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COOPETITIVE GAMES AND GLOBAL GREEN ECONOMY

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Abstract

The paper aims at providing a Game Theory model of cooptation which addresses the problem of the *global Green Economy*. The *Green Economy* is a theoretical model of sustainable development. This sustainable development model should lead to reduce emissions of greenhouse gases, determine the reduction of global pollution and the establishment of a sustainable and lasting *global Green Economy*, using mainly renewable resources.

The paper applies the notion of cooptation, originally devised at microeconomic level, at a country level. The country has to decide whether it wants to collaborate with the rest of the world in getting an *efficient Green Economy*, even if the country is competing in the global scenario.

The model provides a win-win solution, that shows the convenience for each country to participate actively to a program of *sustainability and efficient resource allocation* within a cooptative framework.

Keywords: cooptation, game theory, differentiable Pareto analysis, natural resources, green economy, macroeconomic and global interaction.

JEL Classification: B21, C7, C71, C72, C78, F19, Q50

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1 Introduction

A *Green Economy* is an economy based on sustainable development and a knowledge of ecological economics. Thus the *Green Economy* is one that results in improved human well-being and social equity, while significantly reducing environmental risks and ecological scarcities.

In this paper we apply the notion of cooptition, devised by Branderburger and Nalebuff (1995) (Stiles (2001)) in the field of strategic management, to the green economy. The notion of cooptition is a complex construct, since according to this concept, the economic agents (i.e. firms or countries) must seek to change the game and find a *win-win solution*, that indicates a situation in which each agent thinks about both cooperative and competitive ways to change the game. The *win-win solution* is therefore a situation in which each agent must cooperate and compete at the same time.

In the present work we apply a cooptitive model at a macroeconomic level to find appropriate solutions for a green economy, the notion of cooptition is then applied *at country level*, instead of microeconomic firm level. The country has to decide whether it wants to collaborate with the rest of the world in getting an *efficient Green Economy*, even if the country is competing in the global scenario.

Our model will provide a win-win solution, which is going to show the convenience for each country to participate actively to a *program of sustainability and efficient resource allocation* within a cooptitive framework.

The three main variables of our cooptitive model are:

- x representing the strategy of any country c ;
- y representing the strategy of the rest of the world w ;
- z representing the *cooptitive sustainability strategy*.

In this paper we suggest an original analytical model of cooptitive games applied at the global environment, with the aim to enrich the set of tools for environmental policies.

The paper aims at demonstrating the strategies that could bring to feasible solutions in a cooptitive perspective between a given country and the rest of the world, by offering a win-win outcome for both players and to establish a true efficient resource allocation for a Green Economy at a global level.

2 An Analytical Framework of Cooptitive Games

In this section we provide a general analytical framework of cooptition, a model of cooptitive game introduced by David Carfi in the last two years. This suggested analytical framework enables us to wide the set of possible solutions in a cooptitive context and it allows us “to share the pie fairly” in a win-win scenario. At the same time, it permits to examine the range of possible economic outcomes along a cooptitive

dynamic path. Finally, it limits the space within which the cooperative solutions can be determined.

2.1 *Coopetitive games*

The basic definition we propose of cooperative game is the following one, recently introduced by D. Carfi.

Definition (of cooperative game). Let E , F and C be three nonempty sets. We define *two players cooperative gain game carried by the strategic triple* (E, F, C) any pair of the form $G = (f, >)$, where f is a function from the Cartesian product $E \times F \times C$ into the real Euclidean plane and $>$ is the usual order of the Cartesian plane, defined, for every couple of points p, q , by $p > q$ iff $p_i > q_i$, for each index i .

Remark. The difference among a two person normal-form gain game and a two person cooperative game is simply the presence of the third strategy Cartesian-factor C .

Terminology and notation. Let $G = (f, >)$ be a two players cooperative gain game carried by the strategic triple (E, F, C) . We will use the following terminologies:

- the function f is called the *payoff function of the game* G ;
- the first component f_1 of the payoff function f is called the *payoff function of the first player* and analogously the second component f_2 is called the payoff function of the second player;
- the set E is said the *strategy set of the first player*, the set F the *strategy set of the second player*;
- the set C the *cooperative strategy set of the two players*.
- the Cartesian product $E \times F \times C$ is called the *cooperative strategy space of the game* G .

