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Skill Investment, Farm Size Distribution and Agricultural Productivity

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Abstract

Cross-country differences in agricultural productivity dwarf those in the aggregate. This paper presents a theory in which low aggregate productivity distorts skill formation of farmers. A model in the style of [Lucas \(1978\)](#) is extended to allow skill accumulation. Differences in skill magnify the differences in agricultural productivity. The model is calibrated to the US to reproduce the size distribution of farms and the time allocation of farmers. Given exogenous differences in total factor productivity (TFP), the model explains 45-50% of the differences in agricultural output per worker. Moreover, differences in farmer's skill accounts for about 30% of the variation in agricultural productivity. The model also maps the latent distribution of skill to a size distribution of farms, which is observable. The model is able to resemble salient features of the cross-country distribution of farm size.

Keywords: Agricultural productivity, skill investment, farm size distribution, income differences.

1. Introduction

Cross country differences in agricultural productivity are much larger than the differences in aggregate income per worker. This feature is first noted in [Kuznets \(1971\)](#). For a larger set of countries, [Caselli \(2005\)](#) documents that between the 90th percentile country and the

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10th percentile country of the world income distribution, the ratio of PPP output per worker in agriculture exceeds a factor of 45, compared to 22 in GDP per worker and merely 5 in non-agricultural output per worker. Given that agriculture typically employs most of the labor force in low income countries, understanding why agriculture is so unproductive in these countries is clearly of first order importance.

This paper presents a theory in which unmeasured skill of farmers plays an important role in agricultural production. I construct a model featuring an agricultural sector displaying decreasing returns to scale, and a non-agricultural sector displaying constant returns to scale. Skill is modeled as the ability to manage a farm, in the style of [Lucas \(1978\)](#). Given the same amount of input, a more skilled individual is able to produce more agricultural output. Hence agricultural productivity critically depends on the skill component of individuals who choose to become farmers and produce in agriculture. Such division of labor across occupations arises endogenously in equilibrium. Moreover, it is shown to vary with the level of aggregate productivity.

A key element of this paper is to allow skill to grow over time. This feature is introduced to capture productivity growth over operator's life cycle, as observed in farm level data in the US. Detailed description of the data is postponed until section 2. While the data suggest that older farmers can be as much as three times more productive than their younger peers, such productivity difference is not assumed to be a mere byproduct of working and aging. Instead, productivity growth is the result of explicit skill-enhancing investment committed earlier in their life. In fact, younger farm operators in the US tend to spend more time on non-farm activities, relative to their older peers. Consistent with these empirical observations, a skill accumulation technology is posited with time as an input. Individuals choose the profile of investment over their life cycle, in addition to the selection into different occupations.

The key result is that economies with low TFP comprise of a large pool of low skill farmers. In economies with high TFP, farmers are rare but possess high skill. This is true

despite the fact that economies differ only in their levels of TFP, which is sector-neutral . Moreover, the differences in the skill composition of farmers arise from two distinct margins. Through the extensive margin, the subsistence need of agricultural good dictates that more people, even those with low skill, produce in agriculture. This margin is similar to the one emphasized in [Lagakos and Waugh \(2010\)](#). The authors show that subsistence need induces less specialization in low TFP countries, and offers a neat explanation of cross-country productivity differences at the sector level. The intensive margin operates through on-the-job skill investment. Farmers in low productivity economies invest less in skill due to high financing cost - in the form of high equilibrium interest rate. In addition to a larger pool of farmers, each farmer faces a flatter life-cycle skill profile in these economies. Skill accumulation hence offers an extra channel through which TFP affects the skill of farmers, and is a key innovation of this paper.

The intensive margin is particularly interesting. In a standard human capital model with only time input in the human capital technology, it is generally the case the differences in levels of TFP have no impact on the optimal allocation of time. Recent advances in the literature stress the key role of resources input in the human capital technology. [Manuelli and Seshadri \(2005\)](#), [Cordoba and Ripoll \(2007\)](#), and [Erosa, Koreshkova, and Restuccia \(2010\)](#) are excellent examples along this line. In these papers, optimal investment bundle varies with levels of TFP, and offers an additional source of differences in human capital. In the current paper, resources input is absent in the skill accumulation technology. Nonetheless, optimal time investment varies with levels of TFP. This is because low TFP is accompanied by high equilibrium interest rate, which renders skill investment less profitable.

When fed exogenously the level of TFP and land endowment, the model has predictions about the allocation of labor across sectors, time allocation of farmers between investment and production, and output per worker in agriculture and non-agriculture. In equilibrium, the model also yields a non-degenerate size distribution of farms. This is particularly useful

because there is a unique mapping between the distribution of farmer's skill and the size distribution of farmers. Given the latent nature of farmer's skill, the observable size distribution of farms serves as a disciplining device on model's prediction about cross-country differences in skill of farmers.

To be used to explain cross-country differences in agricultural productivity, the model is calibrated to the US as a benchmark economy. In particular, model parameters are chosen such that the model reproduces the size distribution of farms, time allocation of farmers, and other macroeconomic statistics in the US. By varying country-specific TFP to reproduce PPP output per worker in non-agriculture, the model is able to explain 45-50% of the variation in agricultural output per worker across countries. While cross-country difference in agricultural productivity is mostly a story of TFP difference - difference in TFP accounts for 50% of the productivity difference - the importance of skill is too large to be ignored. The differences in skill of farmers are found to account for 30% of the differences in agricultural productivity. Further decomposition reveals that most of the variation in skill stems from the extensive margin. The intensive margin accounts for less than 5% of the variation in agricultural productivity across countries.

Consistent with the prediction that farmers possess low skill in low TFP countries, the model predicts that farms are much smaller in size in developing countries. This prediction accords well with data on the size distribution of farms across countries. Using data from the World Census of Agriculture, the correlation between PPP output per worker in agriculture and mean farm size is 0.45. In the model, it is 0.6. The model also successfully captures the fact that farms in low income countries are predominantly small. For the poorest 5% countries, for example, the share of farms with less than 5 hectares of land is 89% in the data. The model predicts a share of 87%, which is almost identical to that in the data. For some individual countries, the endogenous size distribution of farms almost perfectly matches their empirical counterpart.

On the prediction of cross-country size distribution of farms, this paper is related to [Adamopoulos and Restuccia \(2011\)](#). Both papers use a span-of-control framework to produce a nondegenerate size distribution and aim at explaining international productivity difference in agriculture. However, there are subtle yet important differences. Firstly, [Adamopoulos and Restuccia \(2011\)](#) offers an interesting application of the span-of-control framework. Unlike in this paper, they do not consider the allocation of labor between occupations. Such specification allows them to isolate effect of idiosyncratic policy distortions on total output in agriculture. As a result, the origins of productivity differences are different between these two papers. In their paper, productivity is low in agriculture because the most productive farms are not operating at the optimal scale, due to distortions. In the current paper, low productivity is due to a large share of unproductive farmers who do not invest to enhance their skill. All farmers, productive or not, are producing at their optimal scale. Secondly, these two papers also differ in their implications of the farm size distribution. In [Adamopoulos and Restuccia \(2011\)](#), the size distribution of farms is used to infer the distribution of idiosyncratic policy distortions. In this paper, the size distribution is a mapping from the distribution of farmer's skill.

This paper fits into an expanding literature that emphasizes the key role of agriculture in understanding cross-country productivity differences. [Gollin, Parente, and Rogerson \(2004\)](#) show that in a model with home production, investment distortions reduce *measured* productivity much more in agriculture, relative to non-agriculture. [Restuccia, Yang, and Zhu \(2008\)](#) argue convincingly that barriers to intermediate inputs in agriculture can substantially reduce productivity. High transportation cost is shown to adversely impact productivity in [Adamopoulos \(2006\)](#) and [Gollin and Rogerson \(2010\)](#). Unlike these papers that focus on specific distortions, this paper considers an environment in which countries only differ in sector-neutral TFP, and all allocations are efficient.

