BNP	Banks
Carrefour	CA	Food Retails and Wholesalers
AXA	CS	Full Line Insurance
Dassault Systemes	DSY	Software
Total	FP	Integrated Oil and Gas
Lafarge	LG	Building Materials and Fixtures
Sanofi-Aventis	SAN	Pharmaceuticals
Peugeot	UG	Automobiles
Panel B: Indexes		
SBF 120	SBF120	Paris Index

Note: The table depicts the names, abbreviated names and the setors of the selected firms.

Table 5.2

Basic statistics for return series

Panel A: Descriptive statistics	Air France	Alcatel-Lucent	BNP Paribas	Carrefour	AXA	Dassault Systèmes	Total	Lafarge	Sanofi-Aventis	Paugeot	SBF120
Mean (%)	-0.032	-0.130	0.007	-0.022	-0.040	-0.015	-0.0001	-0.024	-0.016	-0.024	-0.015
Std.dev.	0.027	0.035	0.024	0.018	0.029	0.025	0.017	0.022	0.018	0.022	0.015
Maximum	0.154	0.340	0.189	0.094	0.198	0.168	0.127	0.150	0.136	0.129	0.103
Minimum	-0.225	-0.194	-0.189	-0.116	-0.202	-0.200	-0.096	-0.121	-0.109	-0.152	-0.093
Skewness	-0.305	0.145	0.320	-0.150	0.357	0.113	0.137	-0.0005	-0.008	0.044	0.049
Kurtosis	5.096	7.430	9.637	3.976	6.731	5.488	5.940	4.866	4.354	4.010	5.617
JB test$_{p-value}$	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
LB$_{p-value}$	0.023*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
Panel B: Pearson correlation among variables											
Air France	1.000	0.470	0.520	0.413	0.549	0.417	0.358	0.492	0.315	0.477	0.611
Alcatel-Lucent		1.000	0.501	0.457	0.562	0.535	0.431	0.476	0.361	0.466	0.684
BNP Paribas			1.000	0.507	0.728	0.450	0.568	0.588	0.411	0.556	0.786
Carrefour				1.000	0.588	0.415	0.554	0.466	0.498	0.428	0.701
AXA					1.000	0.490	0.630	0.624	0.475	0.575	0.848
Dassault Systèmes						1.000	0.397	0.385	0.324	0.376	0.604
Total							1.000	0.525	0.505	0.487	0.795
Lafarge								1.000	0.352	0.561	0.709
Sanofi-Aventis									1.000	0.344	0.616
Peugeot										1.000	0.656
SBF120											1.000

Notes: This table reports the basic statistics of return series, including mean (Mean), standard deviation (Std. dev), Skewness and Kurtosis. JB test$_{p-value}$ is the Jarque-Bera statistic for test of normality. LB$_{p-value}$ is Ljung-Box statistic for serial correlations of up to 36 orders in returns series. Significance at the 5% is given by *. In this table column 1 of panel B shows names of stocks of some firms listed in France SBF 120 Index. The remaining columns show the correlation between stock prices of the ten firms and SBF 120 Index used in the analysis.

We study the stationarity properties of the series by performing three standard unit root tests: Augmented Dickey-Fuller (ADF), Phillips-Perron (P-P), and Kwiatkowski et al. (KPSS) tests. The ADF and P-P tests are based on the null hypothesis of a unit root, while KPSS test considers the null of no unit root. The obtained results in Table 5.3 shows that all of the return series are stationary at 1% significance level.

Table 5.2 summarizes selected basic statistics and correlation matrix for return series. On average, the *Total* stock price experienced higher returns than all others stocks. SBF 120 stock market index has the smallest standard deviation. This shows that all stocks used in our study have higher volatility than SBF 120 stock market index. Skewness is positive in most cases and the Jarque-Bera test statistic (JB) strongly rejects the hypothesis of normality.

From panel B in Table 5.2 correlations between stocks and SBF 120 index are all positive and generally high. In addition, we showed that *AXA* stock has the highest degree of comovement with SBF 120 stock market index (0.848).