Memento. The first component f_1 of the payoff function f of a cooperative game G is the function of the strategy space of the game G into the real line defined by $f_1(x, y, z) = \text{pr}_1(f(x, y, z))$, analogously we proceed for the second component f_2 .

Interpretation. We have two players, each of them has a strategy set in which to choose his strategy; moreover, the two players can *cooperatively choose a strategy* z in a third set C . The two players will choose their cooperative strategy z to maximize (in some sense) the gain function f .

Bargaining solutions of a cooperative game. The payoff function of a two person cooperative game is (as in the case of normal-form game) a vector valued function with values belonging to the Cartesian plane \mathbf{R}^2 ; so that we should consider the maximal Pareto boundary of the payoff space $\text{im}(f)$ as an appropriate zone for the bargaining solutions.

The family of normal form games associated with a cooperative game. For any cooperative strategy z selected in the cooperative strategy space C there is a corresponding normal form game

$$G_z = (f_z, >)$$

upon the strategy pair (E, F) and with payoff function the section

$$f(\cdot, z) : E \times F \rightarrow \mathbf{R}^2,$$

of the payoff function f of the cooperative game (the section is defined, as usual, on the competitive strategy space $E \times F$ by

$$f(\cdot, z)(x) = f(x, z),$$

for every bi-strategy x in the bi-strategy space $E \times F$).

2.2 General solutions of a cooperative game

The two players should choose the cooperative strategy z in order that:

- the Nash equilibria of G_z are “better” than the Nash equilibria in each other game G_z ;
- the supremum of G_z is greater than the supremum of any other game G_z ;
- the Pareto maximal boundary of G_z is “higher” than that of any other game G_z ;
- the Nash bargaining solution is better in G_z than that in G_z ;

and so on, fixed a common kind of solutions, for any game G_z , say $S(z)$ the set of these kind of solutions, we can consider the problem to find the optimal solutions in set valued path S , defined on the cooperative strategy set C .

We note the fundamental circumstance that, in general, the above criteria are multi-criteria and so they generate multi-criteria optimization problems.

For the formal definitions of the basic kind of solutions see Carfi-Schilirò “A model of Cooperative game and the Greek crisis”.

Let us formalize the concept of normal-form game-family associated with a cooperative game.

Definition (the family associated with a cooperative game). Let $G = (f, >)$ be a two person cooperative gain game carried by the strategic triple (E, F, C) . We naturally can associate with the game a family of competitive games $G = (G_z)_{z \in C}$, which we will denote by the same symbol G and which we call the family of normal-form games associated with the cooperative game G .

Applicative remark. It is clear that with any family of normal form games $G = (G_z)_{z \in C}$ we can associate

- a family of payoff spaces $(\text{im}(f_z))_{z \in C}$,
- a family of Pareto maximal boundary $(\text{bd}^*G_z)_{z \in C}$;
- a family of suprema $(\text{sup } G_z)_{z \in C}$;

and so on.

And we can interpret any of the above families as set-valued paths in the strategy space $E \times F$.

It is just the study of these induced families which becomes of great interest in the study of a cooperative game G .

2.3 A model of cooperative games

The cooperative model we propose hereunder must be interpreted as normative models, in the sense that it will show the more appropriate solutions of a win-win strategy chosen within a cooperative perspective.

The main variables of the two models are:

strategies x of a certain country C (*the investment in agricultural and food production*), which directly influence both pay-off function;

strategies y of the rest of the world (*the investment in agricultural and food production*) which increase both pay-off function;

a shared strategy z which is determined together by the two countries, c and the rest of the world w : z is the level of investment for environmental and natural resources.

Therefore, in the model we assume that c and w define the set of cooperative strategies.