The remaining of the paper is organized as follows. Section 2 presents a set of facts

that motivate this paper. Section 3 describes the economic environment. Section 4 defines a competitive equilibrium, elaborates on equilibrium conditions, and proves some results. Section 5 presents the calibration strategies and the main results. Section 6 concludes. All proofs are relegated to the Appendix.

2. Facts

The main source of data is Census of Agriculture of various years in the US. The unit of observation is the average of all farms under the management of operators in a particular age group. First, I show that there is substantial productivity variation across operators of different ages. Two measures of productivity are constructed, and presented in Table 1. In both measures, the productivity of the youngest operator is normalized to unity. Measure I is output (net of government transfer) per operator. With measure I, older farmers are significantly more productive than their younger peers. For example, an operator aged 35-44 produces three times more output than an operator aged less than 25. While it is true that older farmers tend to have more resources under their management, it turns out that observed productivity difference can not be completely accounted for by differences in factors of production. I consider four factors of production: intermediate inputs, land, capital and hired labor. Intermediate input include feed, seed, chemical and fertilizers. The elasticities of each factor of production is read from Table 5 and 6 in [Herrendorf and Valentinyi \(2008\)](#). Then productivity is computed as a residual and reported as Measure II in the last row of Table 1.² An operator aged 35-44 remains 42% more productive than an operator aged less than 25. In summary, both measures of productivity suggest substantially variation across operators of different ages.

Perhaps not surprising, older farmers also operate a larger farm. The average size of

²Solow residual of operator i is computed as $\frac{y_i}{k_i^{\alpha_k} \cdot x_i^{\alpha_x} \cdot \ell_i^{\alpha_\ell} \cdot h_i^{\alpha_h}}$, where y is output per operator. Factors of production $j = k, x, \ell, h$ are capital, intermediate, land and labor. α_j are the respective elasticity.

Age	< 25	25 – 34	35-44	45-54	55-64
Measure I	1	2.02	3.00	2.88	1.96
Measure II	1	1.26	1.42	1.37	1.21

Table 1: Productivity Profile of Farm Operators

farms under the management of operators aged under 25 is 324 acres, which is about 1/3 the mean size of farms managed by operators aged 35-44. If one computes output per worker by size of a farm, it is likely that a larger farm will have higher measured productivity than a smaller one. In fact, this observation has been highlighted in [Adamopoulos and Restuccia \(2011\)](#) for the US, and in [Cornia \(1985\)](#) for a larger set of developing countries.

It is also possible that the observed cross-section productivity differences reflect differences in education attainment of operators. Unfortunately, education attainment is not reported in the census of agriculture. Instead, I offer indirect evidence suggesting that education attainment is not likely to be the story behind these productivity differences. I compute operator’s productivity using measure I with data from 1997 and 2002 Census of Agriculture. The results are reported in Table 2. For the moment, assume that the productivity difference between operators aged 25-34 and those aged 35-44 are driven solely by education attainment. Then in the 2002 cross section, one should expect to observe the same order of difference in productivity between operators aged 35-44 and those aged 45-54, because education attainment is fixed over time. However, We don’t observe that. Similar calculations for other age groups deliver similar results. Although this calculation is simple in nature, and in particular ignores the entry of farmer (who might have different education than the incumbents) over time, it does suggest that the difference in level of education is not likely to account for, at least not completely, the difference in productivity.

Age	< 25	25 – 34	35-44	45-54	55-64
2007	1	2.02	3.00	2.88	1.96
2002	1	2.04	2.39	2.52	1.86
1997	1	1.66	2.18	2.19	1.54

Table 2: Operator Productivity Profile: Various Census Years

3. Model

3.1. Environment

Each period a continuum of measure one individuals are born, and live for T periods. Individuals of the same cohort constitute a household, with all decisions made by a hypothetical household head. Two goods are produced each period. One is called agricultural good, denoted by c_a . The other is called non-agricultural good, denoted by c_n . The representative household derives utility from consumption of two goods

$$U(c_a, c_n) = \eta \cdot \log(c_a - \bar{a}) + (1 - \eta) \cdot \log(c_n)$$

Parameter \bar{a} is the subsistence in agricultural consumption. $\bar{a} > 0$ implies an income elasticity of agricultural consumption less than unity.

Each member is endowed with one unit of physical time. The fixed stock of land, denoted by \bar{L} , is equally owned by households. There is no population growth or lifetime uncertainty. Total measure of population at any point in time is T .

3.2. Skill Accumulation

When born, individuals within a household draw independently their skill type, $z \in \mathfrak{R}^+$, from a known, time invariant distribution $G(z)$. The initial draw is fixed for any individual. Throughout the paper, an individual with initial draw z is referred to as type z . While skill type is exogenously drawn, the level of skill can grow over time through investment according

to the following technology

$$z_{t+1} = z_t + z_t \cdot s_t^\theta, \quad s_t \in [0, 1] \quad (1)$$

where s_t is the fraction of time committed to skill enhancing activities. It is not difficult to map Equation (1) to farming in the real world. To improve skill, a farmer has to spend time on various tasks including, but not limited to, experimenting with different seeds/crops/fertilizers, updating on the most recent available technologies, learning new equipments etc. While other skill-enhancing tasks may require input other than time - e.g., purchasing a new computer, such possibilities are ruled out here for several reasons. First, it allows for closed-form solutions and clearer expositions. Second, data on time allocations of farm operators are available to discipline relevant parameters. Lastly, data on resources investment by farm operators in skill accumulation are limited, if available at all.

3.3. Technology, Occupation, and Household Optimization

Conditional on skill type, the household head decides whether a member should be a worker or a farmer. This division of labor is assumed to be fixed over one's life. This restriction is innocuous in a steady state. All workers, regardless of skill type, supply labor inelastically and earn the same wage rate. A farmer produces agricultural goods using a technology displaying decreasing returns to scale, and retains profit as rent. The technology is standard Cobb-Douglas

$$y_a = A \cdot (z(1 - s))^{1-\gamma} (h_a^\alpha \cdot \ell^{1-\alpha})^\gamma$$

where h_a, ℓ denote labor and land. Note that the amount of skill used in production is a product of skill level z and time allocated to production $(1-s)$. Variable A denotes the total factor productivity in this economy.

There are competitive rental markets of labor and land at prices w and q , and farm output are sold in competitive markets at price p . All prices are expressed relative to the price of nonagricultural output. A farmer solves

$$\max_{\{h_a, \ell\}} p \cdot y_a - w \cdot h_a - q \cdot \ell$$

For later reference, denote $h_a(z), \ell(z)$ the optimal demand of labor and land. The profit, denoted by $\pi(z)$, is retained by the farmer as rent. It is straightforward to show that profit is linear in the skill input, i.e.,

$$\pi(z, s) = z(1 - s) \cdot (1 - \gamma) \cdot (p \cdot A)^{\frac{1}{1-\gamma}} \left(\gamma \left(\frac{\alpha}{w} \right)^\alpha \left(\frac{1 - \alpha}{q} \right)^{1-\alpha} \right)^{\frac{\gamma}{1-\gamma}}$$

Let Y_t denote household income in period t , given the division of labor and the allocation of time between skill accumulation and production. The household head then solves the standard consumption-saving problem to maximize discounted utility

$$\begin{aligned} \max_{\{c_{at}, c_{nt}\}} & : \sum_{t=1}^T \beta^t U(c_{at}, c_{nt}) \\ \text{s.t.} & : p_t c_{at} + c_{nt} + a_{t+1} = a_t R_t + Y_t \end{aligned}$$

where R_t denotes the return on asset holdings a_t .