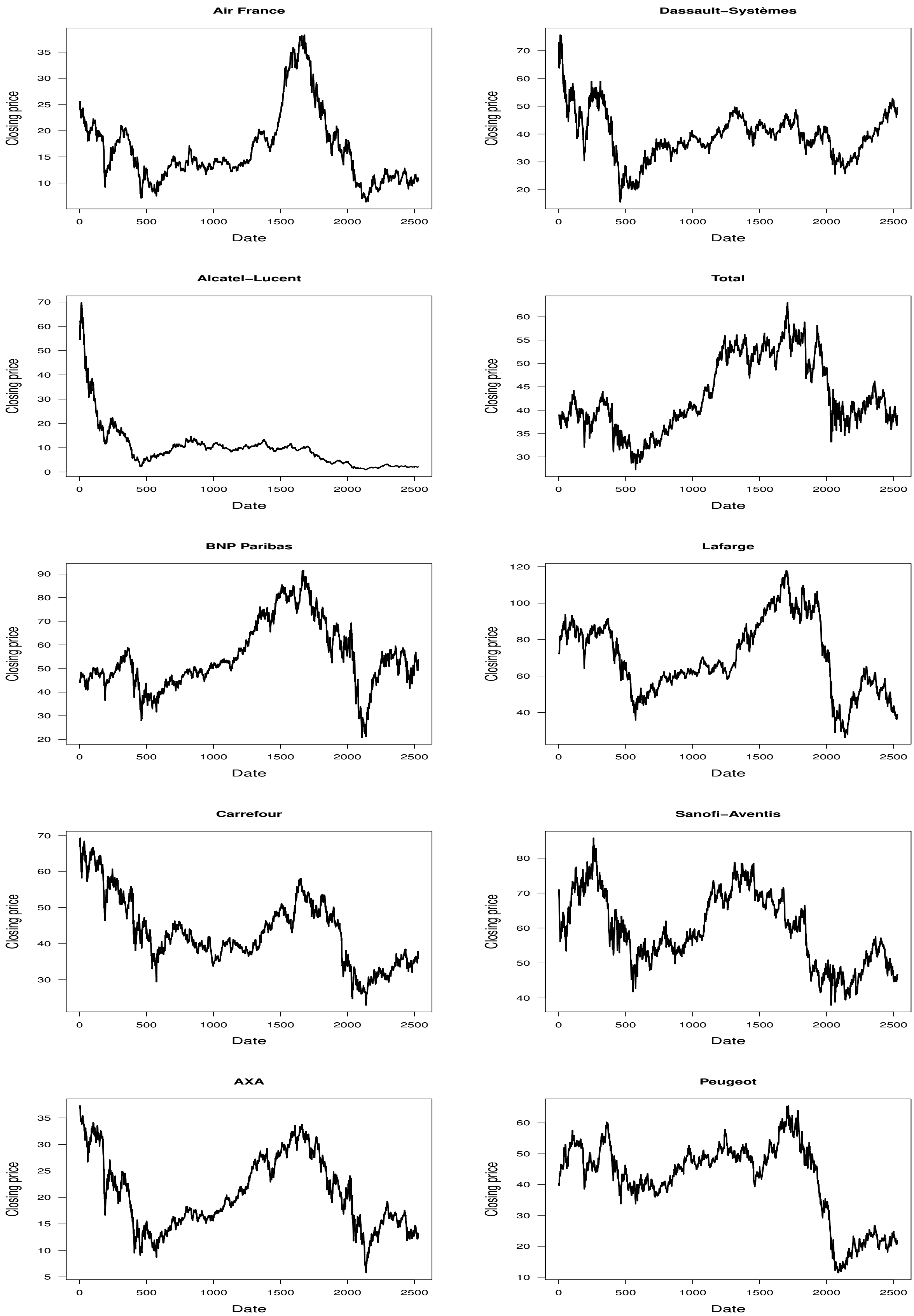

Figure 5.1 Stock Price Time Series

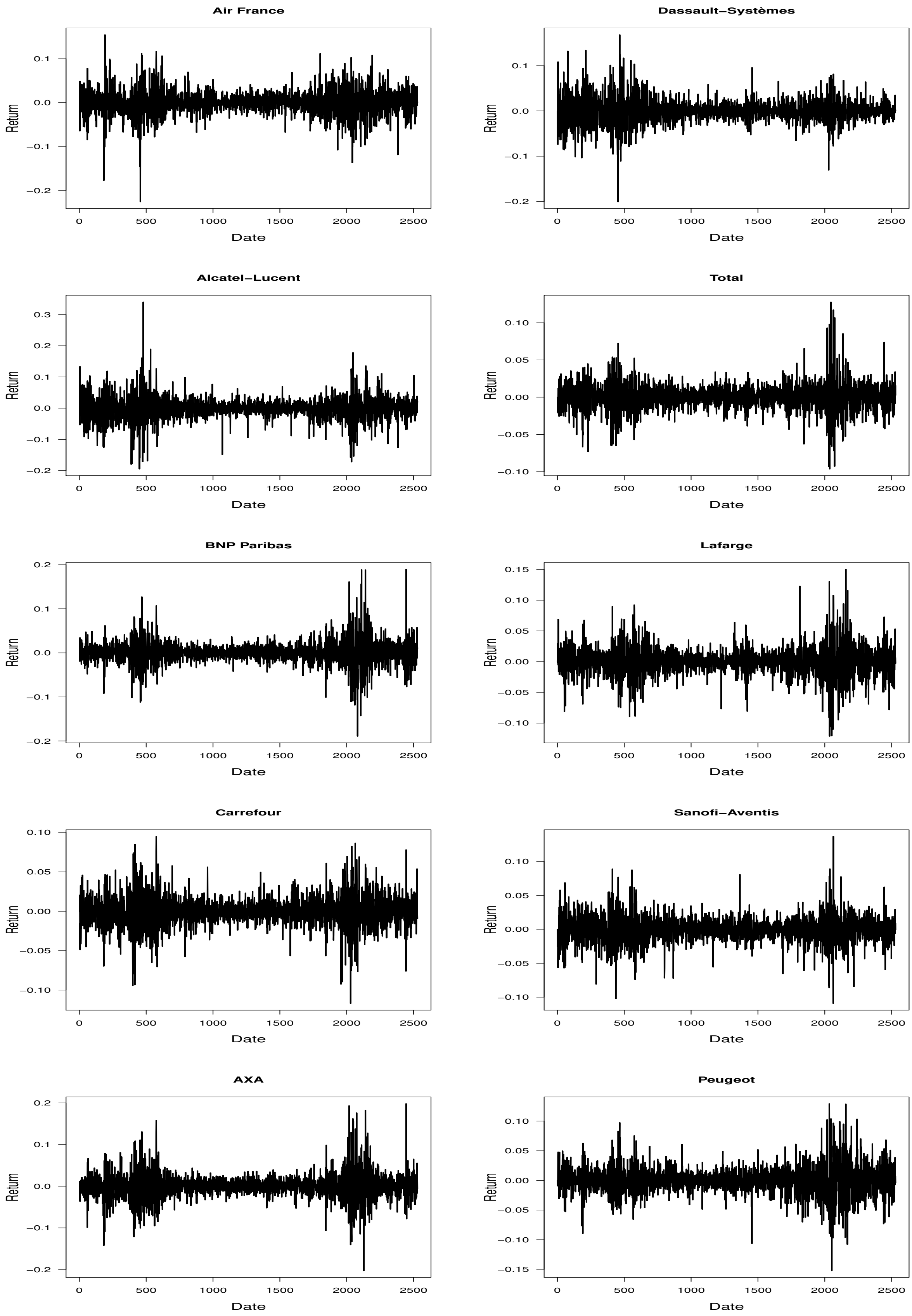

Figure 5.2 Return Stock Price Time Series

Table 5.3
Unit root tests of time series.

	Level			Returns		
	ADF	PP	KPSS	ADF	P-P	KPSS
Air France	-1.617^{a}	-1.718^{b}	2.068^{b*}	-13.807^{a*}	-47.873^{b*}	0.090^{b}
Alcatel-Lucent	-10.619^{a*}	-6.285^{b*}	1.101^{b*}	-11.575^{a*}	-49.853^{b*}	0.166^{b}
BNP Paribas	-1.757^{a}	-2.013^{b}	2.626^{b*}	-15.155^{a*}	-50.276^{b*}	0.052^{b}
Carrefour	-2.426^{a}	-2.821^{b}	1.729^{b*}	-14.183^{a*}	-52.909^{b*}	0.083^{b}
AXA	-2.269^{a}	-2.394^{b}	2.184^{b*}	-13.558^{a*}	-48.914^{b*}	0.090^{b}
Dassault Systèmes	-3.824^{b**}	-3.796^{b**}	1.144^{c*}	-14.436^{a*}	-50.982^{b*}	0.174^{b}
Total	-1.652^{a}	-2.015^{b}	2.966^{b*}	-15.542^{a*}	-54.680^{b*}	0.077^{b}
Lafarge	-1.440^{a}	-1.490^{b}	1.998^{b*}	-14.073^{a*}	-50.033^{b*}	0.131^{b}
Sanofi-Aventis	-2.273^{a}	-2.623^{b}	1.868^{b*}	-14.376^{a*}	-52.329^{b*}	0.040^{b}
Peugeot	-1.980^{a}	-2.084^{b}	2.773^{b*}	-13.944^{a*}	-48.234^{b*}	0.138^{b}
SBF120	-1.878^{a}		2.266^{b*}	-13.329^{a*}	-52.277^{b*}	0.169^{b}

Notes: ADF is the Augmented Dickey–Fuller test, P-P the Phillips–Perron test, and KPSS the Kwiatkowski–Phillips–Schmidt–Shin test. [a] Indicates a model without constant or deterministic trend. [b] Model with constant and deterministic trend. * Denotes rejection of the null hypothesis at the 1% level. ** Denotes rejection of the null hypothesis at the 5% level. *** Denotes rejection of the null hypothesis at the 10%level.