3 A sustainable cooperative model of economy

Main Strategic assumptions. We assume that any real number x , in the canonical unit interval $U = [0,1]$, is a possible investment of the country c in agricultural and food production and any real number y , in the same unit interval U , is the analogous investment of the rest of the world w . Moreover, a real number z , again in U , is the total investment of c and w for sustainability of natural resources and for the environmental protection. Let us assume that the country c and the rest of the world w contribute, for the common investment z , according to the pair of percentages (q, r) , in such a way that we have $z = qz + rz$.

We also consider, as payoff functions of c and w , two Cournot type payoff functions.

Payoff function of c

We assume that the payoff function of the country c is the function f_1 of the unit cube U^3 into the real line, defined by

$$f_1(x, y, z) = x(1 - x - y) + mz,$$

for every triple (x, y, z) in the cube U^3 , where m is a characteristic positive real number depending upon the country c .

Payoff function of w

We assume that the payoff function of w is the function f_2 of the cube U^3 into the real line, defined by

$$f_2(x, y, z) = y(1 - x - y) + nz,$$

for every triple (x, y, z) in the cube U^3 , where n is a characteristic positive real number depending upon w .

Payoff function of the cooperative game

We so have build up a cooperative gain game $G = (f, >)$ with payoff function f given by

$$\begin{aligned} f(x, y, z) &= (x(1 - x - y) + mz, y(1 - x - y) + nz) \\ &= (x(1 - x - y), y(1 - x - y)) + z(m, n), \end{aligned}$$

for every triple (x, y, z) in the cube U^3 .

4. Study of the game $G = (p, >)$.

Note that, fixed a cooperative strategy z in U , the game $G(z) = (p(z), >)$ with payoff function $p(z)$, defined on the square U^2 by

$$p(z)(x, y) = f(x, y, z),$$

is the translation of the game $G(0)$ by the vector $v(z) = z(m, n)$, so that we can study the game $G(0)$ and then we can translate the various information of the game $G(0)$ by the vector $v(z)$.

So let us consider the game $G(0)$. The last game G_0 has been studied completely by D. Carfi in *Topics in Game Theory*, Gabbiano 2011. The conservative part in payoff space (the part of the payoff space greater than the conservative bi-value $(0,0)$) is the canonical 2-simplex T , convex envelope of the origin and of the canonical basis e of the Euclidean plane \mathbf{R}^2 .

Payoff space and Pareto Boundary of the payoff space of $G(z)$

The Pareto boundary of the payoff space of $G(z)$ is the segment $[e_1, e_2]$, with end points the two canonical vectors of the plane \mathbf{R}^2 , translated by the vector $v(z) = z(m, n)$.

The payoff space of the cooperative game G , the image of the payoff function f , is the union of the family of payoff spaces

$$(\text{im } p(z))_{z \in C},$$

that is the convex envelope of the of points $0, e_1, e_2$, and of their translations by the vector $v(1) = (m, n)$.

The Pareto maximal boundary of the payoff space $f(S)$ of the cooperative game G is the segment $[P', Q']$, where the point P' is the translation $e_1 + v(1)$ and the point Q' is the point $e_2 + v(1)$.

5 Solutions of the model and conclusions

1) ***Properly cooperative solution.*** In a purely cooperative fashion, the solution of the game in the payoff space is the translation of the Nash payoff $(1/9, 1/9)$ by the vector (m, n) ; that is, in the strategic cube S the solution $(1/3, 1/3, 1)$. This solution is obtained by cooperating on the set C and competing *a la Nash* in the game $G(1)$.

2) The Nash bargaining solution and the Kalai-Smorodinsky bargaining solution, with respect to the infimum of the Pareto boundary, coincide with the medium point M of the segment $[P', Q']$. This point M represents a win-win solution with respect to the initial (shadow maximum) supremum $(1, 1)$ iff m and n are greater than 1.

3) A best Pareto fair division is the division according to the pair (p, q) i.e. the point

$$pP' + qQ',$$

which is nothing but the translation $(p, q) + v(1)$. Note that, as before, we have

$$(p+m, q+n) > (1,1),$$

if m and n are greater than 1.

4) Since the Pareto boundary is a segment of straight line with slope -1, the Pareto transferable utility boundary of the game G contains the Pareto boundary itself. So that a natural cooperative solution is again the Pareto fair division according to the pair (p,q) .

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