3.4. Non-agriculture

There is a representative firm that produces nonagricultural output with a linear technology $Y_n = A \cdot H_n$. In this technology, H_n denotes labor hours and does not embed skills.³

³Alternatively, the technology can be specified as $y = A \cdot (zh_n)$. In this case, the mapping between TFP in the model and non-agricultural output per worker in the data is lost.

The representative firm solves

$$\max_{\{H_n\}} A \cdot H_n - w \cdot H_n$$

4. Equilibrium

This economy admits a stationary equilibrium, in which prices and the division of labor are constant over time. A formal definition is given below.

A stationary competitive equilibrium is a collection of prices (w, p, q, R) , consumption and investment allocations $(c_{at}, c_{nt}, s_t, a_t)_{t=1}^T$, factor demand $h_a(z), \ell(z), H_n$ such that: (1) given prices, $(c_{at}, c_{nt}, s_t, a_t)_{t=1}^T$ solve household maximization problem; (2) given prices, $h_a(z), \ell(z)$ solve farmer's profit maximization, and H_n solve non-agricultural firm's profit maximization; (3) prices are competitive; (4) all markets clear.

In equilibrium, workers work full time because the return to skill investment is zero. The discounted lifetime income of a worker is simply $Y_w = \sum_{t=1}^T \{w \cdot R^{1-t}\}$. Since farmer's rent is strictly increasing in skill input, it is always profitable for a farmer to invest non-trivial amount of time to accumulate skill. Moreover, optimal time investment of farmers is independent of skill type. Lemma 1 states this result formally.

Lemma 1. *Optimal time investment is independent of initial skill draw*

Lemma 1 implies a common slope of skill profile for all farmers, and the level is determined by the initial draw. It is convenient to define variable x_t as follows

$$x_t = \begin{cases} 1, & t = 1 \\ x_{t-1} \cdot (1 + s_{t-1}^\theta), & t = 2, \dots, T \end{cases}$$

$\{x_t\}_{t=1}^T$ summarize the level of skill at time t relative to the initial draw. Clearly, $\{x_t\}$ is independent of skill type. This allows a simple expression of lifetime discounted income of a

type z farmer

$$Y_f(z) = \pi(z) \cdot \sum_{t=1}^T \{x_t \cdot (1 - s_t) \cdot R^{1-t}\}$$

Note that $Y_f(z)$ is linear and strictly increasing in skill type z . Recall that discounted lifetime income of a worker (Y_w) is independent of skill type z . This leads to Lemma 2.

Lemma 2. *There exists a cut-off level of skill type, \bar{z} , such that household members with skill type $z < \bar{z}$ become workers, and household members with skill type $z > \bar{z}$ become farmers.*

The most able members will manage farms and utilize their skills. The less able members will supply inelastically one unit of labor to the market, and forgo their endowed skills. The marginal farmer, whose skill type is \bar{z} , is indifferent between two occupations. Income of a household consists of labor income, farm profit, and rental income from land. The present value of household income is simply

$$Y = G(\bar{z}) \cdot Y_w + \int_{\bar{z}} Y_f(z) dG(z) + q \cdot \bar{L}/T \cdot \sum_{t=1}^T R^{1-t}$$

To solve the model, I begin by solving for prices (p, q). Equation (2) below states the indifference condition of the marginal farmer. Equation (3) below states the land market clearing condition, which utilizes the fact that land demand is linear in skill input.

$$\pi(\bar{z}) \cdot \sum_{t=1}^T \{x_t \cdot (1 - s_t) \cdot R^{1-t}\} = \sum_{t=1}^T \{w \cdot R^{1-t}\} \quad (2)$$

$$\int_{\bar{z}} \ell(z) dG(z) \cdot \sum_{t=1}^T \{x_t \cdot (1 - s_t)\} = \bar{L} \quad (3)$$

Dividing (2) by (3) yields an expression for the rental price of land

$$q = \left[\frac{\sum_{t=1}^T \{x_t \cdot (1 - s_t)\}}{\sum_{t=1}^T \{x_t \cdot (1 - s_t) \cdot R^{1-t}\}} \right] \cdot \left[\frac{\gamma \cdot (1 - \alpha) \cdot \left(\sum_{t=1}^T \{w \cdot R^{1-t}\} \right)}{(1 - \gamma) \cdot \bar{L}} \right] \cdot \frac{\int_{\bar{z}} z dG(z)}{\bar{z}} \quad (4)$$

Substituting (4) into (3) yields the relative price of agricultural good

$$p = \left[\frac{\sum_{t=1}^T \{w \cdot R^{1-t}\}}{\bar{z} \cdot (1 - \gamma) \cdot \sum_{t=1}^T \{x_t \cdot (1 - s_t) \cdot R^{1-t}\}} \right]^{1-\gamma} \cdot \left(\gamma \left(\frac{\alpha}{w} \right)^\alpha \left(\frac{1 - \alpha}{q} \right)^{1-\alpha} \right)^{-\gamma} \cdot \frac{1}{A} \quad (5)$$

Note the relative price of agricultural good is strictly decreasing in TFP. Agricultural good is more expensive relative to non-agricultural good in countries with low productivity. Solving for optimal consumption bundles and aggregating over generations yields the aggregate demand of two consumption goods

$$C_a = \sum_{t=1}^T c_{at} = \left[\sum_{t=1}^T (\beta R)^{t-1} \right] \cdot \left[\frac{Y - p \cdot \bar{a} \sum_{t=1}^T R^{1-t}}{\sum_{t=1}^T \beta^{t-1}} \right] \cdot \frac{\eta}{p} + T \cdot \bar{a} \quad (6)$$

$$C_n = \sum_{t=1}^T c_{nt} = \left[\sum_{t=1}^T (\beta R)^{t-1} \right] \cdot \left[\frac{Y - p \cdot \bar{a} \sum_{t=1}^T R^{1-t}}{\sum_{t=1}^T \beta^{t-1}} \right] \cdot (1 - \eta) \quad (7)$$

In each household, the measure of workers is $G(\bar{z})$. Given constant prices, the division of labor does not change across cohorts. Hence the total measure of worker in the economy is simply $T \cdot G(\bar{z})$. The measure of workers demanded in agricultural production is obtained by first integrating over farmers within a household, and then summing over generations

$$H_a = \left[\sum_{t=1}^T x_t (1 - s_t) \right] \cdot \int_{\bar{z}} h_a(z) dG(z)$$

Similarly, aggregate agricultural output is given by

$$Y_a = \left[\sum_{t=1}^T x_t (1 - s_t) \right] \cdot \int_{\bar{z}} y_a(z) dG(z)$$

Imposing labor market clearing, the measure of workers in the nonagricultural sector is $H_n = T \cdot G(\bar{z}) - H_a$. The output in the nonagricultural sector is $Y_n = A \cdot H_n$. Goods market clearing conditions require $C_a = Y_a, C_n = Y_n$. Loan market clears by Walras' law.

Finally nonagricultural firm's optimization implies $w = A$. Hence the two goods market clearing conditions constitute two equations with two unknowns (\bar{z}, R) that can be solved numerically. Once the cut-off skill and interest rates are known, rest of the equilibrium variables can be recovered easily. Proposition 1 states an important result.

Proposition 1. *Economy with lower TFP has a lower cut-off skill level, and a higher interest rate.*

Proposition 1 states that low aggregate productivity adversely impacts the productivity of farmers through both the extensive margin and the intensive margin. On the one hand, low TFP implies a larger pool of farmers, as suggested by the lower threshold skill level. Moreover, the new farmers are less productive because they are of lower skill types. On the other hand, financing skill investment is more costly in low TFP economy. High interest rate renders increase in future income less attractive. As a result, the skill profile is less steep. Both margins lead to lower average productivity of farmers, which translates into lower measured labor productivity and smaller scale.