5.2 Empirical results

Firstly, we decomposed the return series into their time-scale components using MODWT. The filter used in the decomposition is the Daubechies least asymmetric (LA) wavelet filter of length $L = 8$, or LA(8) wavelet filter, while our decomposition goes to scale 8 (scale $J \leq \log_2 N$, N is the length of the time series). The LA(8) wavelet decomposition of daily returns for SBF 120 and weighted portfolio are shown in Figures 5.3 and 5.4. We plotted the returns of SBF 120 index and the weighted portfolio constructed from the ten assets and their corresponding details and smooths. We used daily data, the first level detail *D1* represents the variations within two days or four, while the next level details *D2-D8* represent the variations within 2^j days horizon (see Table 5.4 for explanation).
We performed a wavelet variance analysis on a scale-by-scale basis, thus we plotted the wavelet variance coefficients against scales $\lambda_j,\ j = 1,\ldots,8$ and we determine which scale contributes more in the variance of the process.[9]
As we can see in Figure 5.5, all stock returns show similar movements of wavelet variance. We can observe an approximate linear relationship between the wavelet variance and the wavelet scale. The variance of stock returns decreases as the wavelet scale increases, indicating that stock return variances are high in short terms (high frequencies) and low in long terms (low frequencies). As shown in Kim and In (2005), the decrease in wavelet variance implies that an investor with a short investment horizon has to respond to every fluctuation in realized returns, while for an investor with a much longer horizon, the long run risk is significantly less.
To show the degree of association between stock market index and the constructed portfolio across scales, we focused in cross-correlation analysis.[10] Hence, the cross-correlation function provides the degree of relationship between two time series as a function of time lag (h). This function measures the linear synchronization between the stock market index and the portfolio construced from the ten chosen assets. The results of wavelet cross-correlation between stock market index and our portfolio are reported in Figure 5.6. We observed that the first two scales, associated with periods of 2-4 days and approximately one week, indicate only a small number of lags (mostly arround zero) where the wavelet cross-correlation is different from zero (for scale 1 and for lags -1, 0 and 1 the values are -0.556, 0.920 and -0.515, respectively). We observed also that the wavelet cross-correlation appear to be roughly symmetric about zero lag from scale 1 up to scale 4. The asymmetry of cross-correlation function between stock market index and the portfolio becomes more pronounced as the scale increases. For instance, for scale 3 the values are 0.060 and 0.086 for lags -15 and 15, respectively, and for scale 5 the values are -0.436 and -0.515 for the same lags. On scales 6, 7 and 8 the symmetry disappear completely and we observed a positive correlation at scale 7 and scale 8 (128-256 days and 256-512 days). This indicates that when we are dealing with long run (more than two months), the prices of the

[9]The confidence interval is based on a Chi-squared distribution, an approximate $(1-\alpha)$ confidence interval for the wavelet variance can be defined as follows: $\left[\xi\tilde{\sigma}_x^2(\lambda_j)/K_{\xi,1-\frac{\alpha}{2}}, \xi\tilde{\sigma}_x^2(\lambda_j)/K_{\xi,\frac{\alpha}{2}}\right]$, where ξ is the degree of freedom, $K_{\xi,1-\frac{\alpha}{2}}$ and $K_{\xi,\frac{\alpha}{2}}$ are lower and upper $\alpha/2$ quantiles. For a detailed explanation on how to construct the confidence interval of wavelet variance, see Gençay et al. (2002).

[10]The wavelet cross-correlations are computed from the equation (2.13) and the confidence interval for the wavelet cross-correlation (see Gençay et al. (2002)) is defined as follows: $\left(\tanh\left\{h\left(\hat{\rho}_{XY,\tau}(\lambda_j)\right) \pm \Phi^{-1}(1-\rho)/\sqrt{\hat{N}_j - 3}\right\}\right)$, where $h(\rho) \equiv tanh^{-1}(\rho)$, $\hat{N}_j$ is the number of wavelet coefficients associated with scale λ_j computed via the MODWT, $\Phi^{-1}(1-\rho)$ is the $(1-\rho)\times 100\%$ point for the normal distribution.

portfolio are positively correlated with future prices of stock market index.
As a summary, the wavelet cross-correlation analysis between the given portfolio at different time scales shows that the links among stock market returns vary with wavelet scales, moreover wavelet cross-correlation values are low at the lowest scales and high at highest ones.

Table 5.4
Interpretation of time scales.

Scale	Component	Frequency resolution
Scale 1	D1	2 - 4 days
Scale 2	D2	4 - 8 days: approximately one week
Scale 3	D3	8 - 16 days: approximately tow weeks
Scale 4	D4	16 - 32 days: approximately one month
Scale 5	D5	32 - 64 days: approximately 2 months
Scale 6	D6	64 - 128 days: approximately 4 months
Scale 7	D7	128 - 256 days: approximately 1 year
Scale 8	D8	256 - 512 days: approximately 2 years

Notes: The lowest level detail *D1* captures frequencies $1/4 \leq f \leq 1/2$ (i.e. any oscillation with a period length of 2 to 4 days), and the highest level detail *D8* captures frequencies $1/512 \leq f \leq 1/256$ (i.e. any oscillation with a period length of 256 to 512 days).

5.2.1 Systematic risk analysis

Lintner (1965) and Sharpe (1963, 1964) showed that in asset pricing model a company's total risk consists of two types of risk: Unsystematic risk (idyosincratic risk) and systematic risk. [11] In our study, we only based on the changes of systematic risk over time horizons. The systematic risk, as denoted by β_i in the asset pricing model, is a measure of the slope of the regression in equation (4.6). The estimated Beta is therefore the measurement of systematic risk and represents a proxy for the true Beta. The systematic risk can differ from period to period and can be changed depending upon the management of each company. Moreover, managerial decisions about investments and financing influence the performance of the firm. Thus, the Beta could provide investors and company managers with various implications about a firm's financial and investment policies.
To show the instability of systematic risk over time, we measured the Beta of various stocks over different time horizons using MODWT wavelet filter.
Table 5.5 reports the results of estimating Beta coefficient and R^2 goodness of fit coefficient for the equation (4.6). Hence, the estimated coefficient $\beta_i(\lambda_j),\ j = 1,\ldots,8$ measures the contribution of scale λ_j movements in stock market index to assets. As shown in Table 5.5 all the Beta coefficients are positive, indicating the positive correlation between assets and stock market index. We observed also, that the market risk of some stocks such as *Air France*, *AXA*, *Dassault-Systèmes*, *Total*, *Lafarge* and *Peugeot* increases with scales. For instance, from scale 1 up to scale 6 *Air France* and *AXA* systematic risk increased from 0.957 to 1.708 and 1.624 to 1.943, respectively, and from 0.965 and 1.435 for *Dassault-Systèmes* between scale 1 to scale 5.
We observed also, that the systematic risk of almost stocks and for the proposed portfolio are less than one for the 2-4 days time horizon (short term). This, indicating that the market movements have a reduced impact on assets which provides less risk than market index in short term dynamics.
Figure 5.7 depicts the systematic risk variations of the ten assets with time scales. We observed that Beta seem to slightly decrease from the lowest to the highest scale for some stocks (for instance, *BNP Paribas*, *Total*, *Sanofi-Aventis* and *Carrefour* firms) and seem to slightly increase between lowest and highest scale for some others (for instance, *Alcatel-Lucent* and *Air France* firms). Generally, when looking the plots of Beta of stocks, we remark that the systematic risk change non-monotocally with time scale, implying the different trading strategies of traders. We observed that long-term traders are averse to risk; at scale 7, *AXA*, *Sanofi-Aventis* and *Peugeot* stocks are interrested by long-term traders than short-term traders (these assets have the smallest Beta at scale 7). At scales 1 and 2, Air France and Alcatel-Lucent stocks are interrested by short-term traders (these assets have small Beta). Therefore, we can say that short-term traders are "amateur" of risk.