5. Quantitative Analysis

5.1. Calibration

In this section the model is parameterized to answer quantitative questions. Model period is 10-years. Individuals are born at the age of 25 and live for 5 periods. Some model parameters are either standard or can be inferred without solving the model. Discount rate β is set at $(0.96)^{10}$. For the US, TFP is normalized to be 1. \bar{L} is assigned a value of 1.62, which is the arable land (in hectares) per worker in the US. The span-of-control parameter γ is approximated by the return to farm operators. Over the period 1980-1999, the average share of agricultural output accruing to farm operators is 40%, implying $\gamma = 0.6$. This paper is certainly not the first one to estimate the span-of-control parameter. However, existing works either focus on the aggregate economy as in [Guner, Ventura, and Yi \(2008\)](#)

and Restuccia and Rogerson (2008) or the manufacturing sector as in Atkeson and Kehoe (2005) and Gollin (2008). The value of the span-of-control parameters from these studies range from 0.8 to 0.9. A value of 0.6 for the agricultural sector is thus a conservative choice as a higher γ tends to strengthen the quantitative results. Elasticity parameter α is chosen such that labor share is consistent with the ones estimated in Hayami and Ruttan (1970).

Exogenous skill type distribution is assumed to take a log-normal form with mean μ and standard deviation σ . Given values of $(\beta, A, \gamma, \alpha)$, the remaining six parameters $(\bar{a}, \eta, \mu, \sigma, \theta)$ are chosen simultaneously to match moments of the US economy in 1985. From the World Development Indicator, agriculture employs 3% of the labor force. Following the strategies in Restuccia, Yang, and Zhu (2008), η is chosen to target a long run agricultural employment share of 0.5%. This corresponds to the asymptotic agricultural employment share when the subsistence consumption share of income is effectively zero. Parameter θ governs the time allocation between skill accumulation and production. Direct observation on the time split between these two activities are not available. However, the model implies that farmers of generation i spend $(1 - s_i)$ fraction of labor hours producing in their farms. Hence the cross-section distribution of hours supplied by operators of different ages are simply given by $\frac{1-s_i}{\sum_{i=1}^T 1-s_i}$. Census of agriculture reports working days of farmers in 5 different age groups: 25-34, 35-44, 45-54, 55-64, 65+, which allows me to calculate the distribution of hours in the data. I choose θ to best match the empirical distribution. Finally, the model is calibrated to reproduce the observed size distribution of farms in the US. Figure B.4 plots the calibrated size distribution against data. In addition, as depicted in Figure B.5, the model also implies a land distribution that fits the data very well, even though it is not targeted. These figures are presented in Appendix B. Table 3 offers a summary of parameter values and how they are selected.

Parameter	Description	Value	Source
A	TFP	1	Normalization
T	Life cycle	5	Model period = 10 years. Born 25, die 75
β	Discount	$(0.96)^{10}$	Common value
\bar{L}	Land endowment	1.62	Arable land per worker
γ	Span-of-control	0.6	Income share of farm operator
α	Labor share, agriculture	0.8	Hayami and Ruttan (1970)
η	Expenditure share, agriculture	0.01	Restuccia, Yang, and Zhu (2008)
\bar{a}	Subsistence	0.2	Share of labor in agriculture
θ	Time elasticity in skill accumulation	0.33	Hour distribution of farmers
μ	Skill distribution, mean	-2.45	Size distribution of farms
σ	Skill distribution, stdev	4.16	Size distribution of farms

Table 3: Parameter Description, Value, and Source of Identification

5.2. Results

Our sample consists of 50 countries with complete information on sector output per worker, sector labor share, land endowment, and size distribution of farms. The first three are directly available from [Restuccia, Yang, and Zhu \(2008\)](#). The last is compiled from the World Census of Agriculture, a database maintained by the Food and Agriculture Organization. These two sets of data, however, are not directly comparable because of time period differences. The data in [Restuccia, Yang, and Zhu \(2008\)](#) pertain to the year 1985. World Census of Agriculture is a collection of national agriculture censuses administered independently in each member country - possibly in different years. Most of the countries indeed have their censuses conducted around 1990, and hence can serve as good proxies. It is unlikely that the composition of farms will undergo drastic changes over a period of five years.

Countries differ in their Total Factor Productivity (A) and land endowment (\bar{L}), and are otherwise identical. The central question of interest is to quantitatively assess how differences in aggregate productivity (and endowment) affects the skill of farmers, and consequently agricultural productivity. This inquiry falls along the line with those in [Gollin \(2008\)](#) and [Lagakos and Waugh \(2010\)](#). The former asks how aggregate productivity affects the composition of labor across countries, and the latter explores how differences in aggregate

efficiency can be translated into asymmetric differences in sector labor productivity. Countries are assumed to face the same distribution of skill types *ex-ante*. Finally, utilizing the linear technology in non-agriculture, TFP of country i can be inferred as

$$A_i = \frac{yln_i}{yln_{us}}$$

where yln_i is PPP non-agricultural GDP per worker of country i .

How does the model perform in explaining cross-country variation in agricultural productivity? Two statistics are constructed to help answer this question. The first one, following [Caselli \(2005\)](#), computes the ratio of log-variance of model generated productivity series to that of the data. This ratio is 0.45, suggesting that the model explains about 45% of the variation in agricultural output per worker across 50 countries. The second one is the R^2 statistics of OLS regression of model series on data. A very similar value (0.49) is obtained. [Figure 1](#) plots model agricultural productivity against that in the data, both re-scaled such that productivity in the US is equal to unity. A visual outlier is Nepal, for which the model over-predicts its agricultural productivity by a lot. The reason is that Nepal, despite a very high employment share in agriculture, has a very productive non-agricultural sector. This maps into high TFP, and leads to a counterfactually high productivity in agriculture.

Low agricultural productivity is mainly due to low total factor productivity, as opposed to deficiency in land endowment. Consider, for concreteness, the poorest country in the sample (Burkina Faso). Compared to the US, Burkina Faso is 4.8 times less productive overall and has 2.6 times less land per worker. Imagine that Burkina Faso has the US land endowment, but its own TFP. This can not help Burkina Faso catch up with US in agricultural productivity - the gap shrinks from a factor 28 to 22. If, however, Burkina Faso has the US level of TFP and its own land endowment, its productivity in agriculture is increased big time - the gap shrinks to merely a factor of 1.2. Productivity differences

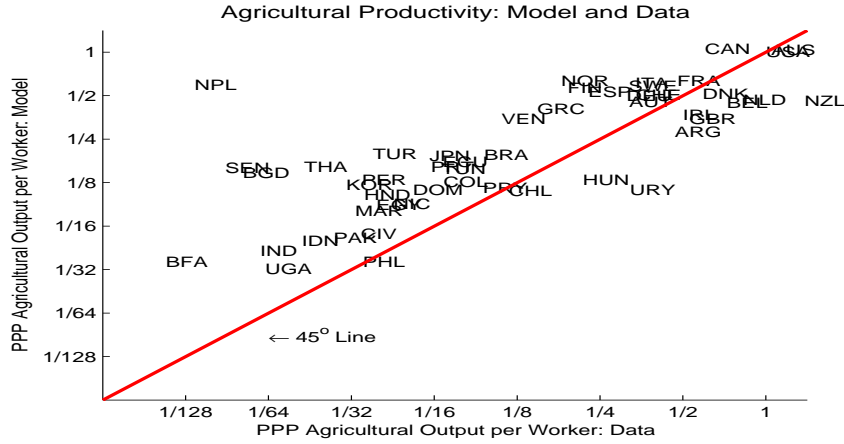


Figure 1: Agricultural Productivity: Model and Data

across countries is mainly a story of TFP differences. This is true not only at the aggregate level as highlighted in Prescott (1998), Hall and Jones (1999) and Caselli (2005), but also in agriculture.⁴

It is worth pointing out that countries in the sample do not differ a lot in terms of PPP output per worker in non-agriculture. Between US and the least productive country, the gap does not exceed a factor of 6. As a result, countries are not assumed to differ a lot in terms of aggregate TFP in the quantitative analysis. Hence, the much larger productivity differences in agriculture generated from the model are not manufactured exogenously, but rather reflect the mechanisms highlighted in the paper, namely, low TFP produces a large pool of unskill farmers, which produce on small scale and low efficiency. For a clearer exposition, first note that total output in agriculture can be written as

$$Y_a = A \cdot \left(\sum_{t=1}^T x_t(1 - s_t) \cdot \int_{\bar{a}} z dG(z) \right)^{1-\gamma} \cdot (H_a^\alpha \cdot \bar{L}^{1-\alpha})^\gamma \quad (8)$$

Let M denote the measure of farmers, then output per worker is simply $yala = \frac{Y_a}{M+H_a}$.

⁴Using measures at the sector level, Caselli (2005) concludes TFP is more important in agriculture than in the aggregate.

Substituting equation (8) into this expression and rearranging terms yields the following expression of agricultural productivity

$$yala = \underbrace{A}_{TFP} \cdot \underbrace{\left(\sum_{t=1}^T x_t(1-s_t) \right)^{1-\gamma}}_{\text{Intensive Margin}} \cdot \underbrace{\left(E(z|z > \bar{z}) \right)^{1-\gamma}}_{\text{Extensive Margin}} \cdot \underbrace{(\bar{L})^{(1-\alpha)\gamma}}_{\text{Land Per Worker}} \cdot \underbrace{\frac{(H_a)^{\alpha\gamma} \cdot M^{-\gamma}}{1 + H_a/M}}_{\text{Labor Composition}} \cdot T^{\gamma-1} \quad (9)$$

Equation (9) offers a nice decomposition of output per worker in agriculture. First of all, Low TFP and poor land endowment decreases agricultural productivity directly. Moreover, TFP also affects the skill profile of farmers, and does so through two distinct margins. The term $E(z|z > \bar{z})$ is the average skill type of farmers, and captures the extensive margin. As more and more farmers produce in agriculture, the average skill is driven down because the marginal farmer is less and less productive. The term $\sum_{t=1}^T x_t(1-s_t)$ summarizes the steepness of the skill profile, and captures the intensive margin. By proposition 1, both terms increase with aggregate productivity, and leverages the impact of TFP on agricultural productivity. To quantify the contribution of each component, consider again the comparison between Burkina Faso and the US. The last column of Table 4 reports how each component contributes to the agricultural productivity differences between these two countries.

Variable	United States	Burkina Faso	Contribution (%)
TFP	1	0.21	47.4
Extensive Margin	4.59	0.61	24.3
Intensive Margin	16.9	12.9	3.3
Land Per Worker	1.62	0.74	2.8
Labor Composition	0.18	0.11	22.2
Agricultural Productivity	109	4	100

Table 4: Decomposition of Agricultural Output per Worker

Between these US and Burkina Faso, differences in TFP explain about 50% of the differences in agricultural output per worker. Differences in farmer's skill, extensive margin and

intensive margin combined, explains about 30% of the differences in agricultural productivity. Most of the variation in farmer's skill comes through the extensive margin. Exogenous differences in land endowment accounts for less than 3% of the differences in agricultural productivity. The results also suggest that these two countries differ substantially in the composition of agricultural workers. In addition to operating with a bigger plot of land, each farm in high TFP countries employs more worker. This appears consistent with the observation the majority of farms in poor countries are small family farms, with limited input of hired labor.

Given its latent nature, differences in farming skill are unobservable to an economist. Instead of trying to construct reasonable measures of farming skill from the data, a different route is taken here. In the model, there is a unique mapping between the *equilibrium* distribution of skills and the size distribution of farms. Although the former is not observable, the latter is. Moreover, more skilled farmers operate a bigger farm, other things equal. Hence, two observations should follow if farmers in poor countries indeed have lower skill. First, mean farm size should be smaller in poor countries. Indeed, it is. A typical farm in Burkina Faso is only 1/20 the size of a typical farm in the US. Figure 2 plots mean farm size in the model and in the data. The model successfully reproduces the positive correlation between output per worker in agriculture and mean farm size. In the Appendix, the size distributions of some selected countries are plotted against their empirical counterparts. Although the model is not expected to replicate the empirical distribution exactly, it does reproduce one salient feature of the data, namely, the size distribution of farms in low income countries is more skewed to the left. This comes from the fact there is a larger share of low skill farmers in these countries. The discrepancy between the model distribution and the empirical distribution - the part that a model of skill investment is unable to explain - might

imply farm level distortions that are prevalent in developing countries⁵ and highlighted in Adamopoulos and Restuccia (2011).

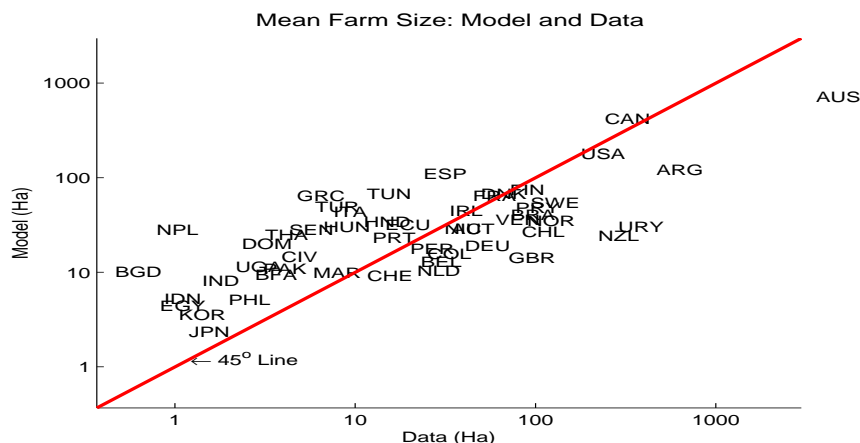


Figure 2: Mean Farm Size: Model and Data

Agriculture, despite its low productivity, absorbs most of the labor force in poor nations. The model performs less well explaining this stylized fact. Figure 3 plots the share of agricultural labor in the model and in the data. For low income countries, the model generally predicts a smaller agricultural labor share than in the data. This is not surprising given that the model is a highly stylized one. It is not at all difficult to list features that can generate a larger share of agricultural labor if included in the model. Among other things, barriers to sectoral labor movements are particularly important to the question posted here. Such barriers are prevalent in developing nations as evidenced by substantial disparities in rural-urban earnings. One famous example is the *Hukou* system in China that imposes institutional restrictions on immigration from rural villages to urban cities. However, accurate measures of such distortions might be difficult, if at all possible, to obtain for a large set of countries, making the quantitative analysis less rewarding.

Agriculture’s share of total output declines as income rises - a macroeconomic implication

⁵For example, restriction on land holding, or subsidy to small farmers.