[11] The total risk of an asset or a portfolio is measured by the variance or standard deviation of stock return. The idyosincratic risk is the firm-specific volatility caused by firm-specific events (e.g., strikes, product defects, poor management, ...). The systematic risk represents the risk of a stock relative to the risk of the market portfolio.

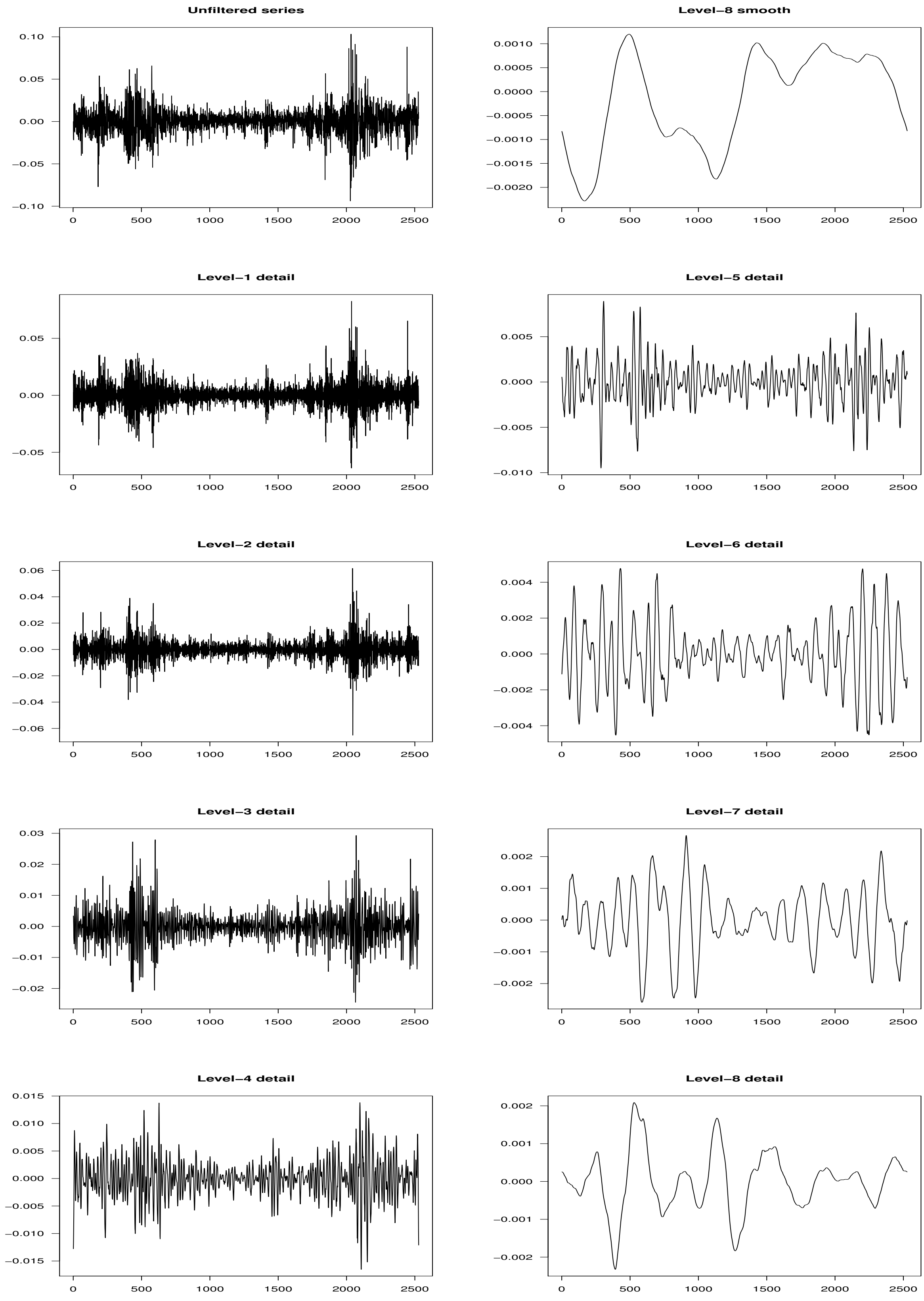

Figure 5.3 Wavelet decomposition of the SBF120 index

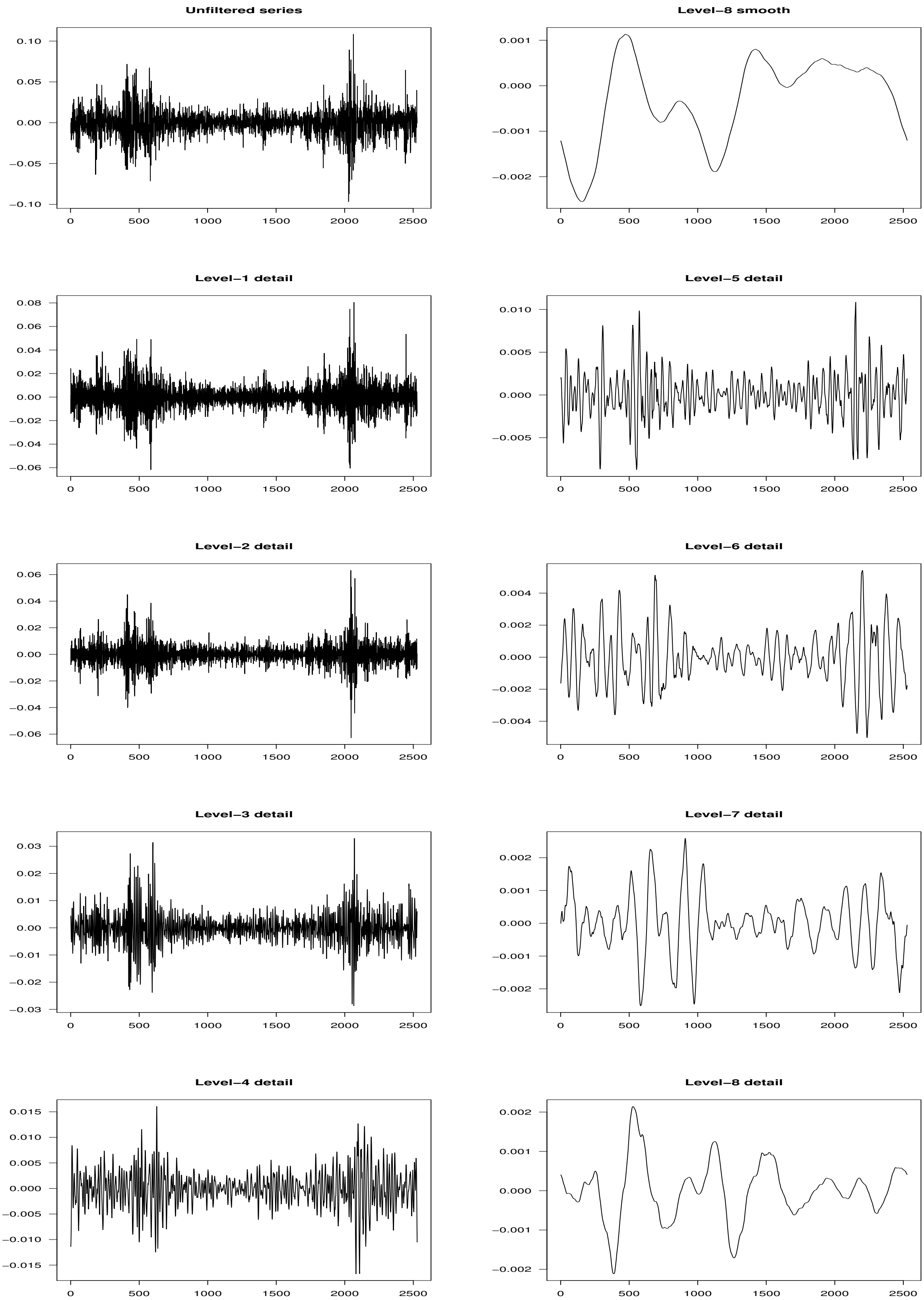