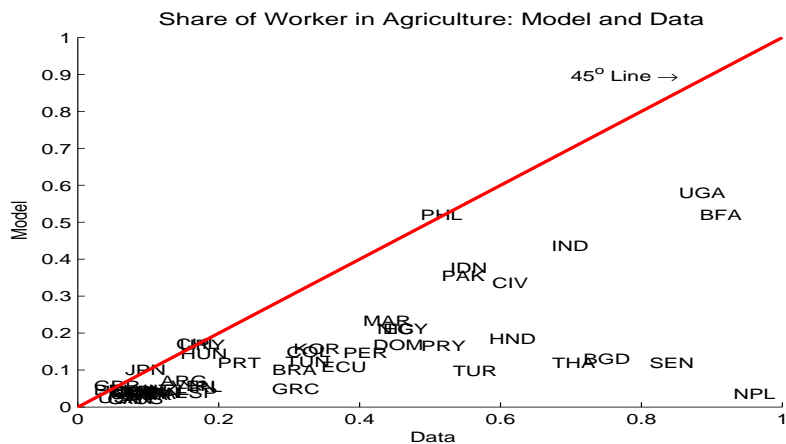


Figure 3: Share of Worker in Agriculture: Model and Data

of Engel’s Law. The model predicts agricultural output to be 10% of the aggregate output in the top quintile countries, and 70% in the bottom quintile countries. In the data, the value is 3% and 30%, respectively. One possible explanation is that the model over-predicts the relative price of agricultural output, resulting in a higher agriculture share of GDP when measured at domestic prices. Using ICP data from the World Bank, I compute the relative price between “agricultural consumption” and “nonagricultural consumption” for all available countries.⁶ The relative price in 2005 is around 4 times higher in the 10th percentile country, compared to the 90th percentile country. In the model, this relative price ratio is 2.8, which is roughly in line with the data.

5.3. Discussion

The model presented in the current paper is a stylized one. In this economy, farming is the job with the highest return. This is at clear odds with the data, even in the most agriculture-advanced countries like the US. However, the focus of this paper is the cross-country comparison of agricultural productivity. It intends to provide an explanation

⁶“Agricultural consumption” is defined as food, non-alcoholic beverage, alcoholic beverage and tobacco. “Nonagricultural consumption” is defined as the rest of individual consumptions plus capital consumption. A similar calculation is done also in [Lagakos and Waugh \(2010\)](#)

of why one country is less productive in agriculture *relative to* another country. In this respect, missing the level of agricultural productivity does not invalidate the comparison across countries.

The key contribution is to present a framework that embeds skill accumulation in a span-of-control model framework. On the one hand, this specification allows a mapping from the equilibrium distribution of skills to the observable size distribution of farms, making model predictions verifiable. On the other hand, while skill investment is not essential to this mapping in the sense that a non-degenerate size distribution arises even without skill investment, it is critical to the identification of the underlying skill distribution in the benchmark economy. A model without skill investment fails to reproduce the farm size distribution of the US, at least within the family of log-normal skill distributions. A similar finding is reported in [Bhattacharya \(2009\)](#), who shows that skill accumulation is critical to quantitatively explain cross-country variation in firm size distribution and income.

The span-of-control parameter is key to the quantitative results. A higher value of γ implies sharper differences in the division of labor in response to given differences in aggregate productivity. As discussed before, existing estimates of this parameter for the manufacturing sector or the aggregate economy are typically in the ballpark of 0.8. In this respect, the preferred value of 0.6 is a conservative choice. To illustrate this, the model is re-calibrated under the case of $\gamma = 0.8$. Then I ask again how much of the differences in agricultural productivity between the US and Burkina Faso can be accounted for by the model, given exogenous TFP and land endowment. The results are summarized in [Table 5](#).

	United States	Burkina Faso	Gap
$\gamma = 0.6$	109	4	27
$\gamma = 0.8$	15.7	0.49	31
Data	19930	128	155

Table 5: Agricultural Productivity Differences Under Different γ

The model is able to explain 15% more of the differences in agricultural productivity between US and Burkina Faso when a larger γ is chosen. However, the relative contribution of exogenous and endogenous variables remains more or less the same. With $\gamma = 0.8$, TFP accounts for 45% of the differences in agricultural productivity, followed by land endowment per farmer (28%). The role of farmer's skill, however, is reduced. This is not surprising given that the elasticities of both the extensive margin and intensive margin from equation (9) is $1 - \gamma$. As a result, a larger γ naturally weakens the importance of skill. Nevertheless, differences in farmer's skill still account for 13% of the differences in output per worker in agriculture.

[Restuccia, Yang, and Zhu \(2008\)](#) explore the impact of intermediate input distortions on agricultural productivity through the *intensive margin*. However, there is evidence suggesting that the *extensive margin* might also be important. [Evenson and Gollin \(2003\)](#) document a substantial lag in the adoption of modern variety in Sub-Saharan Africa during the 1960s and 1970s. There are two ways skill might affect the use of modern inputs. Through the extensive margin, low skill might impede the farmer's learning of the new variety, and delays the decision of adoption. Through intensive margin, low skill farmers might use modern variety to a less extent if skill is complementary to modern varieties. Quantitative explorations from these angles are left for future work.

6. Conclusion

Productivity differences in agriculture are larger than those in the aggregate, and much larger than the differences in non-agriculture. This paper develops a stylized model to quantify the role of unmeasured skill of farmer's in agricultural production. It is shown that in an economy with low TFP (albeit sector-neutral), skill formation of farmers is distorted. Low TFP economy is populated with a large pool of farmers with low skill, and the opposite is true in a high TFP economy. Differences in farmer's skill magnify the differences in

agricultural productivity.

The agricultural sector characterized in this paper is “poor but efficient”, as articulated in [Schultz \(1964\)](#). Nonetheless, various distortions geared specifically towards agriculture are also important. Distortions such as barriers to sectoral labor movements, and implicit government taxation on agriculture as discussed in [Krueger, Schiff, and Valdes \(1988\)](#) and [Anderson \(2009\)](#), might be key to understand the coexistence of a large labor force and low productivity in agriculture in poor nations. Farm level distortions in the style of [Adamopoulos and Restuccia \(2011\)](#) might be important as well for understanding the dominance of small-scale production in agriculture of poor countries. While eliminating these distortions is important for development in agriculture, public policies favoring better institutions, faster technology adoptions and more efficient markets are of first order importance in improving overall living standards.

Appendix A. Data Appendix

- **World Census of Agriculture:** This is an archive of national agriculture censuses from a wide range of developing and developed countries. FAO processes these national censuses and presents key summary statistics in a common, internationally comparable format. The unit of observation in WCA is a holding - defined as “an economic unit of agricultural production under single management comprising all livestock kept and all land used wholly or partly for agricultural production purposes, without regard to title, legal form, or size”. Throughout this paper, I view a holding as identical to a farm. <http://www.fao.org/economic/ess/ess-data/ess-wca>
- **World Development Indicator:** Available on line through <http://data.worldbank.org/indicator>
- **Factor Shares in U.S Farming:** Data are from National Agriculture Statistics Services administrated by the Department of Agriculture, and can be accessed through <http://www.ers.usda.gov/Data/FarmIncome/FinfidmuXls.htm>.
- **Working Days by Age of Farm Operator:** Panel A reports the number of days *off* the farm. There are 250 working days a year, and the midpoint of the interval is used as the interval average.

Panel A						
	25-34	35-44	45-54	55-64	65+	Total
None	52,938	104,375	110,380	158,629	249,512	675,834
1-99 days	18,015	29,804	25,428	27,061	19,267	119,575
100-199 days	7,872	14,648	14,308	12,423	6,169	55,420
200 days +	10,028	15,565	14,681	11,082	5,087	56,443
Panel B						
Work Days (1000s)	17875	33908	34478	46589	66975	
% Days	0.09	0.17	0.17	0.23	0.34	

Table A.6: Days off Farm by Age of Operator

Appendix B. Model Appendix

Appendix B.1. Proofs

Proof of Lemma 1:

Recall that profit function is linear in skill, i.e.,

$$\pi(z) = z(1 - s) \cdot (1 - \gamma) \cdot (P \cdot A)^{\frac{1}{1-\gamma}} \left(\gamma \left(\frac{\alpha}{w} \right)^\alpha \left(\frac{1 - \alpha}{q} \right)^{1-\alpha} \right)^{\frac{\gamma}{1-\gamma}}$$

In a stationary equilibrium, the optimal sequence of skill investment is the solution to the following problem

$$\begin{aligned} \max_{s_t} : & \sum_{t=1}^T R^{1-t} \cdot z_t \cdot (1 - s_t) \\ \text{s.t.} : & z_{t+1} = z_t(1 + s_t^\theta) \end{aligned}$$

The optimal path of investment can be solved using backward induction. Clearly, $s_T = 0$.

The problem at period T-1 can be written as

$$\max_{s_{T-1}} : z_{T-1}(1 - s_{T-1}) + z_{T-1}(1 + s_{T-1}^\theta) \cdot R^{-1}$$

the optimal time is given by $s_{T-1} = \left(\frac{\theta}{R}\right)^{\frac{1}{1-\theta}}$. Now define $d_{T-1} = (1 - s_{T-1}) + (1 + s_{T-1}^\theta)/R$, the problem at period T-2 can be written as

$$\max_{s_{T-2}} : z_{T-2}(1 - s_{T-2}) + z_{T-2}(1 + s_{T-2}^\theta) \cdot d_{T-1} \cdot R^{-1}$$

The solution has a recursive structure

$$S_T = 0$$

$$d_T = 1$$

$$s_{t-1} = \left(\frac{\theta \cdot d_t}{R}\right)^{\frac{1}{1-\theta}}$$

$$d_{t-1} = (1 - s_{t-1}) + (1 + s_{t-1}^\theta)/R$$

Proof of Proposition 1:

Consider two economies with $A_r = g \cdot A_p, g > 1$, and assume the threshold level of skill and interest rate are the same in these two economies. Equation (4) implies $q_r = g \cdot q_p$ because, from Lemma 1, optimal time s_t depends only on interest rate. It follows from equation (5) implies $p_r = p_p$, and from the definition of household income that $Y_r = g \cdot Y_p$, i.e., household income is proportional to TFP. Aggregate production of agricultural good is also proportional to TFP. However, Equation (6) suggests that demand of agricultural consumption drops by less than a factor of g in low TFP economy. Excess demand in agriculture pushes up the price of agricultural consumption, and reduces the threshold level of skill in low efficiency economy. This implies a higher labor share in agriculture, and a decline in the supply of nonagricultural good. Interest rate must rise to offset the excess demand of non-agricultural good.

Appendix B.2. Calibration

Age	25-34	35-44	45-54	55-64	65+
Data	0.09	0.17	0.17	0.23	0.34
Model	0.08	0.17	0.20	0.26	0.29

Table B.7: Time Share by Age of Operator: Model and Data



Figure B.4: Calibrated Size Distribution

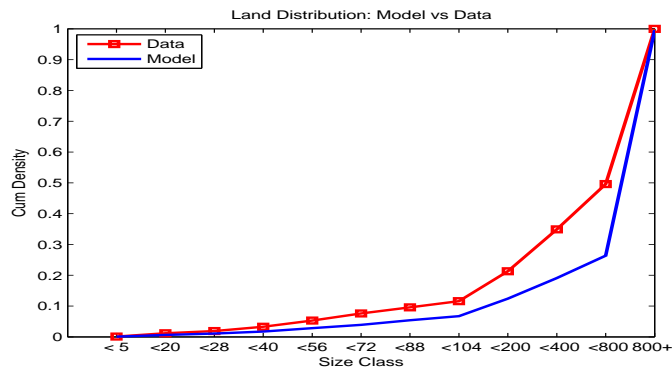
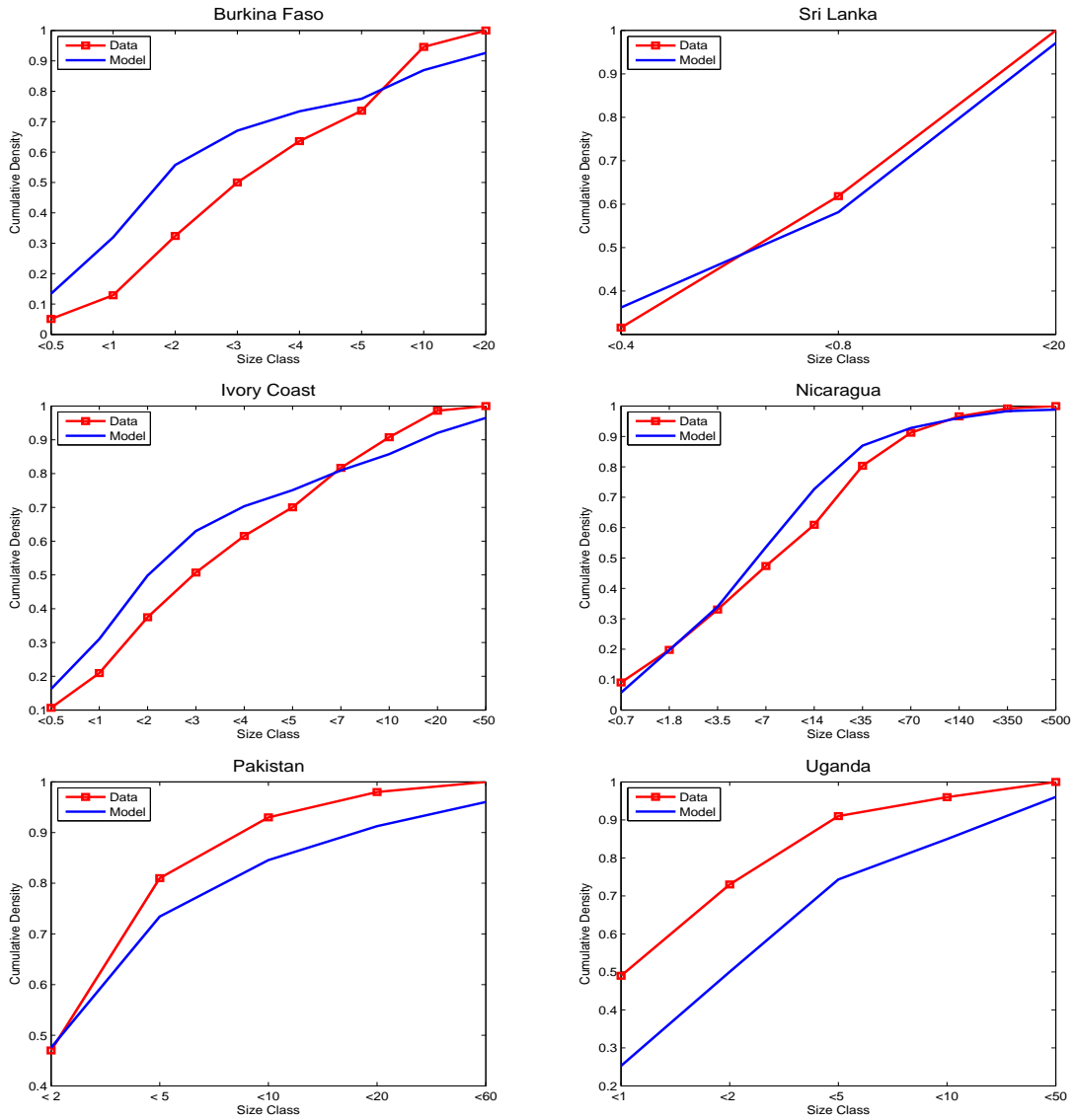
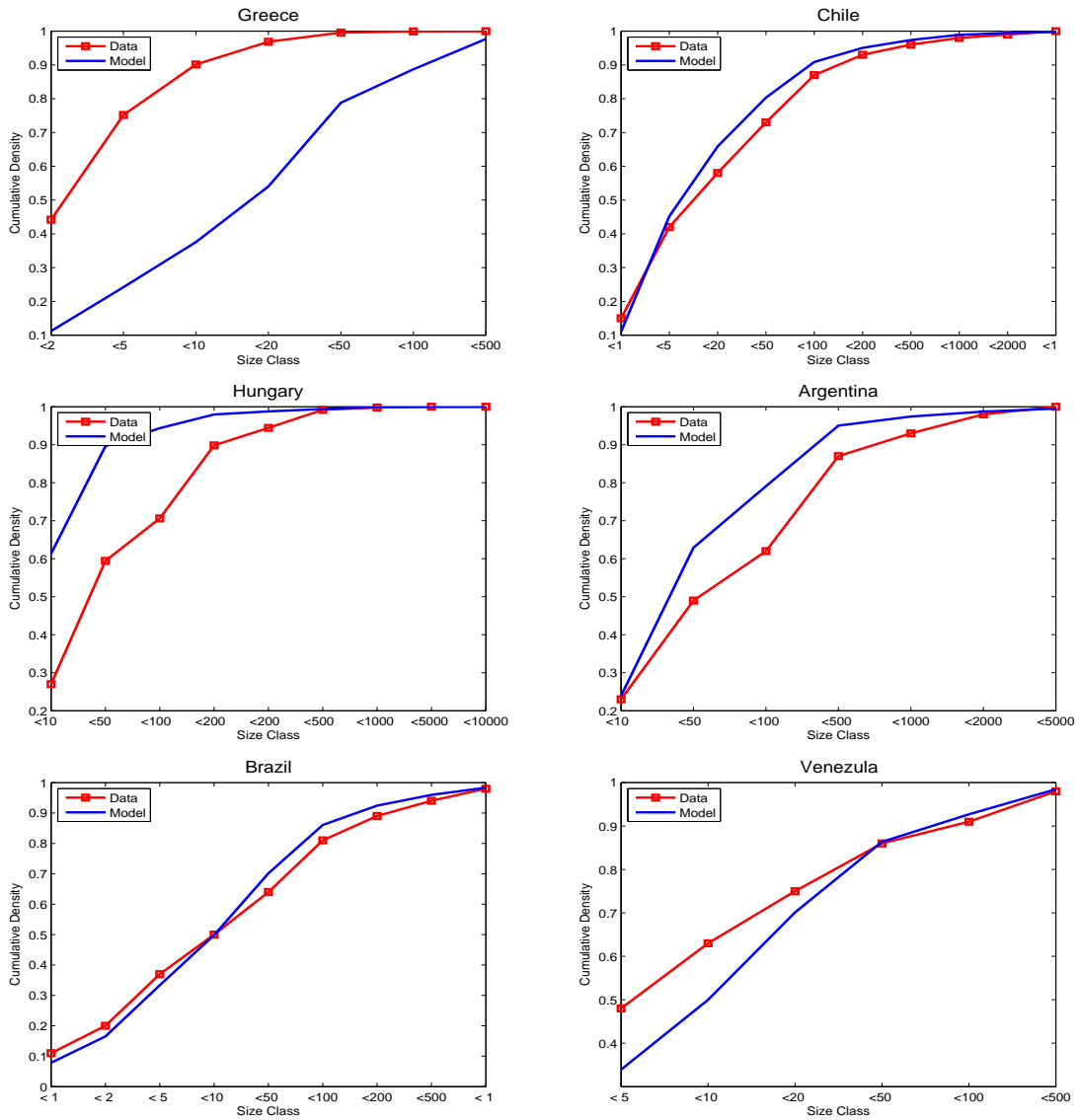