Figure 5.4 Wavelet decomposition of the weighted portfolio constructed from various stocks of firms

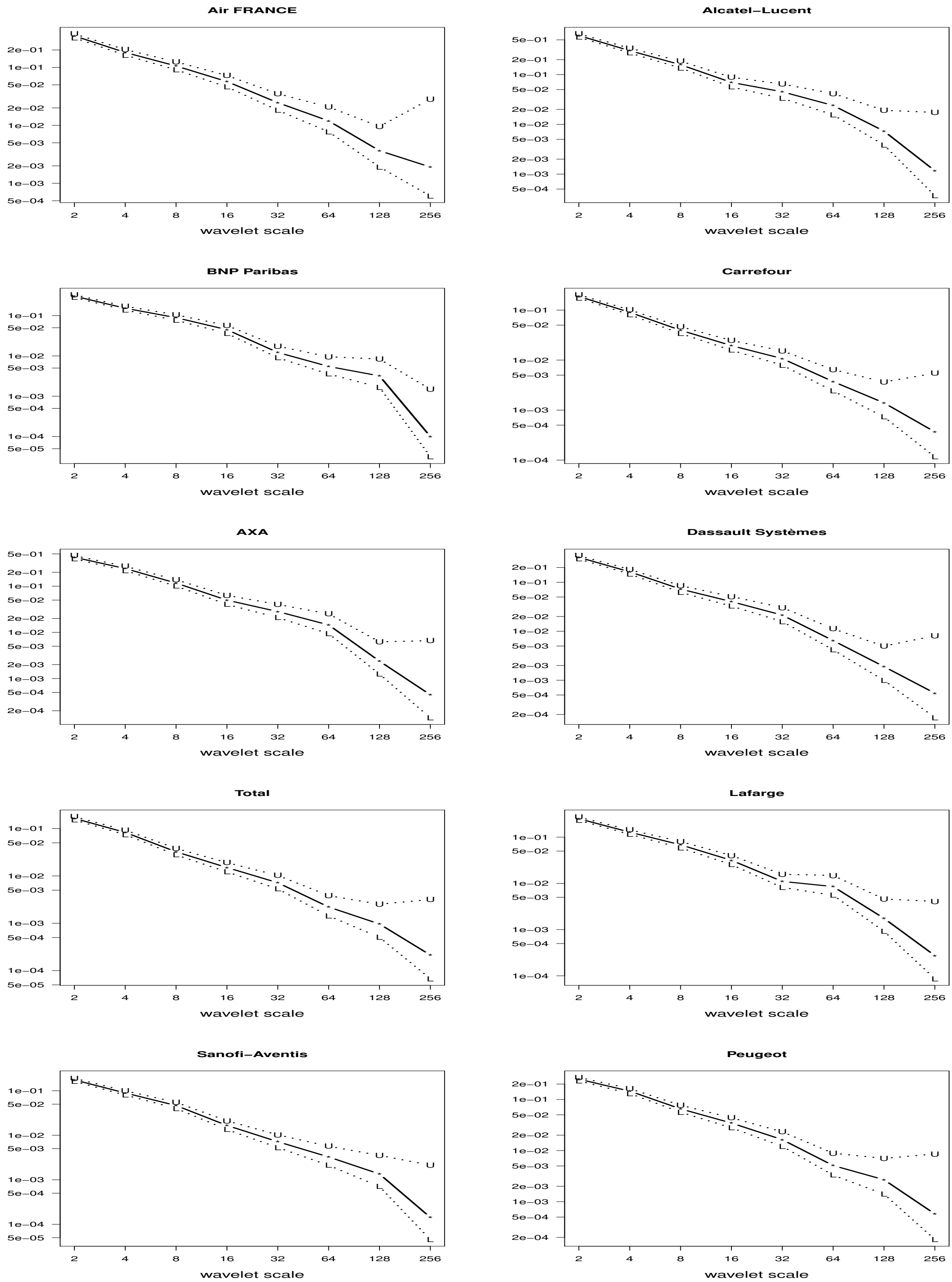

Figure 5.5 Estimated wavelet variance of changes in stock price returns. The MODWT-based wavelet variances have been constructed using the LA(8) wavelet filter. The continuous line indicates the wavelet variance of stock returns and the "U" and "L" dotted lines indicate the upper and lower bounds for 95% confidence interval.