Figure B.5: Implied Distribution of Land

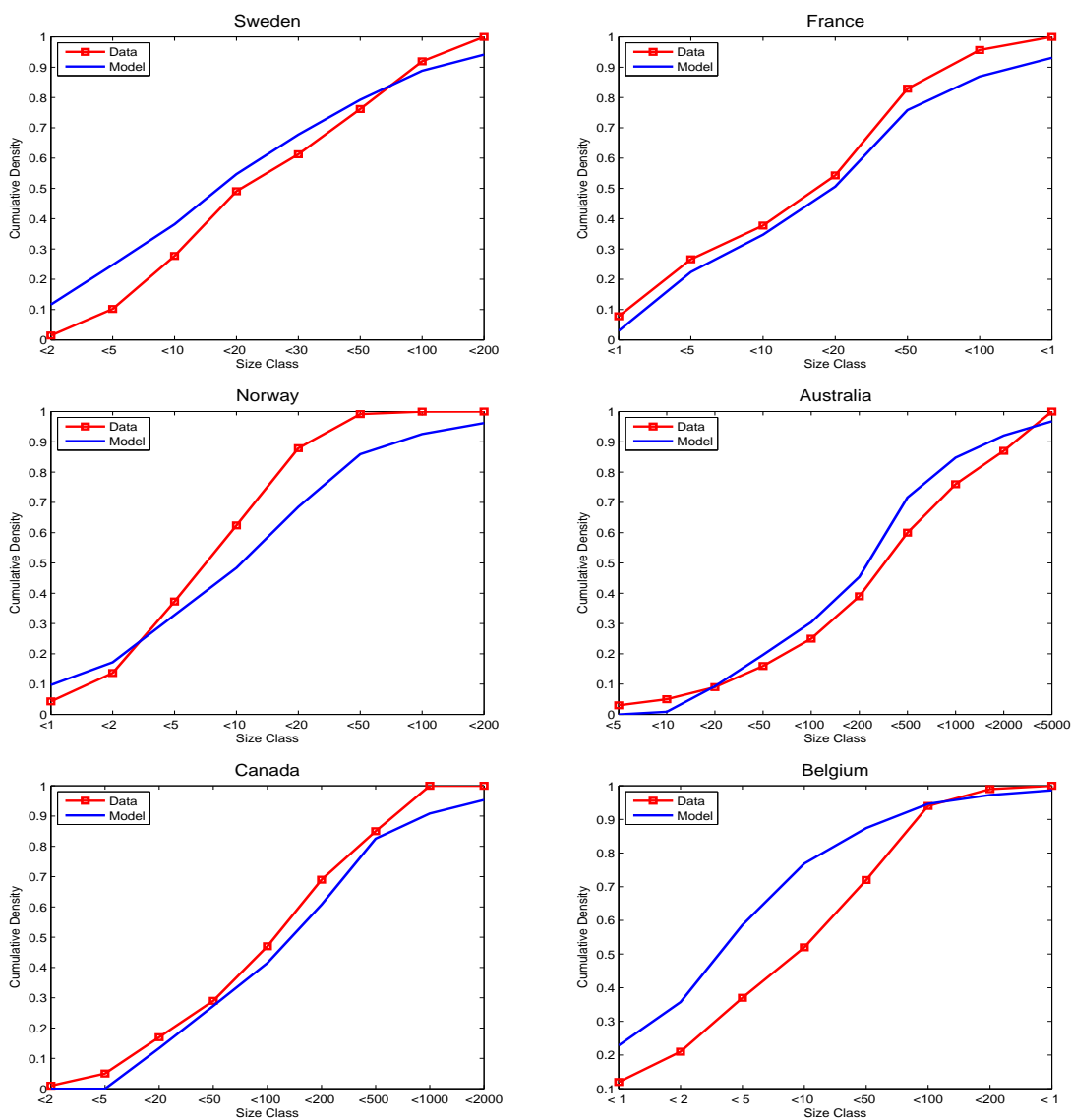
Appendix B.3. Size Distribution of Farms: Model and Data (selected low-income countries)



Appendix B.4. Size Distribution of Farms: Model and Data (selected middle-income countries)



Appendix B.5. Size Distribution of Farms: Model and Data (selected high-income countries)



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