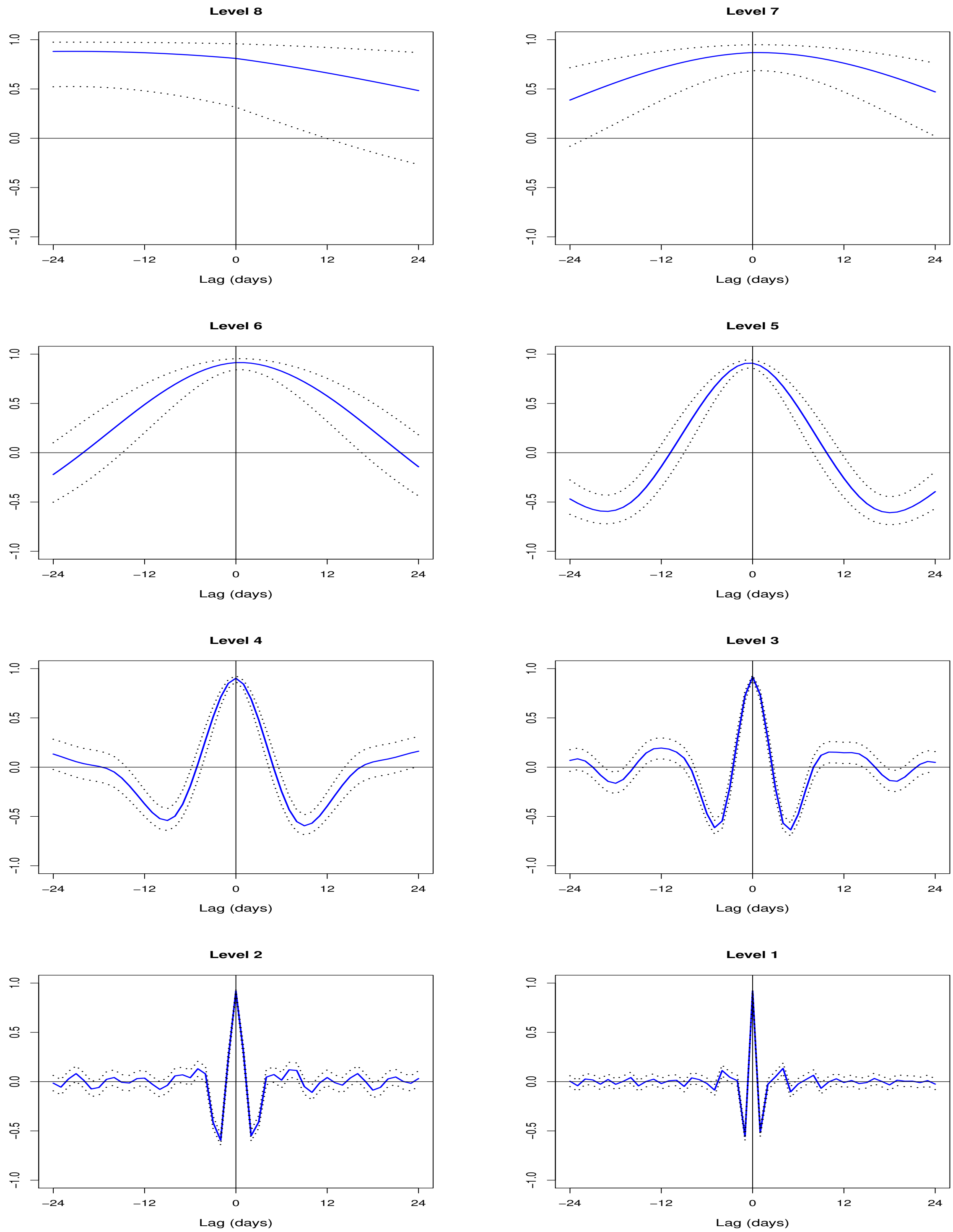

Figure 5.6 Wavelet cross-correlation between the SBF 120 and the Weighted portfolio returns. The individual cross-correlation functions correspond to wavelet scales $\lambda_1, \ldots, \lambda_8$ (i.e. the correlation coefficient of the value of portfolio returns at time t is plotted against the value of the stock market index returns at time $t-h$ and $t+h$ up to 24 days time lags). Diagrams show lags from -24 days to 24 days. The dotted lines bound the approximate 95% confidence interval for the wavelet cross-correlation.

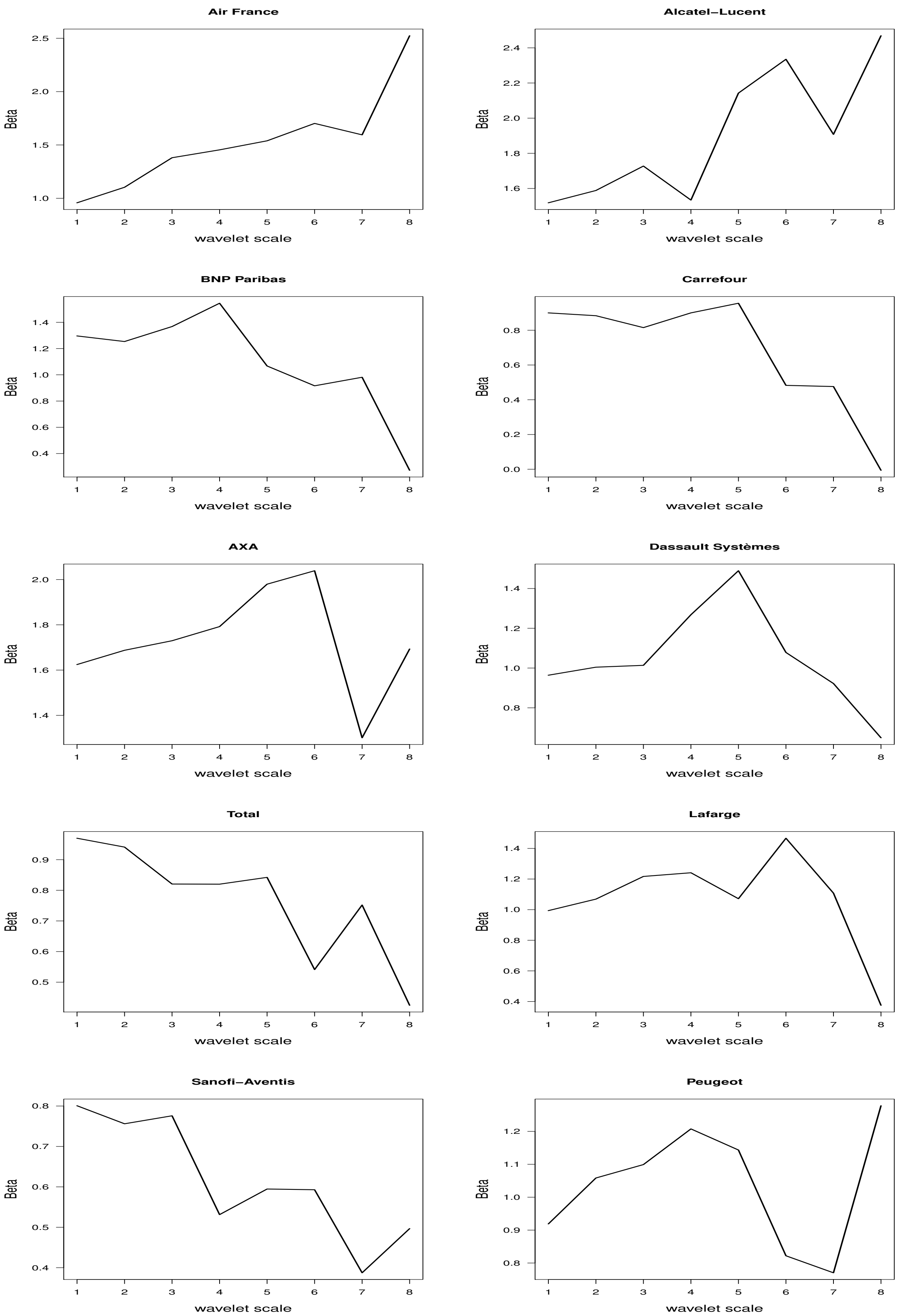

Figure 5.7 Betas of assets as a function of scale. The wavelet-Beta estimate for each stock i, at scale λ_j, was computed as $\hat{\beta}_i(\lambda_j) = \hat{\gamma}_{r_i r_m}(\lambda_j)/\hat{\sigma}^2_{r_m}(\lambda_j)$.

Table 5.5
Market model regression.

	Air France	Alcatel-Lucent	BNP Paribas	Carrefour	AXA	Dassault-Systèmes	Total	Lafarge	Sanofi-Aventis	Peugeot	Portfolio
Panel A: Unfiltered series											
Beta	1.104	1.597	1.294	0.877	1.668	1.013	0.923	1.059	0.756	0.997	0.956
R^2	0.373	0.469	0.619	0.491	0.719	0.365	0.632	0.503	0.380	0.430	0.842
Panel B: Beta											
Scale 1	0.957	1.519	1.297	0.900	1.624	0.965	0.969	0.994	0.800	0.919	0.954
Scale 2	1.099	1.596	1.254	0.879	1.678	1.011	0.939	1.061	0.748	1.058	0.961
Scale 3	1.378	1.723	1.365	0.813	1.722	1.017	0.823	1.211	0.777	1.091	0.980
Scale 4	1.464	1.519	1.535	0.900	1.782	1.231	0.824	1.212	0.548	1.185	0.922
Scale 5	1.551	2.112	1.064	0.927	1.943	1.435	0.842	1.134	0.554	1.163	0.994
Scale 6	1.708	2.260	9.310	0.590	1.952	1.201	0.534	1.275	0.505	0.888	0.865
Scale 7	1.756	2.236	1.117	0.535	1.630	0.978	0.667	0.905	0.403	0.807	0.793
Scale 8	2.021	2.993	1.757	0.437	1.556	1.445	0.586	1.681	0.131	1.425	0.892
Panel C: R^2											
Scale 1	0.306	0.443	0.647	0.506	0.730	0.338	0.671	0.463	0.427	0.397	0.847
Scale 2	0.399	0.497	0.615	0.505	0.699	0.368	0.633	0.522	0.374	0.459	0.840
Scale 3	0.501	0.528	0.591	0.468	0.717	0.395	0.581	0.602	0.363	0.508	0.841
Scale 4	0.439	0.386	0.608	0.471	0.738	0.442	0.515	0.536	0.200	0.473	0.812
Scale 5	0.504	0.507	0.473	0.408	0.705	0.501	0.511	0.539	0.195	0.408	0.796
Scale 6	0.679	0.630	0.472	0.274	0.781	0.533	0.395	0.606	0.260	0.411	0.856
Scale 7	0.636	0.513	0.298	0.205	0.607	0.399	0.477	0.400	0.122	0.190	0.805
Scale 8	0.739	0.635	0.757	0.239	0.789	0.457	0.574	0.602	0.018	0.521	0.894

5.2.2 VaR analysis

After analyzing market risk by estimating the Beta coefficients from regression in equation (4.4), we will measure the market risk of the ten stocks and the portfolio constructed from these stocks by computing the VaR at different time scales. The results are reported in table 5.6 and table 5.7. We computed the contribution to VaR (column 5 in table 5.7) of the portfolio constructed from the ten stocks using the measure:

$$\frac{\hat{\sigma}_m^2(\lambda_j)\left(\sum_{i=1}^N \kappa_i\hat{\beta}_i(\lambda_j)\right)^2+\sum_{i=1}^N \kappa_i^2\hat{\sigma}_{\varepsilon_i}^2(\lambda_j)}{\hat{\sigma}_m^2\left(\sum_{i=1}^N \kappa_i\hat{\beta}_i\right)^2+\sum_{i=1}^N \kappa_i^2\hat{\sigma}_{\varepsilon_i}^2} \tag{5.1}$$

$\hat{\sigma}_{\varepsilon_i}^2(\lambda_j)$ are given by

$$\hat{\sigma}_{\varepsilon_i}^2(\lambda_j)=\hat{\sigma}_i^2(\lambda_j)-\hat{\beta}_i^2(\lambda_j)\hat{\sigma}_m^2(\lambda_j),\ i=1,\ldots,N;\ j=1,\ldots,J \tag{5.2}$$

In Figure 5.8 we depicted Value at Risk of all stock returns and of the weighted diversified portfolio. We observed that all VaRs decrease monotocally from low scale (high frequency dynamics) to high scales (low frequency dynamics). This, indicates that market risk is concentrated at the lower scale of the data (short term dynamics).

6 Extension: Performance measurement

This section extends the analysis to the performance evaluation of a portfolio. There are more measures of performance such as *Sortino ratio* (Sortino and Meer (1991)), the *Treynor ratio* (Treynor (1961)), *Farinelli-Tibiletti ratio* (Tibiletti and Farinelli (2003)), *Sharpe ratio* (Sharpe (1964)) and others. In our analysis we focused in *Sharpe ratio*. This ratio is widely used and is the best known performance measure in the investment strategies. For comparison purposes, we considered both traditional *Sharpe ratio* and modified *Sharpe ratio*. The traditional approach is defined as follows: Suppose we have a portfolio with return $R_{portfolio}$ and observe a benchmark portfolio (market index) with a return $R_{benchmark}$, then the traditional *Sarpe ratio* is

$$SR_{Traditional}=\frac{R_{portfolio}-R_{benchmark}}{\sigma_{portfolio}} \tag{6.1}$$

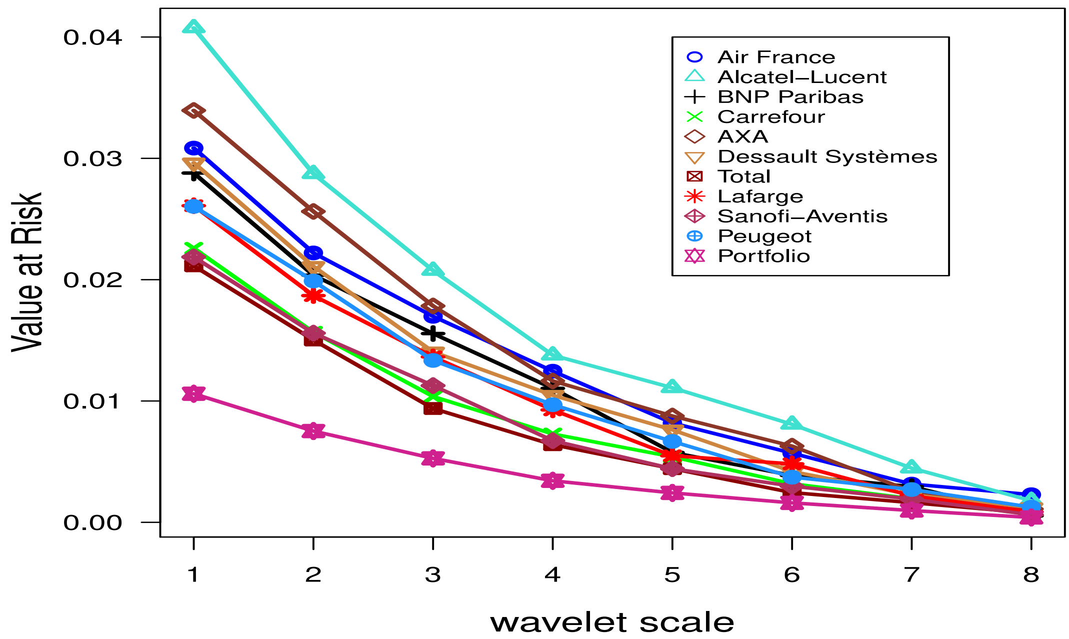

Figure 5.8 Individual VaR and diversified VaR (weighted portfolio constructed from ten stocks of firms) at different time scales. VaR were computed at the 95% significance level.

where $\sigma_{portfolio}$ is the standard deviation of portfolio over a sample period. The modified Sharpe ratio is given by

$$SR_{modified} = \frac{R_{portfolio} - R_{benchmark}}{VaR_{portfolio}} \tag{6.2}$$

where $VaR_{portfolio}$ is the VaR of a given portfolio.
Using wavelet analysis, we computed the *Sharpe ratio* at different time scales.
$SR_{modified}$ and $SR_{Traditional}$ are all negative, indicating that the portfolio constructed from various ten stocks underperformed the market index during scales.

7 Concluding remarks

Using wavelets we examined the dynamics of stock returns of 10 firms from Paris Stock Market during the period between December 29, 2000 and September 06, 2010. The MODWT was applied to decompose the daily returns into multiscale components. Firstly, we provide a multiscale decomposition of the variance and the correlation to identify the time-frequency properties of the stock returns. Wavelet variance analysis shows that all stocks exhibit a long memory behavior, i.e. we observed an approximate linear relationship between the wavelet variance and the wavelet scale, with a slow decreasing in the wavelet variance as the wavelet scale increases. Wavelet cross-correlation analysis is used to measure the degree of association (the dynamic linking) between stocks returns among scales. The results show that at low scales the relationship between stock returns is generally close to zero, while at high scales, the association become stronger. Our findings are consistent with some recent studies, such as, the study of Gallegati (2005) on examining the features of stock returns and aggregate economic activity, the studies of Kim and In (2005, 2007) on the relationship between stock returns and inflation and on the relationship between changes in stock prices and bond yields in the G7 countries. Second, using time-scale analysis of the market model, we estimated the beta coefficient which measures the systematic risk of an asset. We find that beta of all stocks is not stable over time, due to multi-trading startegies of investors. We further support our results by analyzing the impact of different time scales

Table 5.6
Value at Risk at different time scales.

	Air France		Alcatel-Lucent		BNP Paribas		Carrefour		AXA	
	VaR	Contribution to VaR (%)	VaR	Contribution to VaR (%)	VaR	Contribution to VaR (%)	VaR	Contribution to VaR (%)	VaR	Contribution to VaR (%)
Scale 1	0.0308	33.60	0.0407	49.38	0.0287	49.51	0.0225	52.75	0.0339	48.17
Scale 2	0.0222	24.43	0.0287	24.54	0.0203	24.78	0.0156	25.38	0.0256	27.44
Scale 3	0.0169	14.28	0.0207	12.81	0.0155	14.46	0.0103	11.12	0.0178	13.29
Scale 4	0.0124	7.69	0.0137	5.64	0.0110	7.28	0.0072	5.48	0.0116	5.67
Scale 5	0.0081	3.32	0.0110	3.65	0.0057	1.98	0.0053	3.01	0.0087	3.21
Scale 6	0.0057	1.61	0.0080	1.93	0.0038	0.89	0.0031	1.04	0.0062	1.64
Scale 7	0.0031	0.49	0.0044	0.59	0.0029	0.51	0.0019	0.39	0.0025	0.27
Scale 8	0.0022	0.25	0.0017	0.09	0.0005	0.01	0.0009	0.10	0.0010	0.05
Recomposed data	0.0449		0.0579		0.0409		0.0310		0.0489	
Raw data	0.0449		0.0579		0.0409		0.0310		0.0489	

	Dassault Systèmes		Total		Lafarge		Sanofi-Aventis		Peugeot	
	VaR	Contribution to VaR (%)	VaR	Contribution to VaR (%)	VaR	Contribution to VaR (%)	VaR	Contribution to VaR (%)	VaR	Contribution to VaR (%)
Scale 1	0.0296	50.63	0.0211	53.69	0.0260	49.49	0.0218	51.37	0.0260	47.48
Scale 2	0.0210	25.66	0.0150	27.13	0.0186	25.38	0.0155	26.12	0.0198	27.70
Scale 3	0.0140	11.38	0.0093	10.57	0.0136	13.51	0.0112	13.63	0.0133	12.46
Scale 4	0.0104	6.32	0.0063	4.91	0.0092	6.23	0.0067	4.87	0.0096	6.57
Scale 5	0.0076	3.35	0.0044	2.34	0.0054	2.18	0.0044	2.09	0.0066	3.11
Scale 6	0.0041	1.01	0.0024	0.72	0.0048	1.68	0.0029	0.95	0.0037	0.97
Scale 7	0.0022	0.29	0.0016	0.31	0.0021	0.34	0.0019	0.39	0.0027	0.51
Scale 8	0.0012	0.08	0.0007	0.06	0.0008	0.05	0.0006	0.04	0.0012	0.10
Recomposed data	0.0416		0.0288		0.0370		0.0305		0.0377	
Raw data	0.0416		0.0288		0.0370		0.0305		0.0377	

Notes: The contribution to VaR is computed as the ratio of stock returns variances. We computed the wavelet variance using equation (2.7) and we divided them by the total variance.

Table 5.7
VaR at different time scales for weighted portfolio and market index.

	SBF120 ($\alpha = 5\%$)		portfolio ($\alpha = 5\%$)	
	VaR	Contribution to VaR (%)	VaR	Contribution to VaR (%)
Scale 1	0.0178	51.61	0.0105	51.03
Scale 2	0.0127	26.25	0.0075	26.70
Scale 3	0.0087	12.36	0.0052	13.02
Scale 4	0.0055	5.03	0.0034	4.85
Scale 5	0.0037	2.23	0.0024	2.54
Scale 6	0.0026	1.17	0.0016	1.00
Scale 7	0.0014	0.35	0.0009	0.24
Scale 8	0.0005	0.05	0.0004	0.03
Recomposed data	0.0248		0.0259	
Raw data	0.0248		0.0259	

Notes: The VaR of SBF 120 stock market index is computed as $VaR(\lambda_j) = \sigma_m(\lambda_j)F^{-1}()$, where $\sigma_m(\lambda_j)$ is the standard deviation of stock market index return at scale λ_j and $F^{-1}()$ is the quantile of the inverse normal cumulative distribution. The VaR of the portfolio is computed using the equation (4.12). The contribution to VaR computed for SBF 120 is the ratio between wavelet variance and the total variance, and the contribution to VaR of the portfolio (column 5) is computed using the equation (5.1).

on VaR. The results showed that the risk (measured by VaR) is concentrated more at higher frequencies (lower scales). These results are consistent with the results of Gençay et al. (2005), He et al. (2009) and Masih et al. (2010).

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Table 6.1
Comparisons of the traditional and modified ratios.

	Modified Sharpe ratio	Traditional Sharpe ratio
Scale 1	-0.0140	-0.0011
Scale 2	-0.0139	-0.0022
Scale 3	-0.0199	-0.0046
Scale 4	-0.0306	-0.0124
Scale 5	-0.0433	-0.0236
Scale 6	-0.0655	-0.0598
Scale 7	-0.1082	-0.2412
Scale 8	-0.2608	-1.5540
Unfiltered series	-0.0057	-0.6017

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